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Probing nucleon structure by using a polarized proton beam

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Based on work with Collins, Ji, Kang, Kouvaris, Sterman, Vogelsang, and Yuan

Spin of a composite particle

❑ Spin of a nucleus:

❖ Nuclear binding: 8 MeV/nucleon << mass of nucleon

❖ Nucleon number is fixed inside a given nucleus

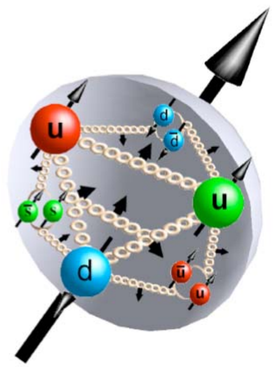
❖ Spin of a nucleus = sum of the valence nucleon spin

❑ Spin of a nucleon – Naïve Quark Model:

❖ If the probing energy << mass of constituent quark

❖ Nucleon is made of three constituent (valence) quark

❖ Spin of a nucleon = sum of the constituent quark spin



❑ Spin of a nucleon – QCD:

❖ Current quark mass << energy exchange of the collision

❖ Number of quarks and gluons depends on the probing energy

Proton spin in QCD

- Angular momentum of a proton at rest:

$$S = \sum_f \langle P, S_z = 1/2 |\, \hat{J}_f^z \,| P, S_z = 1/2 \rangle = \frac{1}{2}$$

- QCD Angular momentum operator:

Energy-momentum tensor

$$J^i_{\rm QCD} = \frac{1}{2}\,\epsilon^{ijk} \int d^3x\; M^{0jk}_{\rm QCD} \quad \longleftarrow \quad M^{\alpha\mu\nu}_{\rm QCD} = T^{\alpha\nu}_{\rm QCD}\, x^\mu - T^{\alpha\mu}_{\rm QCD}\, x^\nu$$

Angular momentum density

- Quark angular momentum operator:

$$\vec{J}_q = \int d^3x \left[\psi_q^\dagger \vec{\gamma}\gamma_5 \psi_q + \psi_q^\dagger (\vec{x} \times (-i\vec{D}))\, \psi_q\right]$$

- Gluon angular momentum operator:

$$\vec{J}_g = \int d^3x \left[\vec{x} \times (\vec{E} \times \vec{B})\right]$$

Also see, X.S. Chen et al PRL100, 232002 (2008)

- Proton state: $|P, S_z = 1/2\rangle$ in terms of quarks and gluons?

Parton's contribution to proton's spin?

- Matrix elements of parton angular momentum operators:

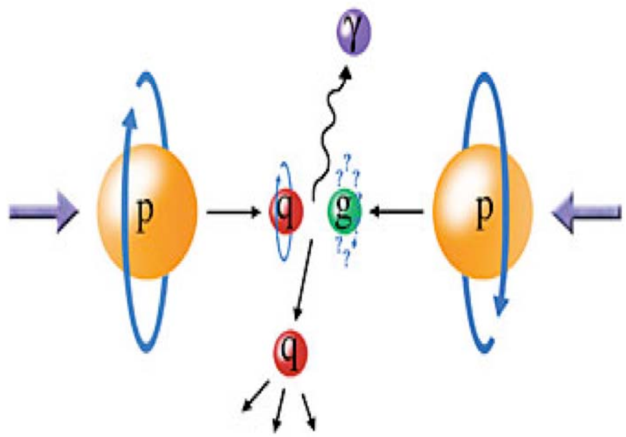
$$\langle P, S_z = 1/2 | J_q^z | P, S_z = 1/2 \rangle \qquad \langle P, S_z = 1/2 | J_g^z | P, S_z = 1/2 \rangle$$

- First principle calculation:
  - Proton wave function in terms of quarks and gluons – unknown
  - Lattice QCD: non-local operators

- Direct measurement:
  - Experiments measure hadronic cross sections
  - Many parton could participate in the hadronic collisions
    – single x-section could depend on many parton matrix elements
  - High energy collision – QCD factorization
    – x-section dominated by matrix elements of single quark/gluon

Approximations and limitations

❑ **High energy collisions – QCD Factorization:**



$$\frac{d\sigma}{dy dp_T^2} = \int \frac{dx}{x} q(x) \int \frac{dx'}{x'} g(x') \frac{d\hat{\sigma}_{qg\to\gamma q}}{dy dp_T^2} + \mathcal{O}(\alpha_s^2) + \mathcal{O}\left(\frac{1}{p_T^\alpha}\right)$$

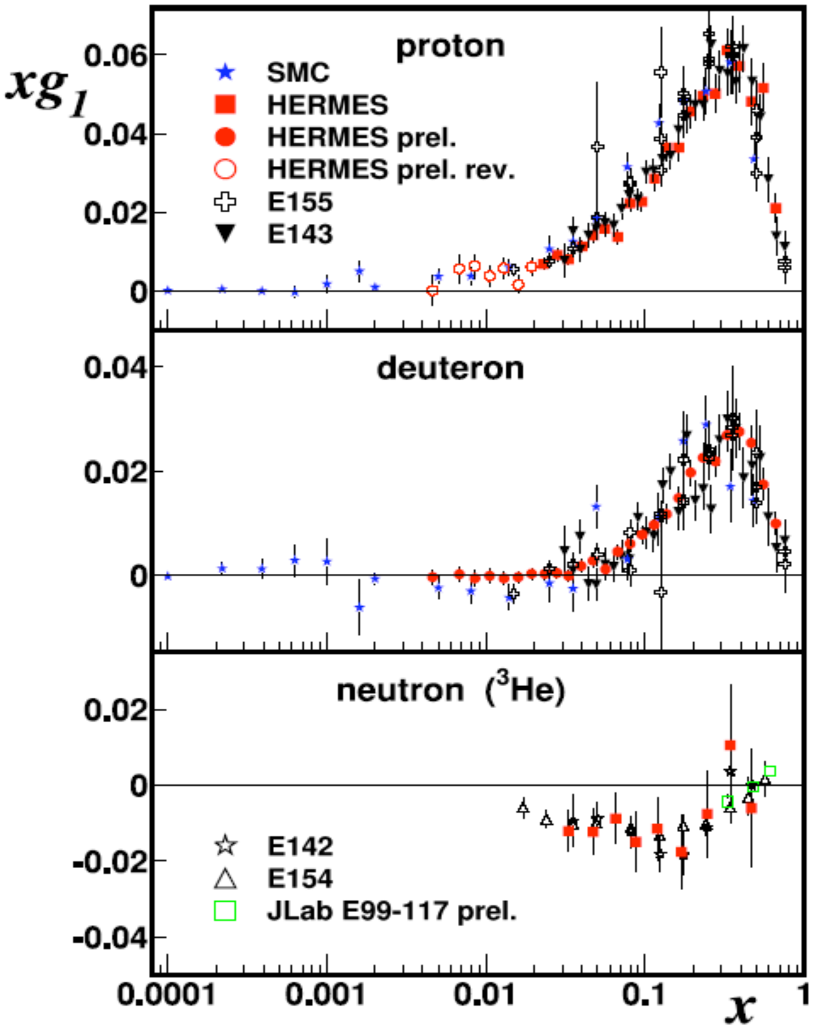
➡ Hadronic matrix elements at $p \neq 0$

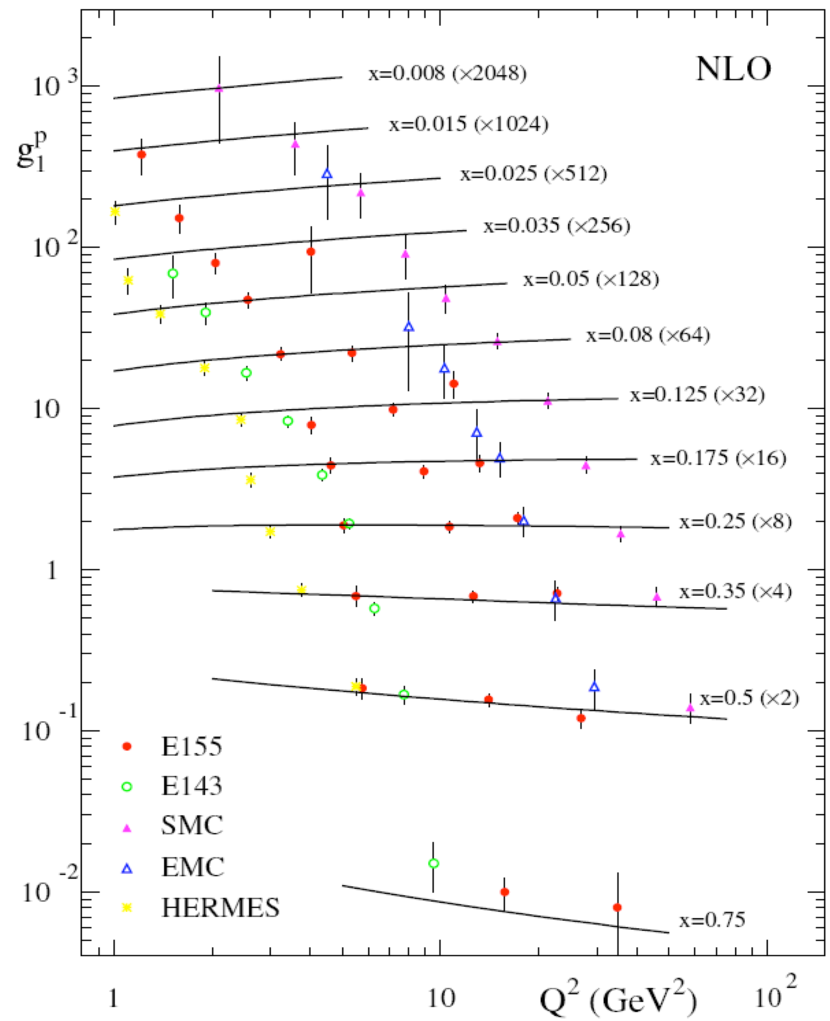
➡ OK for matrix elements independent of p

❑ **Limitations:**

Matrix elements extracted from the cross sections are NOT necessary the same as the matrix elements that define the parton's contribution to proton's spin!

