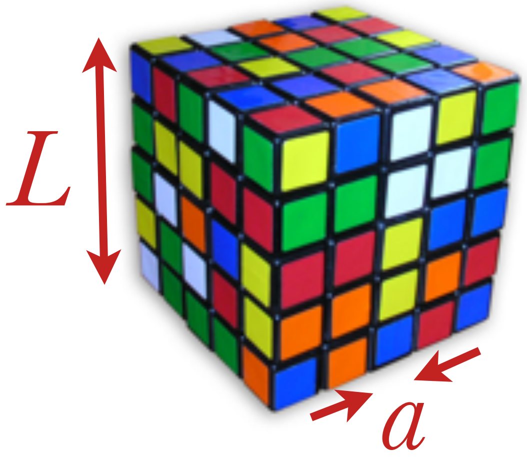
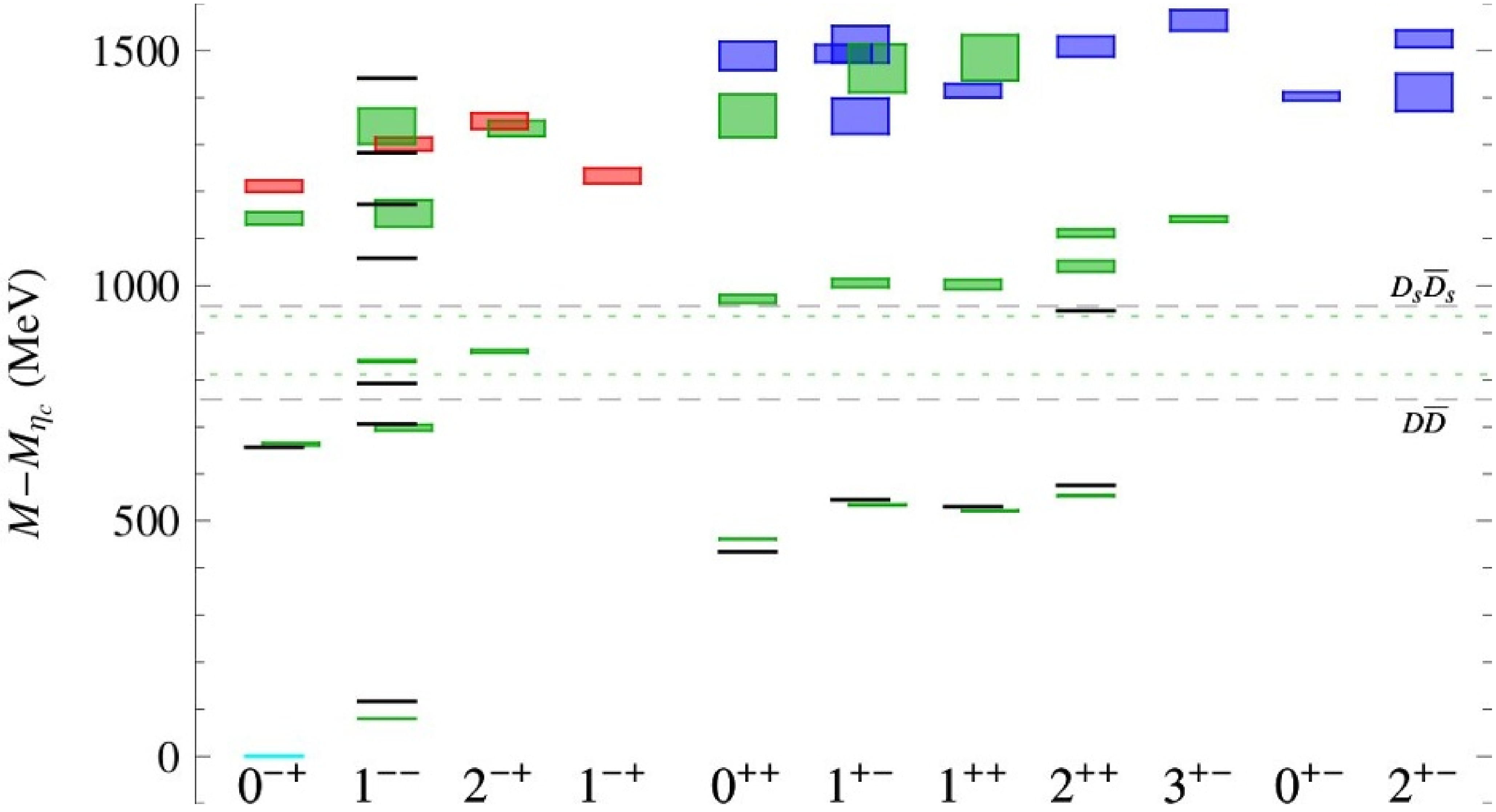
Resonance poles and threshold effects from lattice QCD

David Wilson

A New Era for Hadro-Particle Physics
Thomas Jefferson National Accelerator Facility
23-24 June 2016

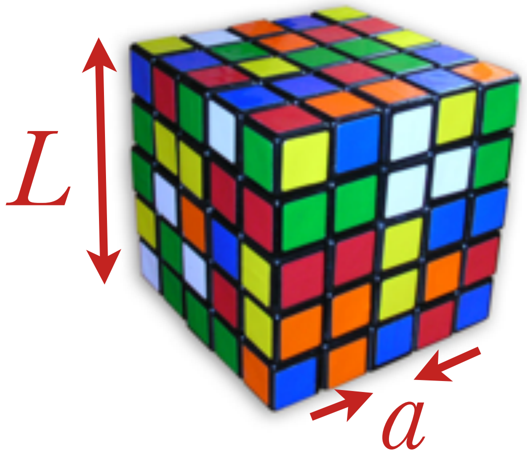
$c\bar{c}$ spectrum from lattice QCD

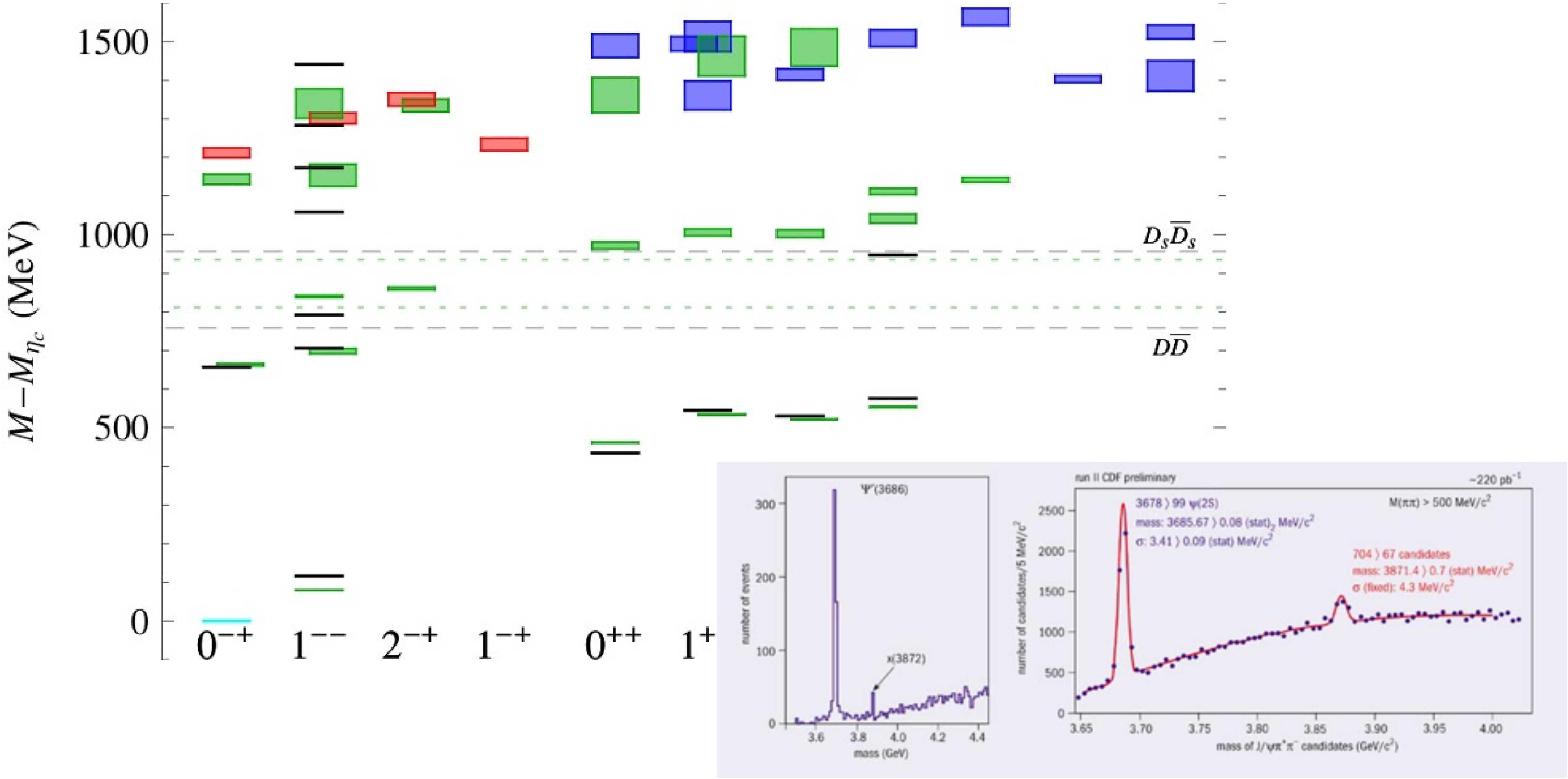




from Liu et al (for the Hadron Spectrum Collaboration), JHEP 1207 (2012) 126.

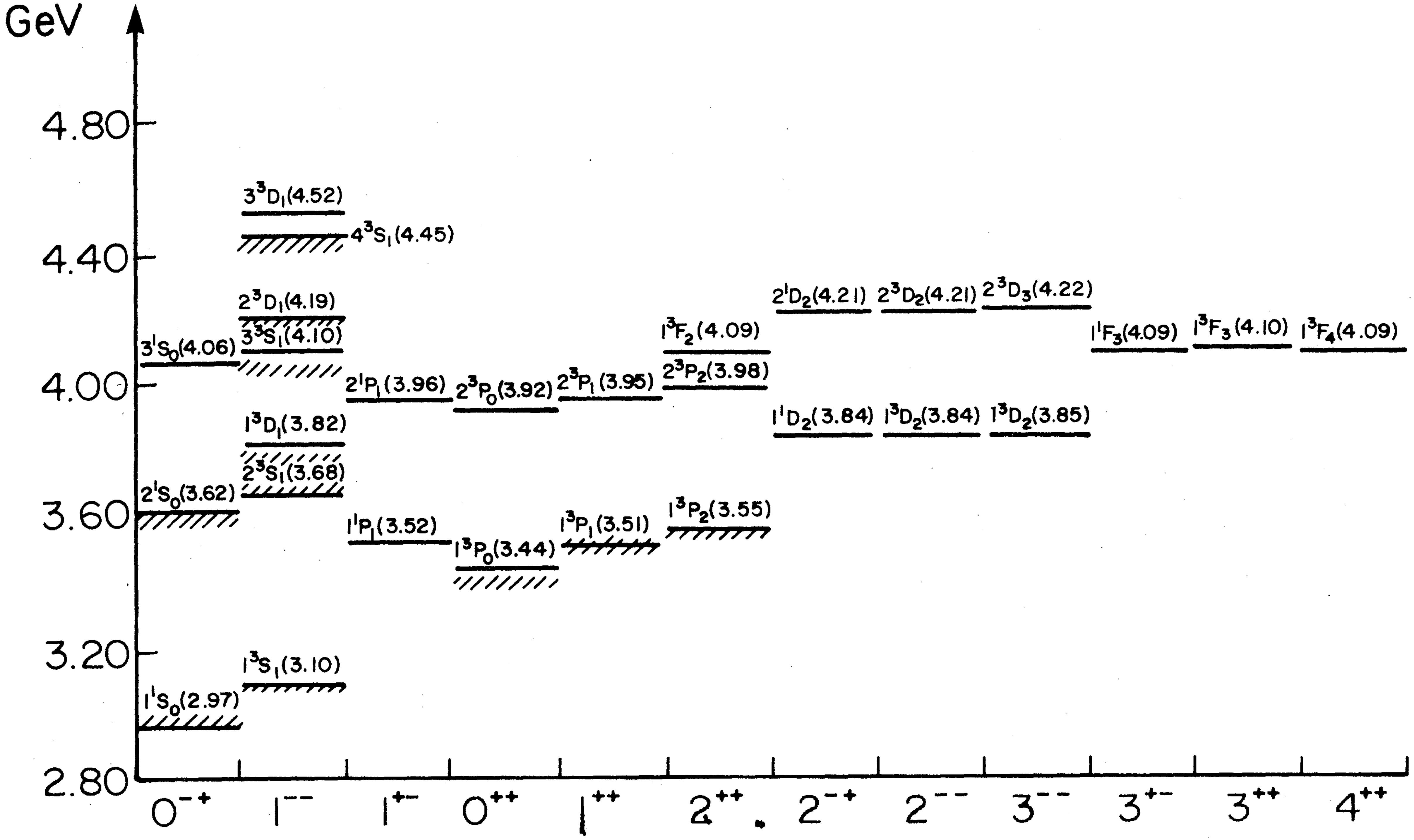
$c\bar{c}$ spectrum from lattice QCD





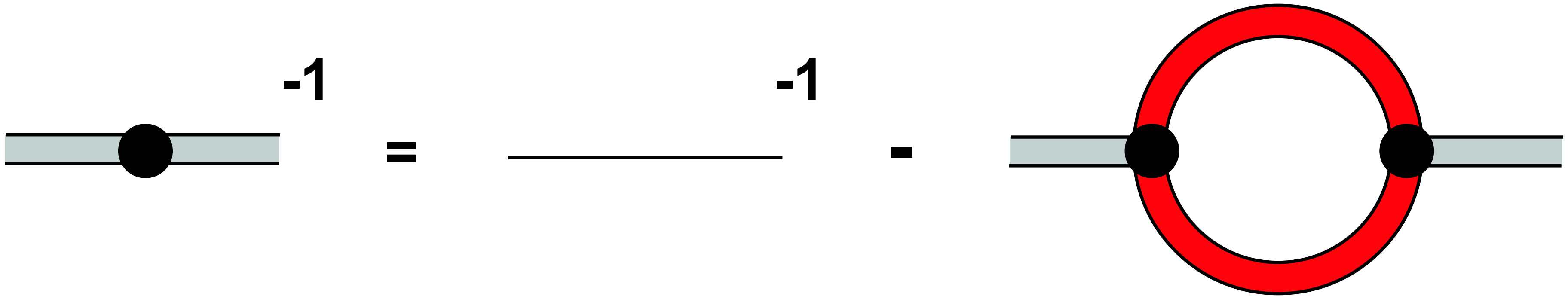
from Liu et al (for the Hadron Spectrum Collaboration), JHEP 1207 (2012) 126.

