Chiral quark model for meson production in the resonance region

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Outline

◆ The “missing baryon resonances” problem
◆ Effective chiral Lagrangian for quark-pseudoscalar-meson interaction
◆ Baryon resonances in pseudoscalar meson photoproduction and meson-nucleon scatterings
◆ Prospects
1. “Missing baryon resonances in $\pi N$ scattering

- The non-relativistic constituent quark model (NRCQM) makes great success in the description of hadron spectroscopy: meson ($q\bar{q}$), baryon ($qqq$).

- However, it also predicted a much richer baryon spectrum, where some of those have not been seen in $\pi N$ scatterings.
  - “Missing Resonances”.

\[ \begin{align*}
\pi, \ 0^- & \quad N^*, \ L_{2I,2J} \\
N, \ 1/2^+ & \quad \left\{ \begin{array}{l}
P_{33}(1232) \ \Delta \\
P_{11}(1440) \\
S_{11}(1535) \\
D_{13}(1520) \\
\ldots
\end{array} \right. 
\end{align*} \]
## PDG2008: 22 nucleon resonances (uud, udd)

<table>
<thead>
<tr>
<th>Particle</th>
<th>$L_{2I,2J}$</th>
<th>Overall status</th>
<th>Status as seen in —</th>
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<td>$N(2700)$</td>
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</tr>
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(* *) not well-established
Dilemma:

a) The NRCQM is **WRONG**: quark-diquark configuration? …

b) The NRCQM is **CORRECT**, but those missing states have only weak couplings to $\pi N$, i.e. small $g_{\pi N^* N}$. (Isgur, 1980)

Looking for “missing resonances” in $N^* \rightarrow \eta N, K\Sigma, K\Lambda, \rho N, \omega N, \phi N, \gamma N$ …

(Exotics …)
Questions:

Should we take the naïve quark model seriously?

How far one can go with it?

What is the success and what is the failure?

… …
The first orbital excitation states in the NRCQM

In the nonstrange sector, NRCQM allows the groundstate $[56, ^28] (p$ and $n)$ to be excited to $[70, ^28]$ and $[70, ^48]$ octets, and $[70, ^210]$ decuplet via single photon absorption.

$\gamma$\n
$\text{EM}$

$\text{N}$

$\text{N^*,Delta^*}$

$|70, ^28, 1, 1, J\rangle \quad \bullet \quad S_{11}(1535) (***)$, $D_{13}(1520) (***)$

$|70, ^48, 1, 1, J\rangle \quad \bullet \quad S_{11}(1650) (***)$, $D_{13}(1700) (***)$, $D_{15}(1675) (***)$

$|70, ^210, 1, 1, J\rangle \quad \bullet \quad S_{31}(1620) (***)$, $D_{33}(1670) (***)$

$|70, ^21, 1, 1, J\rangle \quad \bullet \quad \Lambda(1405) S_{01} (***)$, $\Lambda(1520) D_{03} (***)$

Confirmed recently by JLab Lattice calculation.
(Talk by D. Richards in MENU2010)
The SU(6)$\otimes$O(3) symmetry must be broken due to spin-dependent forces. Thus, state mixings are inevitable.

Several NRCQM selection rules are violated:

- **Moorhouse selection rule** (Moorhouse, PRL16, 771 (1966))

\[
\gamma + p(|56,^28;0,0,1/2\rangle) \nleftrightarrow N^* (|70,^48\rangle) \\
\gamma + n(|56,^28;0,0,1/2\rangle) \leftrightarrow N^* (|70,^48\rangle)
\]

- **Λ selection rule** (Zhao & Close, PRD74, 094014(2006)) in strong decays

\[
N^* (|70,^48\rangle) \nleftrightarrow K(K^*) + \Lambda
\]

- **Faiman-Hendry selection rule** (Faiman & Hendry, PR173, 1720 (1968)).

\[
\Lambda^* (|70,^48\rangle) \nleftrightarrow N(|56,^28;0,0,1/2\rangle) + \bar{K}
\]
2. Effective chiral Lagrangian for quark-pseudoscalar-meson interactions

An effective chiral Lagrangian for quark-pseudoscalar-meson coupling to keep the meson-baryon interaction invariant under the chiral transformation:

\[
\mathcal{L} = \overline{\psi} \left[ \gamma_\mu (i \partial^\mu + V^\mu + \gamma_5 A^\mu) - m \right] \psi + \cdots,
\]

where the vector and axial currents are

\[
V_\mu = \frac{1}{2} \left( \xi^\dagger \partial_\mu \xi + \xi \partial_\mu \xi^\dagger \right),
\]

\[
A_\mu = i \frac{1}{2} \left( \xi^\dagger \partial_\mu \xi - \xi \partial_\mu \xi^\dagger \right),
\]

and the chiral transformation is,

$$\xi = e^{i\phi_m/f_m}, \quad (77)$$

where $f_m$ is the decay constant of the meson. The quark field $\psi$ in the SU(3) symmetry is

$$\psi = \begin{pmatrix} \psi(u) \\ \psi(d) \\ \psi(s) \end{pmatrix}, \quad (78)$$

and the meson field $\phi_m$ is a $3 \otimes 3$ matrix:

$$\phi_m = \begin{pmatrix} \frac{1}{\sqrt{2}}\pi^0 + \frac{1}{\sqrt{6}}\eta & \pi^+ & K^+ \\ \pi^- & -\frac{1}{\sqrt{2}}\pi^0 + \frac{1}{\sqrt{6}}\eta & K^0 \\ K^- & \bar{K}^0 & -\sqrt{\frac{2}{3}}\eta \end{pmatrix}, \quad (79)$$

where the pseudoscalar mesons $\pi$, $\eta$ and $K$ are treated as Goldstone bosons. Thus, the Lagrangian in Eq. (121) is invariant under the chiral transformation. Expanding the nonlinear field $\xi$ in Eq. (77) in terms of the Goldstone boson field $\phi_m$, i.e. $\xi = 1 + i\phi_m/f_m + \cdots$, we obtain the standard quark-meson pseudovector coupling at tree level:

$$H_m = \sum_j \frac{1}{f_m} \bar{\psi}_j \gamma_\mu \gamma_5 \psi_j \partial^\mu \phi_m, \quad (80)$$

where $\psi_j$ ($\bar{\psi}_j$) represents the $j$th quark (anti-quark) field in the nucleon.
• Test of Goldberger-Treiman relation:

The axial vector coupling, \( g_A \), relates the hadronic operator \( \sigma \) to the quark operator \( \sigma_j \) for the \( j \)-th quark,

\[
\langle N_f | \sum_j \hat{I}_j \sigma_j | N_i \rangle \equiv g_A \langle N_f | \sigma | N_i \rangle.
\]

To equate the quark-level coupling to the hadronic level one for the \( \pi NN \) vertex, i.e. axial current conservation, one has

\[
g_{\pi NN} = \frac{g_AM_N}{f_\pi}
\]
Baryon excitations in $\pi^- p \rightarrow \eta n$

The process $\pi^- p \rightarrow \eta n$ can be expressed in terms of the Mandelstam variables:

$$\mathcal{M} = \mathcal{M}_s + \mathcal{M}_u + \mathcal{M}_t.$$ 

The $s$- and $u$-channel transitions are given by

$$\mathcal{M}_s = \sum_j \langle N_f | H_\eta | N_j \rangle \langle N_j | \frac{1}{E_i + \omega_\pi - E_j} H_\pi | N_i \rangle,$$

$$\mathcal{M}_u = \sum_j \langle N_f | H_\pi \frac{1}{E_i - \omega_\eta - E_j} | N_j \rangle \langle N_j | H_\eta | N_i \rangle.$$

\[ \mathcal{M}_s = \sum_j \langle N_f | H_\eta | N_j \rangle \langle N_j | \sum_n \frac{1}{\omega_{\pi}^{n+1}} (\hat{H} - E_i)^n H_\pi | N_i \rangle \]

for any operator \( \mathcal{O} \), one has

\[ (\hat{H} - E_i) \mathcal{O} | N_i \rangle = [\hat{H}, \mathcal{O}] | N_i \rangle \]

Refs.
Zhao, Li, & Bennhold, PLB436, 42(1998); PRC58, 2393(1998);
Zhao, Didelez, Guidal, & Saghai, NPA660, 323(1999);
Zhao, PRC63, 025203(2001);
Zhao, Saghai, Al-Khalili, PLB509, 231(2001);
Zhao, Al-Khalili, & Bennhold, PRC64, 052201(R)(2001); PRC65, 032201(R) (2002);
\[ u\text{-channel} \]

\[ \pi, k \rightarrow \eta, q \]

\[ N, P_i \rightarrow N^{(*)} \Delta^{(*)} \rightarrow N', P_f \]

\[ t\text{-channel} \]

\[ \pi, k \rightarrow \eta, q \rightarrow a_0 \]

\[ N, P_i \rightarrow a_0 \rightarrow N', P_f \]

\[ \mathcal{L}_{a_0 \pi \eta} = g_{a_0 \pi \eta} m_\pi \eta \tilde{\pi} \tilde{a}_0 \]

\[ H_{a_0} = \sum_j g_{a_0 q q} m_\pi \tilde{\psi}_j \psi_j \tilde{a}_0 \]

\[ \mathcal{M}_t = g_{a_0 \pi \eta} m_\pi \langle N_f | H_{a_0} | N_i \rangle \frac{1}{t^2 - m_{a_0}^2} \]
S-channel transition amplitude with quark level operators

Non-relativistic expansion:

\[ H_\pi = \sum_j \frac{I_j}{g_\pi^2} \sigma_j \cdot \left[ A_\pi e^{i k \cdot r_j} + \frac{\omega_\pi}{2m_q} \{ p_j, e^{i k \cdot r_j} \} \right], \]

\[ H_\eta = \sum_j \frac{I_j}{g_\eta^2} \sigma_j \cdot \left[ A_\eta e^{-i q \cdot r_j} + \frac{\omega_\eta}{2m_q} \{ p_j, e^{-i q \cdot r_j} \} \right], \]

with

\[ A_\pi = -\left( \frac{\omega_\pi}{E_i + M_i} + 1 \right) k, \]

\[ A_\eta = -\left( \frac{\omega_\eta}{E_f + M_f} + 1 \right) q. \]
\[ M^s = \sum_n (M^s_3 + M^s_2) e^{-(k^2+q^2)/6\alpha^2}. \]

with

\[ M^s_3 = \langle N_f | \frac{3I_3}{g_A^2} \left\{ \sigma_3 \cdot A_\eta \sigma_3 \cdot A_\pi \sum_{n=0}^\infty \frac{F_s(n)}{n!} \chi^n \right. \]

\[ + \left[ -\sigma_3 \cdot A_\eta \frac{\omega_\pi}{3m_q} \sigma_3 \cdot q - \frac{\omega_\eta}{3m_q} \sigma_3 \cdot k \sigma_3 \cdot A_\pi \right. \]

\[ + \left. \frac{\omega_\eta \omega_\pi \alpha^2}{m_q m_q} \right] \sum_{n=1}^\infty \frac{F_s(n)}{(n-1)!} \chi^{n-1} \]

\[ + \left. \frac{\omega_\eta \omega_\pi}{3m_q 3m_q} \sigma_3 \cdot q \sigma_3 \cdot k \sum_{n=2}^\infty \frac{F_s(n)}{(n-2)!} \chi^{n-2} \right\} |N_i\rangle \]

where \( \chi \equiv k \cdot q/3\alpha^2. \)
\[ M^s = \sum_n \left( M^s_3 + M^s_2 \right) e^{-\left(k^2 + q^2\right)/6\alpha^2} \]

with

\[ M^s_3 = \langle N_f | \frac{3I_3}{g_A^2} \left\{ \sigma_3 \cdot A_\eta \sigma_3 \cdot A_\pi \sum_{n=0}^{\infty} \frac{F_s(n)}{n!} \chi^n \right\} \left| N_i \right\rangle \]

\[ M^s_2 = \langle N_f | \frac{6I_1}{g_A^2} \left\{ \sigma_1 \cdot A_\eta \sigma_3 \cdot A_\pi \sum_{n=0}^{\infty} \frac{F_s(n)}{n!} \frac{\chi^n}{(-2)^n} \right\} \left| N_i \right\rangle \]