Polarized inclusive DIS

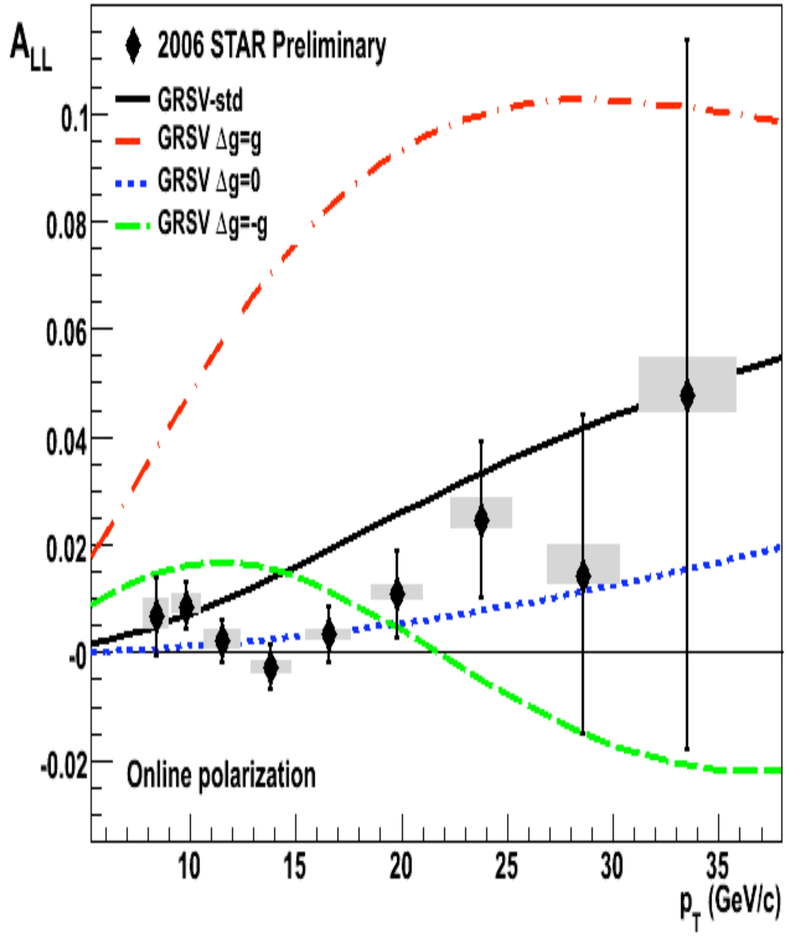




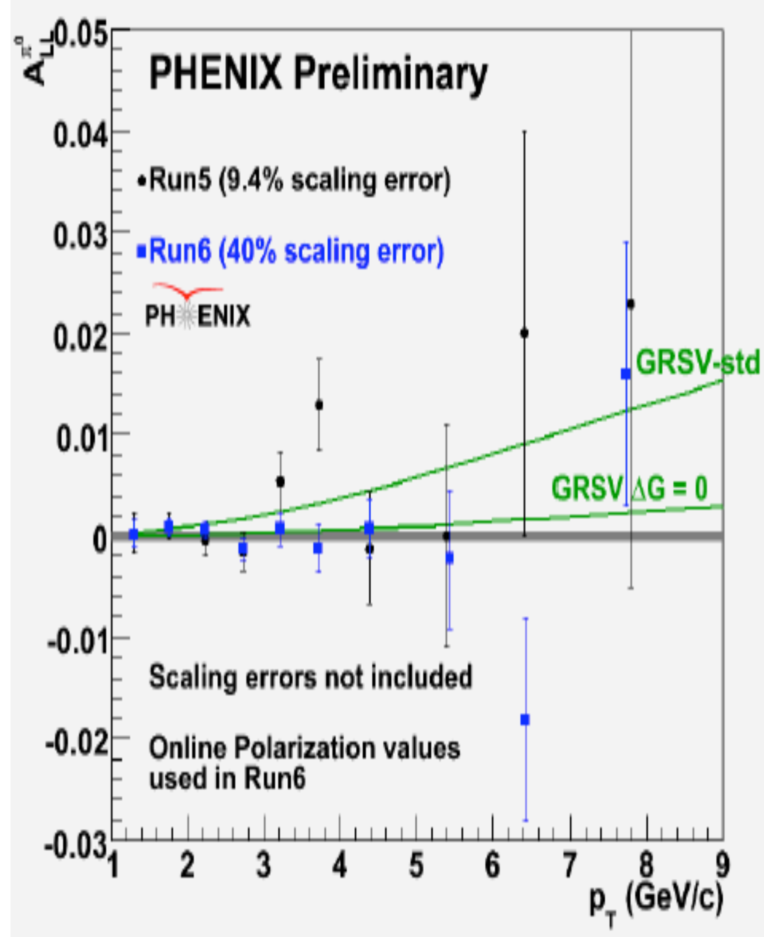
NLO QCD is consistent with the data – improvement at EIC, …

Polarized hadronic collisions

Star jet



Phenix π^0



Small asymmetry leads to small gluon "helicity" distribution

Quark "helicity" to proton spin

❑ **Extracted by the leading power QCD:**

$$\Delta q = \int_0^1 dx\, \Delta q(x) = \langle P, s_{\|} | \overline{\psi}_q(0) \gamma^+ \gamma_5\, \psi_q(0) | P, s_{\|} \rangle$$

❖ Integrated over "ALL" momentum components of active parton

- o parton entering the hard part has only collinear momentum
- o parton in the distribution has all components

❖ Matrix element of a local operator

❑ **NLO QCD global fit - DSSV:**

$$\Delta u + \Delta \bar{u} = 0.813 \qquad \Delta d + \Delta \bar{d} = -0.458 \qquad \Delta \bar{s} = -0.057$$

$$\Sigma = 0.242 \approx 24\% \text{ proton spin}$$

de Florian, Sassot, Stratmann, and Vogelsang
Phys. Rev. Lett. 2008

❖ independent of whose definition of quark contribution to the proton spin – but, does depend on QCD factorization scheme

❑ **Better flavor separation by W-boson production at RHIC**

Gluon "helicity" to proton spin

❑ Extracted by the leading power QCD:

$$\Delta g = \int_0^1 dx\, \Delta g(x) = \langle P, s_{\parallel} | F^{+\mu}(0) F^{+\nu}(0) | P, s_{\parallel} \rangle (-i\epsilon_{\mu\nu})$$

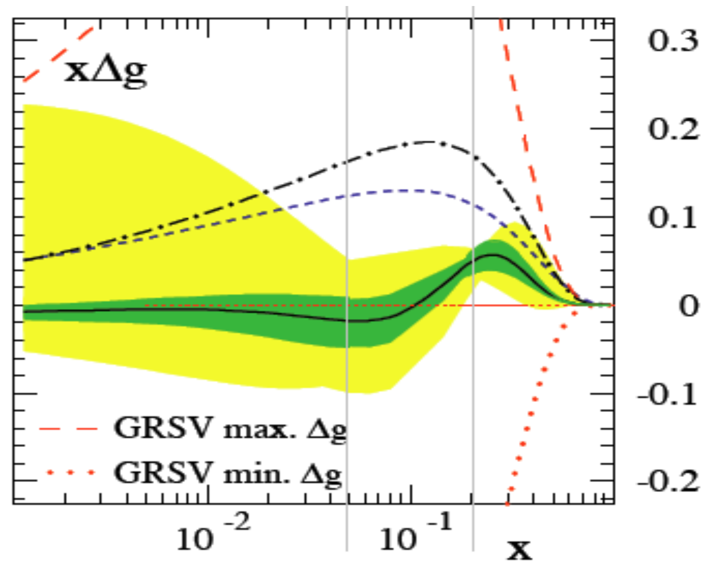
Integrated over "ALL" momentum components of active gluon

❑ NLO QCD global fit - DSSV:

$$\Delta g = -0.084$$ arXiv:0804.0422

❖ $\Delta g(x)$ change sign in RHIC region

❖ Effectively, no contribution to proton spin



NOTE:

❖ The extracted value of $\Delta q(x),\ \Delta g(x)$ is independent of whose definition of quark, gluon contribution to proton's spin

❖ Depend on the scheme, high order corrections in α_s and in 1/Q

Questions

How to go beyond
the probability distributions?

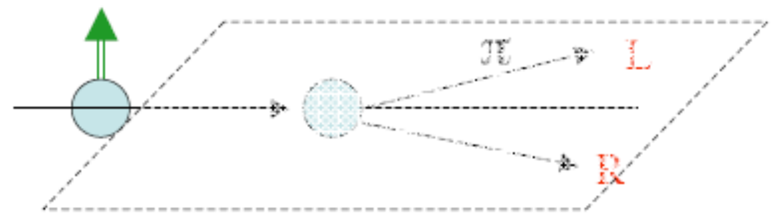
How to directly probe
QCD quantum interference?

Single Transverse-Spin Asymmetry (SSA)

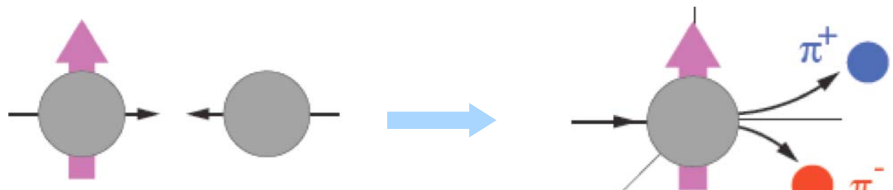
$$A(\ell,\vec{s}) \equiv \frac{\Delta\sigma(\ell,\vec{s})}{\sigma(\ell)} = \frac{\sigma(\ell,\vec{s}) - \sigma(\ell,-\vec{s})}{\sigma(\ell,\vec{s}) + \sigma(\ell,-\vec{s})}$$

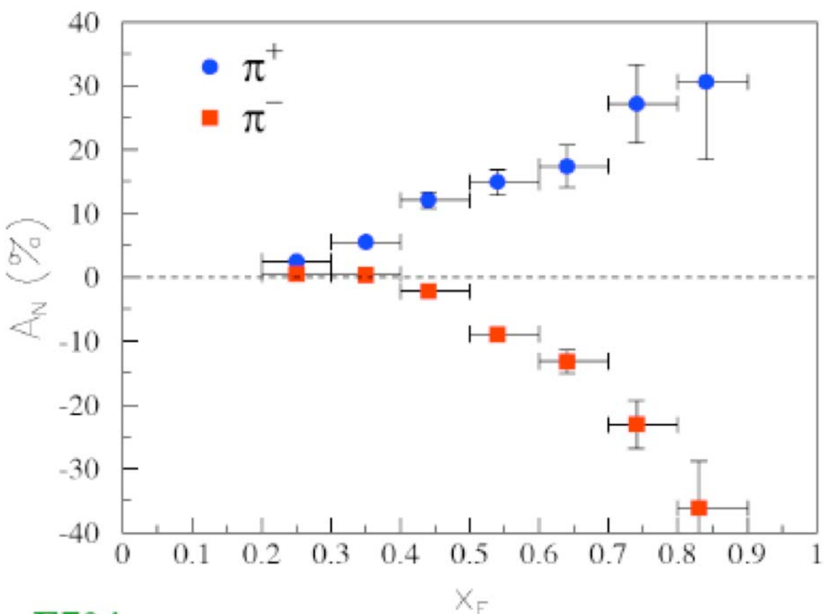
SSA in hadronic collisions

- **Hadronic** $p\uparrow + p \to \pi(l) X$:

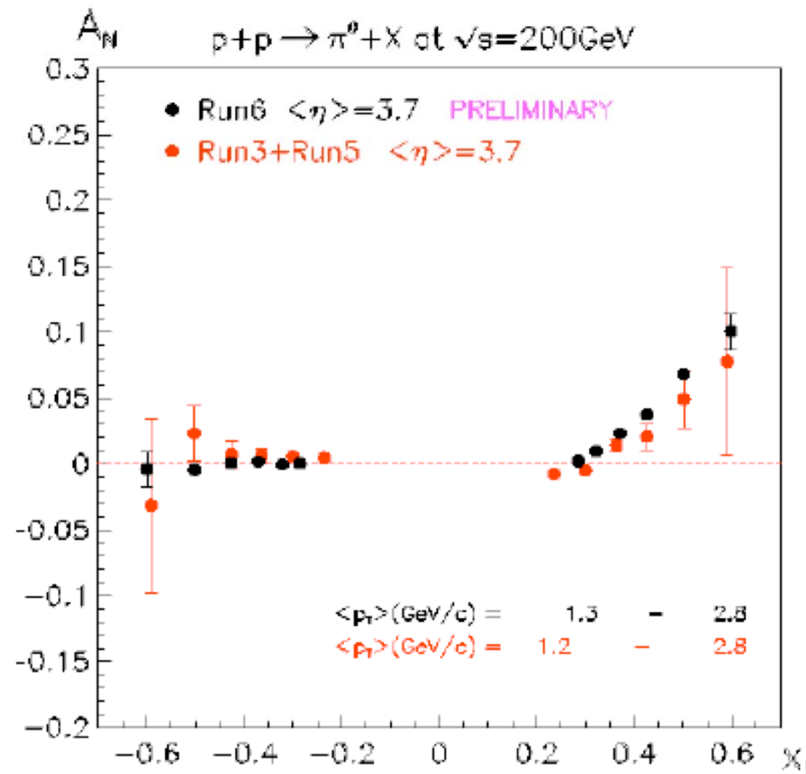


$$A_N = \frac{1}{P_{\text{beam}}} \frac{N^{\pi}_{\text{left}} - N^{\pi}_{\text{right}}}{N^{\pi}_{\text{left}} + N^{\pi}_{\text{right}}}$$