$c\bar{c}$ spectrum from ancient history

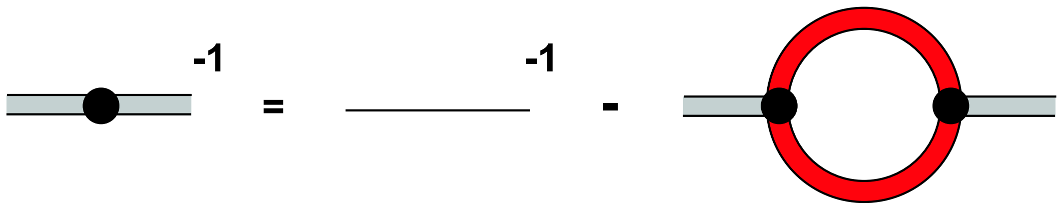


from Godfrey and Isgur PRD32 (1985) 189-231

Meson loops

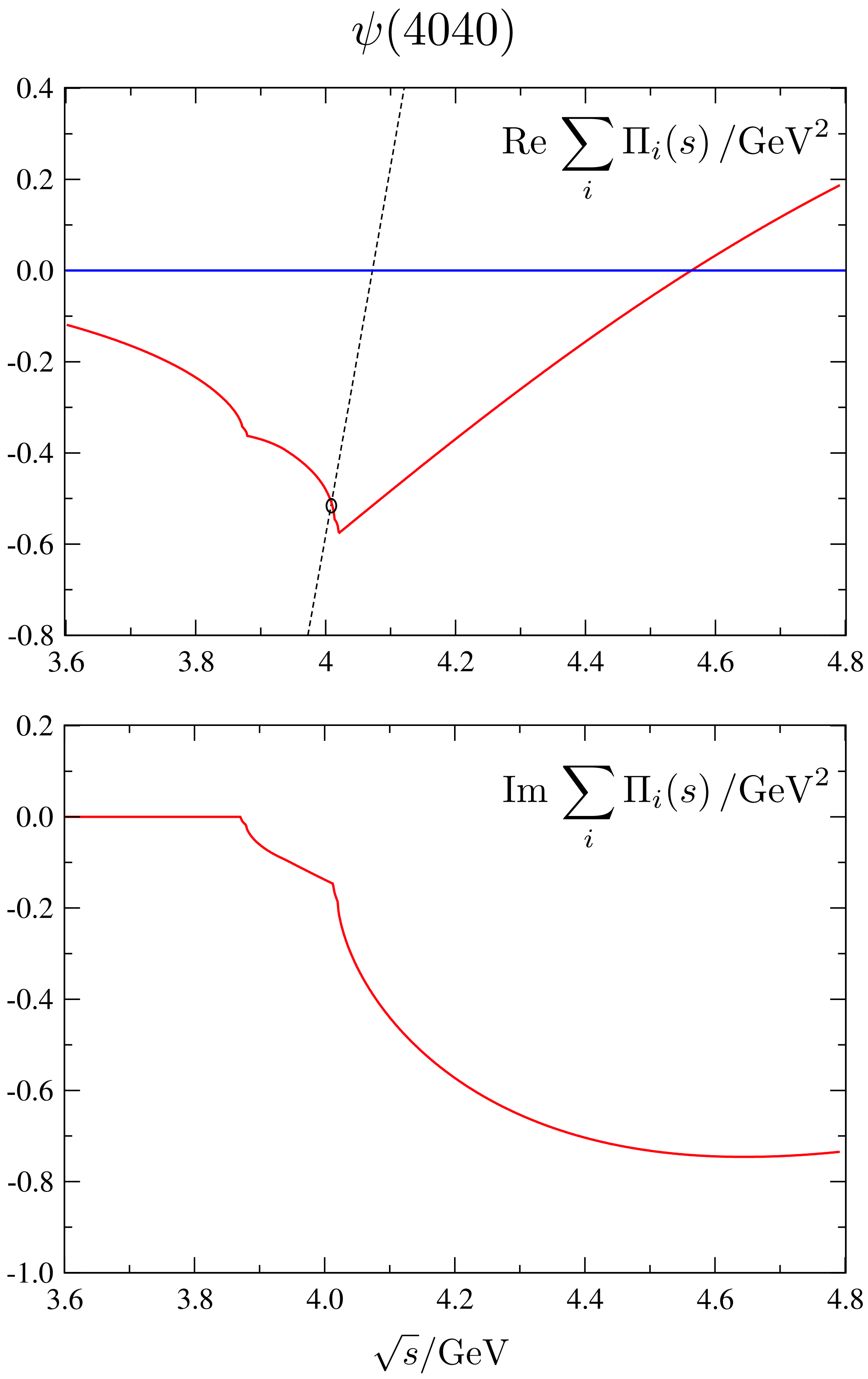


$$\mathcal{P}^{-1}(s) = m_0^2 - s + \sum_i \Pi_i(s)$$

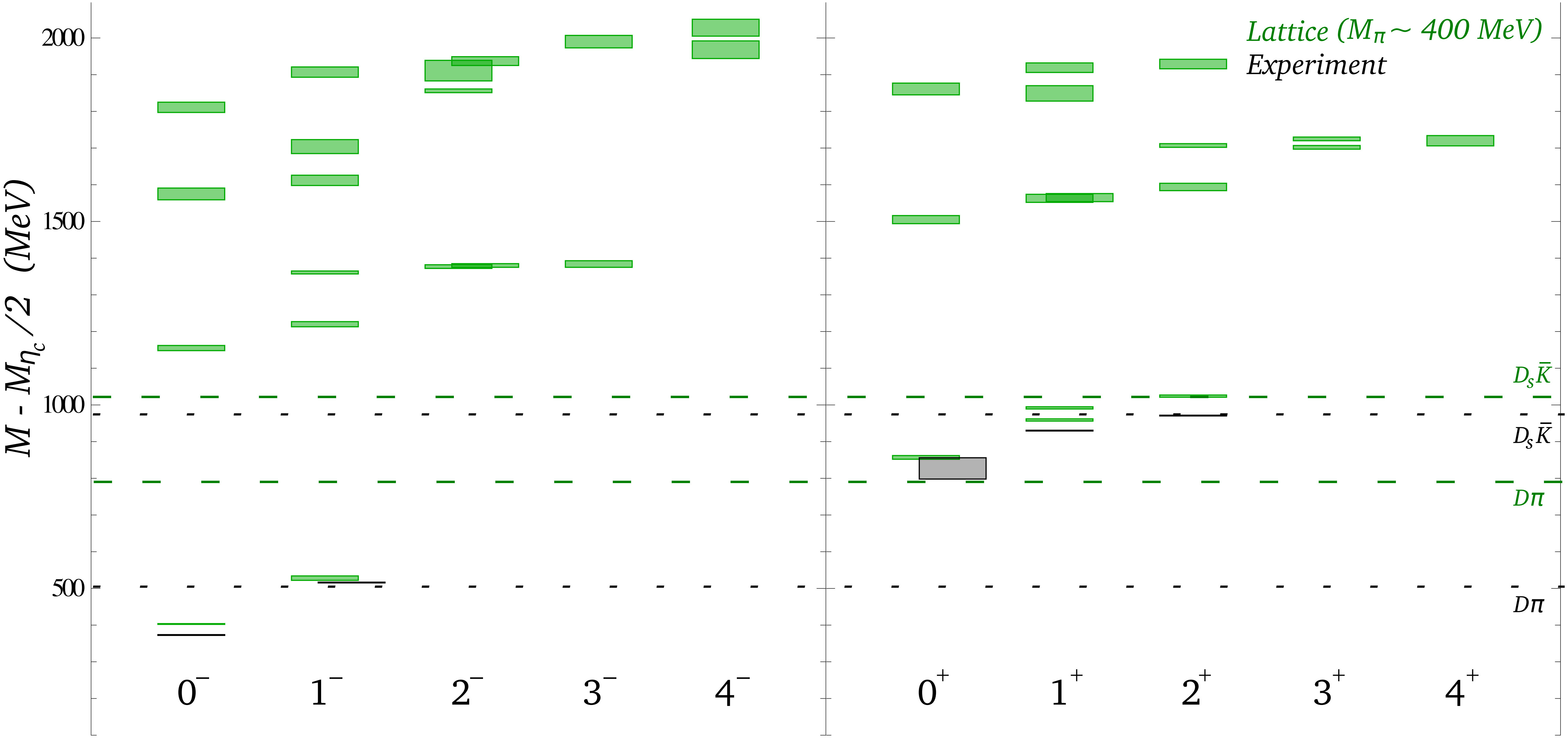


$$\mathcal{P}^{-1}(s) = m_0^2 - s + \sum_i \Pi_i(s)$$

State	Our mass (MeV)	Our Δm (MeV)
2^1S_0	3617.0	-6.9
2^3S_1	3676.5	-14.2
3^1S_0	3924.5	-24.5
3^3S_1	4020.0	-17.0
3^1P_1	3892.0	-30.0
3^3P_0	3818.8	-13.2
3^3P_1	3868.9	-45.1
3^3P_2	3939.4	-7.6
3^1D_2	3813.3	-1.7
3^3D_1	3728.1	-46.9
3^3D_2	3815.0	0.0
3^3D_3	3833.1	-1.9



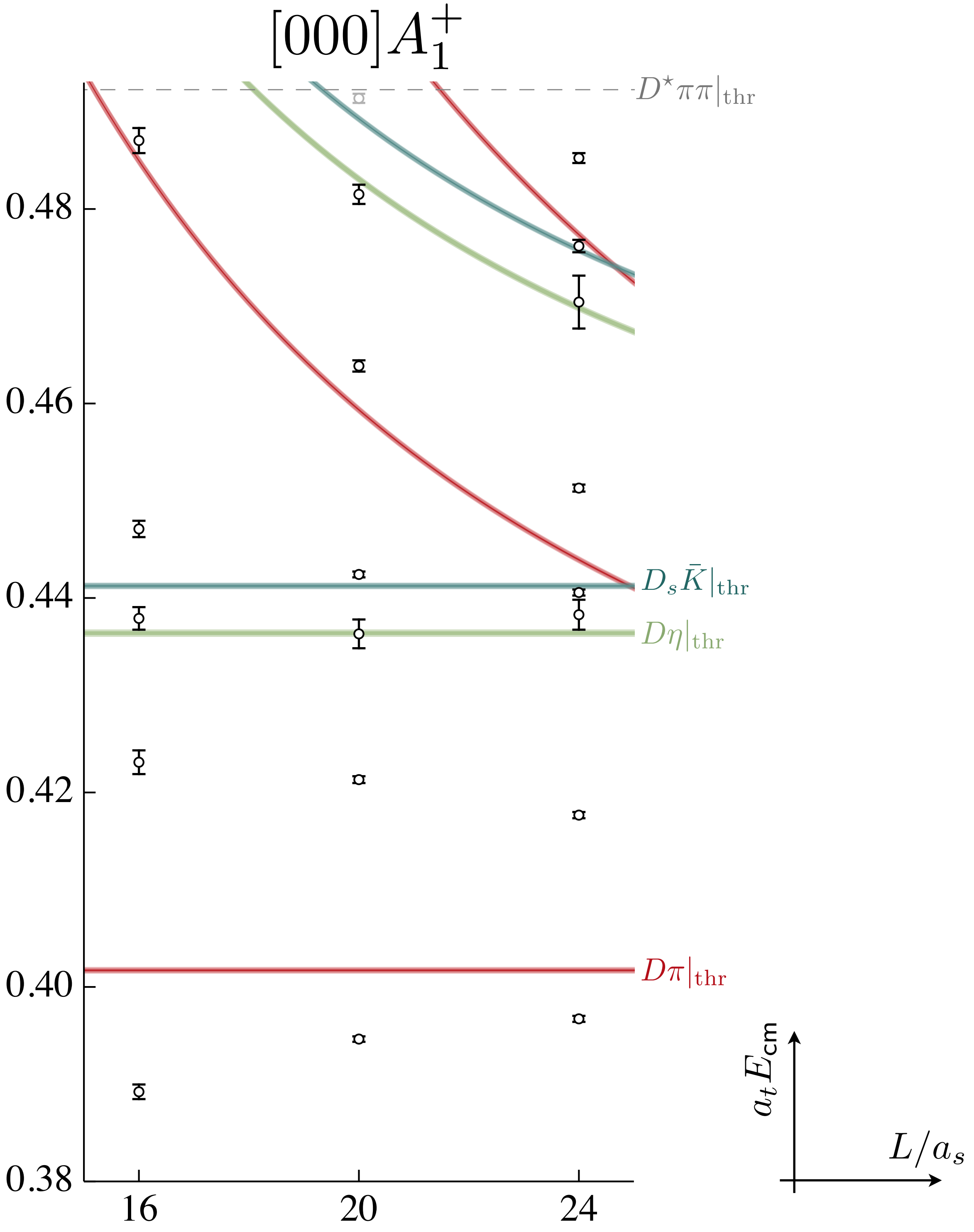
Charm-light spectrum from lattice QCD



from Moir et al (for the Hadron Spectrum Collaboration), JHEP 1305 (2013) 021.

Dπ scattering from the lattice

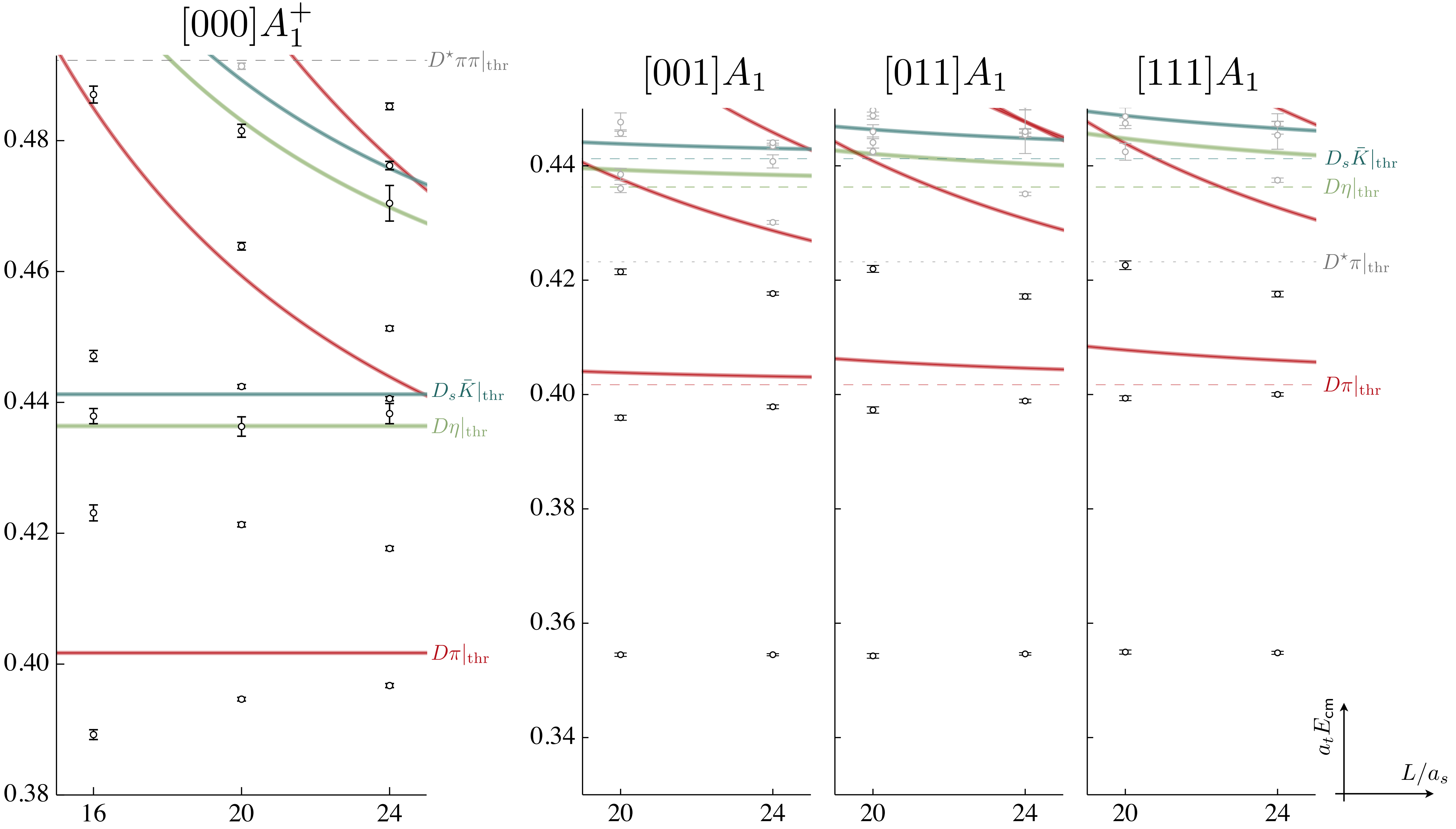
$m_\pi = 391$ MeV



All Dπ work preliminary, G. Moir, DJW and others for the hadron spectrum collaboration, in prep