where

\[ \omega_\pi = \frac{\omega_\pi}{3m_q}, \quad \omega_\eta = \frac{\omega_\eta}{3m_q}, \quad \alpha^2 = \frac{\alpha^2}{m_q m_q}, \quad \sigma_1 \cdot \sigma_3 = \sum_{n=1}^{\infty} \frac{F_s(n)}{(n-1)!} \frac{\chi^{n-1}}{(-2)^n} \]

\[ \sigma_1 \cdot q \sigma_3 \cdot k \sum_{n=2}^{\infty} \frac{F_s(n)}{(n-2)!} \frac{\chi^{n-2}}{(-2)^n} \]
Define $g$-factors:

\[
\mathcal{M}^s = \frac{1}{g_A^\pi} \left\{ A_\eta \cdot A_\pi \sum_{n=0} \left[ g_{s1} + (-2)^{-n} g_{s2} \right] \frac{F_s(n)}{n!} \chi^n \right\}
\]

\[
+ \left( -\frac{\omega_\pi}{3m_q} A_\eta \cdot q - \frac{\omega_\eta}{3m_q} A_\pi \cdot k + \frac{\omega_\eta \omega_\pi \alpha^2}{m_q m_q 3} \right) \sum_{n=1} \left[ g_{s1} + (-2)^{-n} g_{s2} \right] \frac{F_s(n)}{(n-1)!} \chi^{n-1}
\]

\[
+ \frac{\omega_\eta \omega_\pi}{(3m_q)^2} k \cdot q \sum_{n=2} \frac{F_s(n)}{(n-2)!} \left[ g_{s1} + (-2)^{-n} g_{s2} \right] \frac{r_s(n)}{\chi^n}
\]

\[
+ i \sigma \cdot (A_\eta \times A_\pi) \sum_{n=0} \left[ g_{v1} + (-2)^{-n} g_{v2} \right] \frac{r_s(n)}{\chi^n}
\]

\[
+ \frac{\omega_\eta \omega_\pi}{(3m_q)^2} i \sigma \cdot (q \times k)
\]

\[
\times \sum_{n=2} \left[ g_{v1} + (-2)^{-n} g_{v2} \right] \frac{F_s(n)}{(n-2)!} \chi^{n-2} \right\} e^{-\frac{(k^2+q^2)}{6\alpha^2}} \]
Compared with $M^s_3$, amplitude $M^s_2$ is relatively suppressed by a factor of $(-1/2)^n$ for each $n$.

Higher excited states are relatively suppressed by $(k \cdot q/3\alpha^2)^n/n!$

One can identify the quark motion correlations between the initial and final state baryon

Similar treatment can be done for the $u$ channel
Separate out individual resonances

A. $n = 0$ shell resonances

For $n = 0$, only the nucleon pole term contributes to the transition amplitude. Its $s$-channel amplitude is

$$\mathcal{M}^s_N = \mathcal{O}_N \frac{2M_0}{s - M_0^2} e^{-(k^2 + q^2)/6\alpha^2},$$

with

$$\mathcal{O}_N = [g_{s1} + g_{s2}] A_\eta \cdot A_\pi + [g_{v1} + g_{v2}] i\sigma \cdot (A_\eta \times A_\pi),$$

where $M_0$ is the nucleon mass.
B. $n = 1$ shell resonances

For $n = 1$, only $S$ and $D$ waves contribute in the $s$ channel. Note that the spin-independent amplitude for $D$ waves is proportional to the Legendre function $P_2^0(\cos \theta)$ and the spin-dependent amplitude for $D$ waves is in proportion to $\frac{\partial}{\partial \theta} P_2^0(\cos \theta)$. Moreover, the $S$-wave amplitude is independent of the scattering angle.

$$\mathcal{M}^s(S) = \mathcal{O}_S F_s(R) e^{-(k^2+q^2)/6\alpha^2},$$
$$\mathcal{M}^s(D) = \mathcal{O}_D F_s(R) e^{-(k^2+q^2)/6\alpha^2},$$

with

$$\mathcal{O}_S = \left( g_{s1} - \frac{1}{2} g_{s2} \right) \left( |A_\eta||A_\pi| \frac{|k||q|}{9\alpha^2} - \frac{\omega_\eta}{3m_q} A'_\eta \cdot q - \frac{\omega_\eta}{3m_q} A_\pi \cdot k + \frac{\omega_\eta \omega_\pi}{m_q m_q} \frac{\alpha^2}{3} \right),$$

$$\mathcal{O}_D = \left( g_{s1} - \frac{1}{2} g_{s2} \right) |A_\eta||A_\pi|(3 \cos^2 \theta - 1) \frac{|k||q|}{9\alpha^2} + \left( g_{v1} - \frac{1}{2} g_{v2} \right) i \sigma \cdot (A_\eta \times A_\pi) \frac{k \cdot q}{3\alpha^2}.$$
In the SU(6) symmetry limit,

\[
M^s(S) = \left[ g_{S_{11}(1535)} + g_{S_{11}(1650)} \right] M^s(S), \\
M^s(D) = \left[ g_{D_{13}(1520)} + g_{D_{13}(1700)} + g_{D_{15}(1675)} \right] M^s(D).
\]

<table>
<thead>
<tr>
<th>Factor</th>
<th>Value</th>
<th>Factor</th>
<th>Value</th>
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<td>$g_{S_{11}(1535)}$</td>
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<td>$g_2$</td>
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<td>2/3</td>
<td>$g_{S_{11}(1650)}$</td>
<td>-1</td>
<td>$g_{P_{11}(1710)}$</td>
<td>180/619</td>
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<td>5/3</td>
<td>$g_{D_{13}(1520)}$</td>
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<td>$g_{P_{13}(1900)}$</td>
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<td>$g_{D_{13}(1700)}$</td>
<td>-1/10</td>
<td>$g_{P_{11}(2100)}$</td>
<td>-16/619</td>
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<tr>
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<td>5/3</td>
<td>$g_{D_{15}(1675)}$</td>
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<td>5/3</td>
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<td>$g_A^{\eta}$</td>
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<td>$g_{F_{15}(2000)}$</td>
<td>-2/21</td>
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<tr>
<td>$g_1$</td>
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<td>$g_{P_{13}(1720)}$</td>
<td>180/619</td>
<td>$g_{F_{17}(1990)}$</td>
<td>-4/7</td>
</tr>
</tbody>
</table>
Model parameters