E704



STAR (BRAHMS, too)

Role of fundamental symmetries

❑ Fundamental symmetry and vanishing asymmetry:

❖ A_L=0 (longitudinal) for Parity conserved interactions

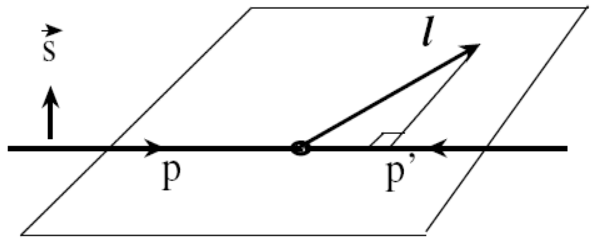
❖ A_N =0 (transverse) for inclusive DIS – Time-reversal invariance

– proposed to test T-invariance by Christ and Lee (1966)

Even though the cross section is finite!

❑ SSA corresponds to a T-odd triple product

$$A_N \propto i\vec{s}_p \cdot \left(\vec{p} \times \vec{\ell}\right) \Rightarrow i\varepsilon^{\mu\nu\alpha\beta} p_\mu s_\nu \ell_\alpha p'_\beta$$

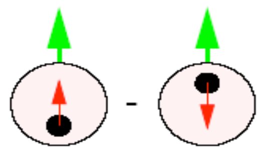


→ Novanishing A_N requires a phase, a spin flip, and enough vectors to fix a scattering plan

SSA in parton model

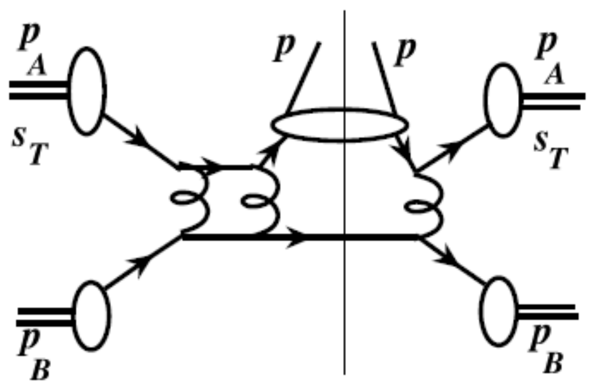
Kane, et al. 1978

- **The spin flip at leading twist – transversity:**

$\delta q(x) =$  $\propto \langle P, \vec{S}_\perp | \overline{\psi}_q \left[\gamma^+ \gamma \cdot \vec{S}_\perp \right] \psi_q | P, \vec{S}_\perp \rangle$

Chiral-odd helicity-flip density

- the operator for δq has even γ's ⟹ quark mass term
- the phase requires an imaginary part ⟹ loop diagram



$\sim m_q \rightarrow 0$

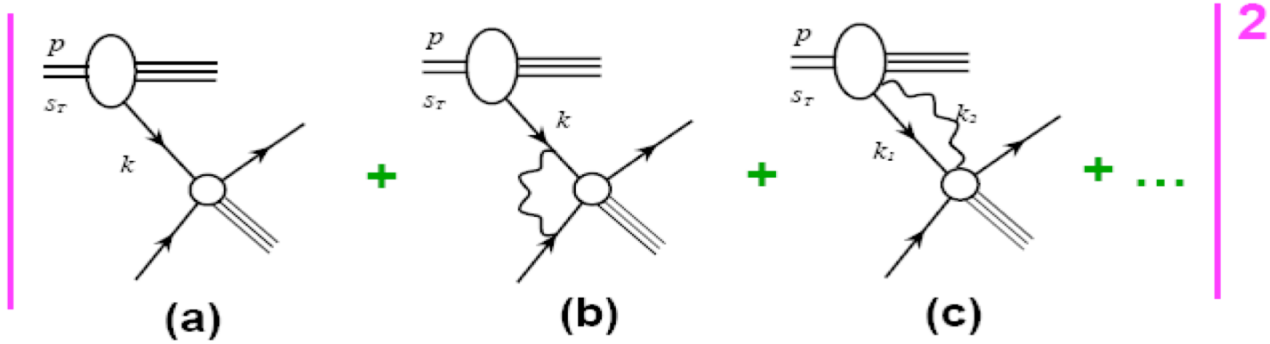
⟹ **SSA vanishes in the parton model**
connects to parton's transverse motion

Asymmetry in QCD collinear factorization

- One large momentum transfer $Q \gg \Lambda_{QCD}$:

Efremov, Teryaev, 1982
Qiu, Sterman, 1991

$\sigma(s_T) \sim$



$$\approx \sigma(s_T)^{(2)}_{|a|^2} + \sigma^{(3)}_{\mathrm{Re}(a)*\mathrm{Im}(b)} + \sigma^{(3)}_{\mathrm{Re}(a)*\mathrm{Im}(c)} + \cdots$$

- Asymmetry – QCD quantum interference:

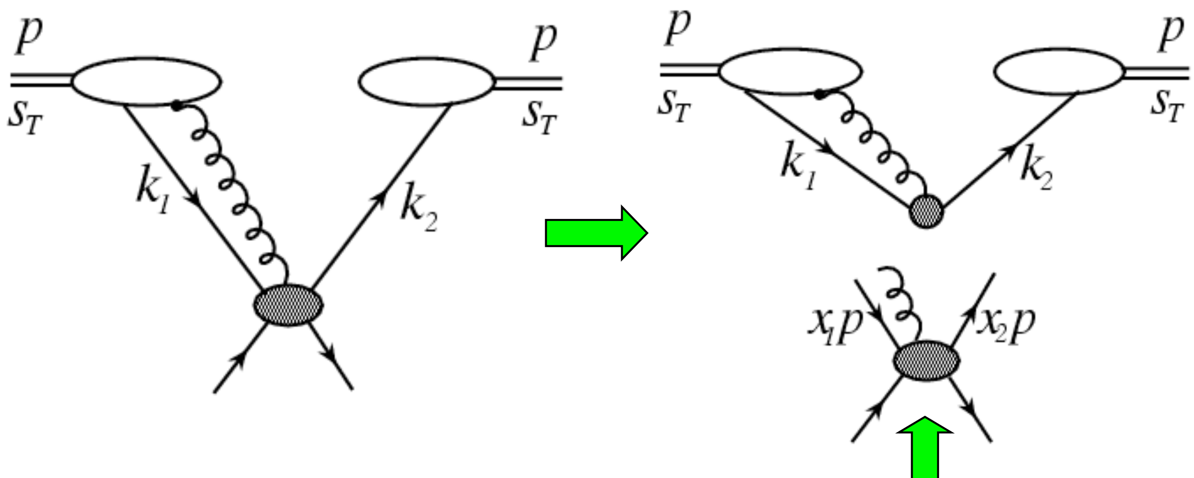
$$\sigma(s_T) - \sigma(-s_T) \approx \sigma^{(3)}_{\mathrm{Re}(a)*\mathrm{Im}(b)} + \sigma^{(3)}_{\mathrm{Re}(a)*\mathrm{Im}(c)} + \cdots$$

- Leading spin dependent part of the cross section
  - ⟶ Interference between amplitudes (a) and (b) or (c)
- The hadronic phase – the "i"
  - ⟶ $\mathrm{Re}[(a)]$ interferes with $\mathrm{Im}[(b)]$ or $\mathrm{Im}[(c)]$
- $\mathrm{Re}[(a)] \times \mathrm{Im}[(b)] \propto m_Q \; \delta q(s_\perp)$ – QCD parton model

A_N from polarized twist-3 correlations

Qiu, Sterman, 1991, 1999

❑ Factorization:



$$T_F(x_1,x_2) \propto \langle \bar{\psi}\gamma^+ F^{+\perp}\psi \rangle$$

$$T_D(x_1,x_2) \propto \langle \bar{\psi}\gamma^+ D_\perp\psi \rangle$$

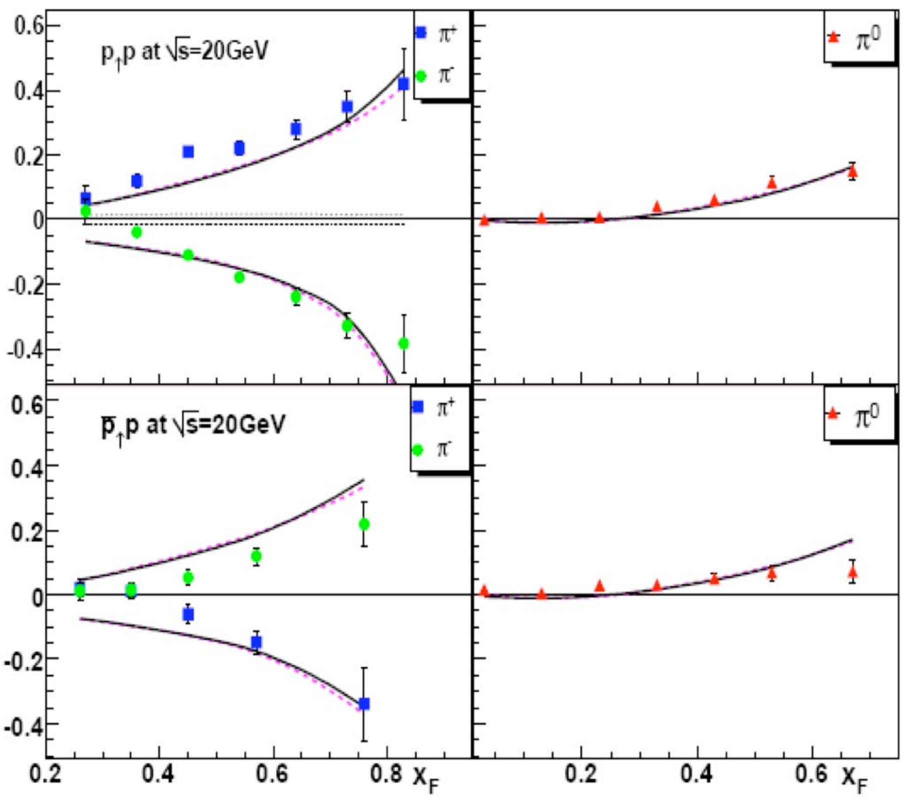
Normal twist-2 distributions

❑ New twist-3 quark-gluon correlation functions:

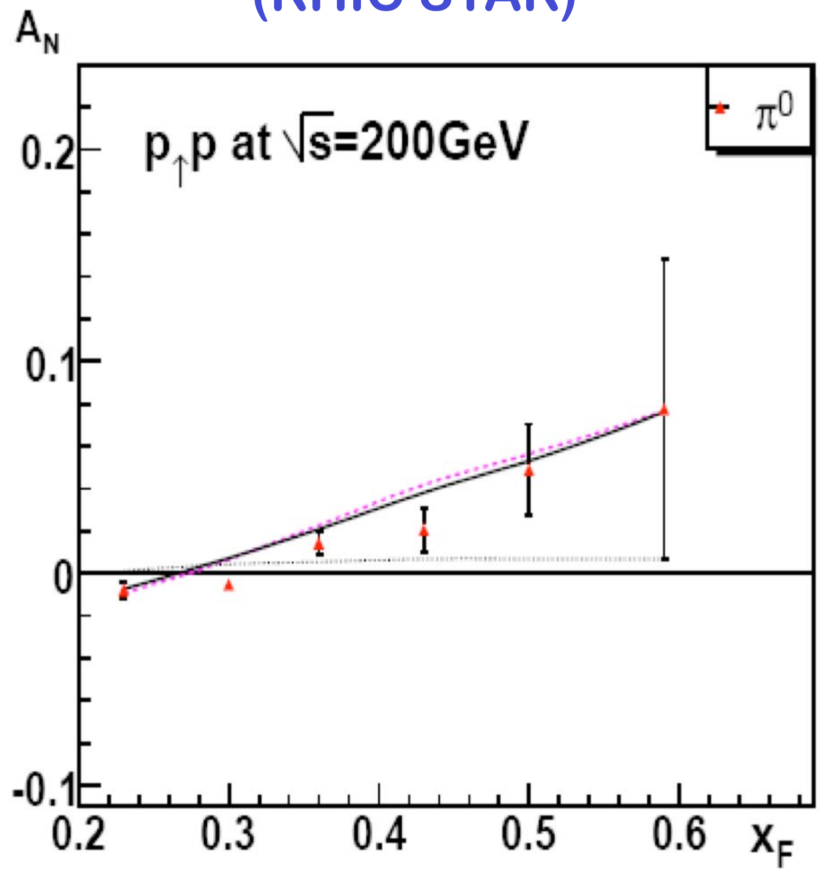
- ❖ $T_F(x_1,x_2)$ and $T_D(x_1,x_2)$ have different properties under the P and T transformation
- ❖ $T_D(x_1,x_2)$ does not contribute to the A_N
- ❖ $T_F(x_1,x_2)$ is universal, $x_1=x_2$ for A_N due to the pole

Asymmetries from the $T_F(x,x)$

(FermiLab E704)



(RHIC STAR)



Kouvaris,Qiu,Vogelsang,Yuan, 2006

Nonvanish twist-3 function ⟶ Nonvanish transverse motion

Scale dependence of correlation functions

- Almost all existing calculations of SSA are at LO:
  - Strong dependence on renormalization and factorization scales
  - Artifact of the lowest order calculation
- Improve/test QCD predictions:
  - Complete set of twist-3 correlation functions relevant to SSA
  - LO evolution for the universal twist-3 correlation functions
  - NLO partonic hard parts for various observables
  - NLO evolution for the correlation functions, …
- Current status:
  - Two sets of twist-3 correlation functions
  - LO evolution kernel for $T_{q,F}(x,x)$ and $T_{G,F}^{(f,d)}(x,x)$ Kang, Qiu, 2009
  - LO evolution kernel for $T_{q,F}(x,x)$ from NLO hard part for SSA of p_T weighted Drell-Yan Vogelsang, Yuan, 2009

Two sets of twist-3 correlation functions

- **Twist-2 distributions:**

  - Unpolarized PDFs:

$$q(x) \propto \langle P|\overline{\psi}_q(0)\frac{\gamma^+}{2}\psi_q(y)|P\rangle$$

$$G(x) \propto \langle P|F^{+\mu}(0)F^{+\nu}(y)|P\rangle(-g_{\mu\nu})$$

  - Polarized PDFs:

$$\Delta q(x) \propto \langle P,S_\parallel|\overline{\psi}_q(0)\frac{\gamma^+\gamma^5}{2}\psi_q(y)|P,S_\parallel\rangle$$

$$\Delta G(x) \propto \langle P,S_\parallel|F^{+\mu}(0)F^{+\nu}(y)|P,S_\parallel\rangle(i\epsilon_{\perp\mu\nu})$$

- **Two-sets Twist-3 correlation functions:**

Kang, Qiu, PRD, 2009

$$\widetilde{T}_{q,F} = \int \frac{dy_1^- dy_2^-}{(2\pi)^2}\, e^{ixP^+y_1^-}\, e^{ix_2P^+y_2^-}\, \langle P,s_T|\overline{\psi}_q(0)\,\frac{\gamma^+}{2}\left[\epsilon^{s_T\sigma n\bar{n}}\, F_\sigma{}^+(y_2^-)\right]\psi_q(y_1^-)|P,s_T\rangle$$

$$\widetilde{T}_{G,F}^{(f,d)} = \int \frac{dy_1^- dy_2^-}{(2\pi)^2}\, e^{ixP^+y_1^-}\, e^{ix_2P^+y_2^-}\, \frac{1}{P^+}\langle P,s_T|F^{+\rho}(0)\left[\epsilon^{s_T\sigma n\bar{n}}\, F_\sigma{}^+(y_2^-)\right]F^{+\lambda}(y_1^-)|P,s_T\rangle(-g_{\rho\lambda})$$

$$\widetilde{T}_{\Delta q,F} = \int \frac{dy_1^- dy_2^-}{(2\pi)^2}\, e^{ixP^+y_1^-}\, e^{ix_2P^+y_2^-}\, \langle P,s_T|\overline{\psi}_q(0)\,\frac{\gamma^+\gamma^5}{2}\left[i\, s_T^\sigma\, F_\sigma{}^+(y_2^-)\right]\psi_q(y_1^-)|P,s_T\rangle$$

$$\widetilde{T}_{\Delta G,F}^{(f,d)} = \int \frac{dy_1^- dy_2^-}{(2\pi)^2}\, e^{ixP^+y_1^-}\, e^{ix_2P^+y_2^-}\, \frac{1}{P^+}\langle P,s_T|F^{+\rho}(0)\left[i\, s_T^\sigma\, F_\sigma{}^+(y_2^-)\right]F^{+\lambda}(y_1^-)|P,s_T\rangle\,(i\epsilon_{\perp\rho\lambda})$$

Leading order evolution equations - I

Kang, Qiu, PRD, 2009

❑ Quark:

$$\frac{\partial T_{q,F}(x,x,\mu_F)}{\partial \ln \mu_F^2} = \frac{\alpha_s}{2\pi}\int_x^1 \frac{d\xi}{\xi}\left\{ P_{qq}(z)\, T_{q,F}(\xi,\xi,\mu_F) + \frac{C_A}{2}\left[\frac{1+z^2}{1-z}\left[T_{q,F}(\xi,x,\mu_F) - T_{q,F}(\xi,\xi,\mu_F)\right] + z\, T_{q,F}(\xi,x,\mu_F)\right] + \frac{C_A}{2}\left[T_{\Delta q,F}(x,\xi,\mu_F)\right] + P_{qg}(z)\left(\frac{1}{2}\right)\left[T_{G,F}^{(d)}(\xi,\xi,\mu_F) + T_{G,F}^{(f)}(\xi,\xi,\mu_F)\right]\right\}$$

❑ Antiquark:

$$\frac{\partial T_{\bar{q},F}(x,x,\mu_F)}{\partial \ln \mu_F^2} = \frac{\alpha_s}{2\pi}\int_x^1 \frac{d\xi}{\xi}\left\{ P_{qq}(z)\, T_{\bar{q},F}(\xi,\xi,\mu_F) + \frac{C_A}{2}\left[\frac{1+z^2}{1-z}\left[T_{\bar{q},F}(\xi,x,\mu_F) - T_{\bar{q},F}(\xi,\xi,\mu_F)\right] + z\, T_{\bar{q},F}(\xi,x,\mu_F)\right] + \frac{C_A}{2}\left[T_{\Delta \bar{q},F}(x,\xi,\mu_F)\right] + P_{qg}(z)\left(\frac{1}{2}\right)\left[T_{G,F}^{(d)}(\xi,\xi,\mu_F) - T_{G,F}^{(f)}(\xi,\xi,\mu_F)\right]\right\}$$

- ❖ All kernels are infrared safe
- ❖ Diagonal contribution is the same as that of DGLAP
- ❖ Quark and antiquark evolve differently – caused by tri-gluon

Leading order evolution equations - II

Kang, Qiu, PRD, 2009

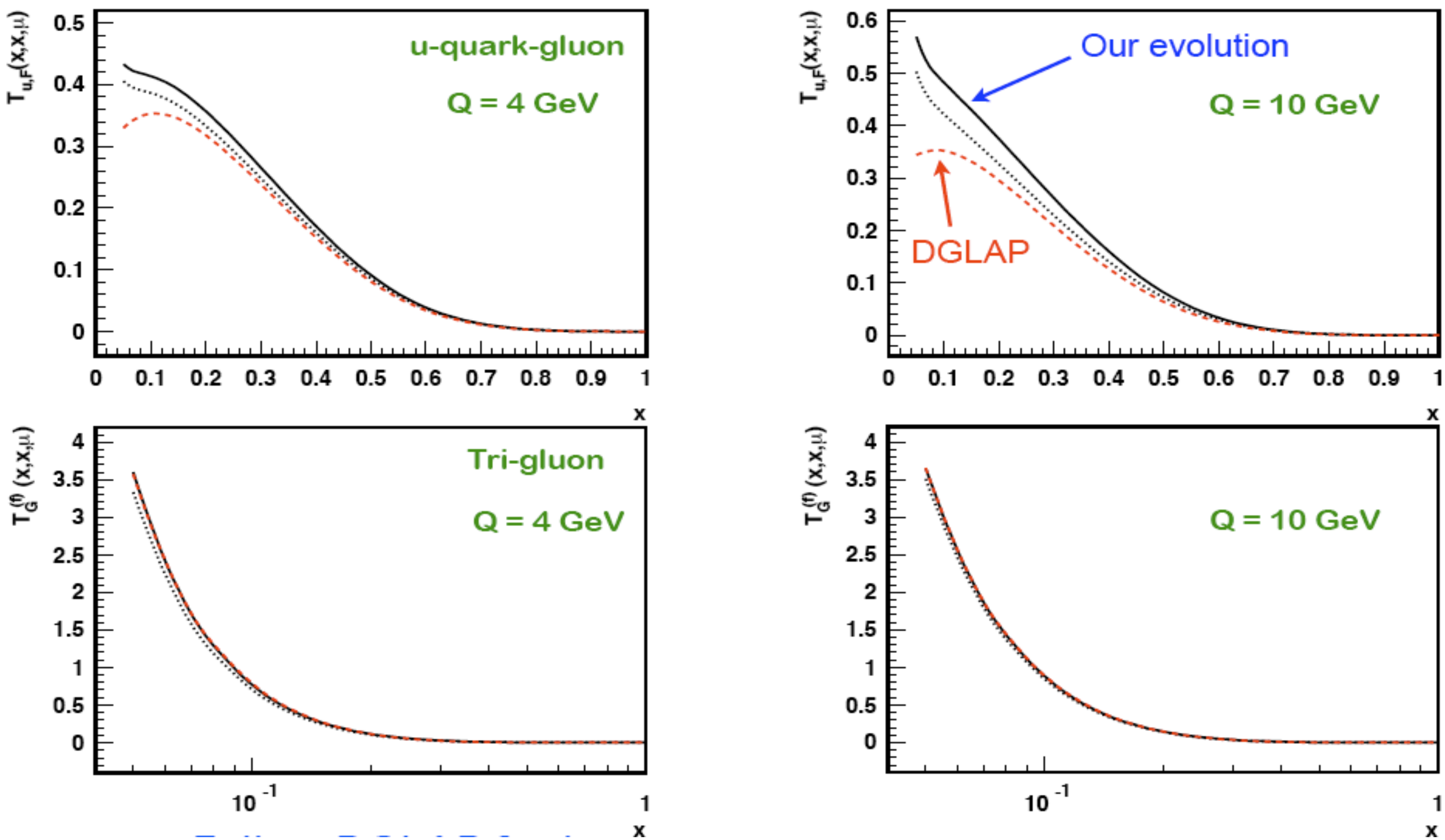
❑ Gluons:

$$\frac{\partial T^{(d)}_{G,F}(x,x,\mu_F)}{\partial \ln \mu_F^2} = \frac{\alpha_s}{2\pi}\int_x^1 \frac{d\xi}{\xi}\Bigg\{ P_{gg}(z)\, T^{(d)}_{G,F}(\xi,\xi,\mu_F)$$
$$+\frac{C_A}{2}\left[2\left(\frac{z}{1-z}+\frac{1-z}{z}+z(1-z)\right)\left[T^{(d)}_{G,F}(\xi,x,\mu_F)-T^{(d)}_{G,F}(\xi,\xi,\mu_F)\right]\right.$$
$$\left.+2\left(1-\frac{1-z}{2z}-z(1-z)\right)T^{(d)}_{G,F}(\xi,x,\mu_F)+(1+z)\,T^{(d)}_{\Delta G,F}(x,\xi,\mu_F)\right]$$
$$+P_{gq}(z)\left(\frac{N_c^2-4}{N_c^2-1}\right)\sum_q\left[T_{q,F}(\xi,\xi,\mu_F)+T_{\bar{q},F}(\xi,\xi,\mu_F)\right]\Bigg\}$$

Similar expression for $T^{(f)}_{G,F}(x,x,\mu_F)$

- ❖ Kernels are also infrared safe
- ❖ diagonal contribution is the same as that of DGLAP
- ❖ Two tri-gluon distributions evolve slightly different
- ❖ $T^{(d)}_{G,F}$ has no connection to TMD distribution
- ❖ Evolution can generate $T^{(d)}_{G,F}$ as long as $\sum_q [T_{q,F}+T_{\bar{q},F}] \neq 0$

Scale dependence of twist-3 correlations



- ❖ Follow DGLAP at large x
- ❖ Large deviation at low x (stronger correlation)

Kang, Qiu, PRD, 2009

Questions

How to probe
parton's transverse motion
at
a given transverse momentum ?

Transverse momentum dependent (TMD)
parton distribution functions (PDFs)

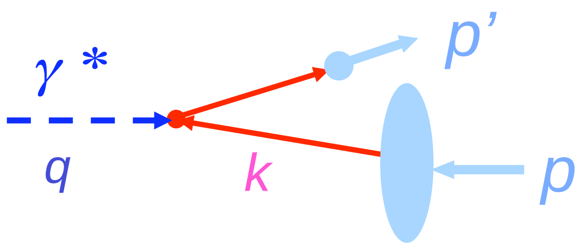
Can SSA be factorized in terms of TMD PDFs ?

TMD factorization

❑ Need processes with two observed momentum scales:

$$Q_1 \gg Q_2 \begin{cases} Q_1 & \text{necessary for pQCD factorization to have a chance} \\ Q_2 & \text{sensitive to parton's transverse motion} \end{cases}$$

❑ Example – semi-inclusive DIS:



❖ Both p and p' are observed

❖ p'_T probes the parton's k_T

❖ Effect of k_T is not suppressed by Q

❑ Very limited processes with valid TMD factorization

❖ Drell-Yan transverse momentum distribution: Q, q_T
- o quark Sivers function
- o low rate

Collins, Qiu, 2007
Vogelsang, Yuan, 2007

❖ Semi-inclusive DIS for light hadrons: Q, p_T
- o mixture of quark Sivers and Collins function

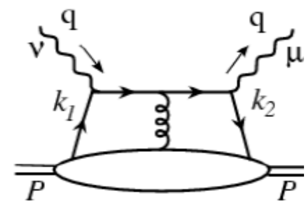
TMD parton distributions

- **SIDIS:**

$$f_{q/h^\uparrow}^{\rm SIDIS}(x,\mathbf{k}_\perp,\vec{S}) = \int \frac{dy^- d^2\mathbf{y}_\perp}{(2\pi)^3}\, e^{ixp^+y^- - i\,\mathbf{k}_\perp\cdot\mathbf{y}_\perp} \langle p,\vec{S}|\overline{\psi}(0^-,\mathbf{0}_\perp)\Phi_n^\dagger(\{\infty,0\},\mathbf{0}_\perp)$$
$$\times\, \Phi_{\mathbf{n}_\perp}^\dagger(\infty,\{\mathbf{y}_\perp,\mathbf{0}_\perp\})\frac{\gamma^+}{2}\Phi_n(\{\infty,y^-\},\mathbf{y}_\perp)\psi(y^-,\mathbf{y}_\perp)|p,\vec{S}\rangle$$

Gauge links:

$$\Phi_n(\{\infty,y^-\},\mathbf{y}_\perp) \equiv \mathcal{P}e^{-ig\int_{y^-}^{\infty} dy_1^- n^\mu A_\mu(y_1^-,\mathbf{y}_\perp)}$$
$$\Phi_{\mathbf{n}_\perp}(\infty,\{\mathbf{y}_\perp,\mathbf{0}_\perp\}) \equiv \mathcal{P}e^{-ig\int_{\mathbf{0}_\perp}^{\mathbf{y}_\perp} d\mathbf{y}'_\perp \mathbf{n}_\perp^\mu A_\mu(\infty,\mathbf{y}'_\perp)}$$



- **Drell-Yan:**

$$f_{q/h^\uparrow}^{\rm DY}(x,\mathbf{k}_\perp,\vec{S}) = \int \frac{dy^- d^2\mathbf{y}_\perp}{(2\pi)^3}\, e^{ixp^+y^- - i\,\mathbf{k}_\perp\cdot\mathbf{y}_\perp} \langle p,\vec{S}|\overline{\psi}(0^-,\mathbf{0}_\perp)\Phi_n^\dagger(\{-\infty,0\},\mathbf{0}_\perp)$$
$$\times\, \Phi_{\mathbf{n}_\perp}^\dagger(-\infty,\{\mathbf{y}_\perp,\mathbf{0}_\perp\})\frac{\gamma^+}{2}\Phi_n(\{-\infty,y^-\},\mathbf{y}_\perp)\psi(y^-,\mathbf{y}_\perp)|p,\vec{S}\rangle$$

- **PT invariance:**

$$f_{q/h^\uparrow}^{\rm SIDIS}(x,\mathbf{k}_\perp,\vec{S}) = f_{q/h^\uparrow}^{\rm DY}(x,\mathbf{k}_\perp,-\vec{S})$$

- **Sivers function:**

$$f_{q/h^\uparrow}(x,\mathbf{k}_\perp,\vec{S}) \equiv f_{q/h}(x,k_\perp) + f_{q/h^\uparrow}^{\rm Sivers}(x,k_\perp)\,\vec{S}\cdot(\hat{p}\times\hat{\mathbf{k}}_\perp)$$

$$\Rightarrow \quad f_{q/h^\uparrow}^{\rm Sivers}(x,k_\perp)^{\rm SIDIS} = -f_{q/h^\uparrow}^{\rm Sivers}(x,k_\perp)^{\rm DY} \quad \Leftarrow$$

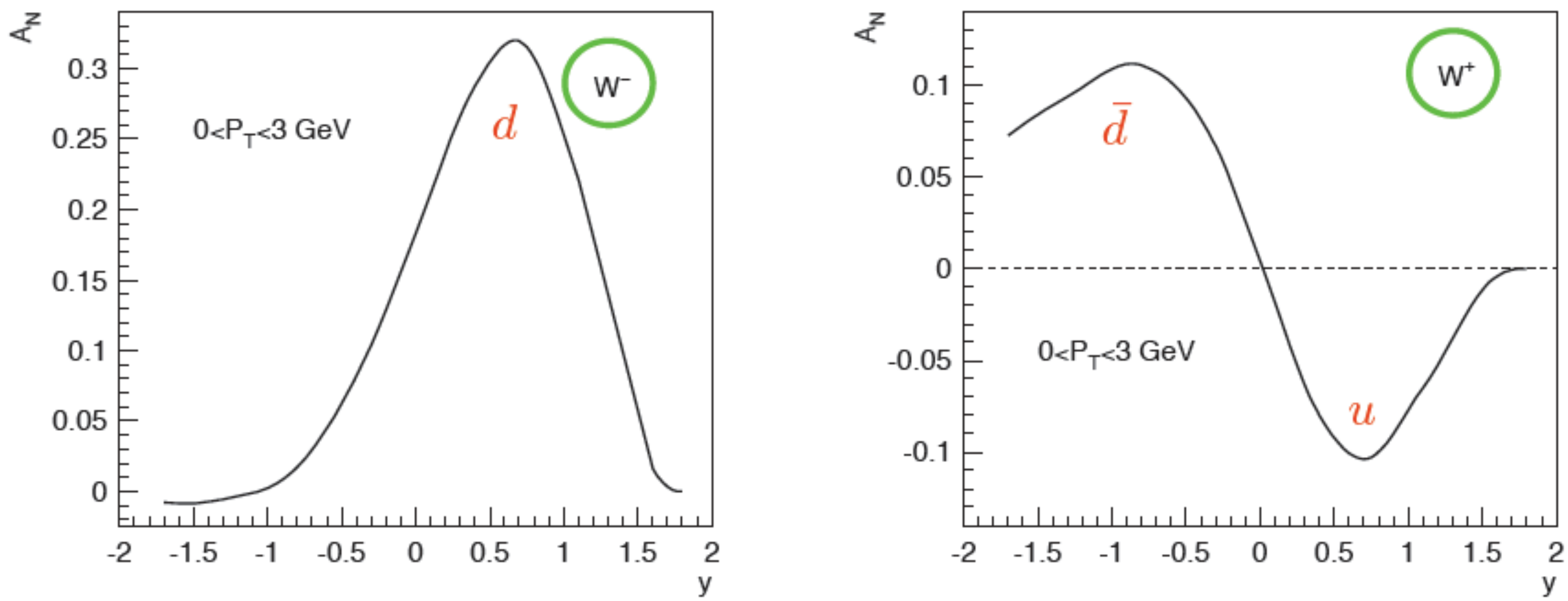
Modified Universality

Test of the modified universality

❑ **SSA of W-production at RHIC :**

Kang, Qiu, 2009

Sivers function same as DY, different from SIDIS by a sign



- flavor separation
- large asymmetry: should be able to see sign change

But, the detectors at RHIC cannot reconstruct the W's

The Sivers functions from Anselmino et al 2009

SSA of lepton from W-decay

- Lepton SSA is diluted from the decay:

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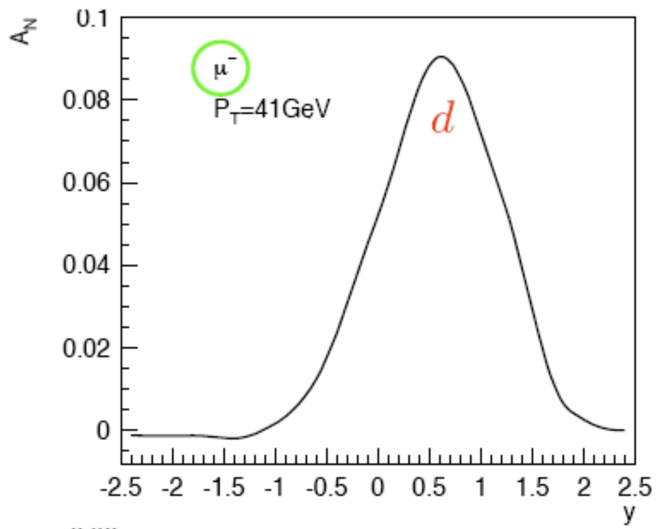
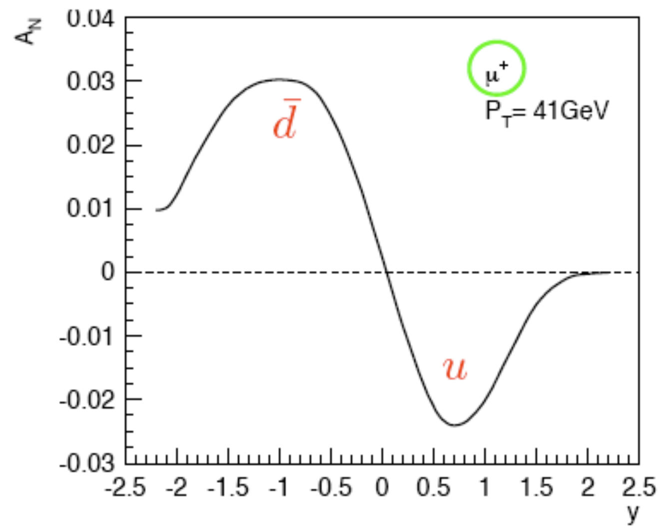
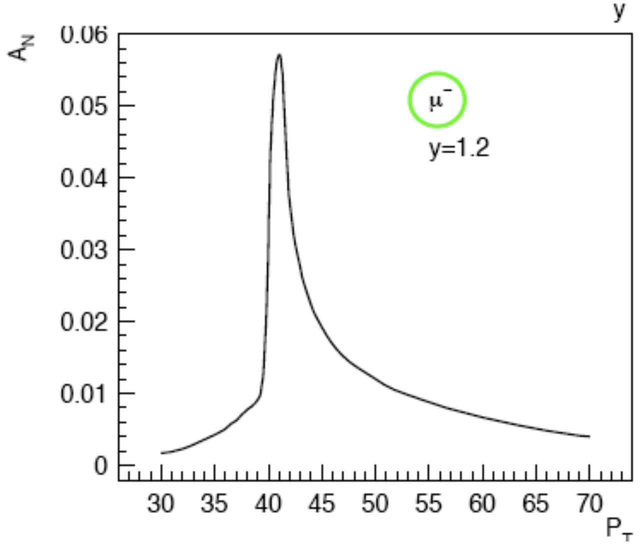






- flavor separation
- asymmetry gets smaller due to dilution
  should still be measurable by current
  RHIC sensitivity

Larger SSA for Z^0 production while the rate is lower

Summary and outlook

- Quark and gluon contribution to proton's spin depend on the "definition" an the scale where we probe them

- Matrix elements extracted from spin asymmetries in HE collisions are NOT necessary equal to the matrix elements defining partonic contribution to proton spin

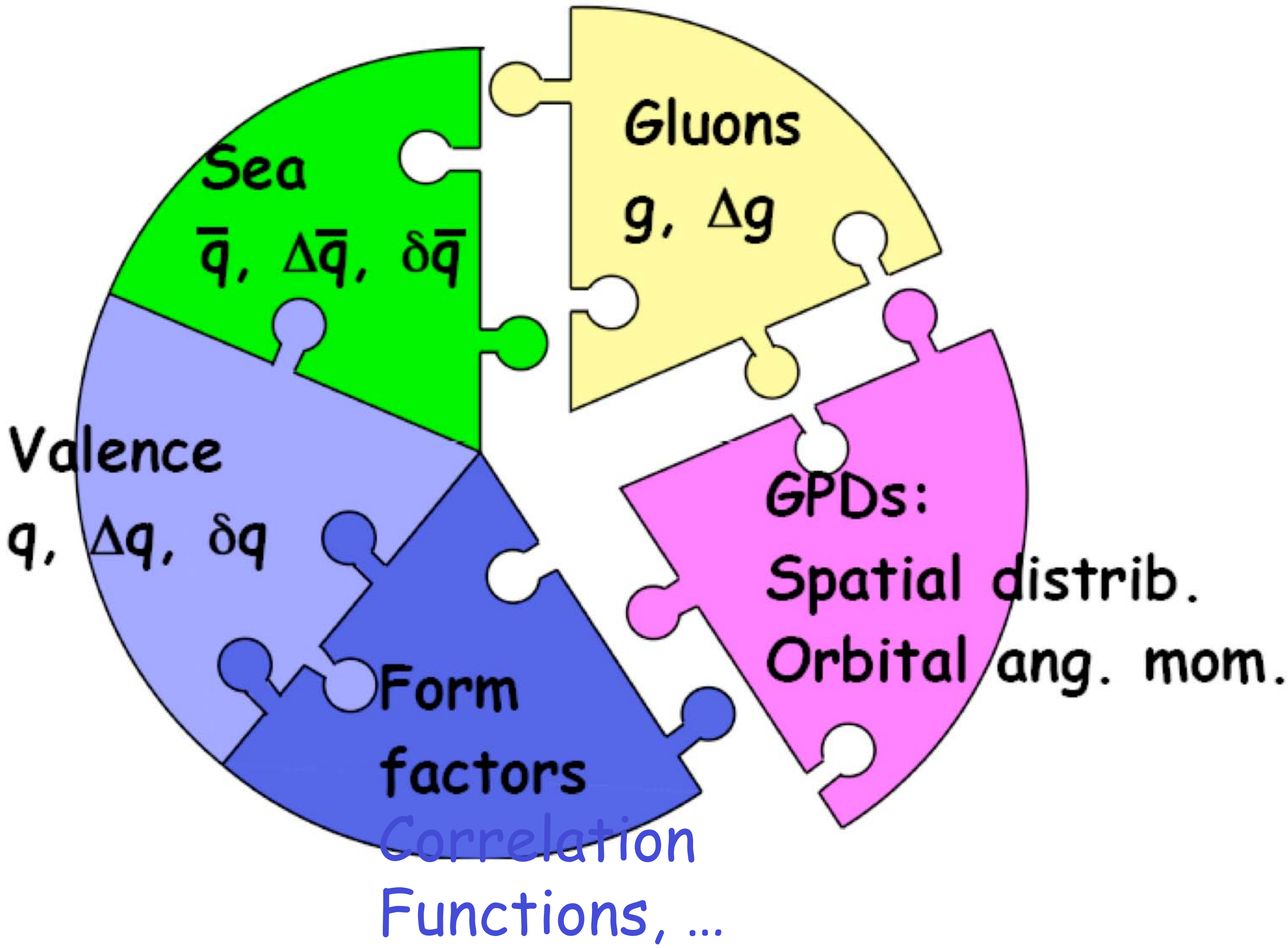
- With RHIC, Jlab upgrade and future EIC, better determination of quark/gluon "helicity" distributions

- Single transverse spin asymmetry opens a whole new meaning to test QCD dynamics!

  Direct measurement of QCD quantum interference, parton's transverse motion, etc.

Thank you!

Challenge: Map out the nucleon



RHIC, Jlab and future EIC spin program will play a key role!

Backup transparencies

Multi-gluon correlation functions

Ji, PLB289 (1992)

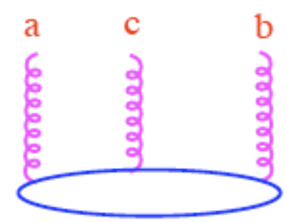
- **Diagonal tri-gluon correlations:**

$$T_G(x,x) = \int \frac{dy_1^- dy_2^-}{2\pi} e^{ixP^+ y_1^-} \times \frac{1}{xP^+} \langle P, s_\perp | F^+{}_\alpha(0) \left[\epsilon^{s_\perp \sigma n \bar{n}} F_\sigma{}^+(y_2^-) \right] F^{\alpha +}(y_1^-) | P, s_\perp \rangle$$

- **Two tri-gluon correlation functions – color contraction:**

$$T_G^{(f)}(x,x) \propto i f^{ABC} F^A F^C F^B = F^A F^C (\mathcal{T}^C)^{AB} F^B$$

$$T_G^{(d)}(x,x) \propto d^{ABC} F^A F^C F^B = F^A F^C (\mathcal{D}^C)^{AB} F^B$$



Quark-gluon correlation: $T_F(x,x) \propto \overline{\psi}_i F^C (T^C)_{ij} \psi_j$

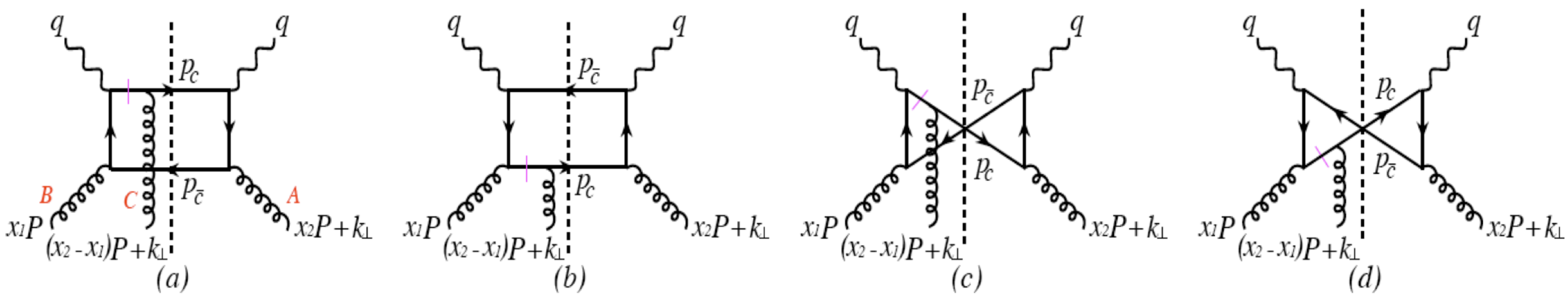
- **D-meson production at EIC:**

  - Clean probe for gluonic twist-3 correlation functions
  - $T_G^{(f)}(x,x)$ could be connected to the gluonic Sivers function

D-meson production at EIC

Kang, Qiu, PRD, 2008

❑ Dominated by the tri-gluon subprocess:



- ❖ Active parton momentum fraction cannot be too large
- ❖ Intrinsic charm contribution is not important
- ❖ Sufficient production rate