Dπ scattering on the lattice

$m_\pi = 391$ MeV



All Dπ work preliminary, G. Moir, DJW and others for the hadron spectrum collaboration, in prep

Scattering in a finite volume

$$\det\left[\boldsymbol{t}^{-1}(E) + i\boldsymbol{\rho}(E) + \boldsymbol{\mathcal{M}}(E,L)\right] = 0$$

infinite volume scattering
t-matrix

$$\boldsymbol{S} = \mathbf{1} + 2i\boldsymbol{\rho}\cdot\boldsymbol{t}$$

diagonal in partial waves,
mixes channels

phase space

known finite-volume
functions

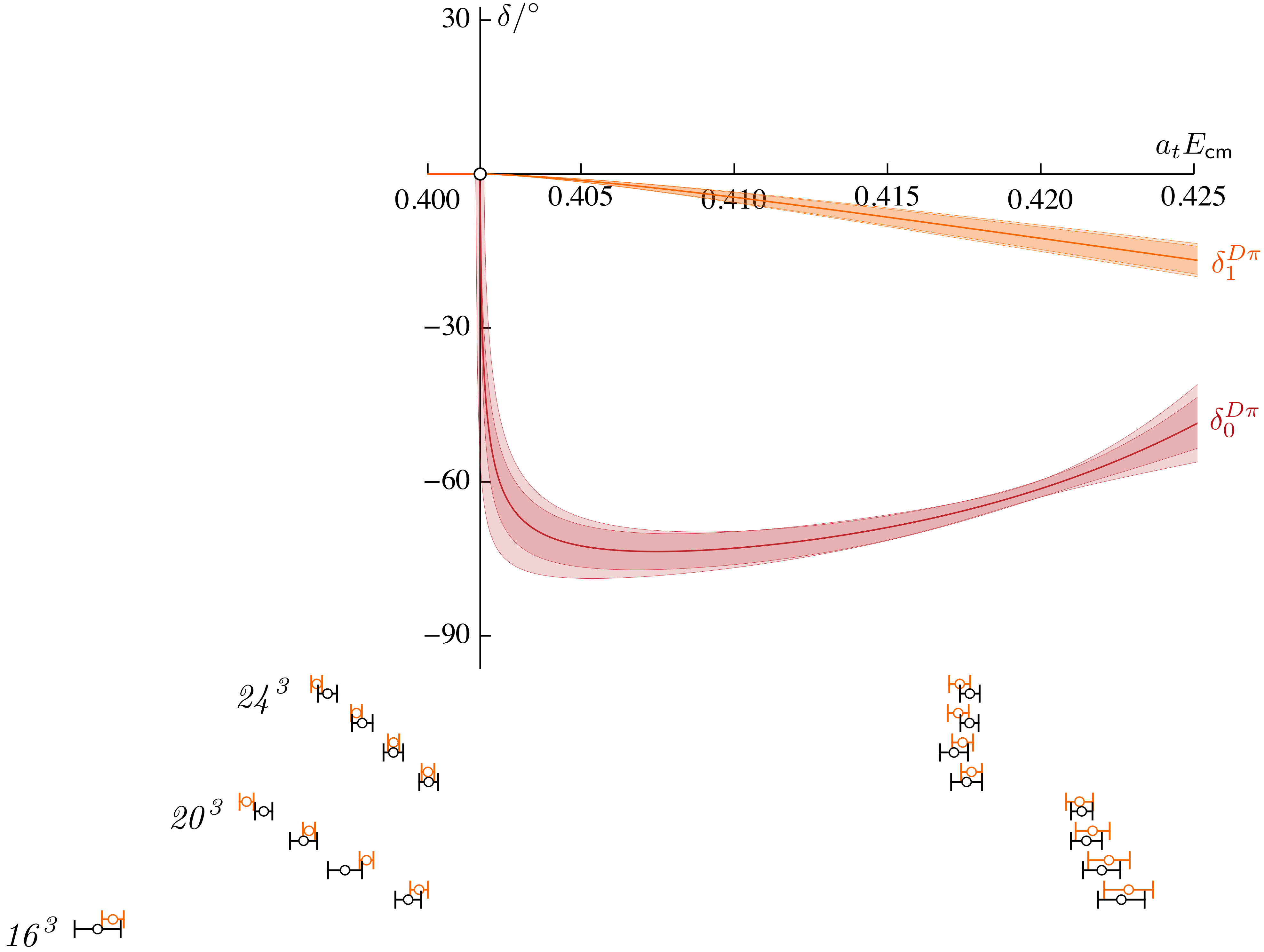
diagonal in channels,
mixes partial waves

K-matrices prove very useful

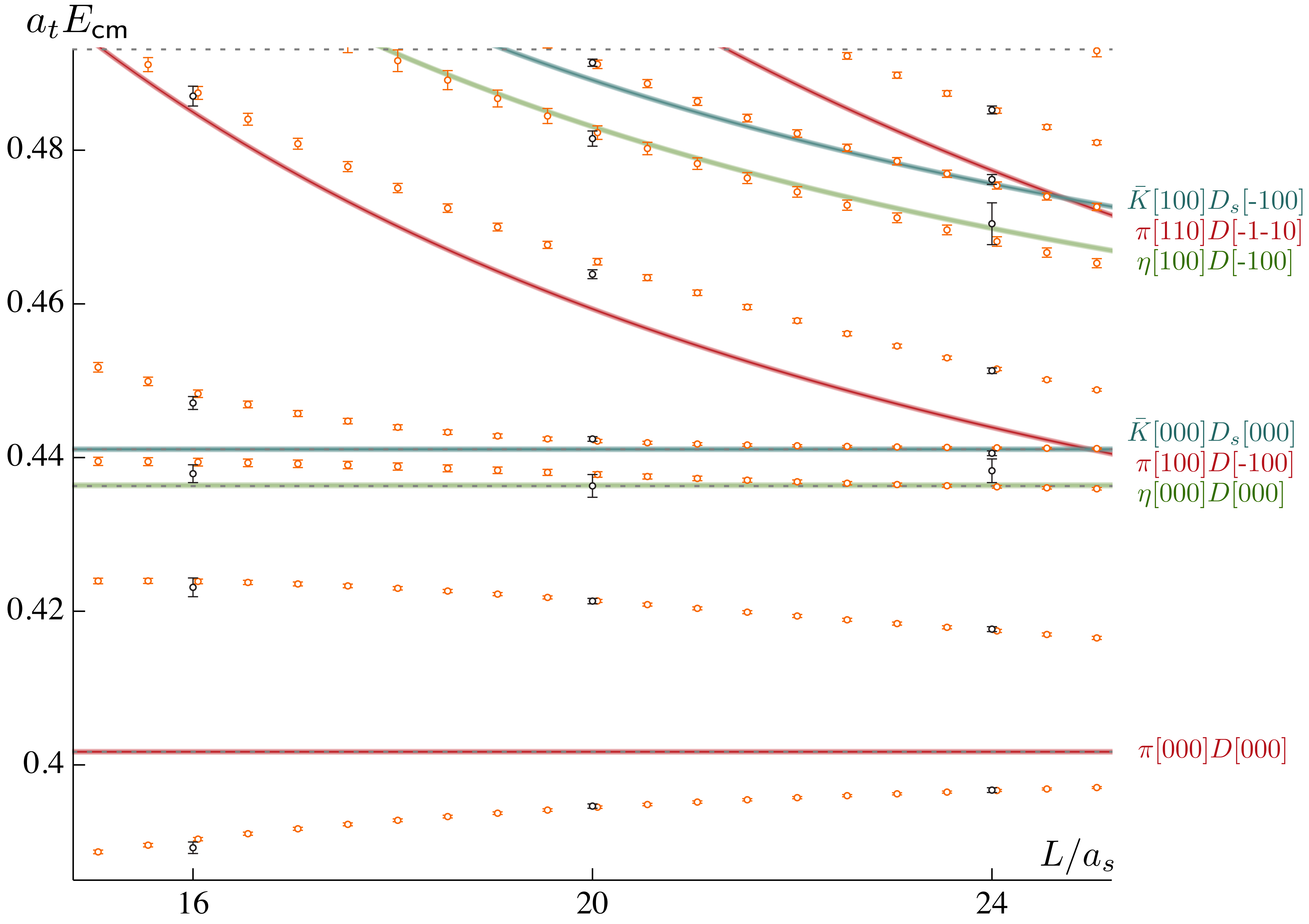
$$\boldsymbol{t}^{-1} = \boldsymbol{K}^{-1} - i\boldsymbol{\rho}$$

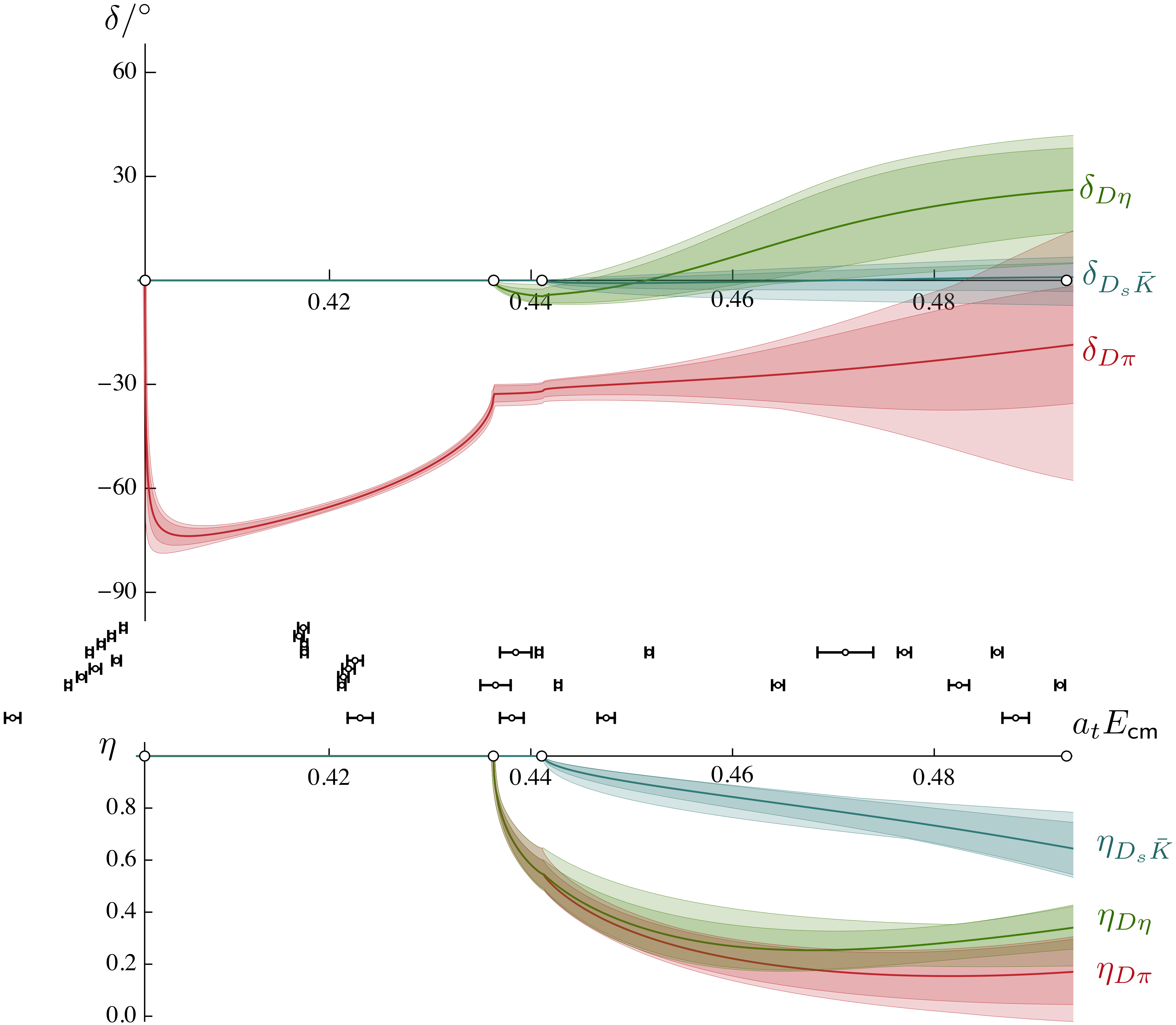
where $\boldsymbol{K}$ is real for real energies