Goldberger-Treiman relation:

\[ g_{mNN} = \frac{g_A^m M_N}{f_m} \]

\[ g_{\pi NN} = 13.48, \]
\[ g_{\eta NN} = 0.81 \]

\[ g_{a_0 NN} g_{a_0 \pi \eta} = 100 \]

\[ m_q = 330 \text{ MeV}, \]
\[ \alpha^2 = 0.16 \text{ GeV}^2. \]

TABLE II. Breit-Wigner masses \( M_R \) (in MeV) and widths \( \Gamma_R \) (in MeV) for the resonances. \( n = 1 \) and \( n = 2 \) stand for the degenerate states with quantum number \( n = 1 \) and \( n = 2 \) in the \( u \) channel.

<table>
<thead>
<tr>
<th>Resonance</th>
<th>( M_R )</th>
<th>( \Gamma_R )</th>
<th>Resonance</th>
<th>( M_R )</th>
<th>( \Gamma_R )</th>
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<tr>
<td>( S_{11}(1535) )</td>
<td>1535</td>
<td>150</td>
<td>( P_{11}(1440) )</td>
<td>1440</td>
<td>300</td>
</tr>
<tr>
<td>( S_{11}(1650) )</td>
<td>1655</td>
<td>165</td>
<td>( P_{11}(1710) )</td>
<td>1710</td>
<td>100</td>
</tr>
<tr>
<td>( D_{13}(1520) )</td>
<td>1520</td>
<td>115</td>
<td>( P_{13}(1720) )</td>
<td>1720</td>
<td>200</td>
</tr>
<tr>
<td>( D_{13}(1700) )</td>
<td>1700</td>
<td>115</td>
<td>( P_{13}(1900) )</td>
<td>1900</td>
<td>500</td>
</tr>
<tr>
<td>( D_{15}(1675) )</td>
<td>1675</td>
<td>150</td>
<td>( P_{11}(2100) )</td>
<td>2100</td>
<td>150</td>
</tr>
<tr>
<td>( n = 1 )</td>
<td>1650</td>
<td>230</td>
<td>( F_{15}(1680) )</td>
<td>1685</td>
<td>130</td>
</tr>
<tr>
<td>( n = 2 )</td>
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<td>300</td>
<td>( F_{15}(2000) )</td>
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<td>200</td>
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<td>–</td>
<td>–</td>
<td>–</td>
<td>( F_{17}(1990) )</td>
<td>1990</td>
<td>350</td>
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</tbody>
</table>
Differential cross sections

Left panel:
- Solid: full calculation
- Dot-dashed: without nucleon
- Born term

Right panel:
- Solid: full calculation
- Dotted lines: exclusive S11(1535)
- Dot-dashed: without S11(1650)
- Dashed: without t-channel
Left panel:
- Solid: full calculation
- Dot-dashed: without nucleon Born term
- Dashed: without D13(1520)

Right panel:
- Solid: full calculation
- Dotted lines: exclusive S11(1535)
- Dot-dashed: without S11(1650)
- Dashed: without t-channel
Total cross sections

- $S_{11}(1535)$ is dominant near threshold. The exclusive cross section is even larger than the data.
- $S_{11}(1650)$ has a destructive interference with the $S_{11}(1535)$, and appears to be a dip in the total cross section.
- States from $n=2$ shell account for the second enhancement around 1.7 GeV.

Zhong, Zhao, He, and Saghai, PRC76, 065205 (2007)
S-channel resonance excitations in $K^-p \rightarrow \Sigma^0 \pi^0$

\[ O_S = [g_{S_{01}(1405)} + g_{S_{01}(1670)}]O_S, \]
\[ O_D = [g_{D_{03}(1520)} + g_{D_{03}(1690)}]O_D, \]

\[ \frac{g_{S_{01}(1405)}}{g_{S_{01}(1670)}} = \frac{\langle N_f | I_3^\pi \sigma_3 | S_{01}(1405) \rangle \langle S_{01}(1405) | I_3^K \sigma_3 | N_i \rangle}{\langle N_f | I_3^\pi \sigma_3 | S_{01}(1670) \rangle \langle S_{01}(1670) | I_3^K \sigma_3 | N_i \rangle} \]

\[ |S_{01}(1405)\rangle = \cos(\theta)|70,^21\rangle - \sin(\theta)|70,^28\rangle \]
\[ |S_{01}(1670)\rangle = \sin(\theta)|70,^21\rangle + \cos(\theta)|70,^28\rangle \]

\[ \frac{g_{S_{01}(1405)}}{g_{S_{01}(1670)}} = \frac{[3 \cos(\theta) - \sin(\theta)][\cos(\theta) + \sin(\theta)]}{[3 \sin(\theta) + \cos(\theta)][\sin(\theta) - \cos(\theta)]} \]

$g_{S_{01}(1405)}/g_{S_{01}(1670)} = -3$ leads to $\theta = 0^\circ$, i.e., no configuration mixing between $[70,^21]$ and $[70,^28]$.

Zhong and Zhao, PRC79, 045202 (2009)
We thus determine the mixing angle by experimental data which requires

\[ \frac{g_{S_{01}(1405)}}{g_{S_{01}(1670)}} \approx -9 \]
$g_{\Lambda(1405)}/g_{\Lambda(1670)} = -9$

$g_{S_{01}(1405)}/g_{S_{01}(1670)} = -3$

$\theta \simeq 41^\circ \text{ and } 165^\circ$
Diff. Xsect. for $K^-p \rightarrow \Sigma^0 \pi^0$
$K^- (s \bar{u})$

$s$-channel

$M_2^s = 0$

$M_3^s$ is the only $s$-channel amplitude

$U$-channel turns to be important

[Diagram with particle interactions and complex expressions]

[Graph showing cross section vs. beam momentum with various channels represented]
Baryon excitations in meson photoproduction

Quark-photon electromagnetic coupling:

$$H_e = - \sum_j e_j \gamma^j A^\mu(k, r)$$

Transition amplitudes in terms of the Mandelstam variables:

$$M_{fi} = M^{sg}_{fi} + M^s_{fi} + M^u_{fi} + M^t_{fi}$$

Zhao et al, PRC65, 065204 (2002)
The seagull term is composed of two parts,

\[ M_{fi}^{sg} = \langle N_f | H_{m,e} | N_i \rangle + i \langle N_f | [g_e, H_m] | N_i \rangle, \]  

(83)

where \( |N_i\rangle\) and \( |N_f\rangle\) are the initial and final state nucleon, respectively, and

\[ H_{m,e} = \sum_j \frac{e_m}{f_m} \phi_m(q, r_j) \overline{\psi}_j \gamma_j^\mu \gamma^5_j \psi_j A^\mu(k, r_j) \]  

(84)

is the contact term from the pseudovector coupling, and

\[ g_e = \sum_j e_j r_j \cdot \epsilon e^{ik \cdot r_j} \]  

(85)

comes from the transformation of the electromagnetic interaction in the s- and u-channel [9, 6].
The $s$- and $u$-channel amplitudes have the following expression:

\[
M^s_{fi} + M^u_{fi} = i\omega_\gamma \sum_j \langle N_f | H_m | N_j \rangle \langle N_j | \frac{1}{E_i + \omega_\gamma - E_j} h_e | N_i \rangle \\
+ i\omega_\gamma \sum_j \langle N_f | h_e \frac{1}{E_i - \omega_m - E_j} | N_j \rangle \langle N_j | H_m | N_i \rangle,
\]

where

\[
h_e = \sum_j e_j r_j \cdot \epsilon (1 - \alpha_j \cdot \hat{k}) e^{i \mathbf{k} \cdot \mathbf{r}_j},
\]

and $\hat{k} \equiv k / \omega_\gamma$ is the unit vector in the direction of the photon momentum.
The nonrelativistic expansions of Eqs. (87) and (80) become [6]:

\[ h_e = \sum_j \left[ e_j r_j \cdot \epsilon - \frac{e_j}{2m_j} \sigma_j \cdot (\epsilon_\gamma \times \hat{k}) \right] e^{i k \cdot r_j}, \]  

(88)

and

\[ H_m^{nr} = \sum_j \left[ \frac{\omega_m}{E_f + M_f} \sigma_j \cdot P_f + \frac{\omega_m}{E_i + M_i} \sigma_j \cdot P_i + \sigma_j \cdot q + \frac{\omega_m}{2\mu_q} \sigma_j \cdot p_j \right] \frac{\hat{I}_j}{g_A} e^{-i q \cdot r_j}, \]  

(89)

where \( M_i \) (\( M_f \)), \( E_i \) (\( E_f \)) and \( P_i \) (\( P_f \)) are mass, energy and three-vector momentum for the initial (final) nucleon; \( r_j \) and \( p_j \) are the internal coordinate and momentum for the \( j \)th quark in the nucleon rest system.
Transition amplitudes in the harmonic oscillator basis

\[ M_{fi}^{sg} = -e^{-(k-a)^2/6\alpha^2} \frac{1}{E_i + M_i} e_m \left[ 1 + \frac{\omega_m}{2} \left( \frac{1}{E_i + M_i} + \frac{1}{E_f + M_f} \right) \right] \sigma \cdot \epsilon, \]

\[ M_{fi}^{t} = e^{-(k-a)^2/6\alpha^2} \frac{e_m (M_f + M_i)}{q \cdot k} \left( \frac{\sigma \cdot q}{E_f + M_f} - \frac{\sigma \cdot k}{E_i + M_i} \right) q \cdot \epsilon, \]

\[ M_{fi}^{s} = (M_2^{s} + M_3^{s}) e^{-(k^2+q^2)/6\alpha^2}, \]

\[ M_{fi}^{u} = (M_2^{u} + M_3^{u}) e^{-(k^2+q^2)/6\alpha^2}, \]
\[ \frac{M_3^s}{g_3^s} = -\frac{1}{2m_q} \left[ ig_v A_s \cdot (\epsilon_\gamma \times k) - \sigma \cdot (A_s \times (\epsilon_\gamma \times k)) \right] \times \frac{M_n}{n!(P_i \cdot k - nM\omega_h)} \left( \frac{k \cdot q}{3\alpha^2} \right)^n \]
\[ + \frac{1}{6} \left[ \frac{\omega_\gamma \omega_m}{\mu_q} (1 + \frac{\omega_\gamma}{2m_q}) \sigma \cdot \epsilon_\gamma + \frac{2\omega_\gamma}{\alpha^2} \sigma \cdot A_s \epsilon_\gamma \cdot q \right] \] \[ A_s = - \left( \frac{\omega_m}{E_f + M_f} + 1 \right) q \]

\[ \frac{M_2^s(-2)^n}{g_2^s} = -\frac{1}{2m_q} \left[ ig_v' A_s \cdot (\epsilon_\gamma \times k) - g_\alpha' \sigma \cdot (A_s \times (\epsilon_\gamma \times k)) \right] \times \frac{M_n}{n!(P_i \cdot k - nM\omega_h)} \left( \frac{k \cdot q}{3\alpha^2} \right)^n \]
\[ + \frac{1}{6} \left[ \frac{\omega_\gamma \omega_m}{\mu_q} (1 + g_\alpha' \frac{\omega_\gamma}{2m_q}) \sigma \cdot \epsilon_\gamma + \frac{2\omega_\gamma}{\alpha^2} \sigma \cdot A_s \epsilon_\gamma \cdot q \right] \]
\[ \times \frac{M_n}{(n - 1)!(P_i \cdot k - nM\omega_h)} \left( \frac{k \cdot q}{3\alpha^2} \right)^{n-1} \]
\[ + \frac{\omega_\gamma \omega_m}{18\mu_q\alpha^2} \sigma \cdot k\epsilon_\gamma \cdot q \frac{M_n}{(n - 2)!(P_i \cdot k - nM\omega_h)} \left( \frac{k \cdot q}{3\alpha^2} \right)^{n-2} \]
Compared with $M_{s3}$, amplitude $M_{s2}$ is relatively suppressed by a factor of $(-1/2)^n$ for each $n$.

Higher excited states are relatively suppressed by $(k \cdot q/3\alpha^2)^n/n!$.

One can identify the quark motion correlations between the initial and final state baryon.

Similar treatment can be done for the $u$ channel.

In principle, all the $s$- and $u$-channel states have been included in the amplitudes, and the quark level operators have been related to the hadronic level ones through $g$-factors defined as follows.

Then, one has to separate out the amplitudes for each single resonance (see Ref. Zhao et al, PRC65, 065204 (2002)).
Some numerical results for pion photoproduction

\[ M_{1^+}^{3/2} = -g_{\pi NN} g_R \left( \frac{1}{2m_q} \left( \frac{\omega_m}{E_f + M_f} + 1 \right) \right) \times \frac{2M_\Delta}{s - M_\Delta^2 + iM_\Delta \Gamma_\Delta} e^{-(k^2 + q^2)/6\alpha^2} \]

\[ g_R \equiv g_3^s g_v + g_2^u g_v' - \mu_i \]

Zhao et al, PRC65, 065204 (2002)
Differential cross sections for $\gamma p \rightarrow \pi^+ n$. 
Polarized beam asymmetry for $\gamma p \rightarrow \pi^+ n$. 
Polarized target asymmetry for $\gamma p \rightarrow \pi^+ n$. 
Recoil polarization asymmetry for $\gamma p \rightarrow \pi^+ n$.

Simultaneous account for $\gamma p \rightarrow \pi^0 p$ and $\gamma n \rightarrow \pi^- p$ reaction and other relevant reactions.

Zhao et al, PRC65, 065204 (2002)
Number of states with the principle quantum number $n \leq 2$:

\[ \gamma n \rightarrow N^* (\Delta^*) \rightarrow \pi N \quad 27 \text{ states} \]

\[ \gamma p \rightarrow N^* (\Delta^*) \rightarrow \pi N \quad 19 \text{ states} \]

\[ \gamma n \rightarrow N^* \rightarrow \eta N \quad 16 \text{ states} \]

\[ \gamma p \rightarrow N^* \rightarrow \eta N \quad 8 \text{ states} \]

\[ \gamma n \rightarrow N^* \rightarrow K \Lambda \quad 8 \text{ states} \]

\[ \gamma p \rightarrow N^* \rightarrow K \Lambda \quad \]

Due to $\Lambda$ selection rule

\[ \Lambda \text{ Selection rule: Zhao & Close, PRD74, 094014(2006)} \]
Prospects - I

1. For the purpose of searching for individual resonance excitations, it is essential to have a quark model guidance for both known and “missing” states. And then allow the data to tell:

i) which state is favored;

ii) whether a state beyond the conventional quark model is needed;

iii) how quark model prescriptions for N*NM form factors complement with isobaric models.
Prospects - II

2. Understanding the non-resonance background

A reliable estimate of the non-resonance background, such as the t- and u-channel. Their interferences with the resonances are essentially important.

3. Unitarity constraint

A coherent study of the pseudoscalar photoproduction and meson-baryon scattering is needed. In particular, a coupled channel study will put a unitary constraint on the theory.

Photoproduction of pseudoscalar mesons ($\pi$, $\eta$, $\eta'$, $K$); and $\pi N \rightarrow \eta N$; $K^- p \rightarrow \pi \Sigma$, and more are coming out soon...

Q. Z., *PRC* 63, 035205 (2001);
Q. Z., J.S. Al-Khalili, Z.P. Li, and R.L. Workman, *PRC* 65, 065204 (2002);
Q. Z., B. Saghai and Z.P. Li, *JPG* 28, 1293 (2002);
Thanks !
A revisit to the S-wave state mixing

The mixing between pure \([70, ^2\!8]\) and \([70, ^4\!8]\) states is defined as

\[
\begin{pmatrix}
    S_{11}(1535) \\
    S_{11}(1650)
\end{pmatrix} =
\begin{pmatrix}
    \cos \theta_S & -\sin \theta_S \\
    \sin \theta_S & \cos \theta_S
\end{pmatrix}
\begin{pmatrix}
    \langle [70, ^2\!8, 1, 1, 1/2^-] \rangle \\
    \langle [70, ^4\!8, 1, 1, 1/2^-] \rangle
\end{pmatrix}
\]

Similarly, the \(D\)-wave mixing can be written as

\[
\begin{pmatrix}
    D_{13}(1520) \\
    D_{13}(1700)
\end{pmatrix} =
\begin{pmatrix}
    \cos \theta_D & -\sin \theta_D \\
    \sin \theta_D & \cos \theta_D
\end{pmatrix}
\begin{pmatrix}
    \langle [70, ^2\!8, 1, 1, 3/2^-] \rangle \\
    \langle [70, ^4\!8, 1, 1, 3/2^-] \rangle
\end{pmatrix}
\]
The physical states should be orthogonal which means:

\[ \pi, \eta, K \ldots \]
\[ S_{11}(1535) \quad S_{11}(1650) \]
\[ N, \Lambda, \ldots \]

\[ = 0 \]

This expectation can be examined by the K-matrix propagator between [70, ²8] and [70, ⁴8] mixing states:

\[
G = \frac{1}{D_a D_b - |D_{ab}|^2} \begin{pmatrix} D_a & D_{ab} \\ D_{ab} & D_b \end{pmatrix}
\]

\[
D_a = s - m_a^2 + i\sqrt{s} \Gamma^a(s)
\]
\[
D_b = s - m_b^2 + i\sqrt{s} \Gamma^b(s)
\]

\[
\Gamma^a(s) = \Gamma^a_{\pi N} + \Gamma^a_{\eta N} + \ldots ,
\]
\[
\Gamma^b(s) = \Gamma^b_{\pi N} + \Gamma^b_{\eta N} + \ldots .
\]

\[
D_{ab} \approx \frac{i}{16\pi} [\rho_{\pi N} g_{S_{11}N\pi}^a g_{S_{11}N\pi}^b + \rho_{\eta N} g_{S_{11}N\eta}^a g_{S_{11}N\eta}^b]
\]
Recalling that

\[ H_{m}^{NR} = \sum_{j} \left\{ \frac{\omega_{m}}{E_{f} + M_{f}} \right\} \]

The N* → NM transition amplitudes can be expressed as

\[
\alpha \equiv \langle \psi_{000}^{s} | q e^{i \sqrt{\frac{2}{3}} q \lambda_{z}} | \psi_{110}^{\lambda} \rangle = i \frac{q^{2}}{\sqrt{3} \alpha_{h}} e^{-q^{2} / 6 \alpha_{h}^{2}},
\]

\[
\beta \equiv \langle \psi_{000}^{s} | e^{i \sqrt{\frac{2}{3}} q \lambda_{z}} \hat{p}_{3-} | \psi_{111}^{\lambda} \rangle = -\langle \psi_{000}^{s} | e^{i \sqrt{\frac{2}{3}} q \lambda_{z}} \hat{p}_{3+} | \psi_{111}^{\lambda} \rangle
\equiv -i \frac{2}{3} \alpha_{h} e^{-q^{2} / 6 \alpha_{h}^{2}},
\]

\[
\gamma \equiv \langle \psi_{000}^{s} | e^{i \sqrt{\frac{2}{3}} q \lambda_{z}} \hat{p}_{3z} | \psi_{110}^{\lambda} \rangle = i \frac{\alpha_{h}}{\sqrt{3}} \left( 1 + \frac{q^{2}}{3 \alpha_{h}^{2}} \right) e^{-q^{2} / 6 \alpha_{h}^{2}},
\]

\[
\mathcal{M}_{S_{11} \rightarrow NM} = \frac{1}{f_{m}} [C_{1} \langle \hat{H}_{1} \rangle \alpha(q) + C_{2} \langle \hat{H}_{2} \rangle (\gamma(q) - \sqrt{2} \beta(q))],
\]

\[
\mathcal{M}_{D_{13}(D_{15}) \rightarrow NM} = \frac{1}{f_{m}} \left[ C_{1} \langle \hat{H}_{1} \rangle \alpha(q) + C_{2} \langle \hat{H}_{2} \rangle \left( \gamma(q) + \frac{\beta(q)}{\sqrt{2}} \right) \right].
\]

with \[ C_{1} = -3 \left( \frac{\omega_{m}}{E_{f} + M_{f}} + 1 \right), \quad C_{2} = \frac{3 \omega_{m}}{2 \mu q}. \]

<table>
<thead>
<tr>
<th>\hat{H}<em>{1}(\alpha), \hat{H}</em>{2}(\gamma - \sqrt{2}\beta)</th>
<th>S_{11}^{+} \rightarrow \Lambda K^{+}</th>
<th>S_{11}^{+} \rightarrow p\eta</th>
<th>S_{11}^{+} \rightarrow n\pi^{+}</th>
<th>S_{11}^{+} \rightarrow p\pi^{0}</th>
<th>S_{11}^{+} \rightarrow \Sigma^{+}K^{0}</th>
</tr>
</thead>
<tbody>
<tr>
<td>\langle N, J_{z} = \frac{1}{2}</td>
<td>\hat{H}_{1}</td>
<td>S_{11}^{+}, J_{z} = \frac{1}{2} \rangle</td>
<td>-\frac{1}{6}</td>
<td>-\cos \theta \cos \frac{\theta}{\sqrt{3}}</td>
<td>-\frac{2 \sqrt{2}}{9 \sqrt{3}}</td>
</tr>
<tr>
<td>\langle N, J_{z} = \frac{1}{2}</td>
<td>\hat{H}_{2}</td>
<td>S_{11}^{+}, J_{z} = \frac{1}{2} \rangle</td>
<td>-\frac{1}{6}</td>
<td>-\cos \theta \cos \frac{\theta}{\sqrt{3}}</td>
<td>-\frac{2 \sqrt{2}}{9 \sqrt{3}}</td>
</tr>
</tbody>
</table>


We can then extract the N*NM form factors given by the chiral effective Lagrangian in the NRCQM, e.g.

\[ \sum_{\text{spin}} |M_{\text{hadron}}|^2 \equiv (E_i + M_i)(E_f + M_f) \sum_{\text{spin}} |M_{\text{quark}}|^2, \]

where

\begin{align*}
N^*(S_{11} \to NM) : & \quad M_{\text{hadron}}^{S_{11}} = g_{S_{11}}NM \bar{u}_N u_R, \\
N^*(D_{13} \to NM) : & \quad M_{\text{hadron}}^{D_{13}} = g_{D_{13}}NM \bar{u}_N \gamma_5 \gamma_\mu u_R \gamma_\nu p_\mu p_\nu, \\
N^*(D_{15} \to NM) : & \quad M_{\text{hadron}}^{D_{15}} = g_{D_{15}}NM \bar{u}_N u_R \gamma_\mu \gamma_\nu p_\mu p_\nu, \\
\end{align*}

\[ M_{\text{quark}}^{S_{11}} = \frac{1}{f_m} [C_1 \alpha(q) + C_2 (\gamma(q) - \sqrt{2}/\beta(q))] \langle \hat{H} \rangle, \]

\[ = \frac{1}{f_m} \frac{i \alpha_h e^{-q^2/6\alpha^2}}{\sqrt{3}} \left[ C_1 \frac{q^2}{\alpha_h^2} + C_2 \left(3 + \frac{q^2}{3\alpha_h^2}\right) \right] \langle \hat{H} \rangle, \]

\[ M_{\text{quark}}^{D_{13}/D_{15}} = \frac{1}{f_m} [C_1 \alpha(q) + C_2 (\gamma(q) + \frac{\beta(q)}{\sqrt{2}})] \langle \hat{H} \rangle, \]

\[ = \frac{1}{f_m} \frac{i q^2 e^{-q^2/6\alpha^2}}{3\sqrt{3} \alpha_h} \langle \hat{H} \rangle. \]
\[ N \text{ threshold} \]

\[ |\xi_{ab}| \equiv \left| \frac{D_{ab}}{D_a} \right| \]

\[ |\xi_{ba}| \equiv \left| \frac{D_{ab}}{D_b} \right| \]
With the data for $S_{11} \rightarrow N\pi$ and $N\eta$ [1], i.e.

\[ Br(S_{11}(1535) \rightarrow N\pi) = 35 \sim 55\% \]
\[ Br(S_{11}(1650) \rightarrow N\pi) = 60 \sim 95\% , \]

\[ Br(S_{11}(1535) \rightarrow N\eta) = 45 \sim 60\% \]
\[ Br(S_{11}(1650) \rightarrow N\eta) = 3 \sim 10\% , \]

$$\theta_S \approx 24.6^\circ \sim 32.1^\circ$$

Similarly, with the data for $D_{13} \rightarrow N\pi$

\[ Br(D_{13}(1520) \rightarrow N\pi) = 55 \sim 65\% \]
\[ Br(D_{13}(1700) \rightarrow N\pi) = 5 \sim 15\% , \]

\[ Br(D_{13}(1520) \rightarrow N\eta) = 0.23 \pm 0.04\% \]
\[ Br(D_{13}(1700) \rightarrow N\eta) = 0.0 \pm 1.0\% , \]

$$\theta_D \approx 6.3^\circ \sim 18.3^\circ.$$
Relative signs for the N*NM couplings are given by the NRCQM.

<table>
<thead>
<tr>
<th>$\theta_S(24.6^\circ \sim 32.1^\circ)$</th>
<th>$S_{11}^+ \rightarrow p\eta$</th>
<th>$S_{11}^+ \rightarrow \Lambda K^+$</th>
<th>$S_{11}^+ \rightarrow n\pi^+$</th>
<th>$S_{11}^+ \rightarrow p\pi^0$</th>
<th>$S_{11}^+ \rightarrow \Sigma^+ K^0$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\mathcal{M}_{N* \rightarrow NM}$</td>
<td>6.86 $\sim$ 7.18</td>
<td>4.32 $\sim$ 4.07</td>
<td>3.29 $\sim$ 2.68</td>
<td>$-2.312 \sim -1.92$</td>
<td>3.32 $\sim$ 3.88</td>
</tr>
<tr>
<td>$g_{S_{11}NM}$</td>
<td>7.03$\sim$7.35</td>
<td>4.42$\sim$4.16</td>
<td>3.37$\sim$2.74</td>
<td>$-2.38 \sim -1.94$</td>
<td>3.41 $\sim$ 3.99</td>
</tr>
<tr>
<td>$g_{S_{11}NM}/g_{S_{11}p\eta}$</td>
<td>1</td>
<td>0.63 $\sim$ 0.57</td>
<td>0.48 $\sim$ 0.37</td>
<td>$-0.34 \sim -0.27$</td>
<td>0.49 $\sim$ 0.54</td>
</tr>
</tbody>
</table>

**TABLE VI:** Strong coupling constants for $S_{11}(1535) \rightarrow NM$.

<table>
<thead>
<tr>
<th>$\theta_S(24.6^\circ \sim 32.1^\circ)$</th>
<th>$S_{11}^+ \rightarrow p\eta$</th>
<th>$S_{11}^+ \rightarrow \Lambda K^+$</th>
<th>$S_{11}^+ \rightarrow n\pi^+$</th>
<th>$S_{11}^+ \rightarrow p\pi^0$</th>
<th>$S_{11}^+ \rightarrow \Sigma^+ K^0$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\mathcal{M}_{N* \rightarrow NM}$</td>
<td>$-2.56 \sim -1.67$</td>
<td>2.0 $\sim$ 2.57</td>
<td>4.06 $\sim$ 4.44</td>
<td>$-2.85 \sim -3.15$</td>
<td>$-4.19 \sim -3.75$</td>
</tr>
<tr>
<td>$g_{S_{11}NM}$</td>
<td>$-2.50 \sim -1.63$</td>
<td>1.96 $\sim$ 2.51</td>
<td>3.96 $\sim$ 4.34</td>
<td>$-2.80 \sim -3.07$</td>
<td>$-4.0 \sim -3.58$</td>
</tr>
<tr>
<td>$g_{S_{11}NM}/g_{S_{11}p\eta}$</td>
<td>1</td>
<td>$-0.78 \sim -1.54$</td>
<td>$-1.58 \sim -2.66$</td>
<td>1.12 $\sim$ 1.88</td>
<td>1.6 $\sim$ 2.2</td>
</tr>
</tbody>
</table>

**TABLE VII:** Strong coupling constants for $S_{11}(1650) \rightarrow NM$.

Indication of a destructive sign between $S_{11}(1535)$ and $S_{11}(1650)$ amplitudes in $\gamma p \rightarrow \eta p$, and $\pi^- p \rightarrow \eta n$. 
It is important to have a correct definition of the common sign of amplitudes and relative sign between helicity amplitudes, i.e. $A_{1/2}, A_{3/2},$ and $S_{1/2}$.

\[
A_{1/2}, A_{3/2}, S_{1/2}:
\]

\[
A_{\frac{1}{2}, \frac{3}{2}} = \zeta A_{\frac{1}{2}, \frac{3}{2}}, \quad S_{\frac{1}{2}} = \zeta S_{\frac{1}{2}}.
\]

\[
\zeta = -\text{sign}(g^*/g)
\]
\[ \gamma^* p \rightarrow S_{11} (1535) : \text{ 3q picture} \]

Combined with the difficulties in the description of large width of \( S_{11}(1535) \rightarrow \eta N \) and large \( S_{11}(1535) \rightarrow \phi N, \Lambda K \) couplings, this shows that 3q picture for \( S_{11}(1535) \) should be complemented.

From I. Aznauryan, Electromagnetic \( N-N^* \) Transition Form Factors Workshop, 2008

\[ A_{1/2} \]

\[ S_{1/2} \]

Opposite sign of \( S_{1/2} !!! \)

Impossible to change in quark model !!!

**LF RQM:**

- Capstick, Keister, PR D51 (1995) 3598
- Pace, Simula et al., PR D51 (1995) 3598
FIG. 1: Helicity amplitude for $\gamma^* p \rightarrow S_{11}(1535)$

FIG. 2: Helicity amplitude for $\gamma^* n \rightarrow S_{11}(1535)$
FIG. 3: Helicity amplitude for $\gamma^* p \to S_{11}(1650)$

FIG. 4: Helicity amplitude for $\gamma^* n \to S_{11}(1650)$