❑ Single transverse-spin asymmetry:

$$A_N = \frac{\sigma(s_\perp) - \sigma(-s_\perp)}{\sigma(s_\perp) + \sigma(-s_\perp)} = \frac{d\Delta\sigma(s_\perp)}{dx_B dy dz_h dP_{h\perp}^2 d\phi} \Bigg/ \frac{d\sigma}{dx_B dy dz_h dP_{h\perp}^2 d\phi}$$

SSA is directly proportional to tri-gluon correlation functions

Features of the SSA in D-production at EIC

- **Dependence on tri-gluon correlation functions:**

$$D - \text{meson} \propto T_G^{(f)} + T_G^{(d)} \qquad \overline{D} - \text{meson} \propto T_G^{(f)} - T_G^{(d)}$$

Separate $T_G^{(f)}$ and $T_G^{(d)}$ by the difference between D and $\bar{D}$

- **Model for tri-gluon correlation functions:**

$$T_G^{(f,d)}(x,x) = \lambda_{f,d} G(x) \qquad \lambda_{f,d} = \pm\lambda_F = \pm 0.07\text{GeV}$$

- **Kinematic constraints:**

$$x_{min} = \begin{cases} x_B\left[1 + \frac{P_{h\perp}^2 + m_c^2}{z_h(1-z_h)Q^2}\right], & \text{if } z_h + \sqrt{z_h^2 + \frac{P_{h\perp}^2}{m_c^2}} \geq 1 \\ x_B\left[1 + \frac{2m_c^2}{Q^2}\left(1 + \sqrt{1 + \frac{P_{h\perp}^2}{z_h^2 m_c^2}}\right)\right], & \text{if } z_h + \sqrt{z_h^2 + \frac{P_{h\perp}^2}{m_c^2}} \leq 1 \end{cases}$$

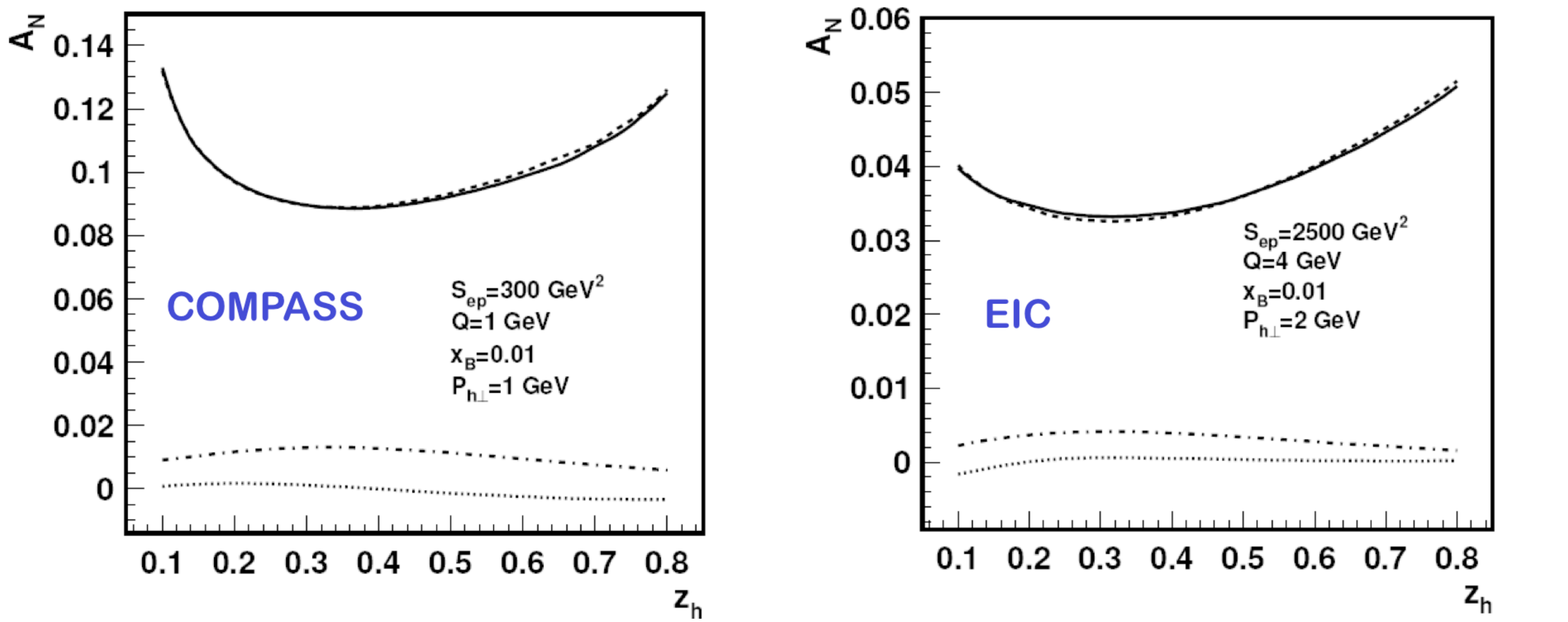
Note: The $z_h(1-z_h)$ has a maximum

SSA should have a minimum if the derivative term dominates

Minimum in the SSA of D-production at EIC

Kang, Qiu, PRD, 2008

- SSA for D^0 production (λ_f only):

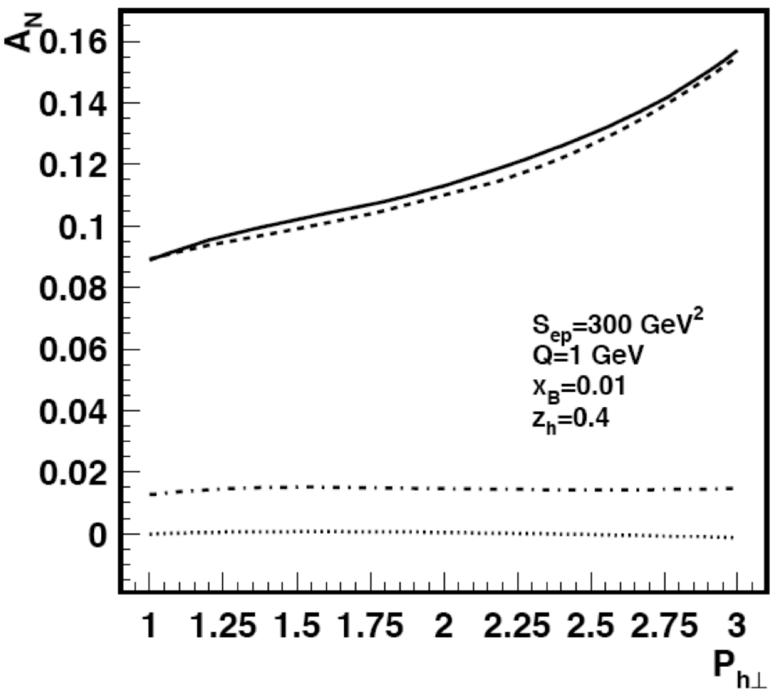


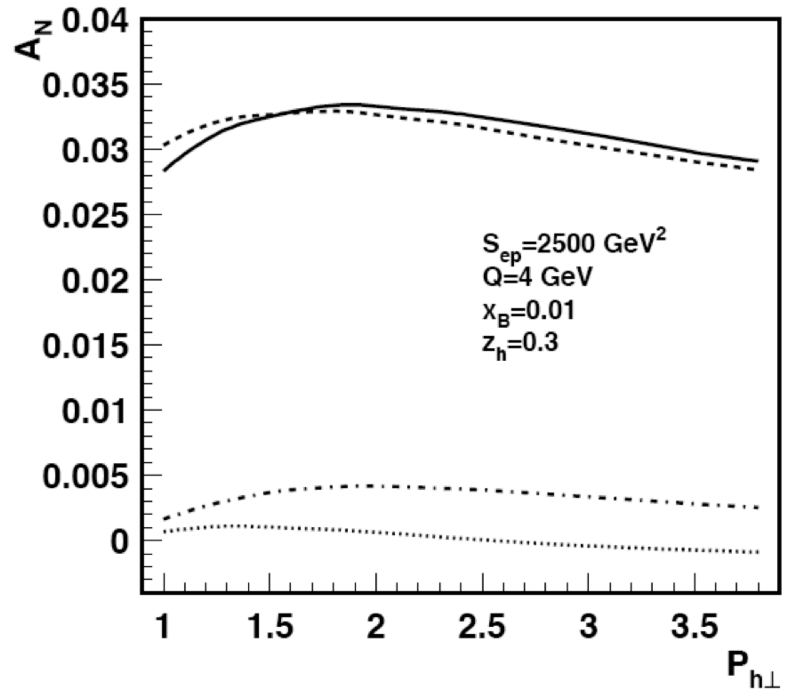
- Derivative term dominates, and small φ dependence
- Asymmetry is **twice** if $T_G^{(f)} = +T_G^{(d)}$, or **zero** if $T_G^{(f)} = -T_G^{(d)}$
- Opposite for the $\bar{D}$ meson
- Asymmetry has a minimum ~ z_h ~ 0.5

Maximum in the SSA of D-production at EIC

Kang, Qiu, PRD, 2008

❑ **SSA for D^0 production (λ_f only):**





❖ The SSA is a twist-3 effect, it should fall off as $1/P_T$ when $P_T >> m_c$

❖ For the region, $P_T \sim m_c$,

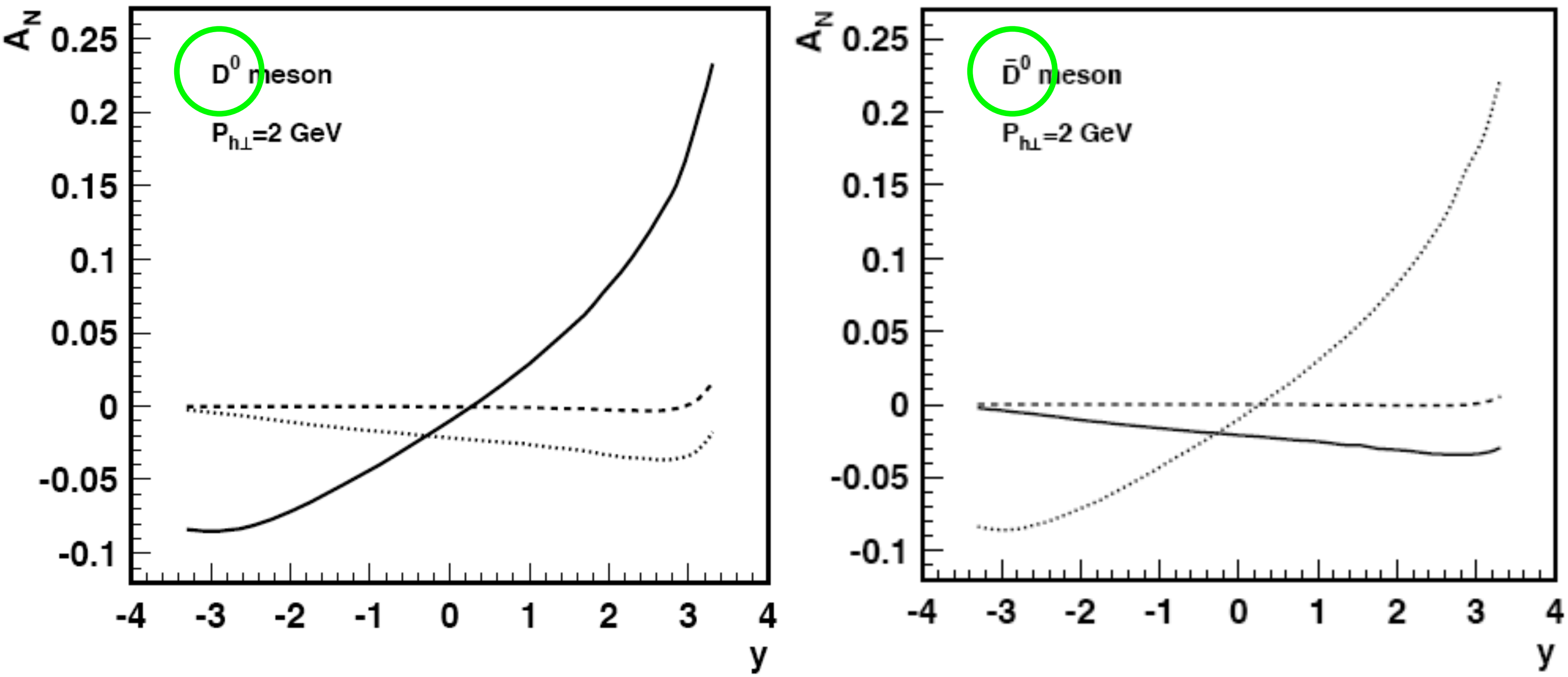
$$A_N \propto \epsilon^{P_h s_\perp n \bar{n}} \frac{1}{\tilde{t}} = -\sin\phi_s \frac{P_{h\perp}}{\tilde{t}}$$