Elastic Dπ phases

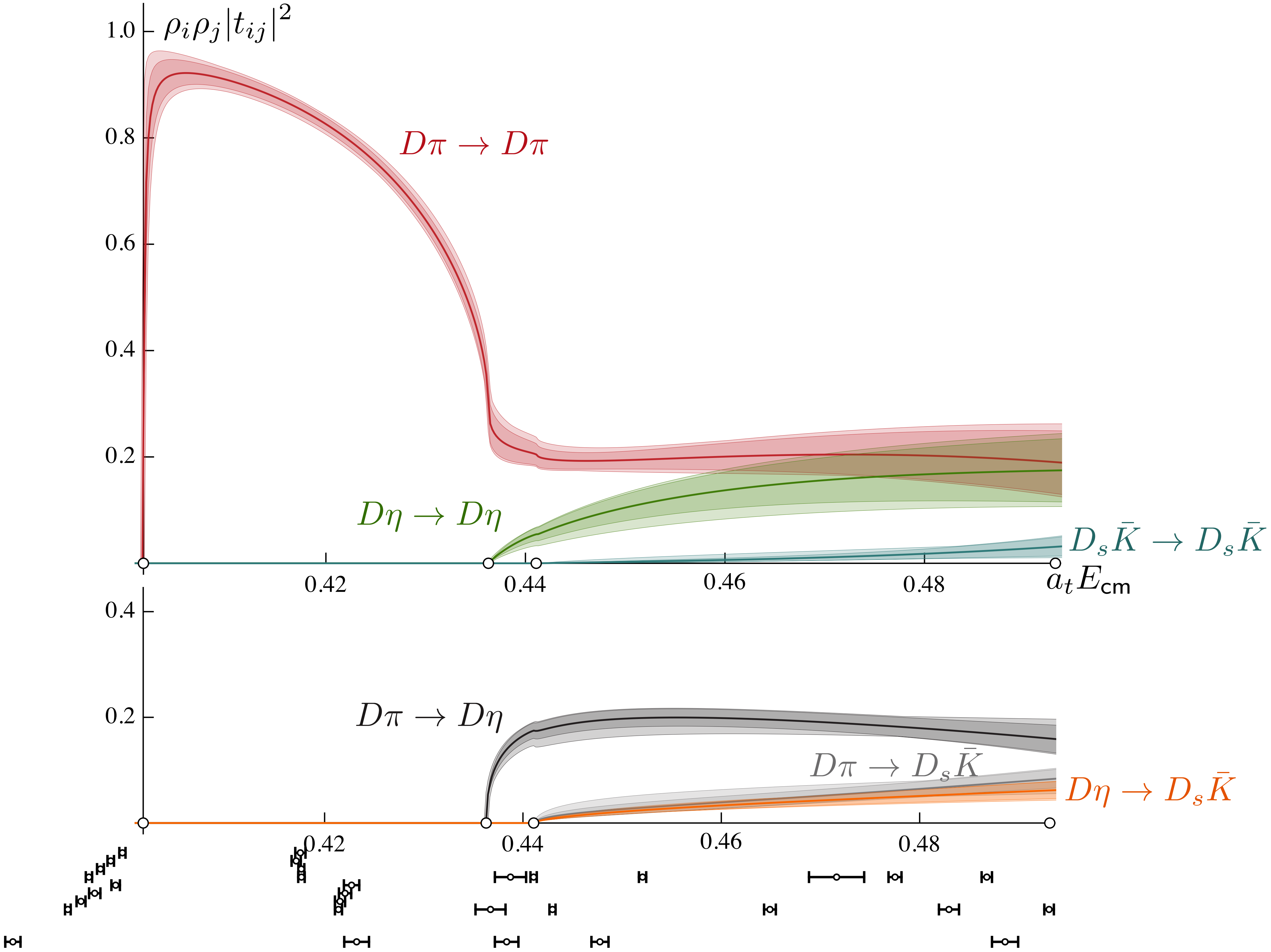


Coupled-channel scattering

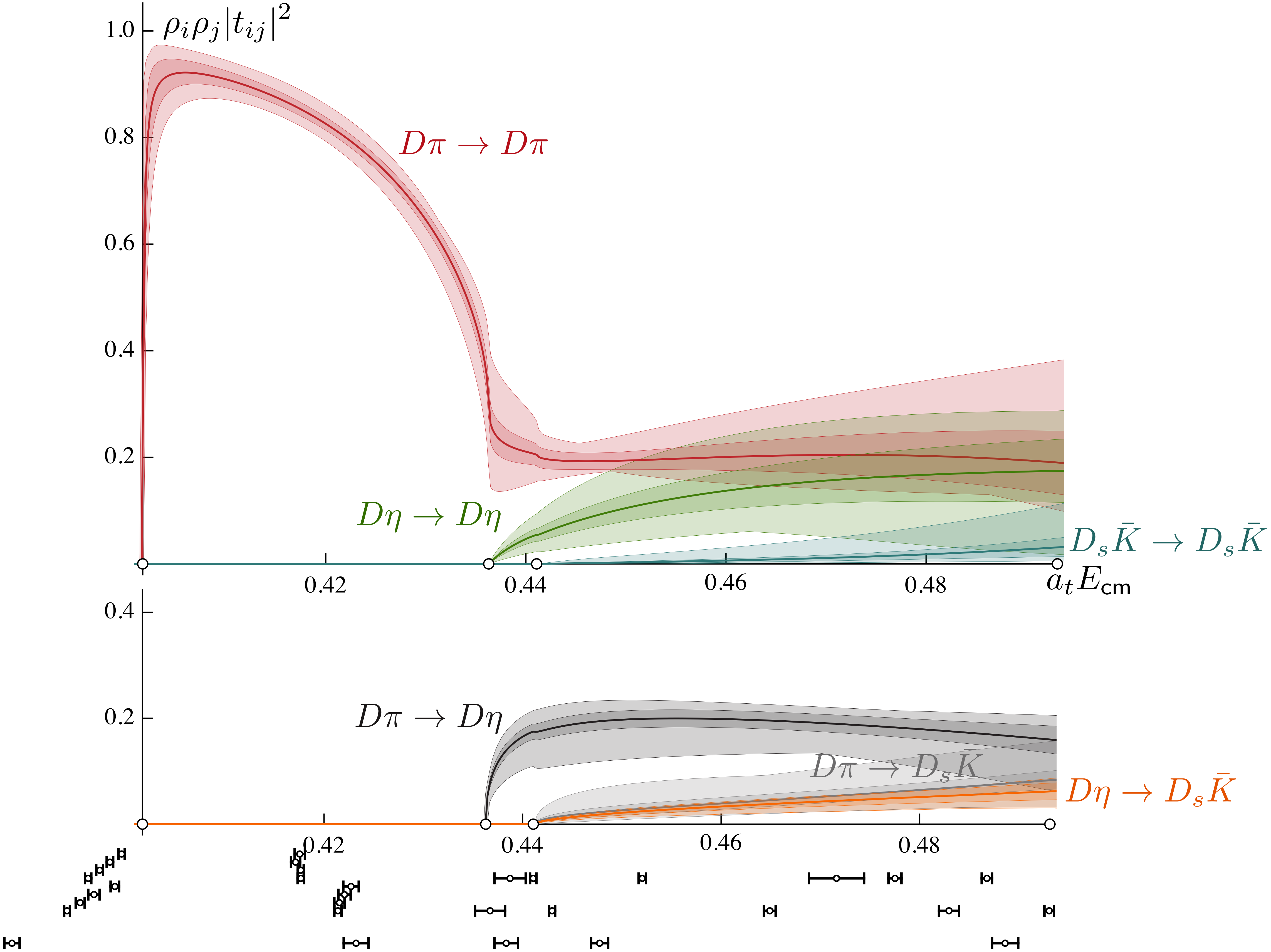


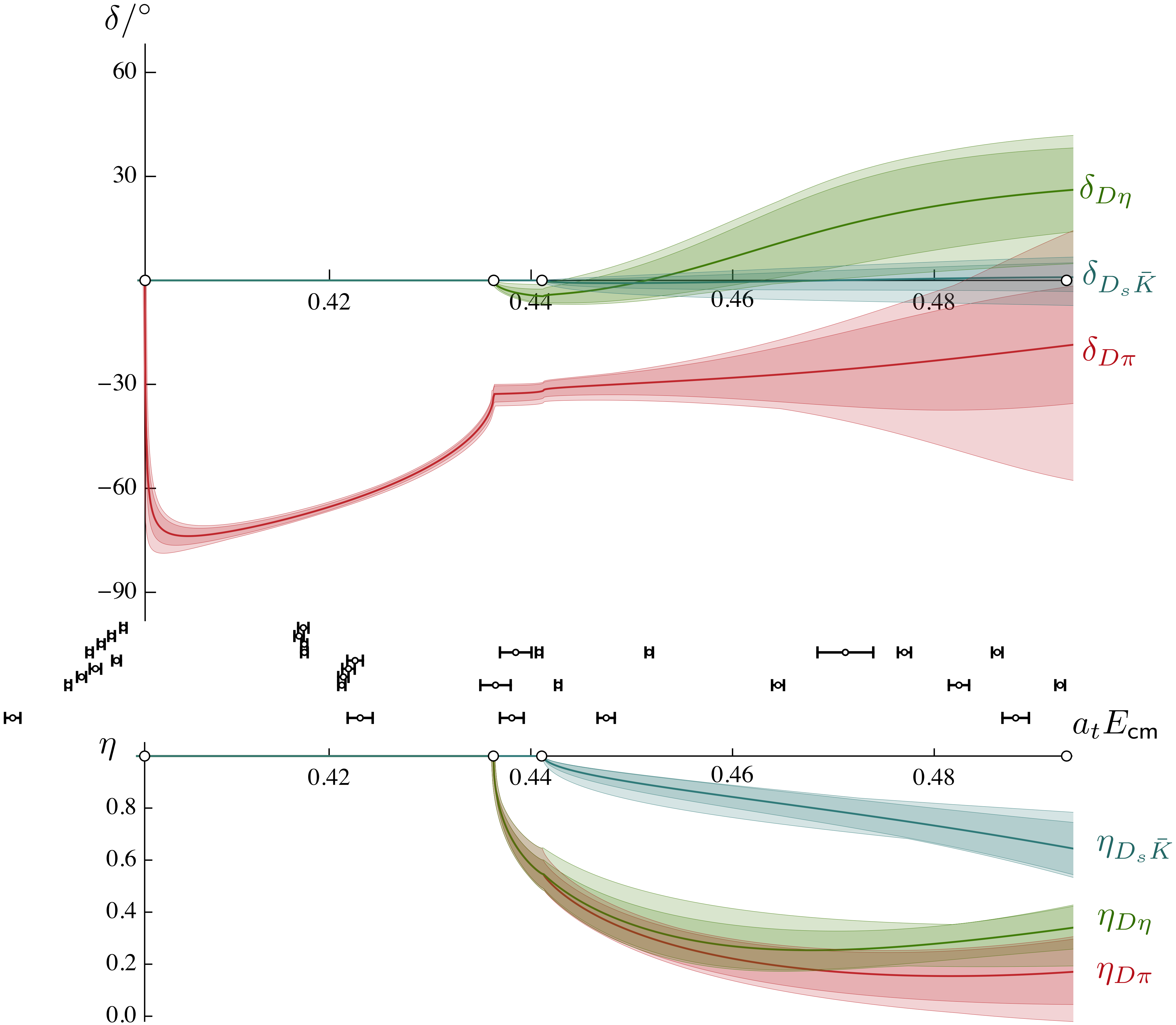

$\delta/°$
60
30
−30
−60
−90
0.42
0.44
0.46
0.48
$\delta_{D\eta}$
$\delta_{D_s\bar{K}}$
$\delta_{D\pi}$
$a_t E_{\mathsf{cm}}$
η
0.8
0.6
0.4
0.2
0.0
$\eta_{D_s\bar{K}}$
$\eta_{D\eta}$
$\eta_{D\pi}$

Dπ scattering on the lattice



Dπ scattering on the lattice




$\delta/°$
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−30
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0.42
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$a_t E_{\mathsf{cm}}$
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$\eta_{D_s\bar{K}}$
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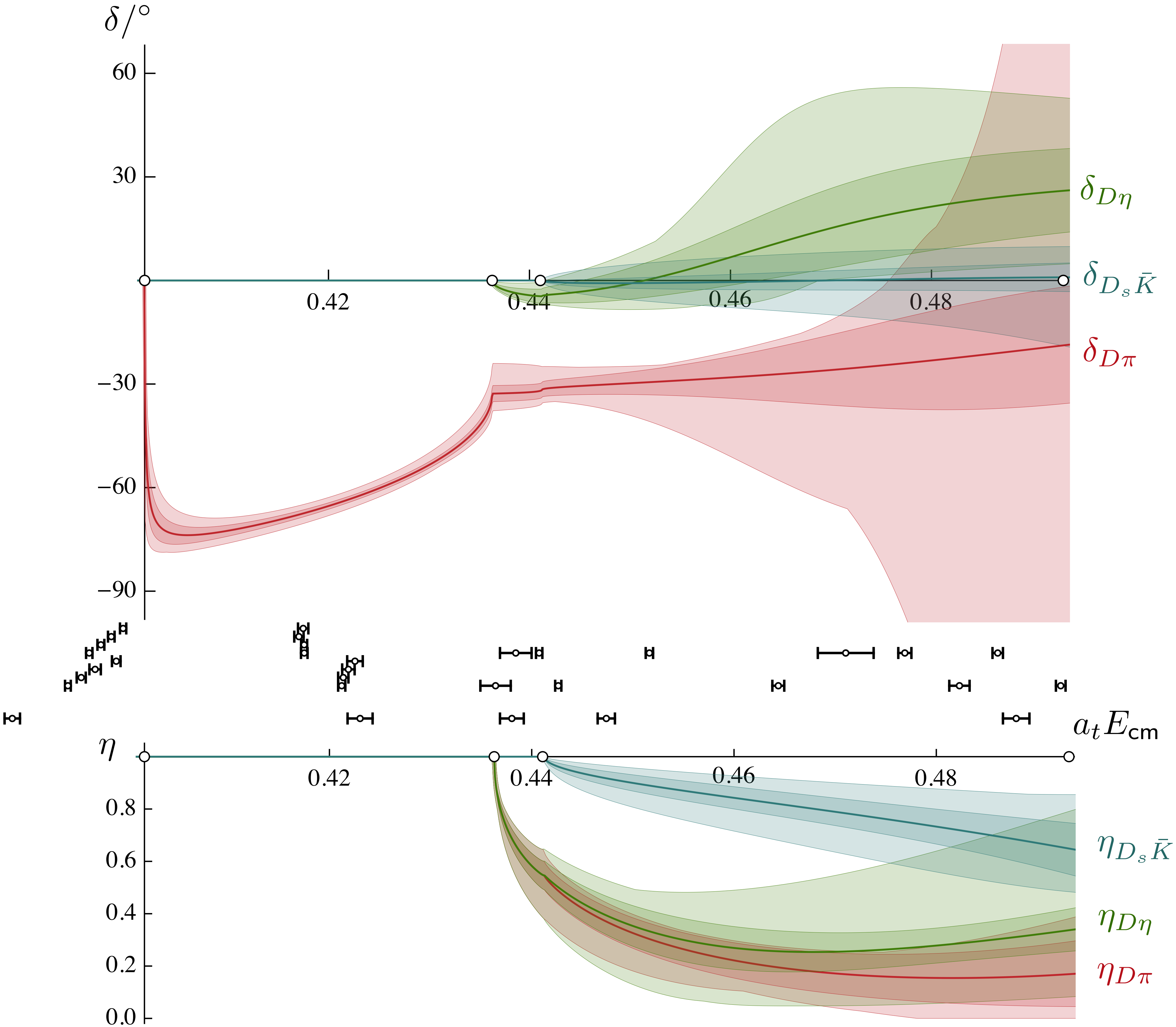
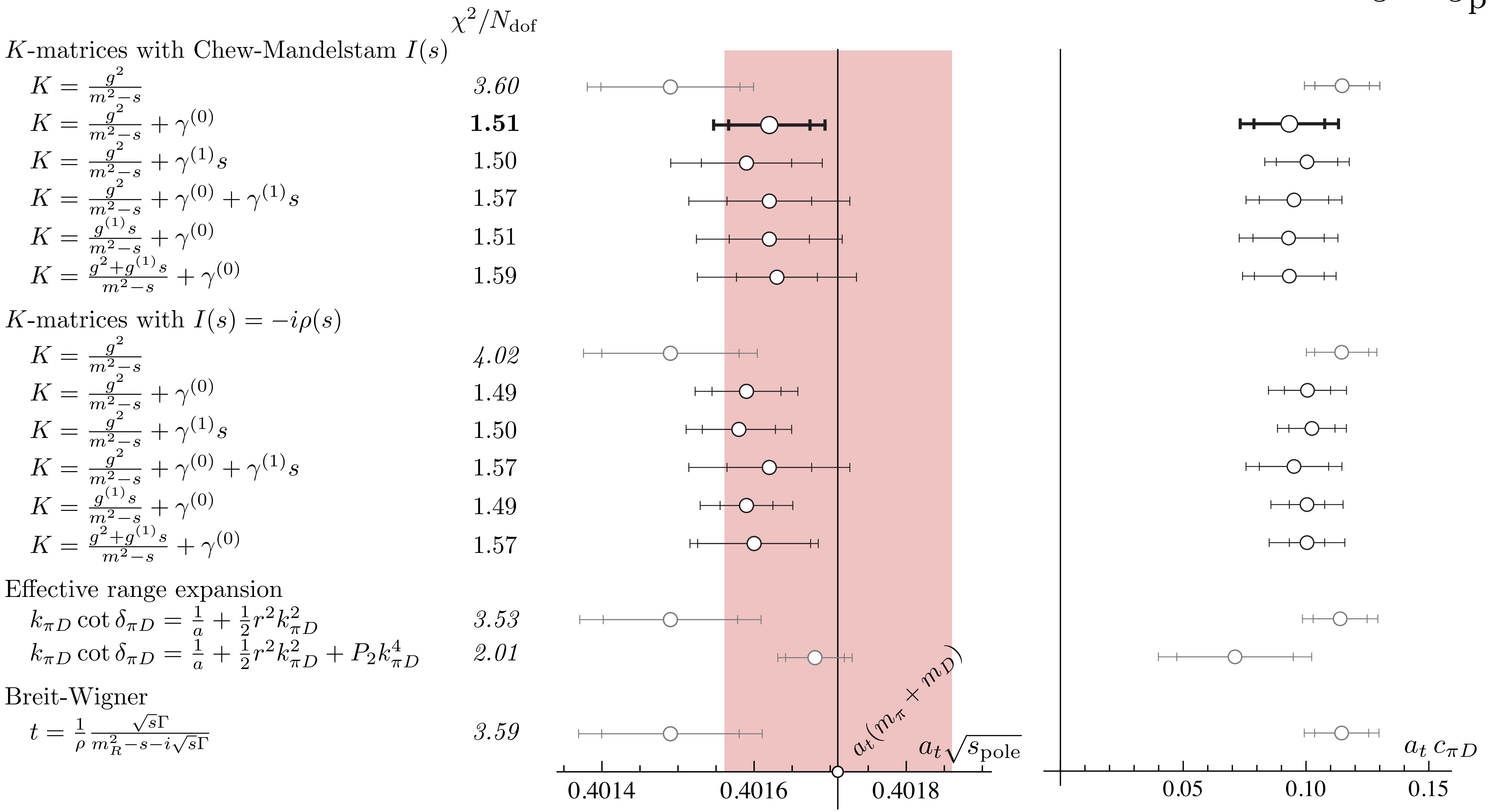

$\delta/^\circ$
60
30
−30
−60
−90
0.42
0.44
0.46
0.48
$\delta_{D\eta}$
$\delta_{D_s\bar{K}}$
$\delta_{D\pi}$
$a_t E_{\mathsf{cm}}$
η
0.8
0.6
0.4
0.2
0.0
$\eta_{D_s\bar{K}}$
$\eta_{D\eta}$
$\eta_{D\pi}$

Amplitude poles

$$t_{ij} \sim \frac{c_i c_j}{s - s_{\rm pole}}$$

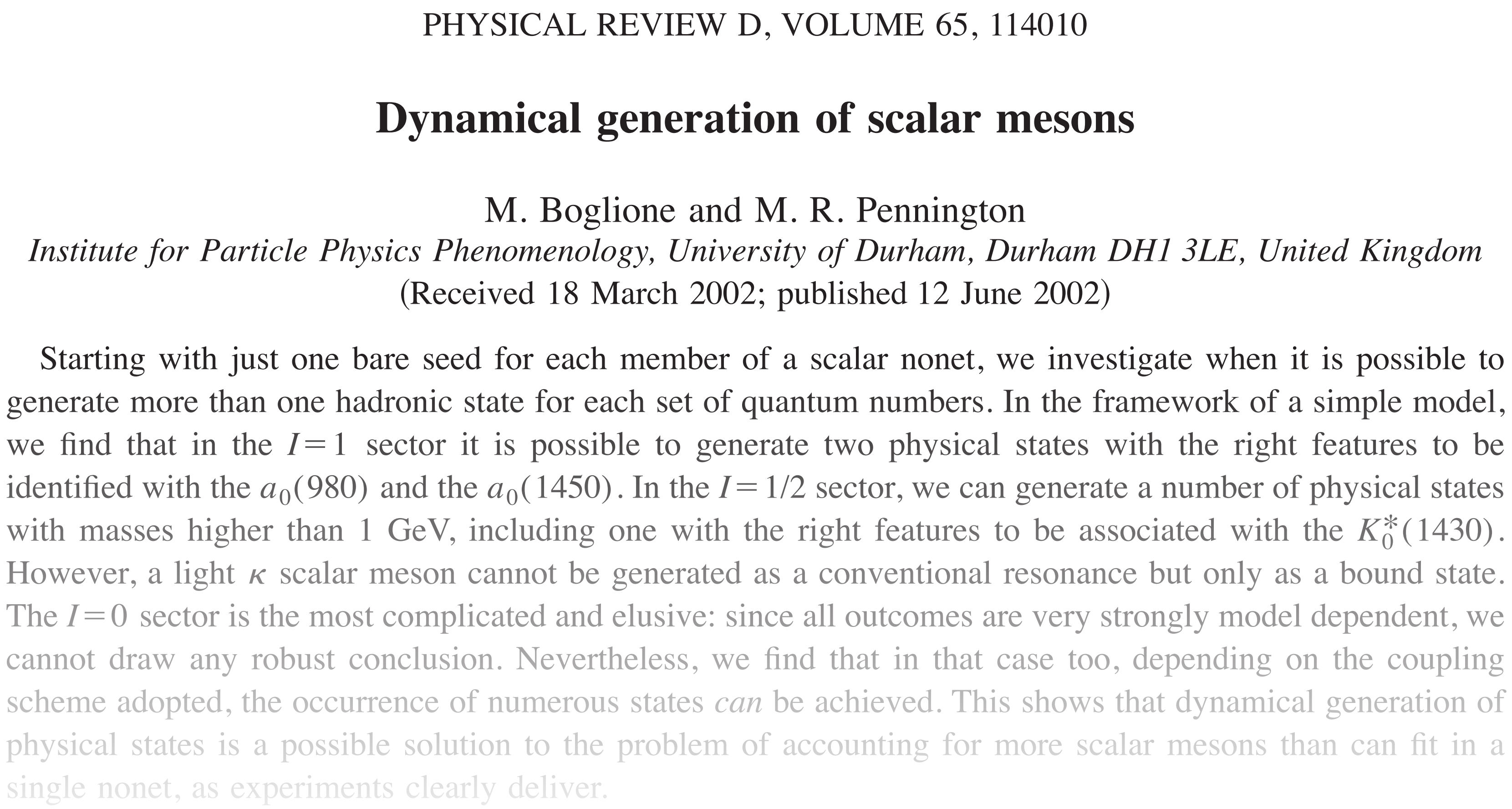


	$\chi^2/N_{\rm dof}$
K-matrices with Chew-Mandelstam $I(s)$	
$K = \frac{g^2}{m^2-s}$	*3.60*
$K = \frac{g^2}{m^2-s} + \gamma^{(0)}$	**1.51**
$K = \frac{g^2}{m^2-s} + \gamma^{(1)}s$	1.50
$K = \frac{g^2}{m^2-s} + \gamma^{(0)} + \gamma^{(1)}s$	1.57
$K = \frac{g^{(1)}s}{m^2-s} + \gamma^{(0)}$	1.51
$K = \frac{g^2+g^{(1)}s}{m^2-s} + \gamma^{(0)}$	1.59
K-matrices with $I(s) = -i\rho(s)$	
$K = \frac{g^2}{m^2-s}$	*4.02*
$K = \frac{g^2}{m^2-s} + \gamma^{(0)}$	1.49
$K = \frac{g^2}{m^2-s} + \gamma^{(1)}s$	1.50
$K = \frac{g^2}{m^2-s} + \gamma^{(0)} + \gamma^{(1)}s$	1.57
$K = \frac{g^{(1)}s}{m^2-s} + \gamma^{(0)}$	1.49
$K = \frac{g^2+g^{(1)}s}{m^2-s} + \gamma^{(0)}$	1.57
Effective range expansion	
$k_{\pi D}\cot\delta_{\pi D} = \frac{1}{a} + \frac{1}{2}r^2k_{\pi D}^2$	*3.53*
$k_{\pi D}\cot\delta_{\pi D} = \frac{1}{a} + \frac{1}{2}r^2k_{\pi D}^2 + P_2k_{\pi D}^4$	*2.01*
Breit-Wigner	
$t = \frac{1}{\rho}\frac{\sqrt{s}\Gamma}{m_R^2 - s - i\sqrt{s}\Gamma}$	*3.59*

Near-threshold bound state, coupled strongly to Dπ
At m_π = 391 MeV, we find a pole at (2274±1) MeV

Experiment finds a very broad S-wave resonance with m = (2318±29) - (267±40)$i/2$
Further studies with m_π = 236 MeV are planned

Back to the $a_0(980)$



PHYSICAL REVIEW D, VOLUME 65, 114010

Dynamical generation of scalar mesons

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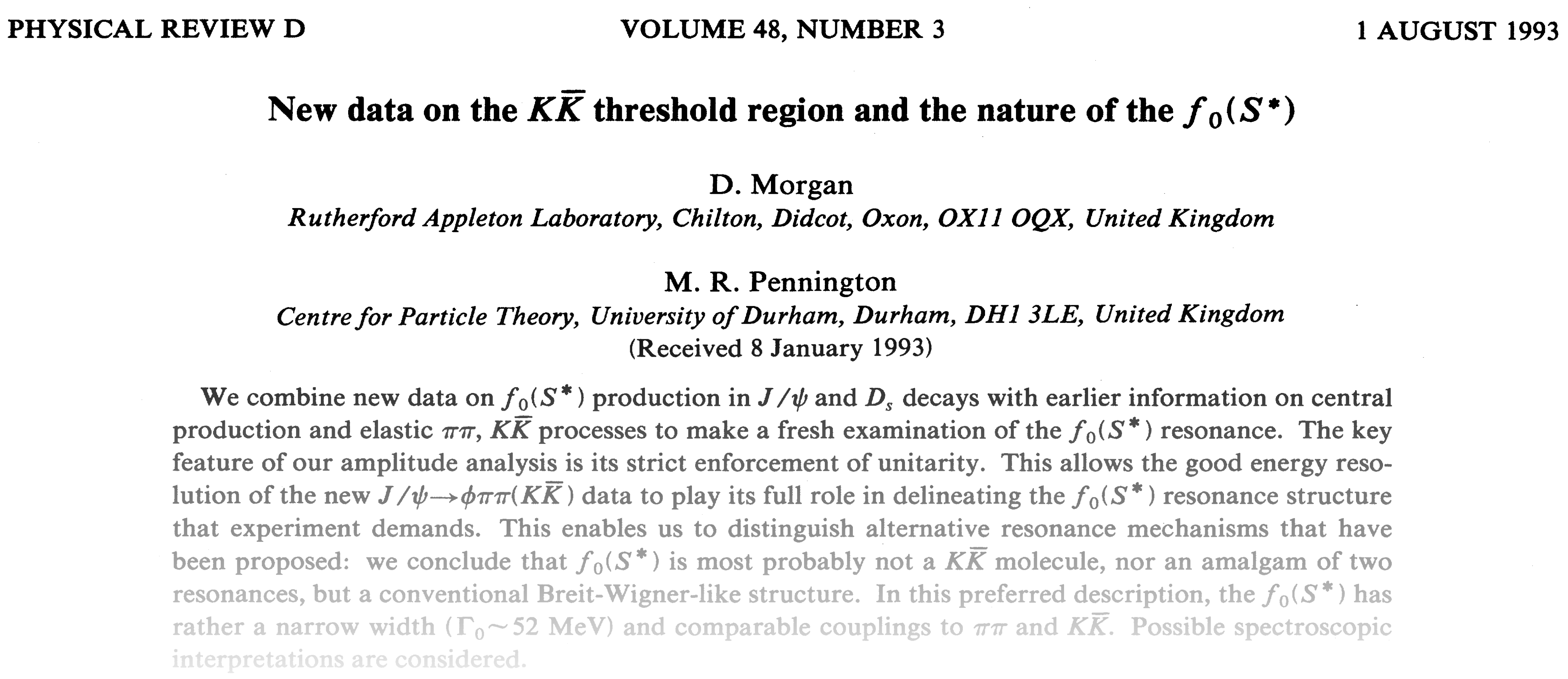
(Received 18 March 2002; published 12 June 2002)

Starting with just one bare seed for each member of a scalar nonet, we investigate when it is possible to generate more than one hadronic state for each set of quantum numbers. In the framework of a simple model, we find that in the $I=1$ sector it is possible to generate two physical states with the right features to be identified with the $a_0(980)$ and the $a_0(1450)$. In the $I=1/2$ sector, we can generate a number of physical states with masses higher than 1 GeV, including one with the right features to be associated with the $K_0^*(1430)$. However, a light κ scalar meson cannot be generated as a conventional resonance but only as a bound state. The $I=0$ sector is the most complicated and elusive: since all outcomes are very strongly model dependent, we cannot draw any robust conclusion. Nevertheless, we find that in that case too, depending on the coupling scheme adopted, the occurrence of numerous states *can* be achieved. This shows that dynamical generation of physical states is a possible solution to the problem of accounting for more scalar mesons than can fit in a single nonet, as experiments clearly deliver.

Back to the ~~$a_0(980)$~~ $f_0(980)$



PHYSICAL REVIEW D VOLUME 48, NUMBER 3 1 AUGUST 1993

New data on the $K\bar{K}$ threshold region and the nature of the $f_0(S^*)$

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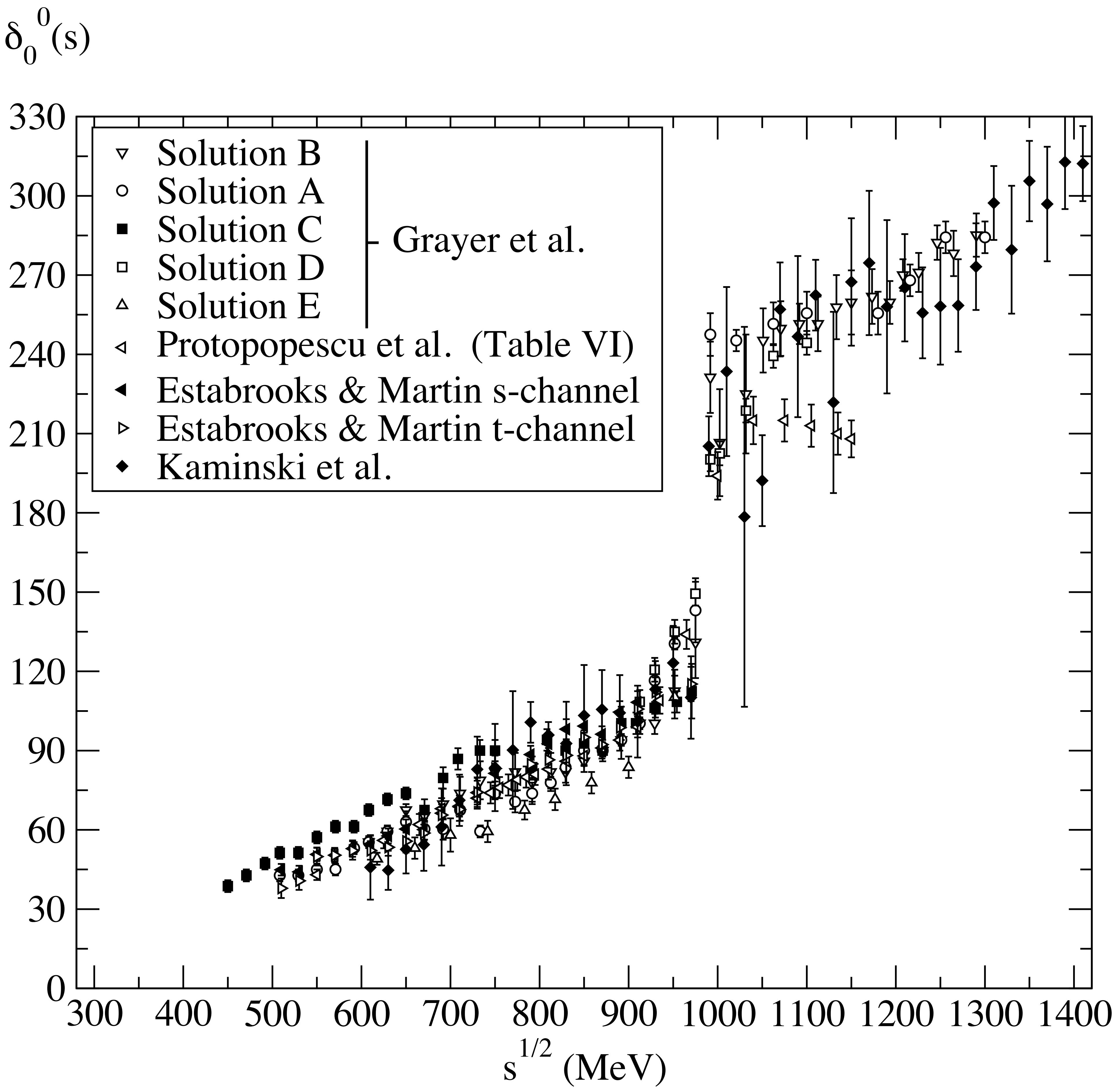
M. R. Pennington
Centre for Particle Theory, University of Durham, Durham, DH1 3LE, United Kingdom
(Received 8 January 1993)

We combine new data on $f_0(S^*)$ production in J/ψ and D_s decays with earlier information on central production and elastic $\pi\pi$, $K\bar{K}$ processes to make a fresh examination of the $f_0(S^*)$ resonance. The key feature of our amplitude analysis is its strict enforcement of unitarity. This allows the good energy resolution of the new $J/\psi\rightarrow\phi\pi\pi(K\bar{K})$ data to play its full role in delineating the $f_0(S^*)$ resonance structure that experiment demands. This enables us to distinguish alternative resonance mechanisms that have been proposed: we conclude that $f_0(S^*)$ is most probably not a $K\bar{K}$ molecule, nor an amalgam of two resonances, but a conventional Breit-Wigner-like structure. In this preferred description, the $f_0(S^*)$ has rather a narrow width ($\Gamma_0\sim 52$ MeV) and comparable couplings to $\pi\pi$ and $K\bar{K}$. Possible spectroscopic interpretations are considered.

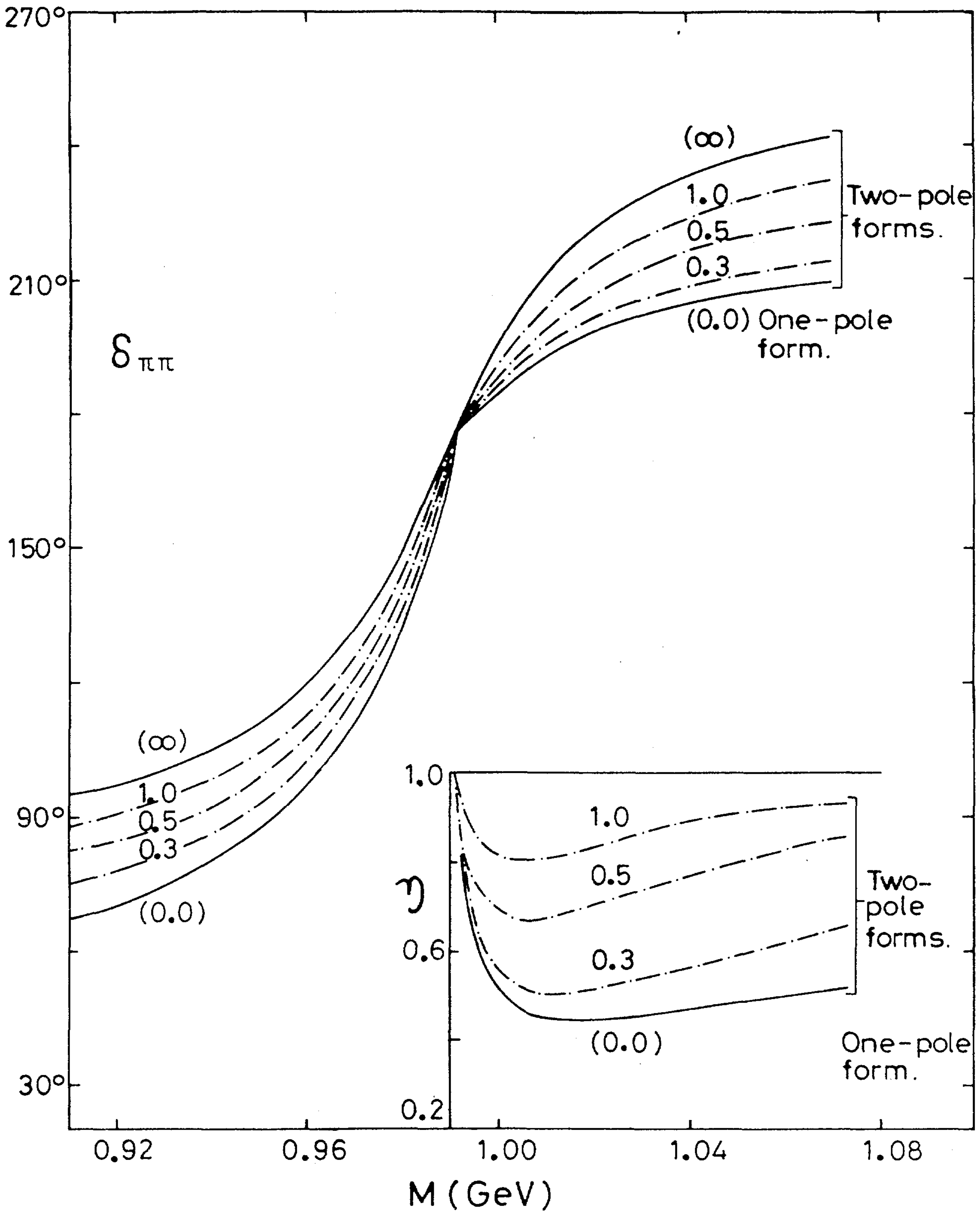

$\delta_0^0(s)$
330
300
270
240
210
180
150
120
90
60
30
0
300
400
500
600
700
800
900
1000
1100
1200
1300
1400
$s^{1/2}$ (MeV)
▽ Solution B
○ Solution A
■ Solution C
□ Solution D
△ Solution E
Grayer et al.
◁ Protopopescu et al. (Table VI)
◀ Estabrooks & Martin s-channel
▷ Estabrooks & Martin t-channel
◆ Kaminski et al.

Pole counting

Weinberg, Morgan & Pennington

Resonance decaying in S-wave
close to threshold

one-pole: extended object ~ meson-meson
two-poles: compact object ~ $q\bar{q}$, …



Jost functions

In general:

$$S_{11} = \frac{\mathfrak{J}(-k_1, k_2)}{\mathfrak{J}(k_1, k_2)}$$

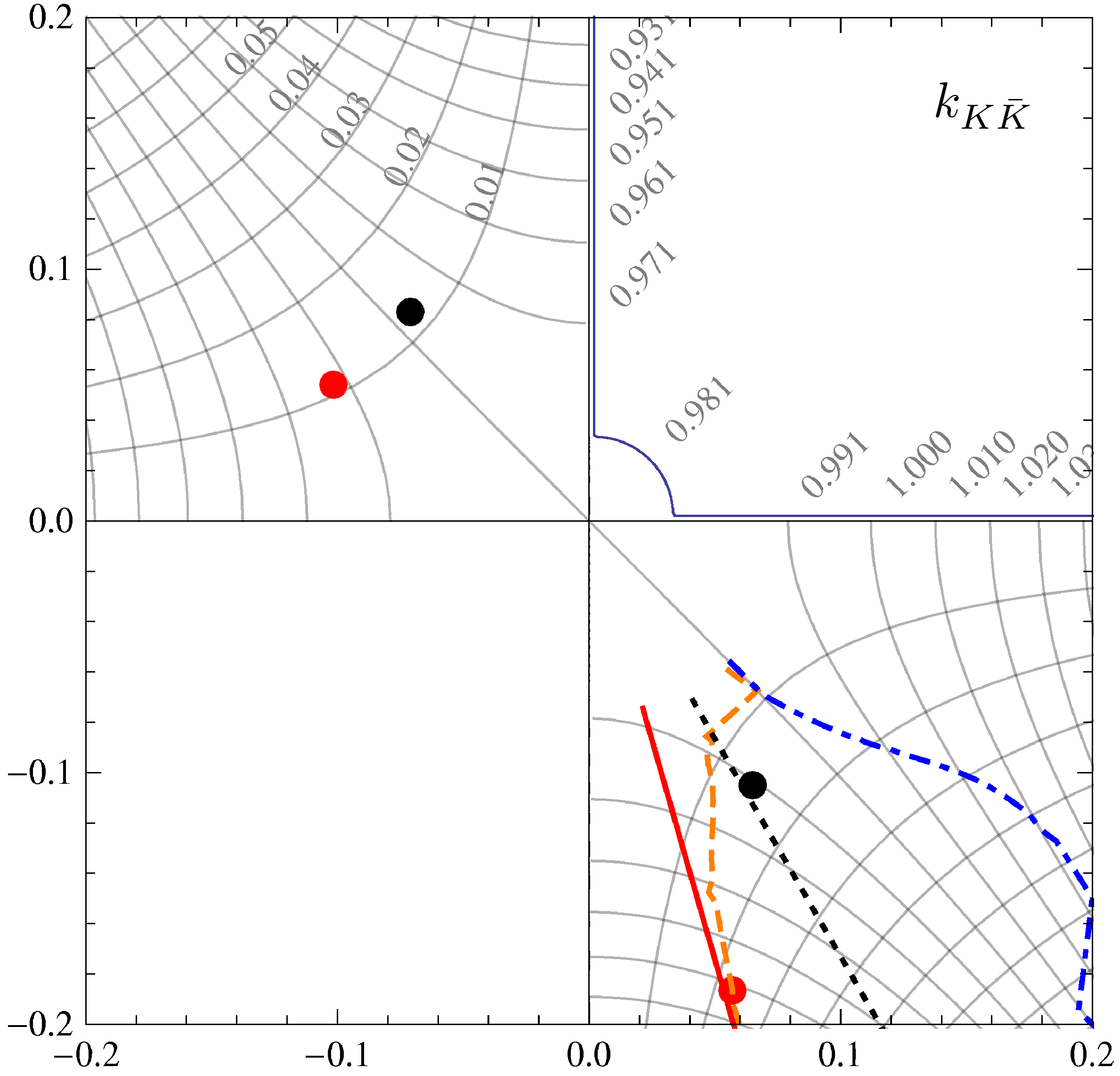
$$S_{22} = \frac{\mathfrak{J}(k_1, -k_2)}{\mathfrak{J}(k_1, k_2)}$$

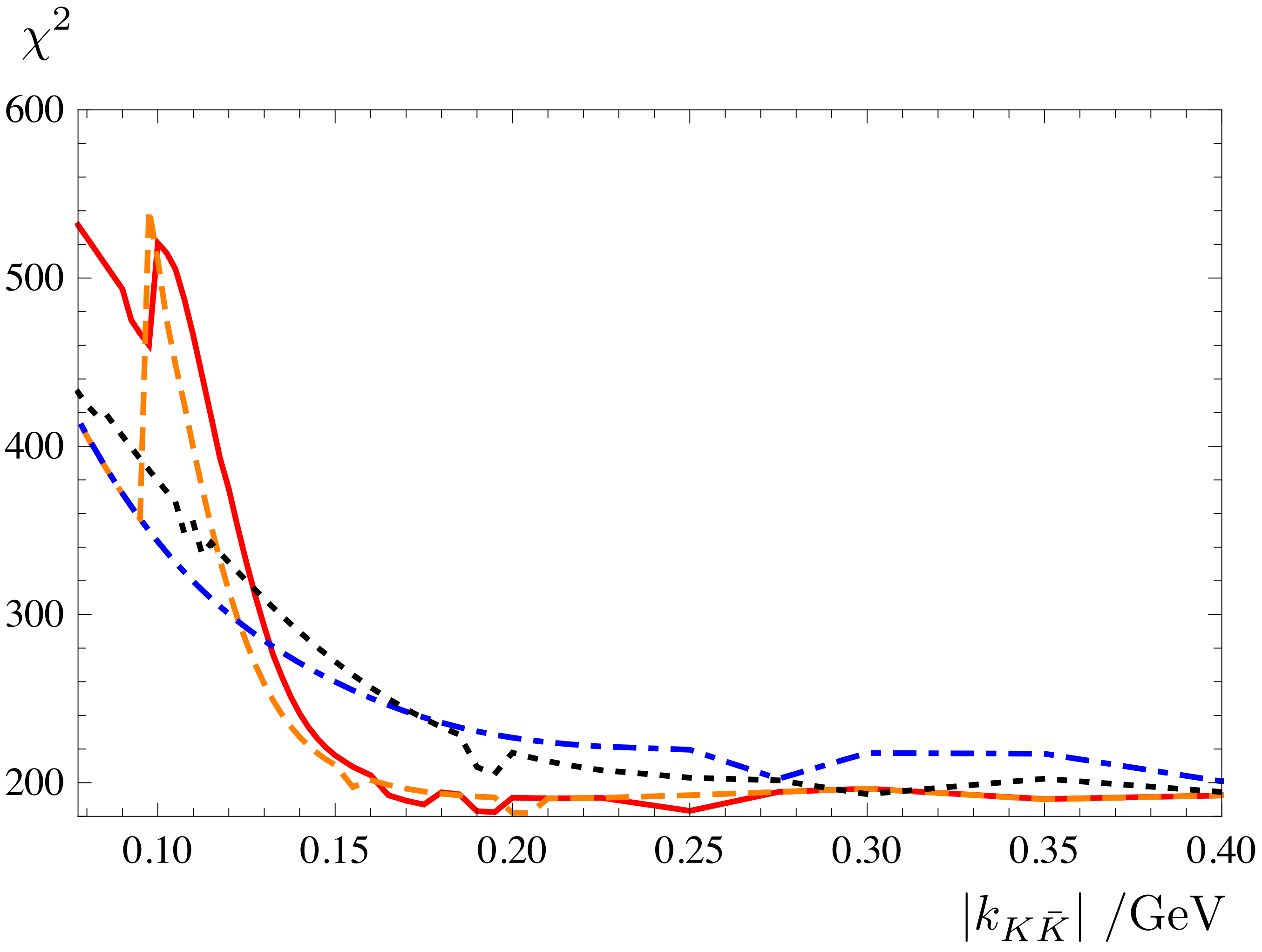
$$\det \mathbf{S} = \frac{\mathfrak{J}(-k_1, -k_2)}{\mathfrak{J}(k_1, k_2)}$$

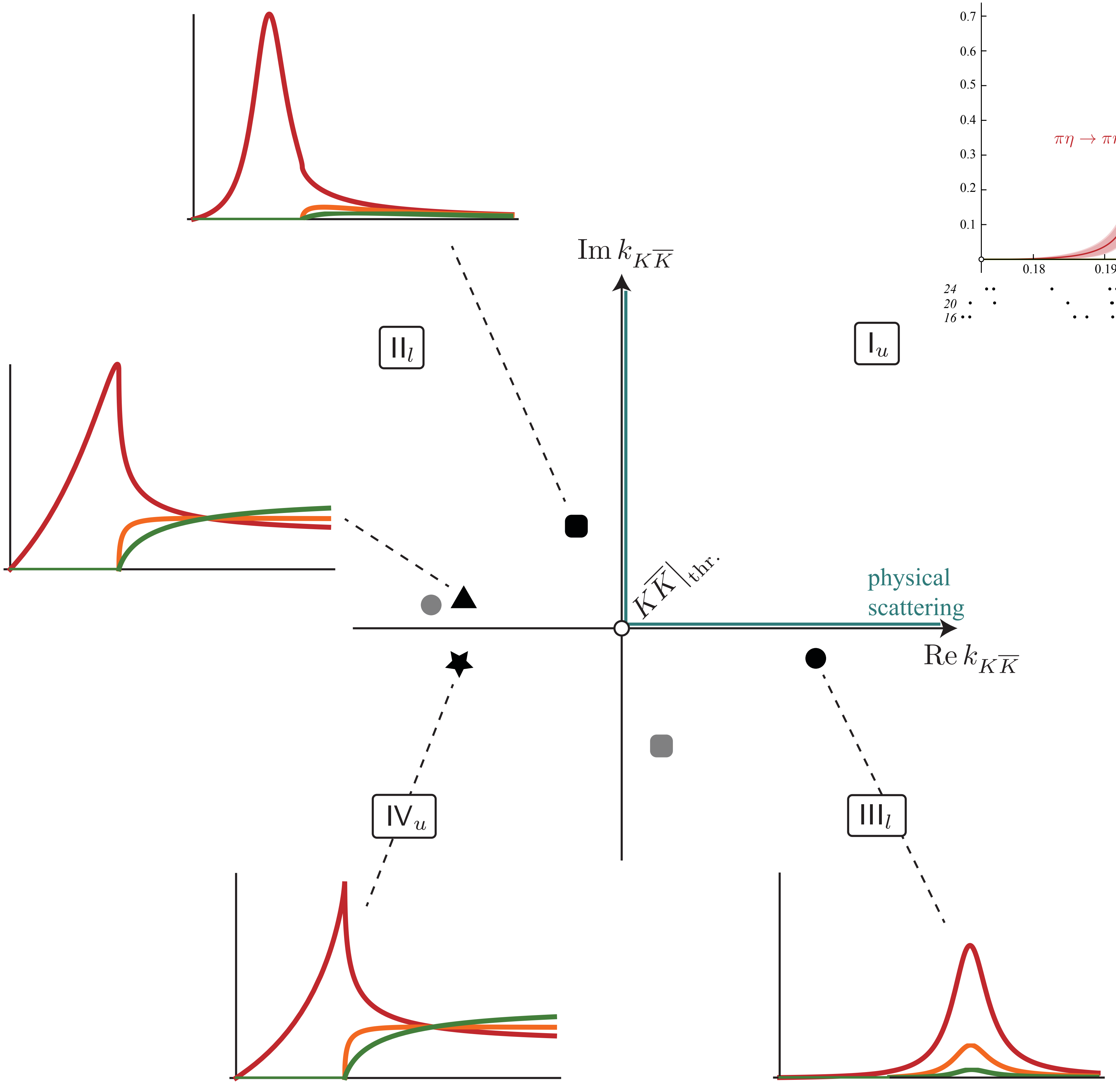
Morgan & Pennington,
far above lowest threshold:

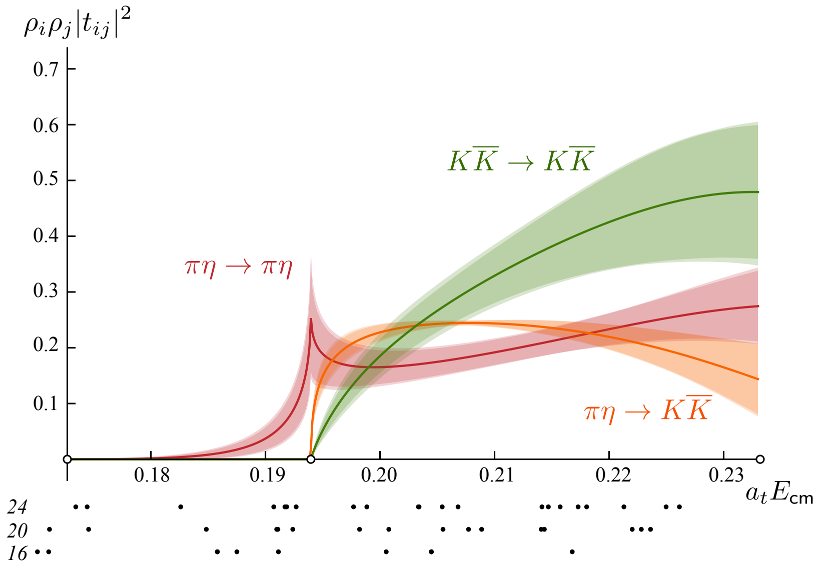
$$S_{11} = \frac{\phi^\star\left(-k^\star_{K\bar{K}}\right)}{\phi\left(k_{K\bar{K}}\right)}$$

$$\phi(k_{K\bar{K}}) = \prod_i \left(1 - \frac{k_{K\bar{K}}}{k_{p_i}}\right) \sum_j \left(c_j k^j_{K\bar{K}}\right)$$


$k_{K\bar{K}}$
0.05
0.04
0.03
0.02
0.01
0.931
0.941
0.951
0.961
0.971
0.981
0.991
1.000
1.010
1.020
1.02
0.2
0.1
0.0
−0.1
−0.2
−0.2
−0.1
0.0
0.1
0.2

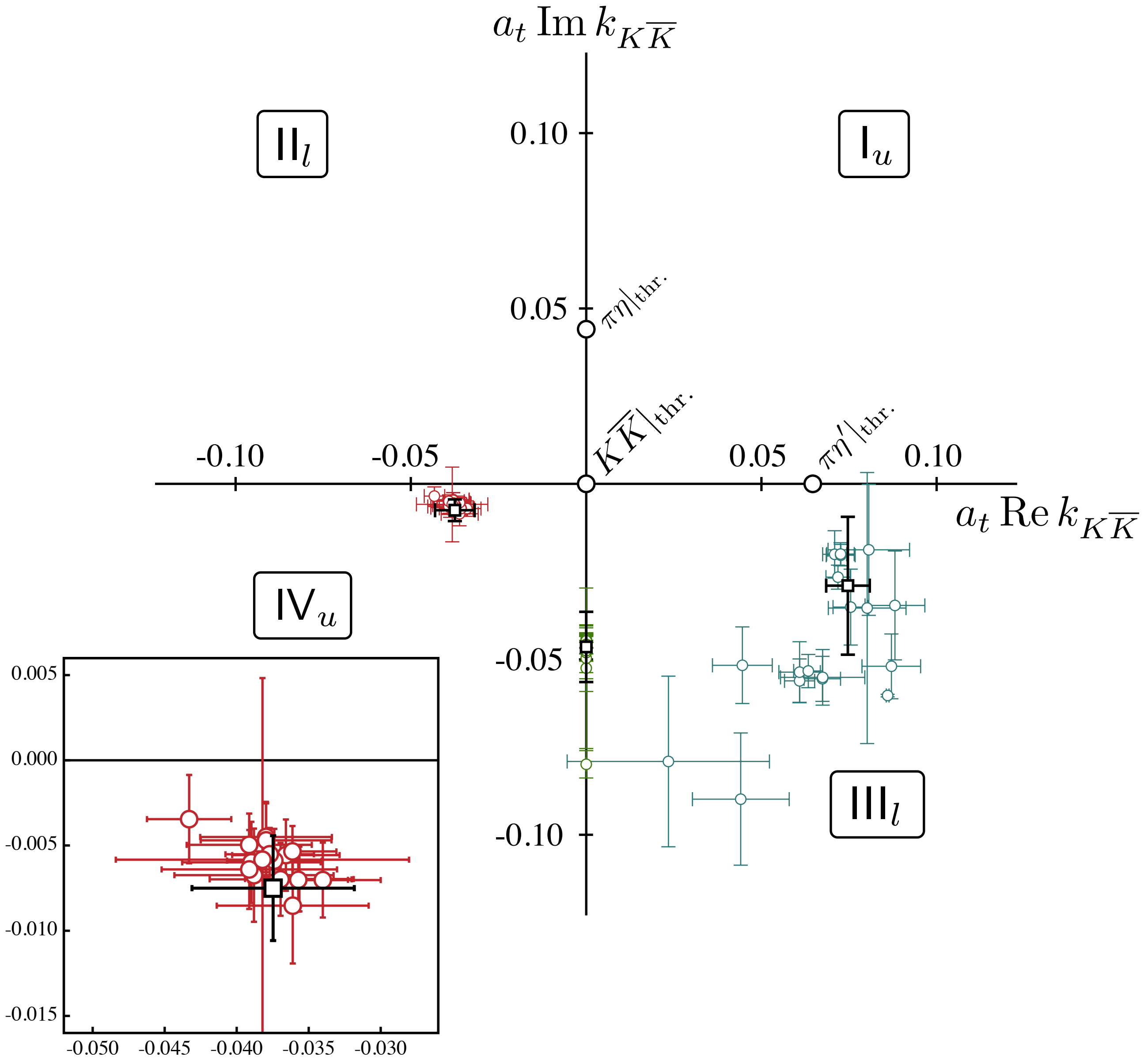

χ^2
600
500
400
300
200
0.10
0.15
0.20
0.25
0.30
0.35
0.40
$|k_{K\bar{K}}|$ /GeV

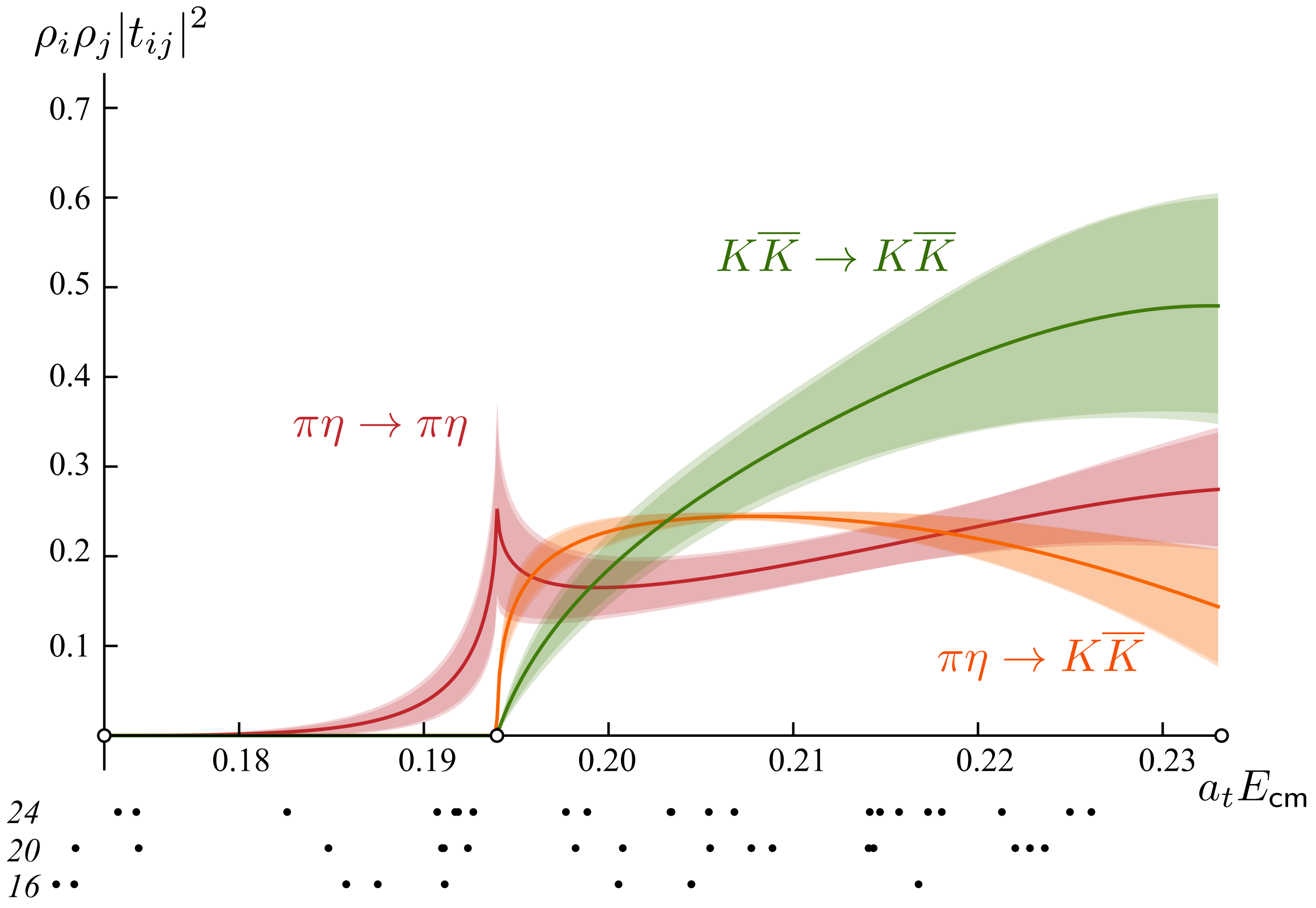




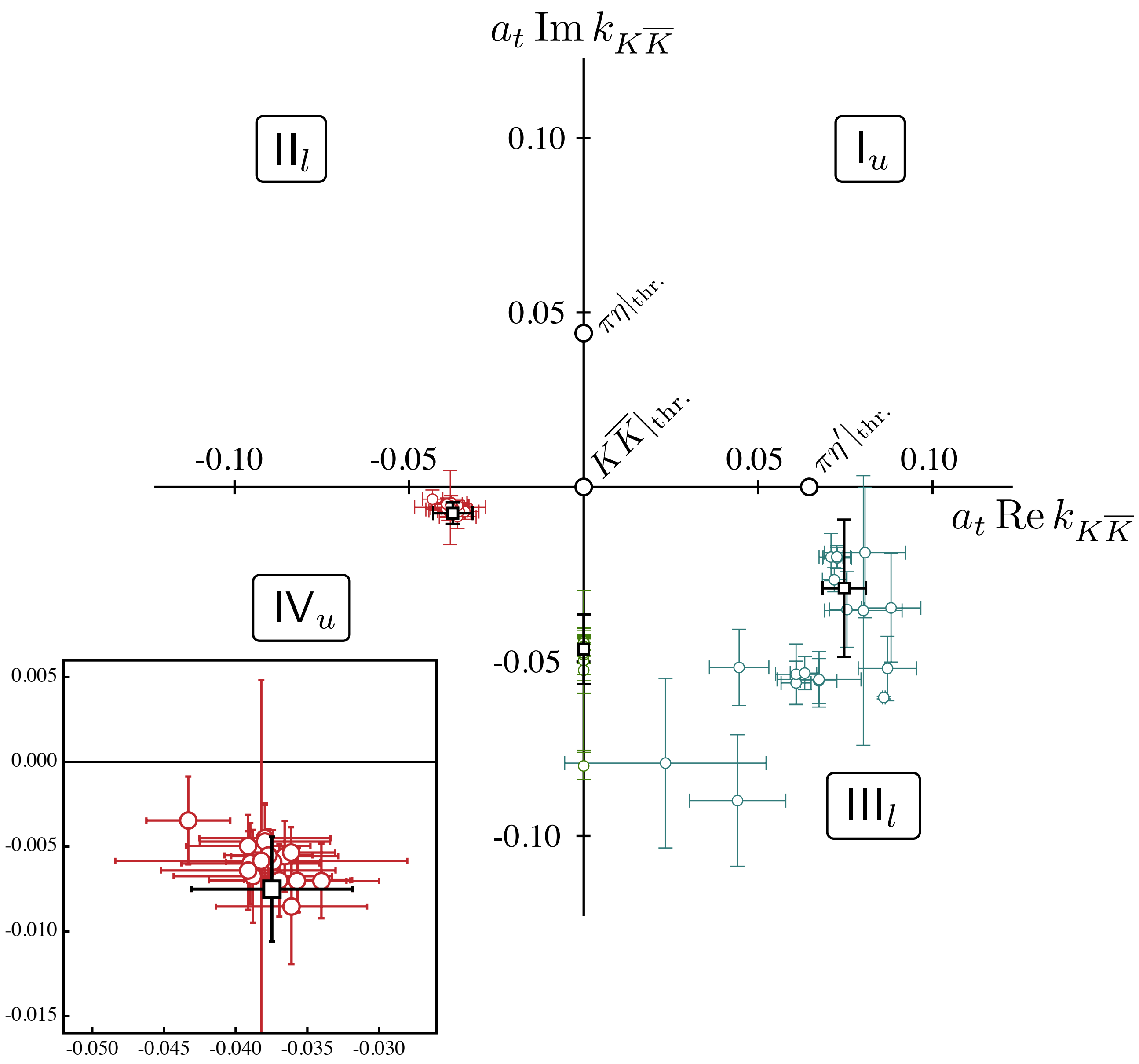
from Dudek et al (for the Hadron Spectrum Collaboration), PRD93 (2016) no.9, 094506.

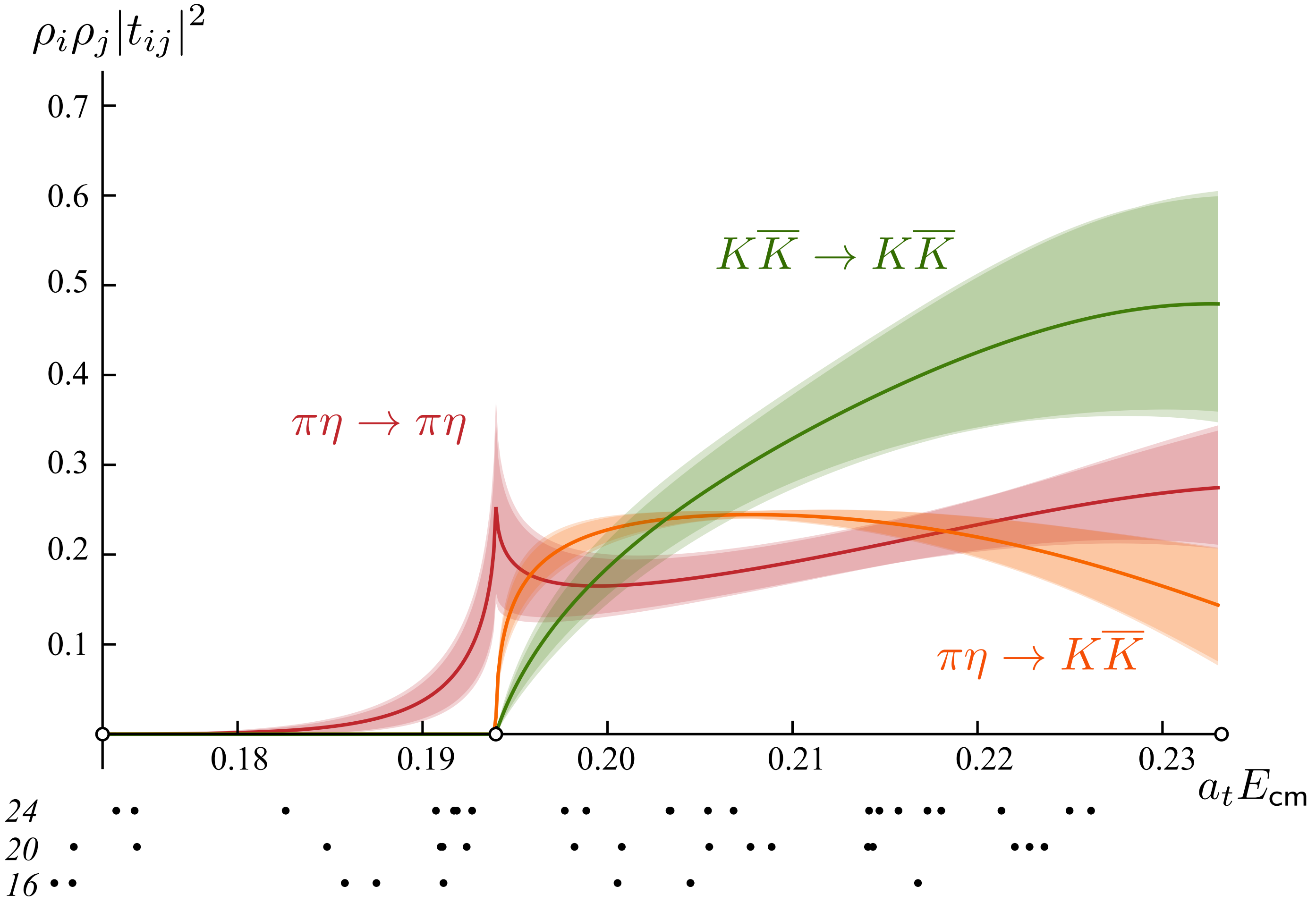
Pole counting



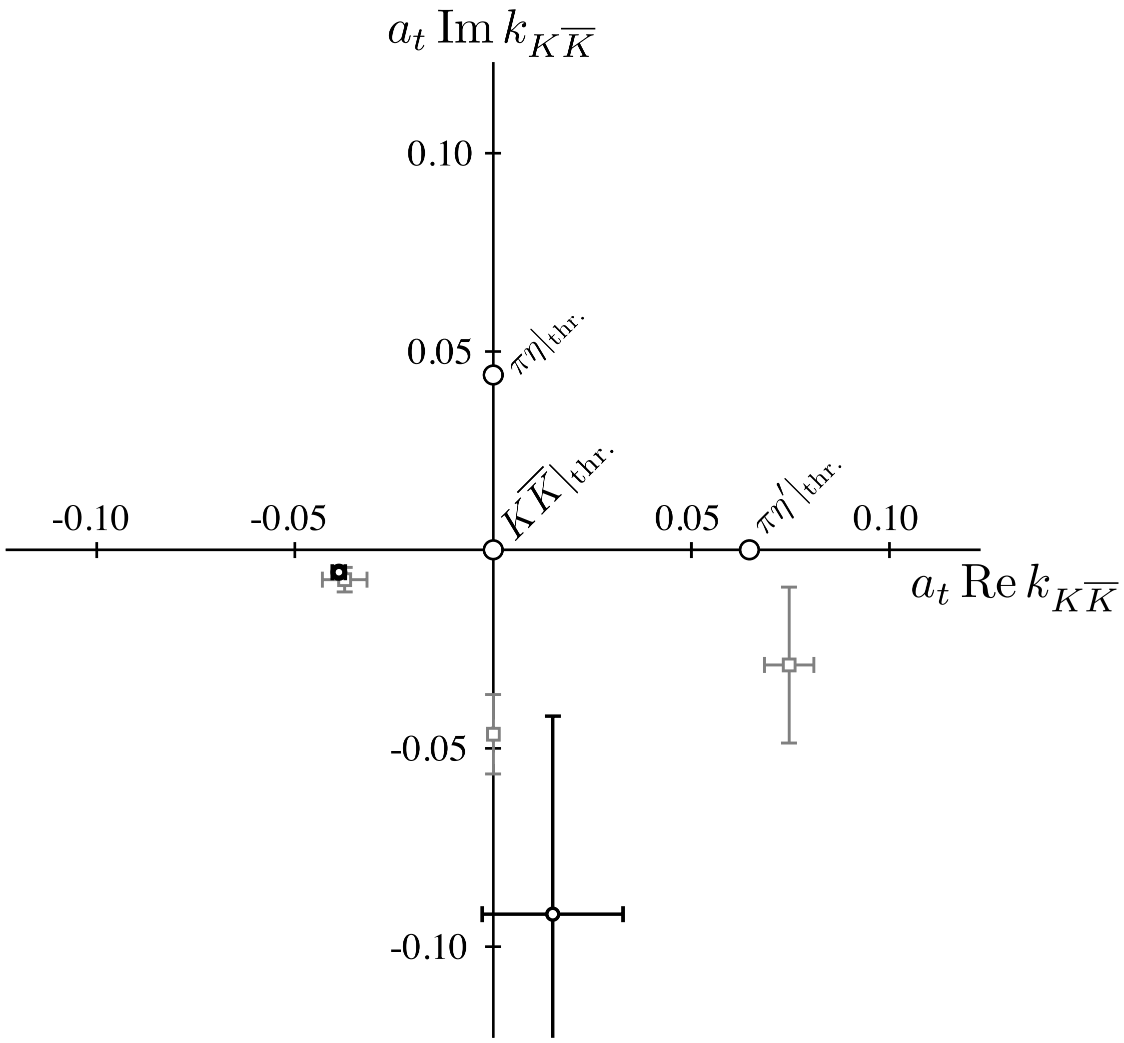


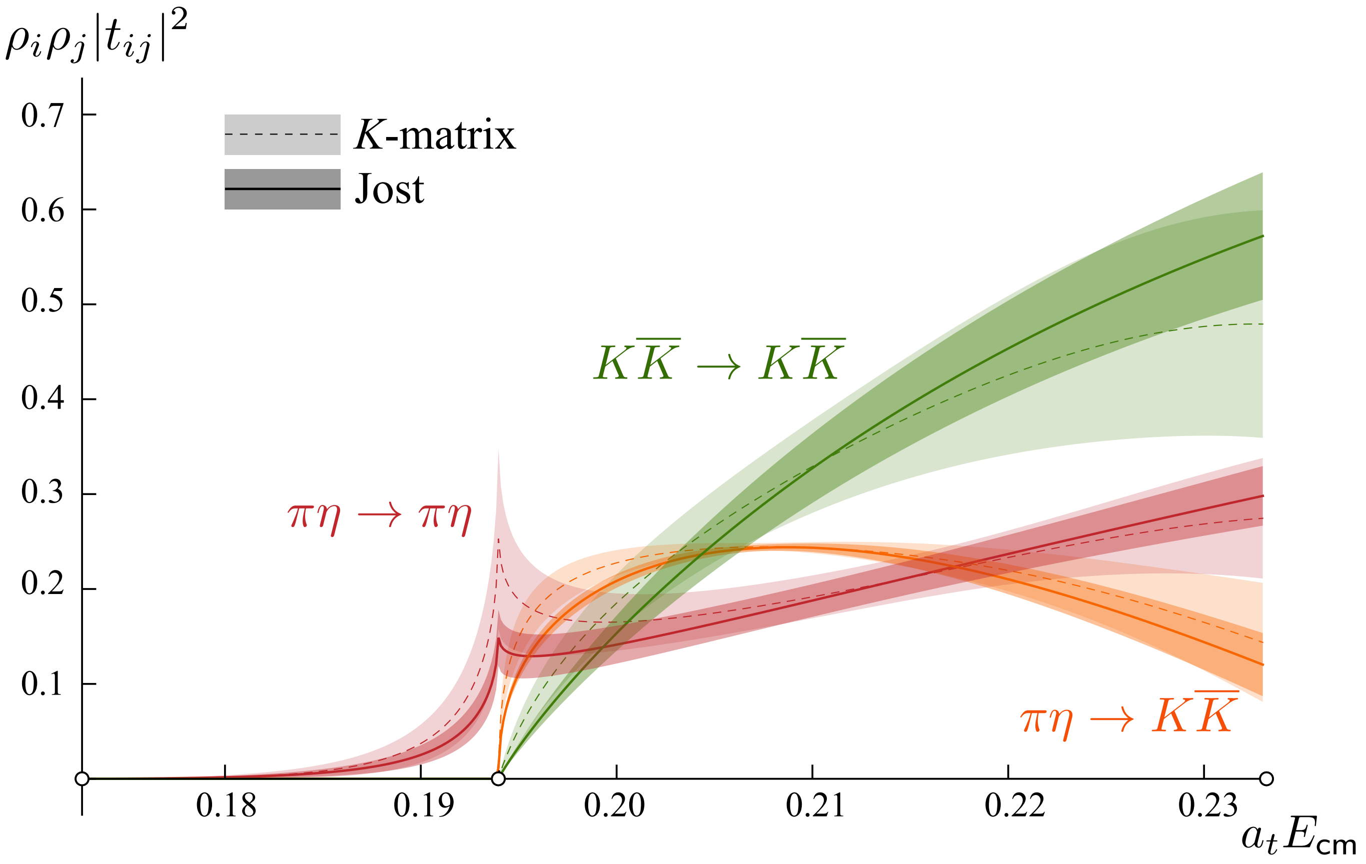
Pole counting





Pole counting

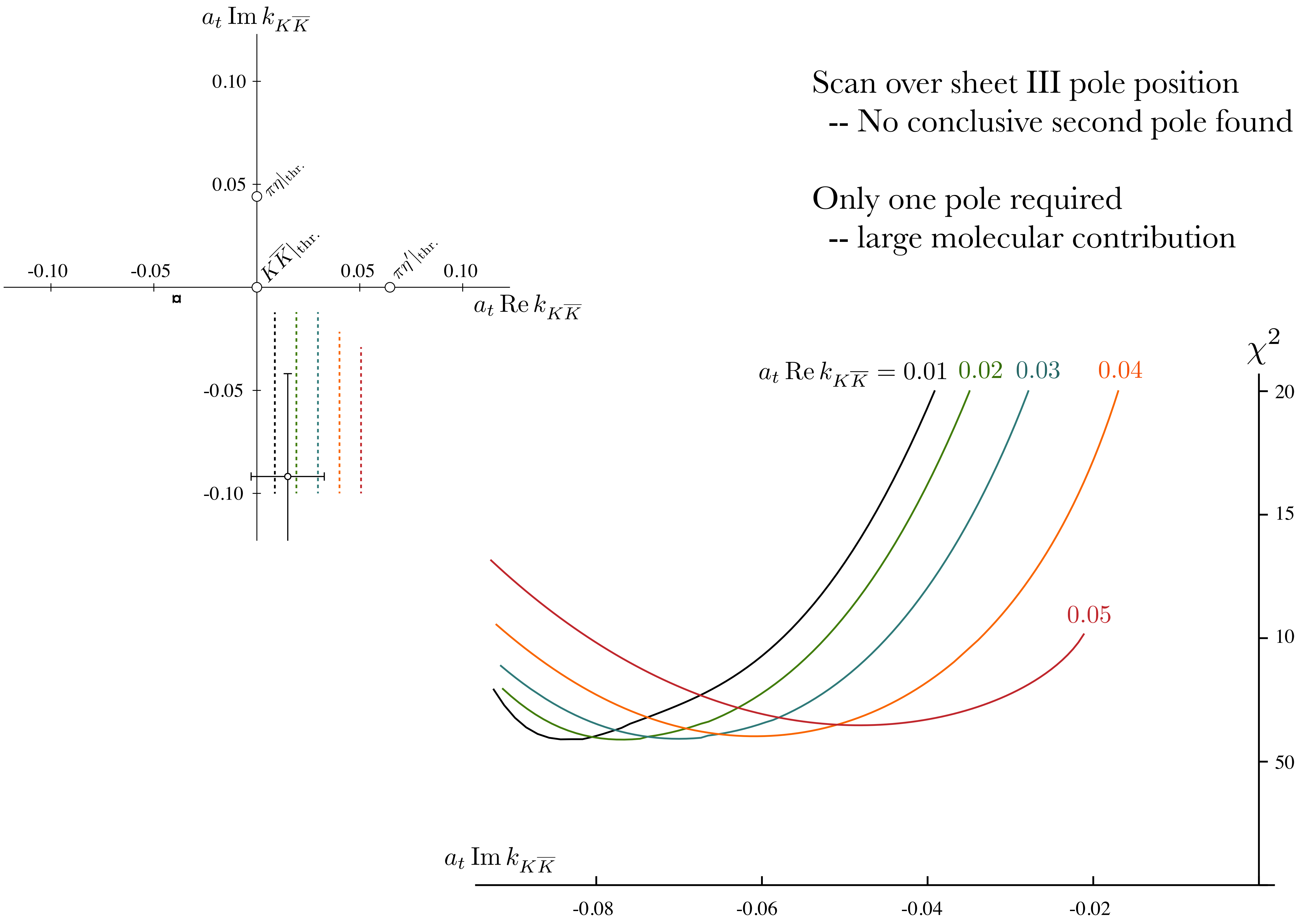




Pole counting

Scan over sheet III pole position
 -- No conclusive second pole found

Only one pole required
 -- large molecular contribution



Summary

Resonance information is now being extracted in systematically improvable, first principles methods using lattice QCD.

Coupled-channel physics is present almost everywhere in the spectrum and recent progress has made extraction of the poles and couplings possible.

Thresholds play an important role, particularly in S-wave where they can introduce sharp effects into the amplitudes.

Careful analyses are needed to extract the pole content.

The methods are widely applicable:
$f_0(980)$, X(3872), hybrids,
$\pi\gamma \rightarrow \pi\pi$, $\pi N \rightarrow \pi N$, $\gamma N \rightarrow \pi N$.