$$\tilde{t} = (p_c - q)^2 - m_c^2 = -\frac{1-\hat{z}}{\hat{x}} Q^2$$

$$\hat{z} = z_h/z, \quad \hat{x} = x_B/x$$

SSA of D-meson production at RHIC

- **Rapidity:** $\sqrt{s} = 200\ \text{GeV} \quad \mu = \sqrt{m_c^2 + P_{h\perp}^2} \quad m_c = 1.3\ \text{GeV}$



Solid: $(1)\ \lambda_f = \lambda_d = 0.07\ \text{GeV} \qquad T_G^{(f)} = T_G^{(d)}$

Dashed: $(2)\ \lambda_f = \lambda_d = 0 \qquad T_G^{(f)} = T_G^{(d)} = 0$

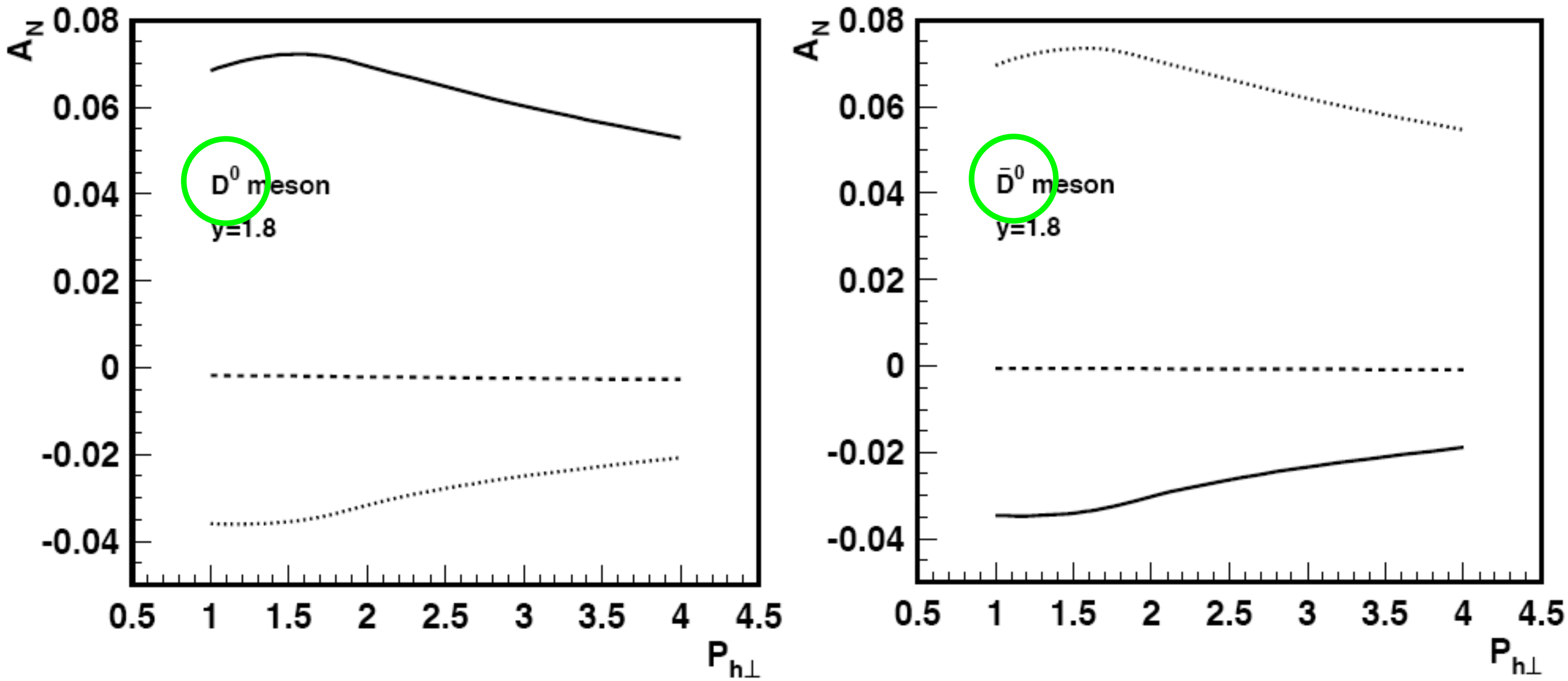
Dotted: $(3)\ \lambda_f = -\lambda_d = 0.07\ \text{GeV} \qquad T_G^{(f)} = -T_G^{(d)}$

No intrinsic Charm included

Kang, Qiu, Vogelsang, Yuan, 2008

SSA of D-meson production at RHIC

- P_T dependence: $\sqrt{s} = 200\ \text{GeV}$ $\mu = \sqrt{m_c^2 + P_{h\perp}^2}$ $m_c = 1.3\ \text{GeV}$



Solid: $(1)\ \lambda_f = \lambda_d = 0.07\ \text{GeV}$ $T_G^{(f)} = T_G^{(d)}$

Dashed: $(2)\ \lambda_f = \lambda_d = 0$ $T_G^{(f)} = T_G^{(d)} = 0$

Dotted: $(3)\ \lambda_f = -\lambda_d = 0.07\ \text{GeV}$ $T_G^{(f)} = -T_G^{(d)}$

No intrinsic Charm included

Kang, Qiu, Vogelsang, Yuan, 2008

What is the $T_F(x,x)$?

- Twist-3 correlation $T_F(x,x)$:

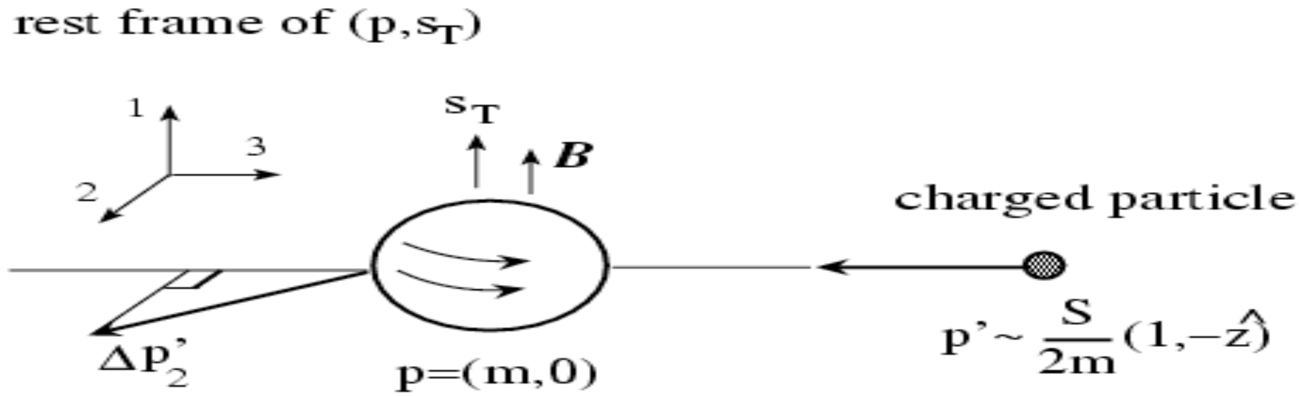
$$T_F(x,x) = \int \frac{dy_1^-}{4\pi} \mathrm{e}^{ixP^+y_1^-}$$
$$\times \langle P, \vec{s}_T | \bar{\psi}_a(0)\gamma^+ \left[\int dy_2^- \, \epsilon^{s_T \sigma n \bar{n}} \, F_\sigma^{\ +}(y_2^-) \right] \psi_a(y_1^-) | P, \vec{s}_T \rangle$$

- Twist-2 quark distribution:

$$q(x) = \int \frac{dy_1^-}{4\pi} \mathrm{e}^{ixP^+y_1^-} \langle P, \vec{s}_T | \bar{\psi}_a(0)\gamma^+ \psi_a(y_1^-) | P, \vec{s}_T \rangle$$

T_F **Represents a fundamental quantum correlation between quark and gluon inside a hadron**

What the $T_F(x,x)$ tries to tells us?



- Consider a classical (Abelian) situation:
  - change of transverse momentum

$$\frac{d}{dt}p_2' = e(\vec{v}' \times \vec{B})_2 = -ev_3 B_1 = ev_3 F_{23}$$

  - in the c.m. frame

$$(m, \vec{0}) \rightarrow \bar{n} = (1, 0, 0_T), \quad (1, -\hat{z}) \rightarrow n = (0, 1, 0_T)$$

$$\Longrightarrow \frac{d}{dt}p_2' = e\, \epsilon^{s_T \sigma n \bar{n}}\, F_\sigma{}^+$$

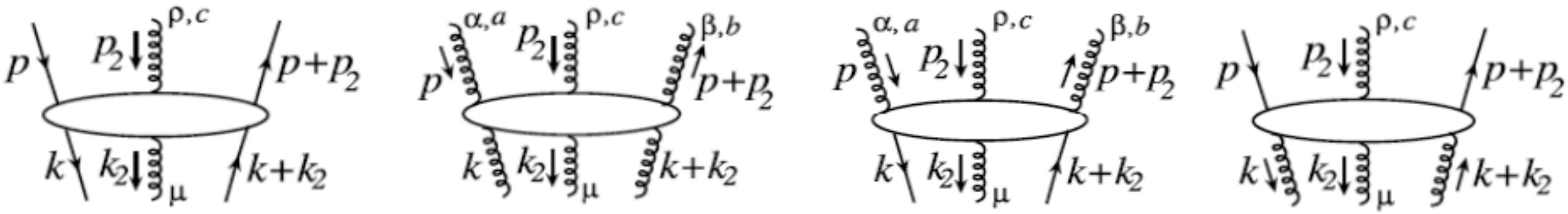
  - total change: $\Delta p_2' = e \int dy^- \epsilon^{s_T \sigma n \bar{n}}\, F_\sigma{}^+(y^-)$

$T_F(x,x)$ probes a net asymmetry in parton transverse momentum caused by a color Lorentz force inside a spinning proton

Evolution Kernels

Kang, Qiu, PRD, 2009

❑ Feynman diagrams:



❑ LO for flavor non-singlet channel: