Hadron Structure from Lattice QCD

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Outline

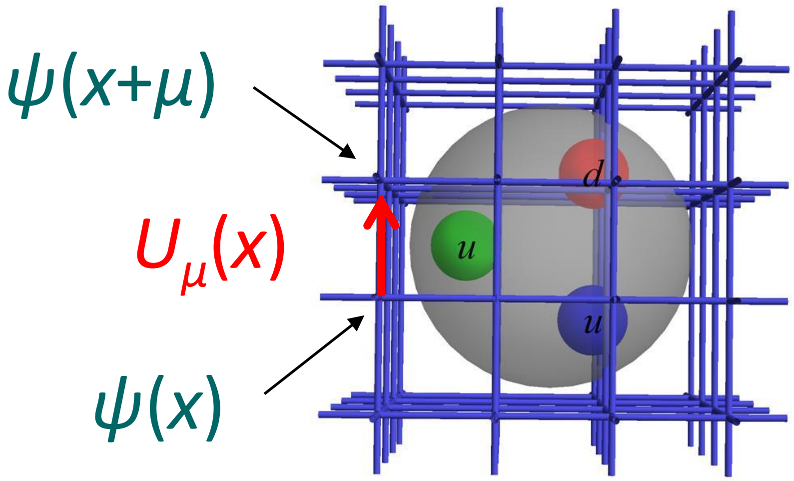
- § Lattice QCD Overview
- § Nucleon Structure
  - PDF, form factors, GPDs
- § Hyperons
  - Axial coupling constants, charge radii...
- § Summary

Lattice QCD

- QCD observables are calculated from the path integral

$$\langle 0|O(\overline{\psi},\psi,A)|0\rangle = \frac{1}{Z}\int [dA][d\overline{\psi}][d\psi]\, O(\overline{\psi},\psi,A) e^{i\int d^4x\, \mathcal{L}^{\mathrm{QCD}}(\overline{\psi},\psi,A)}$$

- Strong-coupling regions: expansions no longer converge
- Lattice QCD is a discrete version of continuum QCD



- Numerical integration to calculate the path integral
- Take $a \to 0$ and $V \to \infty$ in the continuum limit

Lattice Actions

Symanzik Improvement

- Order-by-order in a improvement of the action and operators
- Systematic error due to discretization under control

Gauge Actions

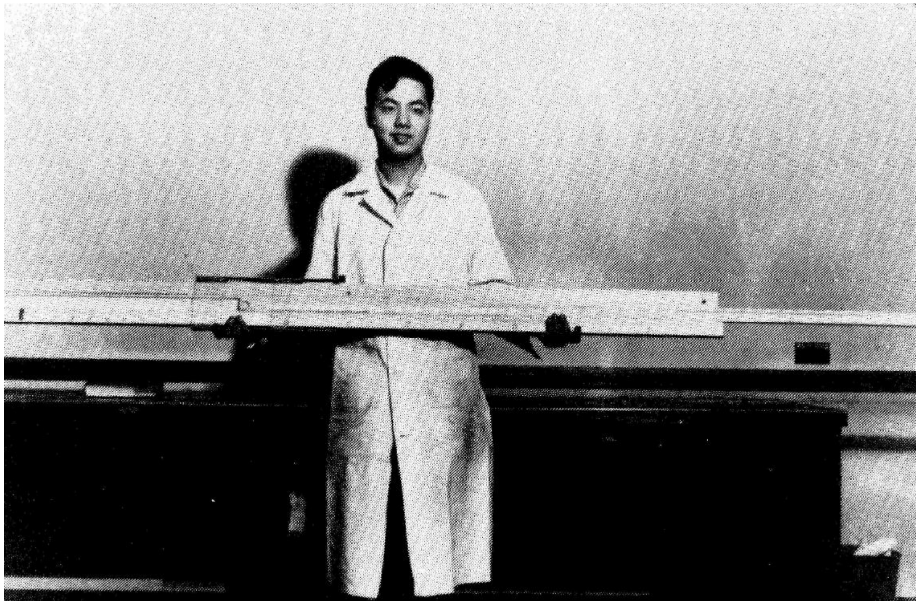
- Most gauge actions used today are $O(a^2)$ improved
- Small discretization effects ($\sim O(\Lambda_{\mathrm{QCD}}^3 a^3)$) due to gauge choices
  <1% systematic with current a

Fermion Actions

- Most fermion actions are only $O(a)$ improved ($O(\Lambda_{\mathrm{QCD}}^2 a^2)$) ~4%
- Differences are benign once all systematics are included
- Different choices of fermion action are confined by limits of computational and human power + by personal interest
- Commonly used actions:
  domain-wall fermions, overlap fermions,
  Wilson/clover fermions, twisted-Wilson fermions, staggered fermions

Computational Requirements

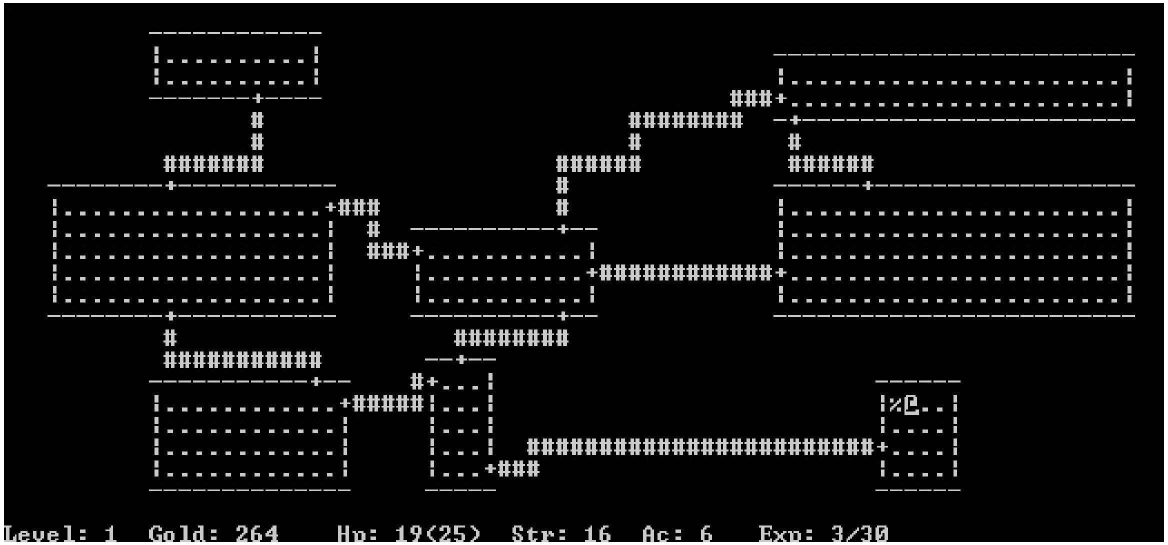
- A variety of first-principles QCD calculations can be done:
- In 1970, Wilson wrote down the first actions
- Progress is limited by computational resources
  - But assisted by advances in algorithms
- 2 generations ago:



To calculate stellar radiative transfer equations, T.D. Lee uses an “analog computer”

Computational Requirements

- A variety of first-principles QCD calculations can be done:
- In 1970, Wilson wrote down the first actions
- Progress is limited by computational resources
  - But assisted by advances in algorithms
- Computer power available for gaming in the 1980s:



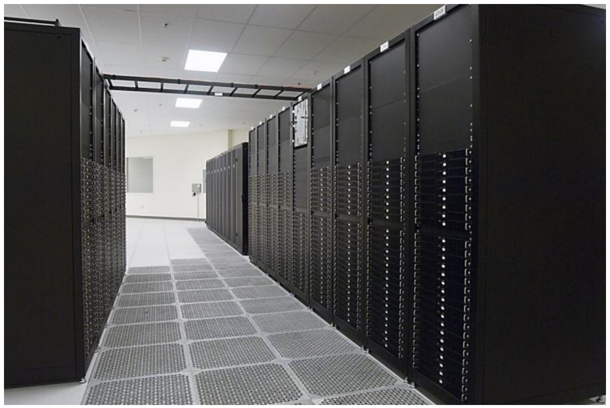
Computational Requirements

- A variety of first-principles QCD calculations can be done:
- In 1970, Wilson wrote down the first actions
- Progress is limited by computational resources
  - But assisted by advances in algorithms
- Computer power available today:

- Exciting progress during the last decade

Computational Requirements

- USQCD facilities: JLab, Fermilab, BNL



- Non-lattice resources open to USQCD: ORNL, LLNL, ANL



- NSF supercomputer and worldwide increase in computer facilities

Computational Requirements

- Gauge generation costs with the latest algorithms scale like
  - Cost factor (DWF): a^{-6}, L^5, M_π^{-3} Norman Christ, LAT07

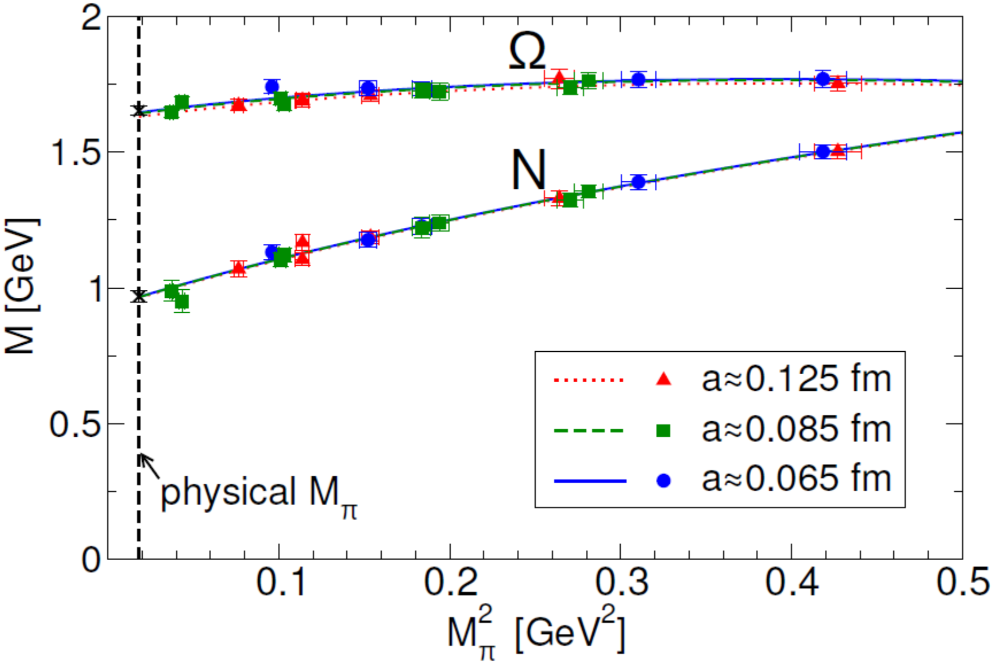
- Most of the major 2+1-flavor gauge ensembles:

  $M_\pi < 300$ MeV
  - MILC (staggered): $M_\pi \sim$ **217** MeV
  - PACS-CS (Clover action): $M_\pi \sim$ **156** MeV (but small volume)
  - The Budapest-Marseille-Wuppertal (BMW) Collaboration: $M_\pi \sim$ **193** MeV, 3 lattice spacing, multiple volumes
  - The Hadron Spectrum Collaboration (anisotropic clover): $M_\pi \sim$ **180, 220** MeV (on-going)
  - RBC/UKQCD (DWF), $M_\pi \sim$ **210** MeV (on-going)
  - Most of them have *multiple lattice spacings and volumes*
- Pion-mass extrapolation $M_\pi \rightarrow (M_\pi)_{\text{phys}}$

  (Bonus products: Low-Energy Constants)

Precision Extrapolation

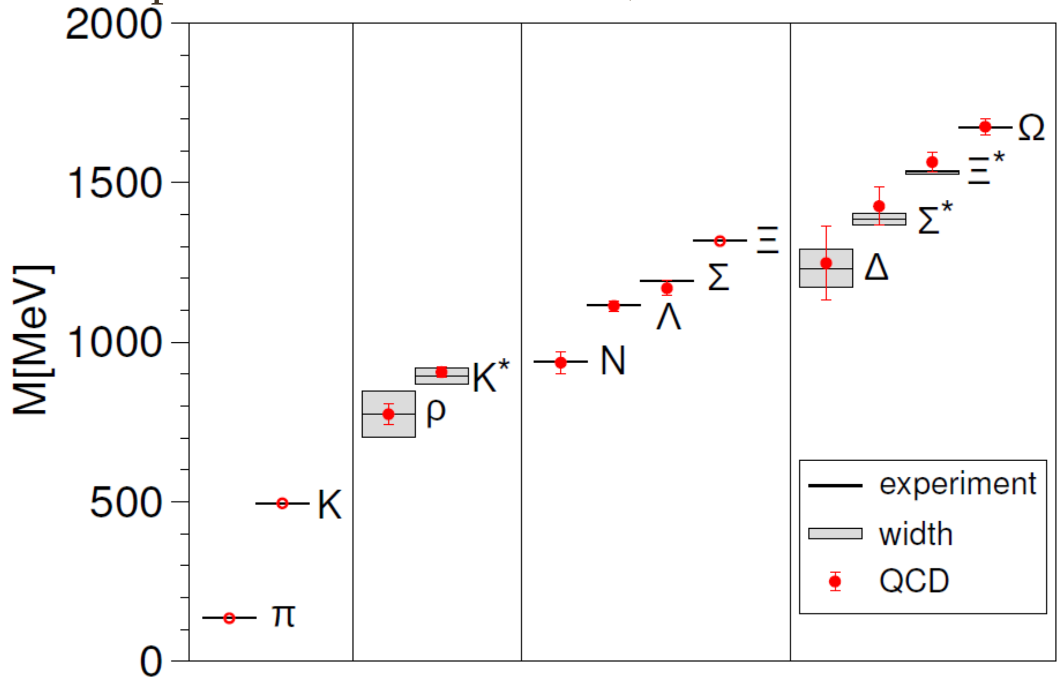
- Currently, not at the physical pion-mass point
  - XPT uncertainty (parameters used in XPT, etc.)
  - Example: BMW Collaboration, LAT2008



- Small range of chiral extrapolation reduces systematic uncertainties
- Less dependence on chiral perturbation theory

Lattice in the News (in Science)

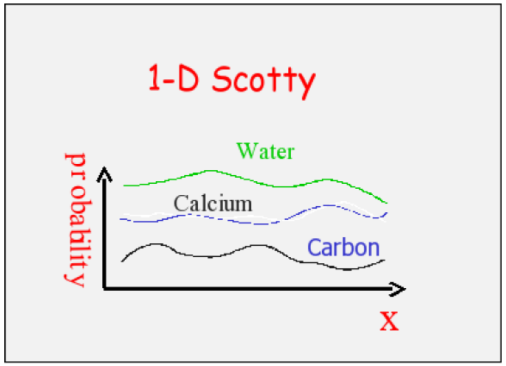
- Post-dictions of well known quantities
  - Example: BMW Collaboration, LAT2008



- Proves all the systematics are under control
- Next look at experimentally less-known quantities: excited-baryon spectrum, hybrid mesons, etc.

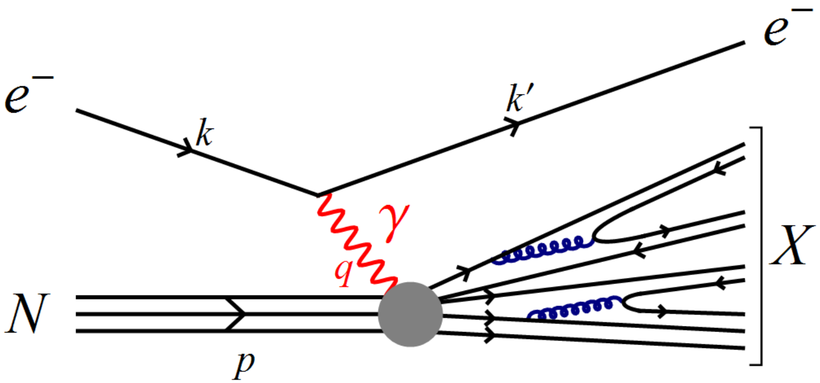
Nucleon Structure

- Structure function/distribution functions
  - deep inelastic scattering
  - $\langle x^n \rangle_q$, $\langle x^n \rangle_{\Delta q}$, $\langle x^n \rangle_{\delta q}$



Deep Inelastic Scattering

- Probing nucleon structure



$$\sigma \sim L^{\mu\nu} W_{\mu\nu},$$
$$W_{\mu\nu} = i\int d^4x e^{iqx}\langle N|T\{J^\mu(x), J^\nu(0)\}|N\rangle$$

- The symmetric, unpolarized, spin-averaged

$$W^{\{\mu\nu\}}(x,Q^2) = \left(-g^{\mu\nu} + \frac{q^\mu q^\nu}{q^2}\right)F_1(x,Q^2) + \left(p^\mu - \frac{\nu}{q^2}q^\mu\right)\left(p^\nu - \frac{\nu}{q^2}q^\nu\right)\frac{F_2(x,Q^2)}{\nu}$$

- The anti-symmetric, polarized

$$W^{[\mu\nu]}(x,Q^2) = i\epsilon^{\mu\nu\rho\sigma}q_\rho\left(\frac{s_\sigma}{\nu}(g_1(x,Q^2) + g_2(x,Q^2)) - \frac{q\cdot s p_\sigma}{\nu^2}g_2(x,Q^2)\right)$$

Moments of the Structure Function

- No light-cone operator has been directly calculated on the lattice
- Operator product expansion
  - Polarized

$$2\int dx\, x^n g_1(x,Q^2) = \sum_{q=u,d} e_{1,n}^{(q)}(\mu^2/Q^2, g(\mu))\langle x^n\rangle_{\Delta q}$$

$$2\int dx\, x^n g_2(x,Q^2) = \frac{n}{(n+1)}\sum_{q=u,d}\Big[2e_{2,n}^{(q)}(\mu^2/Q^2, g(\mu))d_n^q(\mu) + e_{1,n}^{(q)}(\mu^2/Q^2, g(\mu))\langle x^n\rangle_{\Delta q}\Big]$$

  - Unpolarized

$$2\int dx\, x^{n-1} F_1(x,Q^2) = \sum_{q=u,d} c_{1,n}^{(q)}(\mu^2/Q^2, g(\mu))\langle x^n\rangle_q$$

$$\int dx\, x^{n-2} F_2(x,Q^2) = \sum_{q=u,d} c_{2,n}^{(q)}(\mu^2/Q^2, g(\mu))\langle x^n\rangle_q$$

- e_1, e_2, c_1, c_2 are Wilson coefficients
- $\langle x^n\rangle_q$, $\langle x^n\rangle_{\Delta q}$, d_n are the forward nucleon matrix elements

Nucleon Structure Functions

§ Matrix element $\langle P,S \mid O \mid P,S \rangle$

- Density

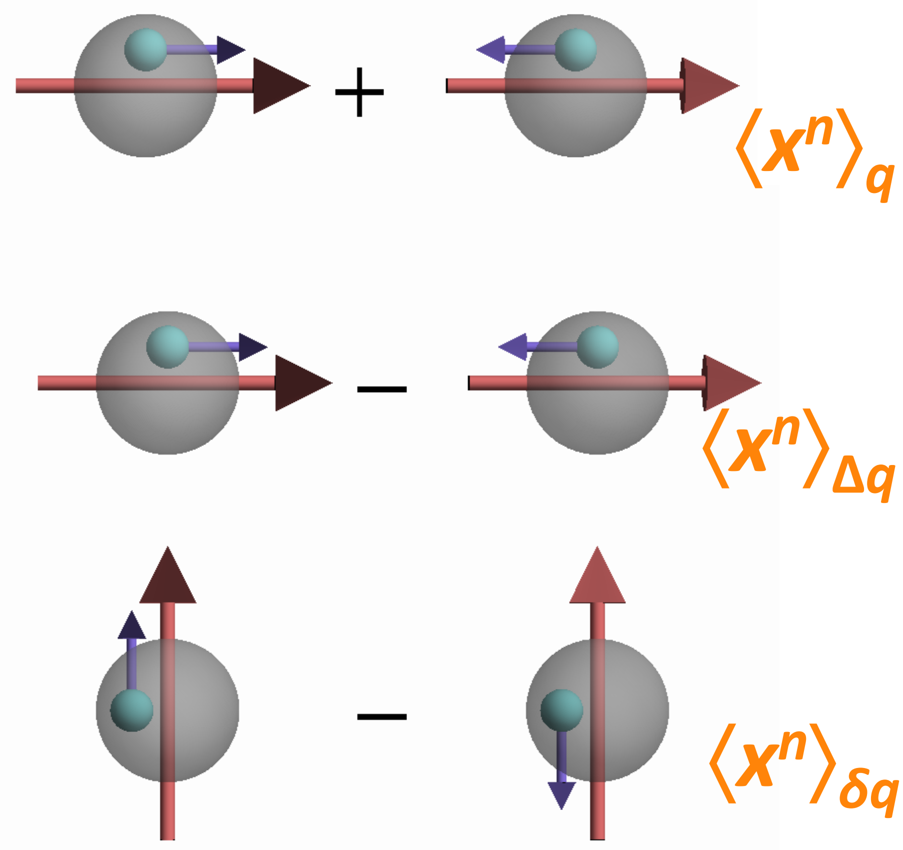
$$\mathcal{O}^{q}_{\mu_1\mu_2\cdots\mu_n} = \left(\frac{i}{2}\right)^{n-1} \bar{q}\gamma_{\mu_1} \overleftrightarrow{D}_{\mu_2} \cdots \overleftrightarrow{D}_{\mu_n}\, q - trace$$

- Helicity

$$\mathcal{O}^{5q}_{\sigma\mu_1\mu_2\cdots\mu_n} = \left(\frac{i}{2}\right)^{n-1} \bar{q}\gamma_{\sigma}\gamma_5 \overleftrightarrow{D}_{\mu_2} \cdots \overleftrightarrow{D}_{\mu_n}\, q - trace$$

- Transversity

$$\mathcal{O}^{\sigma q}_{\rho\nu\mu_1\mu_2\cdots\mu_n} = \left(\frac{i}{2}\right)^{n} \bar{q}\sigma_{\rho\nu} \overleftrightarrow{D}_{\mu_1} \cdots \overleftrightarrow{D}_{\mu_n}\, q - trace$$



Green Functions

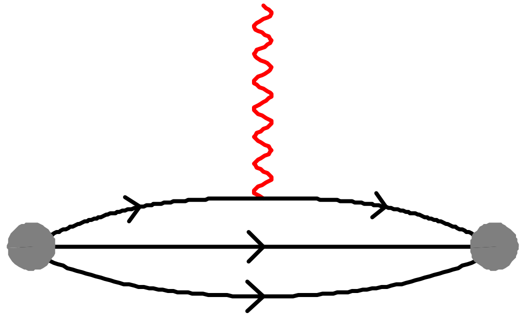
- Three-point function with connected piece only

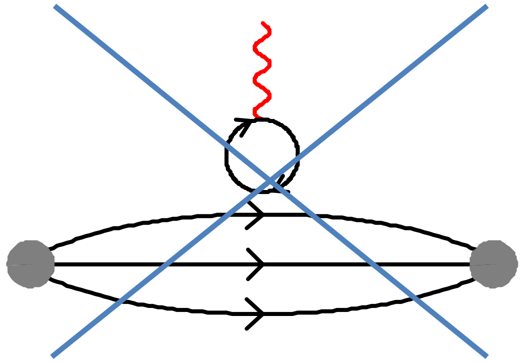
$$C_{\text{3pt}}^{\Gamma,\mathcal{O}}\left(\vec{p},t,\tau\right)=\sum_{\alpha,\beta}\Gamma^{\alpha,\beta}\langle J_\beta\left(\vec{p},t\right)\mathcal{O}(\tau)\overline{J}_\alpha\left(\vec{p},0\right)\rangle$$

O: $V_\mu = \bar{q}\gamma_\mu q$, $A_\mu = \bar{q}\gamma_\mu\gamma_5 q$, or others

$J = \epsilon^{abc}[q_1^{aT}(x)C\gamma_5 q_2^b(x)]q_1^c(x)$

- Two topologies:





- Isovector quantities O^{u-d}

disconnected diagram cancelled

Lattice Operators

- Rotation symmetry is reduced due to discretization

  Lorentz SO(3,1) ⇒ hypercubic H_4 group

  - 20 irreps: 4·**1** + 2·**2** + 4·**3** + 4·**4** + 4·**6** + 2·**8**

- Limit symmetry to prevent operators from mixing with each other
  - Some fermion actions (e.g. Wilson or staggered) cause mixing by breaking either chiral or flavor symmetry at finite *a*

- Choose a specific basis to avoid potential mixing or work out the all the possible mixing matrix elements and subtract them out from the desired ones
  - At larger moments, divergences occur involving lower-dimension operators at finite *a*

    ⇒ limited number of moments accessible to lattice QCD

Axial Charge Coupling

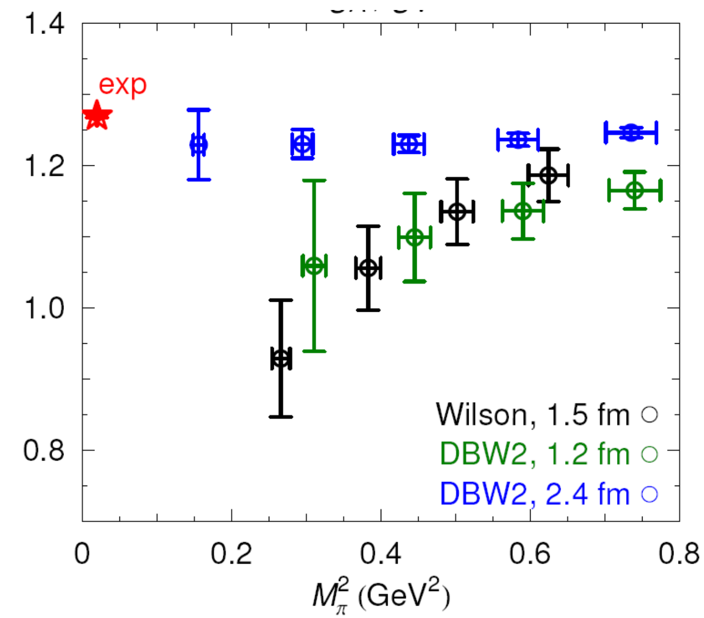
- Axial-vector–current matrix element

$$\langle B\,|A_\mu(q)|\,B\rangle \;=\; \overline{u}_B(p')\left[\gamma_\mu\gamma_5 G_A(q^2)+\gamma_5 q_\nu \frac{G_P(q^2)}{2M_B}\right]u_B(p)$$

and axial charge coupling $g_A = G_A^{u-d}(Q^2{=}0)$

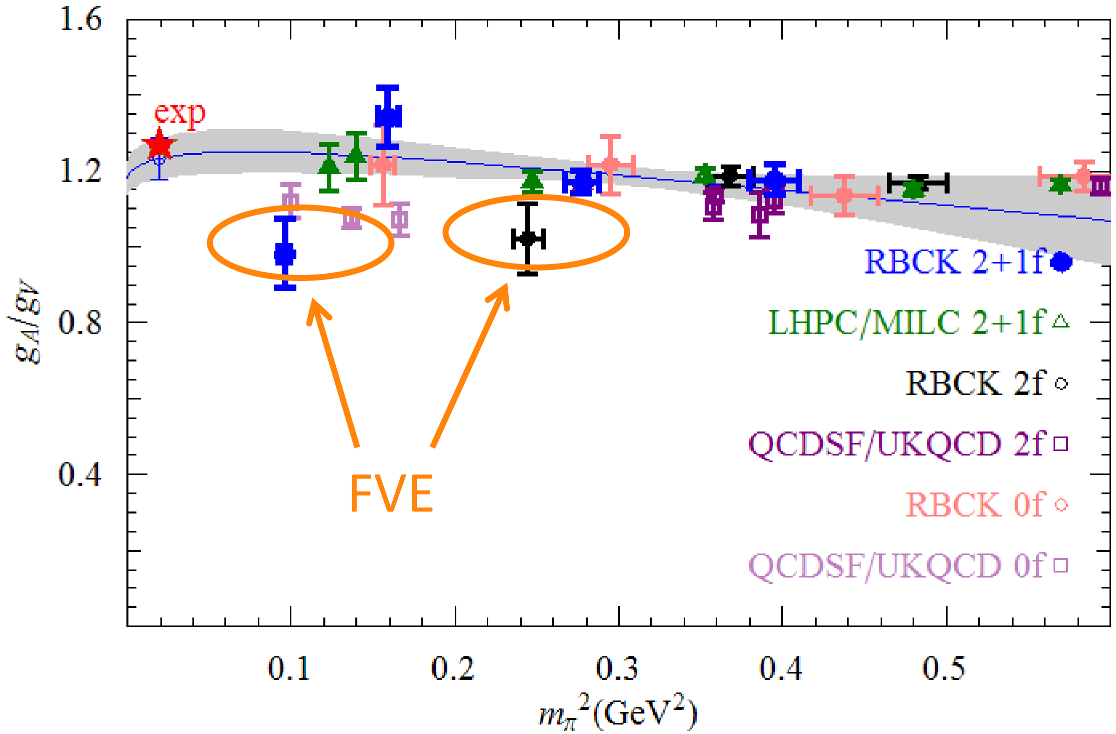
- Well measured experimentally from neutron beta decay
- Should be able to reproduce the experimental number and understand systematic effects

  - Example: finite-volume effect Phys.Rev.D68:054509 (2003)
    - Quenched approximation
    - Pion mass range: 300–650 MeV
    - DWF, $L_s = 16$, $M_5 = 1.8$, $a = 0.15$ fm



Axial Charge Coupling

§ World data: statistical error-bars only



HWL et al., Phys. Rev. D78, 014505 (2008) and Phys. Rev. Lett. 100:171602 (2008);
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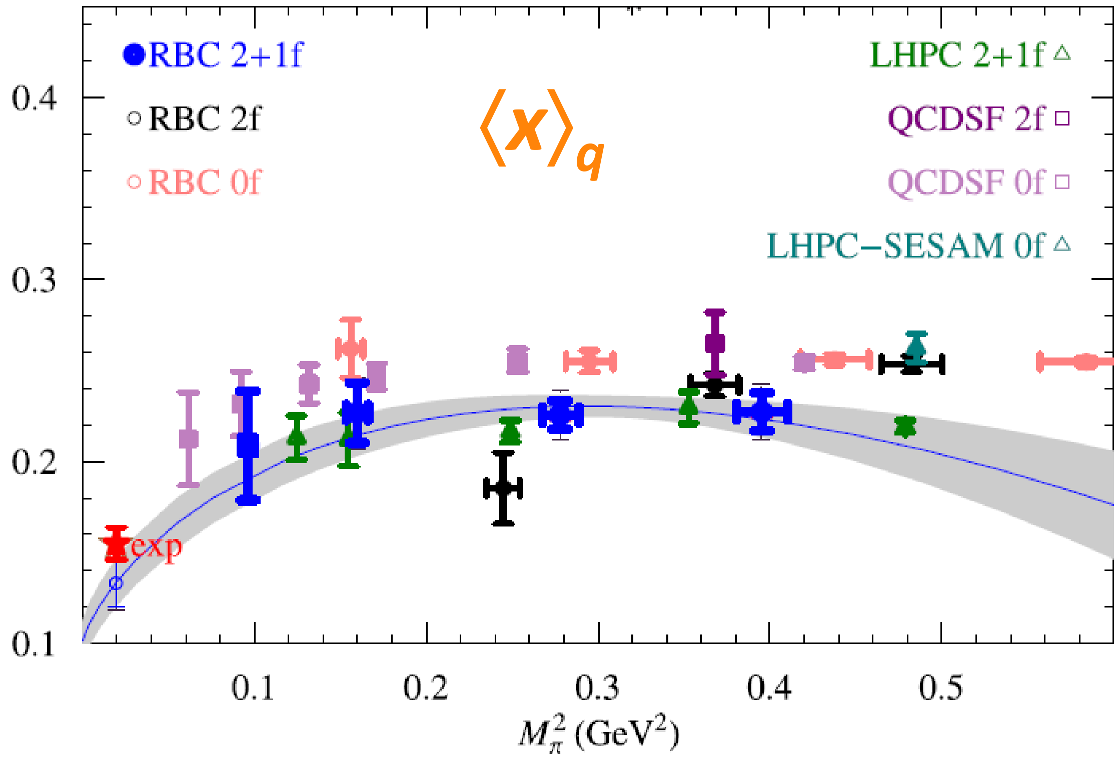
M. Guertler et al., PoS(LAT2006)107; D. Pleiter et al., PoS(LAT2006)120;
D. Renner et al., PoS(LAT2006)121

Nucleon Structure Functions

- World data: the first moment of the quark momentum fraction



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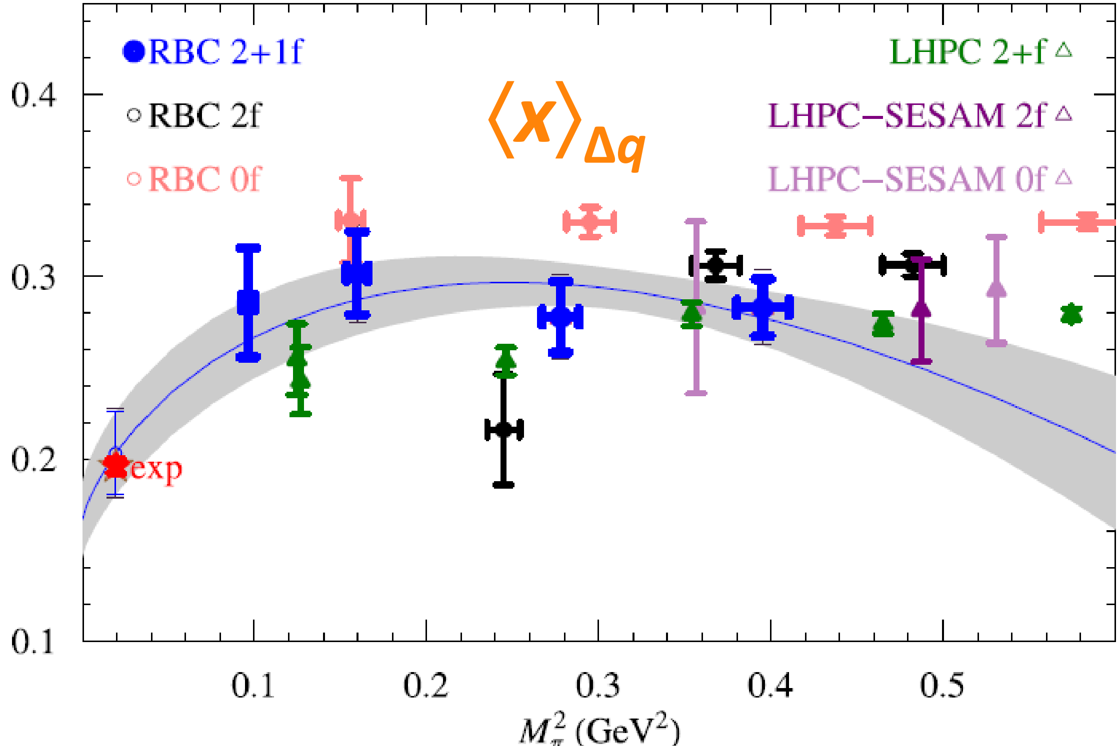
Nucleon Structure Functions

- World data: the first moment of the quark helicity distribution



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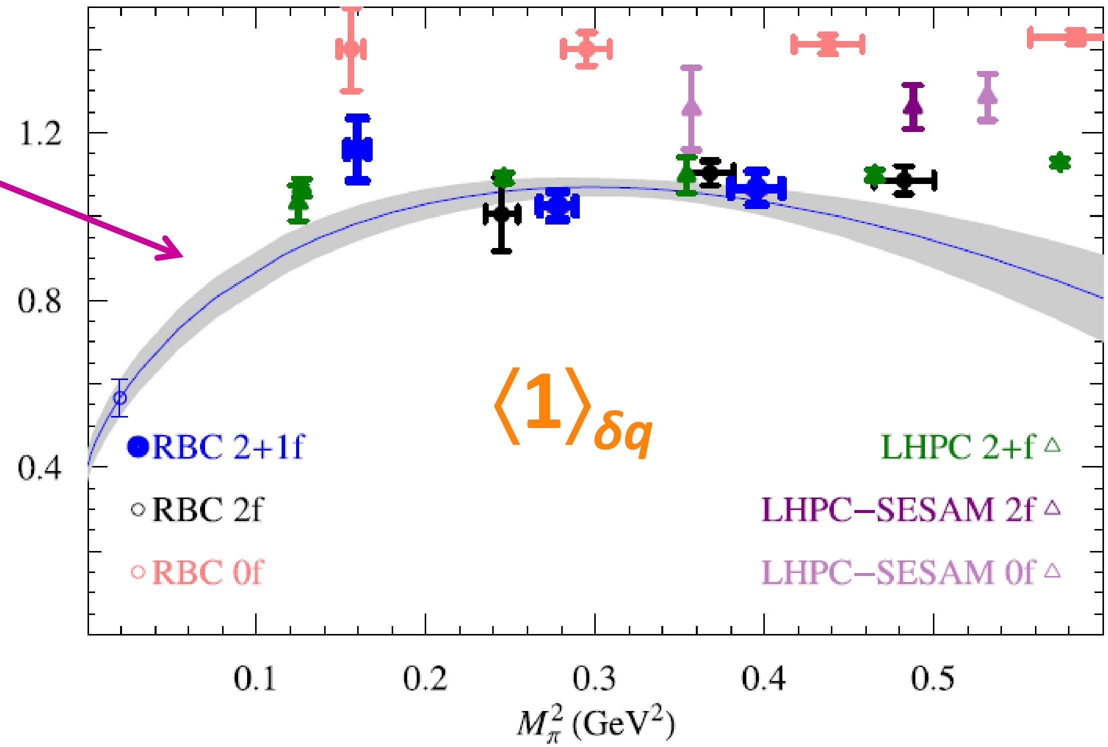
D. Renner et al., PoS(LAT2006)121

Nucleon Structure Functions

§ World data: the zeroth moment of the transversity

Trustworthy?



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M. Guertler et al., PoS(LAT2006)107; D. Pleiter et al., PoS(LAT2006)120;
D. Renner et al., PoS(LAT2006)121

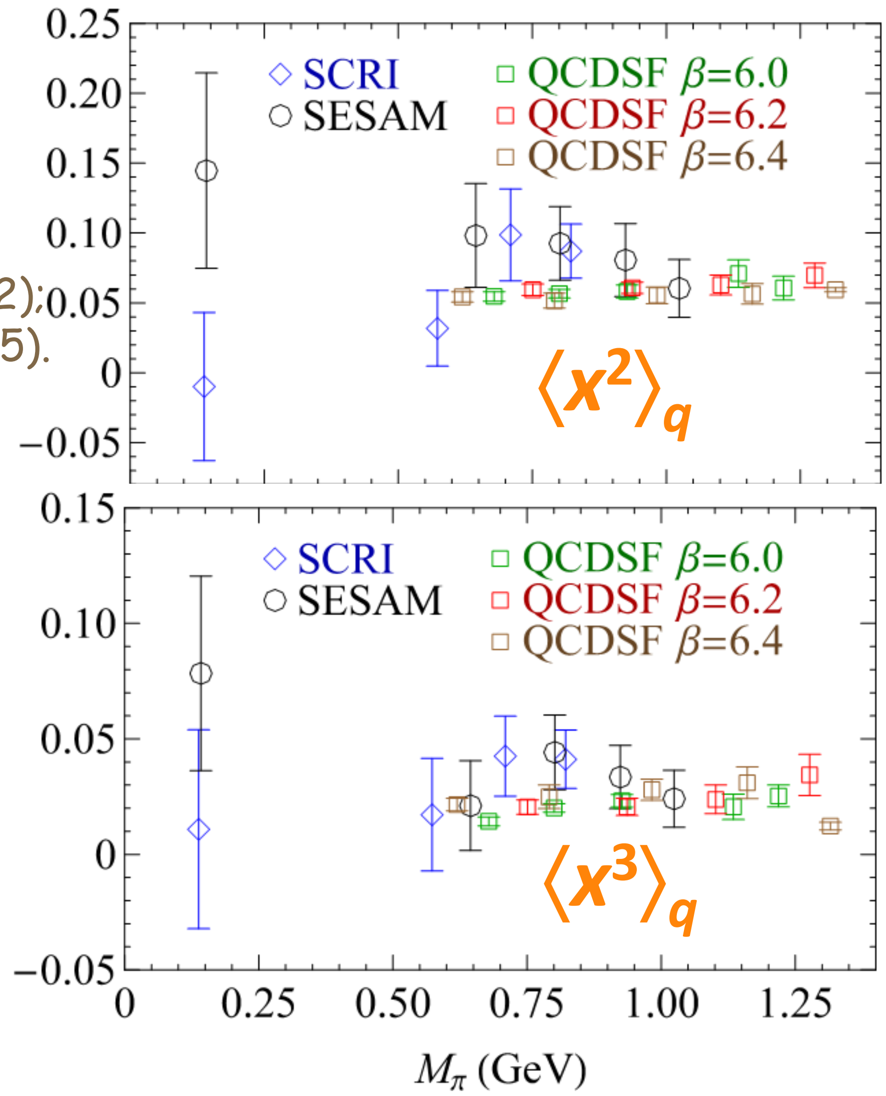
Nucleon Structure Functions

- Example: isovector unpolarized moments

D. Dolgov et al., Phys. Rev. D66, 034506 (2002); M. Gockeler et al. Phys. Rev. D71, 114511 (2005).

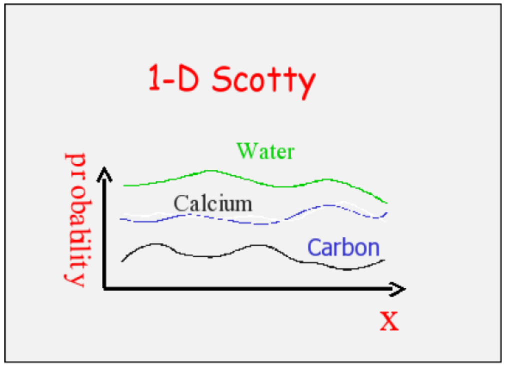
- Data:
  - LHPC (SCRI, SESAM): 2f, Wilson and clover
  - QCDSF: 0f clover, multiple lattice spacings

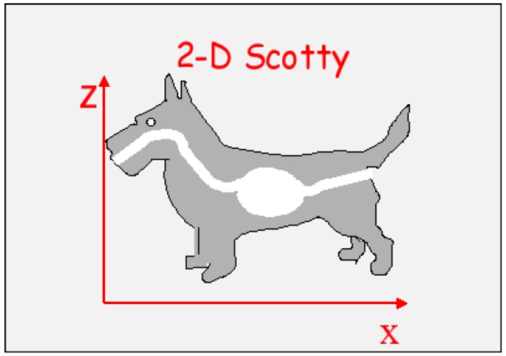
- $n \geq 4$: mixing with lower-dimension operators



Form Factors

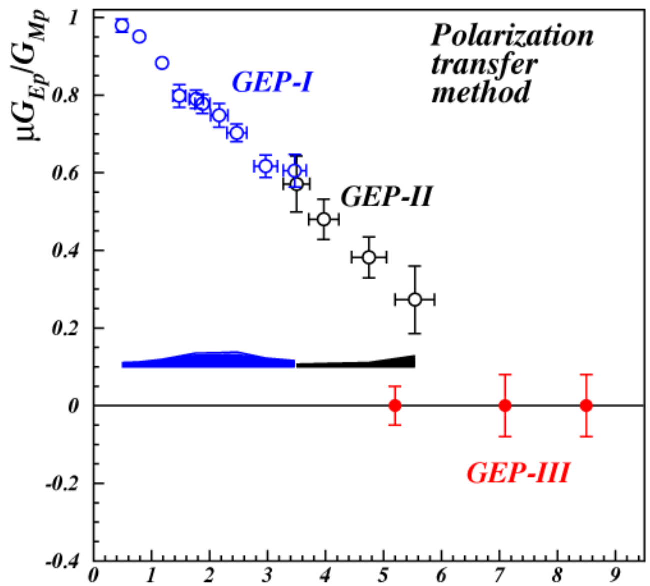
- Structure function/distribution functions
  - deep inelastic scattering
  - $\langle x^n \rangle_q, \langle x^n \rangle_{\Delta q}, \langle x^n \rangle_{\delta q}$
- Form factors
  - elastic scattering
  - $F_1(Q^2), F_2(Q^2), G_A(Q^2), G_P(Q^2)$

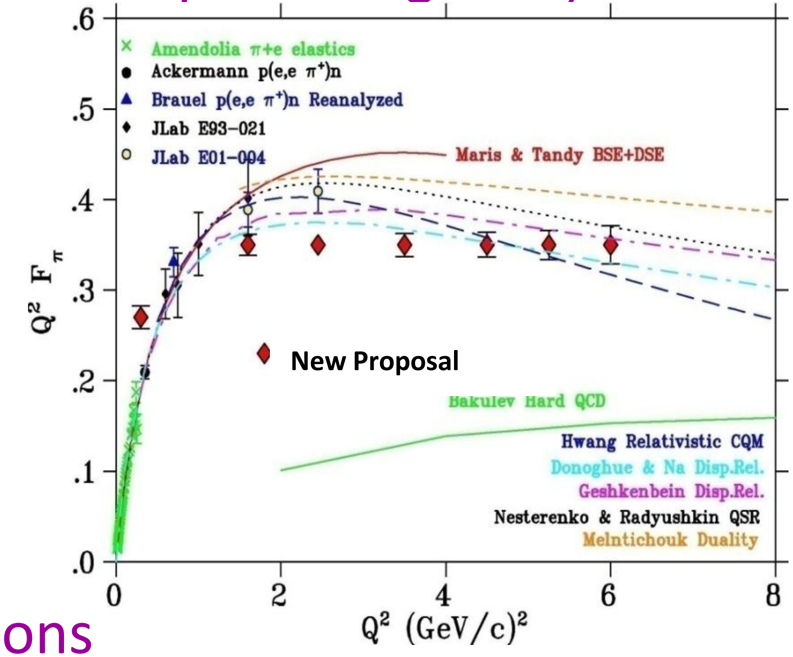




Higher-Q² Form Factors

- Higher-Q^2 data will help us to understand hadrons and challenge QCD-based models
- Experiments are looking...
  (JLab 12-GeV upgrade will provide more promising data)





- Challenge for Lattice QCD calculations
  - Typical Q^2 range for nucleon form factors is < 3.0 GeV²
  - Higher-Q^2 calculations suffer from poor noise-to-signal ratios

Lattice High-Q^2 Form Factors

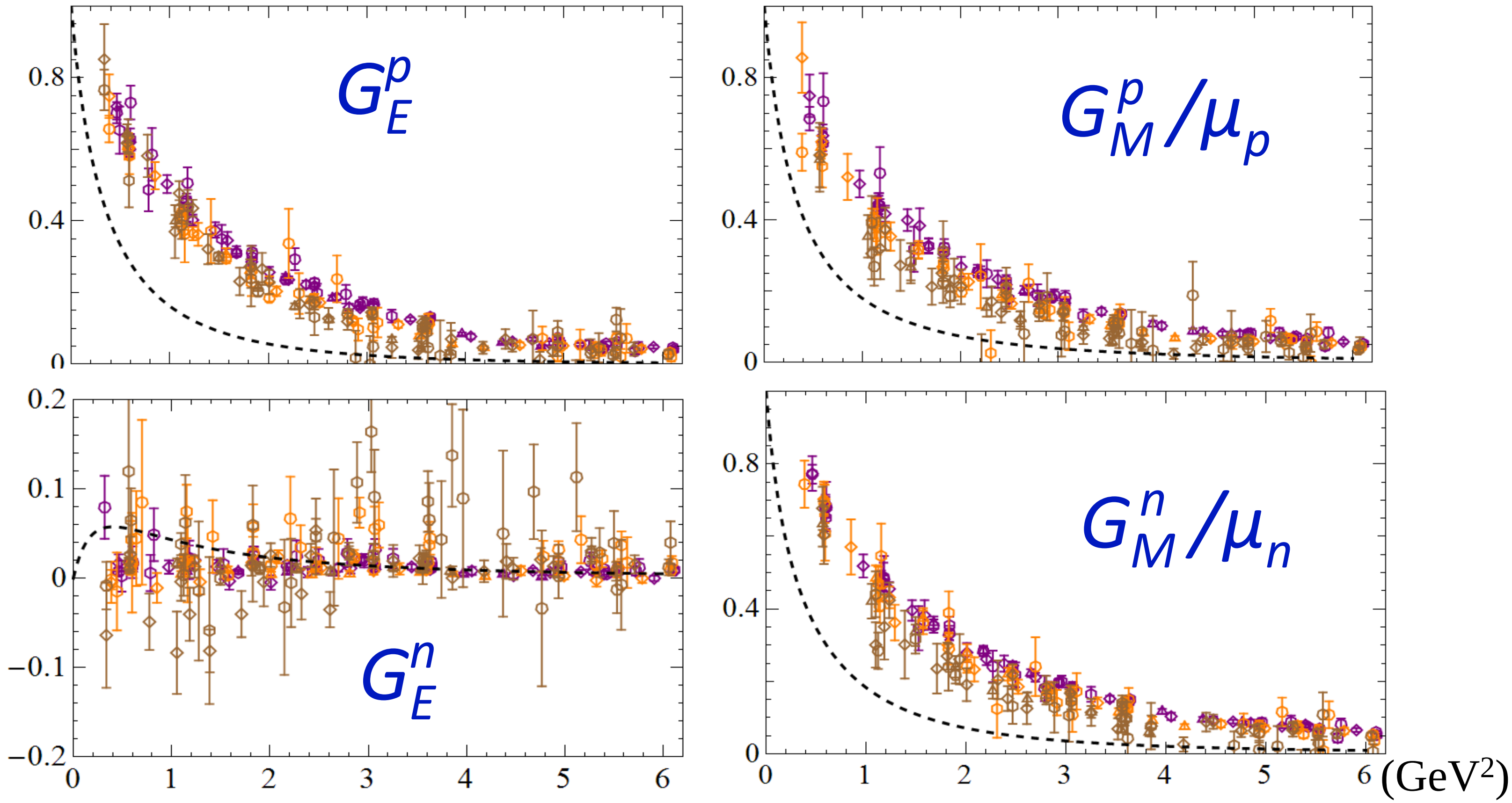
- Problem: traditional approach
  - Start by looking at two-point correlators, masses
  - Tune the parameters and source-sink range to be optimal for the ground state at low momenta
  - Smearings are tuned to eliminate excited-state signals and also all the higher-momentum contributions

- Solution: confront the excited states directly and allow operators to couple to both ground and excited states

- Demonstration: $N_f = 0$ anisotropic lattices using a simple operator with different operator smearings
  - Clean signal for nucleon form factors up to 6 GeV^2
  - N-P_{11} transition form factors

Higher-Q^2 Nucleon Form Factors

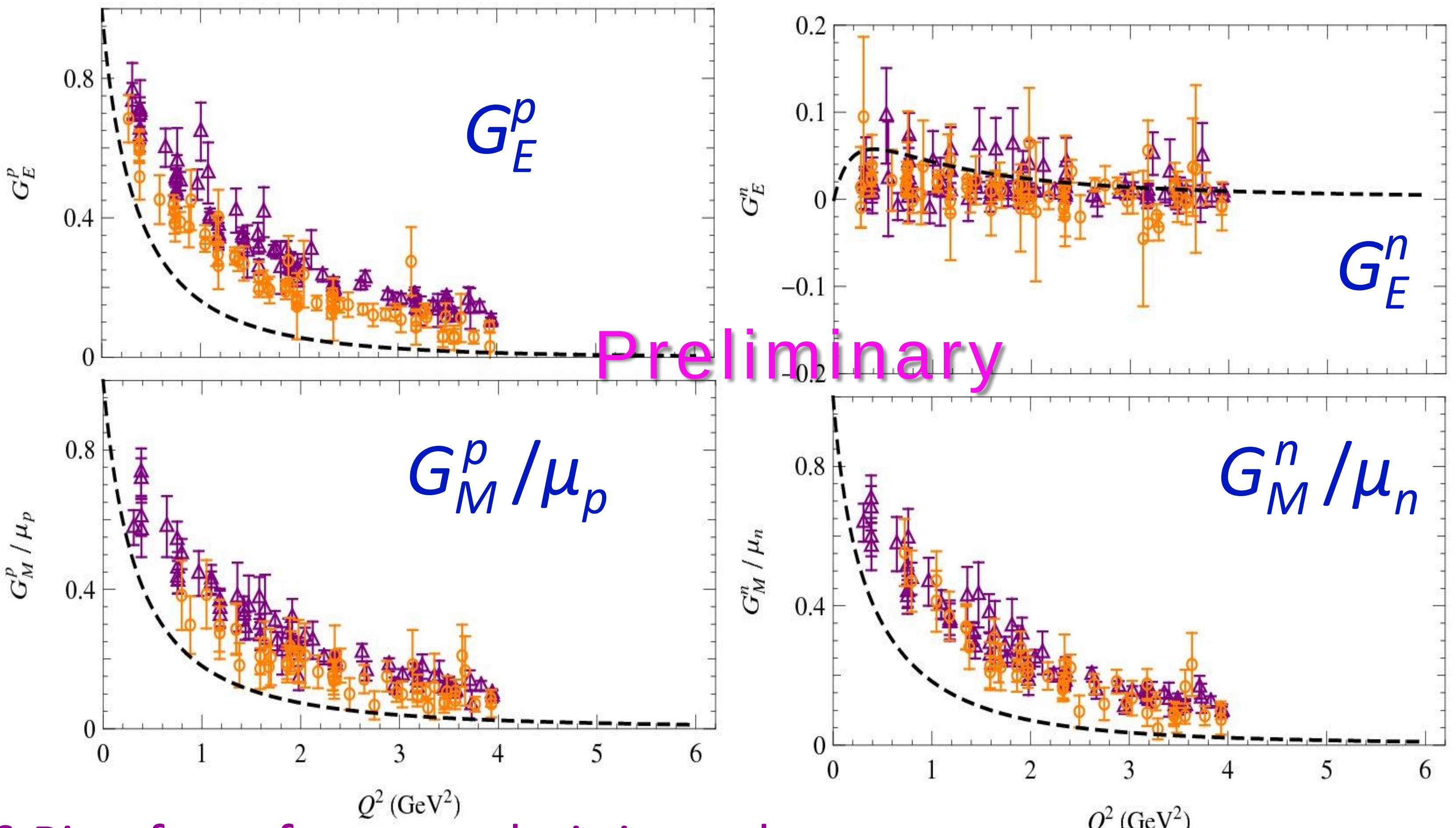
- $N_f = 0$ anisotropic lattices, $M_\pi \approx$ **480**, **720**, **1080** MeV



HWL et al., Phys. Rev. D78, 114508 (2008); HWL et al. [0810.5141]

Higher-Q^2 Nucleon Form Factors

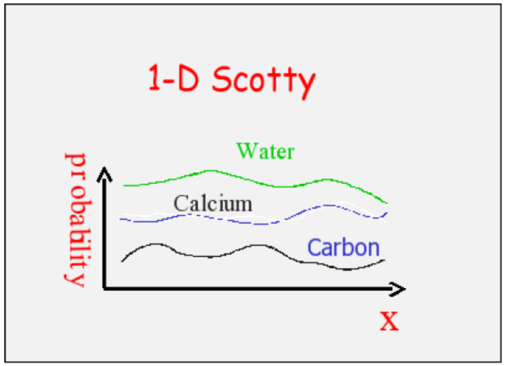
- $N_f = 2+1$ anisotropic lattices, $M_\pi \approx$ **580**, **870** MeV

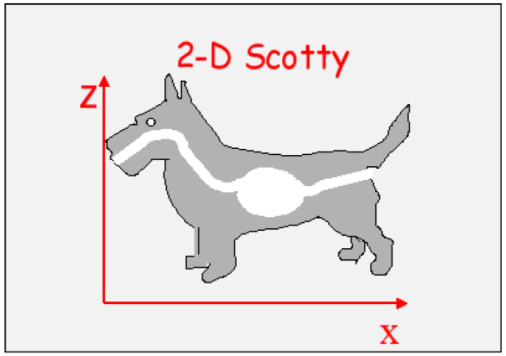


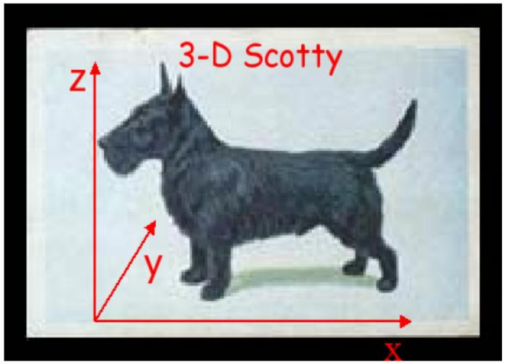
- Pion form-factor analysis is on the way

Generalized Parton Distributions

- Structure function/distribution functions
  - deep inelastic scattering
  - $\langle x^n \rangle_q$, $\langle x^n \rangle_{\Delta q}$, $\langle x^n \rangle_{\delta q}$
- Form factors
  - elastic scattering
  - $F_1(Q^2)$, $F_2(Q^2)$, $G_A(Q^2)$, $G_P(Q^2)$
- Generalized Parton Distribution
  - DVCS
  - $\langle x^{n-1} \rangle_q = A_{n0}(0)$, $\langle x^{n-1} \rangle_{\Delta q} = \tilde{A}_{n0}(0)$, $\langle x^n \rangle_{\delta q} = A_{Tn0}(0)$,
  - $F_1(Q^2) = A_{10}(Q^2)$, $F_2(Q^2) = B_{20}(Q^2)$, $G_A(Q^2) = \tilde{A}_{10}(Q^2)$, $G_P(Q^2) = \tilde{B}_{10}(Q^2)$
  - Nucleon spin







Generalized Parton Distributions

- Cannot directly calculate the matrix element $\langle P', S' | O_\Gamma(x) | P, S \rangle$
- Use operator production expansion (OPE)
- Definition of GFF:

$$\langle P' | \mathcal{O}^{\mu_1} | P \rangle = \langle\langle \gamma^{\mu_1} \rangle\rangle A_{10}(t) + \frac{i}{2m} \langle\langle \sigma^{\mu_1 \alpha} \rangle\rangle \Delta_\alpha B_{10}(t)\,,$$

Only even *n*

$$\langle P' | \mathcal{O}^{\{\mu_1\mu_2\}} | P \rangle = \bar{P}^{\{\mu_1} \langle\langle \gamma^{\mu_2\}} \rangle\rangle A_{20}(t) + \frac{i}{2m} \bar{P}^{\{\mu_1} \langle\langle \sigma^{\mu_2\}\alpha} \rangle\rangle \Delta_\alpha B_{20}(t) + \frac{1}{m} \Delta^{\{\mu_1} \Delta^{\mu_2\}} \boxed{C_{20}(t)\,,}$$

$$\begin{aligned} \langle P' | \mathcal{O}^{\{\mu_1\mu_2\mu_3\}} | P \rangle &= \bar{P}^{\{\mu_1} \bar{P}^{\mu_2} \langle\langle \gamma^{\mu_3\}} \rangle\rangle A_{30}(t) + \frac{i}{2m} \bar{P}^{\{\mu_1} \bar{P}^{\mu_2} \langle\langle \sigma^{\mu_3\}\alpha} \rangle\rangle \Delta_\alpha B_{30}(t) \\ &+ \Delta^{\{\mu_1} \Delta^{\mu_2} \langle\langle \gamma^{\mu_3\}} \rangle\rangle A_{32}(t) + \frac{i}{2m} \Delta^{\{\mu_1} \Delta^{\mu_2} \langle\langle \sigma^{\mu_3\}\alpha} \rangle\rangle \Delta_\alpha B_{32}(t), \end{aligned}$$

with $\overline{P} = (P' + P)/2 \qquad \Delta = P' - P \qquad t = \Delta^2$

- List of GFF:
  - Polarized: $\tilde{A}_{ni}(0)$, $\tilde{B}_{ni}(0)$
  - Unpolarized: $A_{ni}(0)$, $B_{ni}(0)$, $C_n(0)$
  - Transverse (+ pol.): $A_{Tni}(0)$, $B_{Tni}(0)$ ($\tilde{A}_{Tni}(0)$, $\tilde{B}_{Tni}(0)$)

Nucleon Spin

§ Decomposition according to sum rule

Xiang-Dong Ji, Phys. Rev. Lett., 78:610–613, 1997.

$$\frac{1}{2} = \frac{1}{2}\Delta\Sigma^{u+d} + L^{u+d} + J^g$$

quark spin fraction

quark orbital angular momentum

total gluonic contribution

$$\frac{1}{2}\Delta\Sigma^q = \frac{1}{2}\tilde{A}^q_{10}(0)$$

$$J^g = \frac{1}{2} - J^{u+d}$$

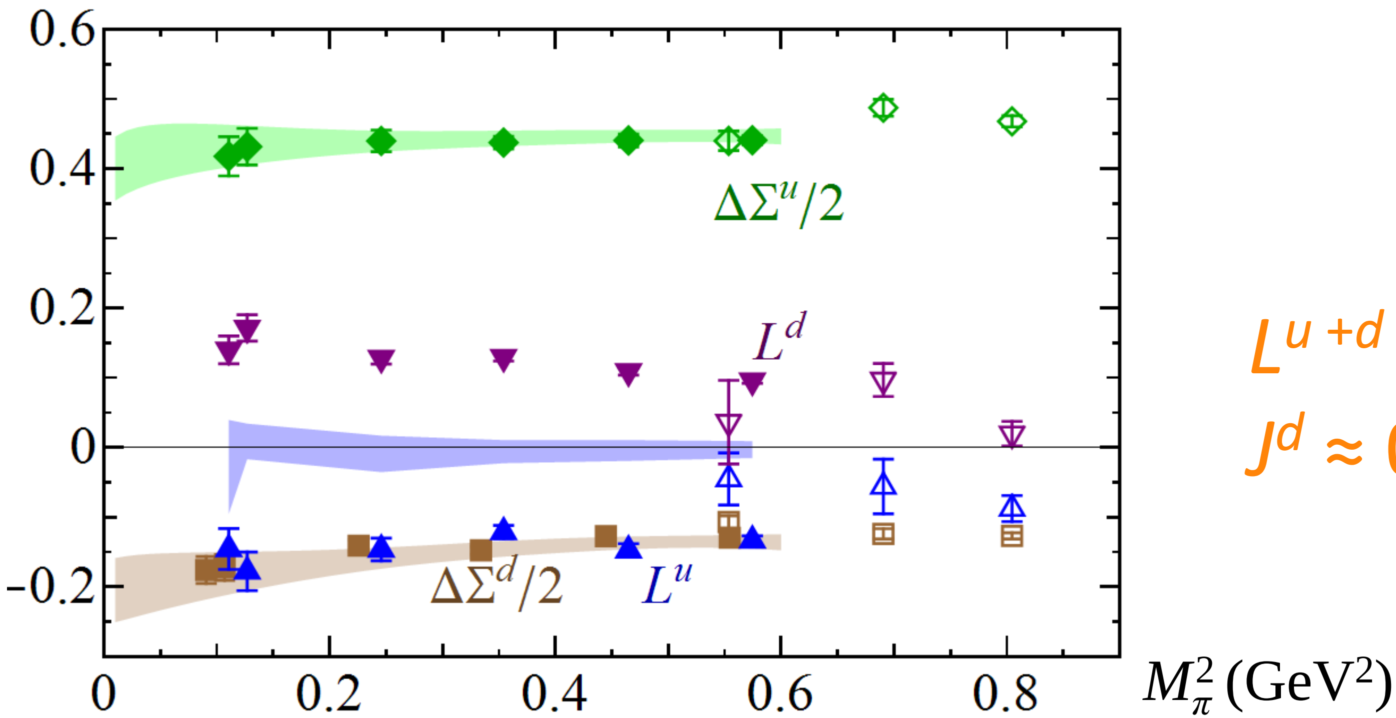
$$L^q = \frac{1}{2}\left[A^q_{20}(0) + B^q_{20}(0)\right] - \frac{1}{2}\Delta\Sigma^q$$

§ List of GFF:

- Polarized: $\tilde{A}_{ni}(0)$, $\tilde{B}_{ni}(0)$
- Unpolarized: $A_{ni}(0)$, $B_{ni}(0)$, $C_n(0)$
- Transverse (+ pol.): $A_{Tni}(0)$, $B_{Tni}(0)$ ($\tilde{A}_{Tni}(0)$, $\tilde{B}_{Tni}(0)$)

Nucleon Spin

- Using GFF and Ji's sum rule — Xiang-Dong Ji, Phys. Rev. Lett., 78:610 (1997)
- Decomposition according to quark flavor:
  - LHPC: $N_f = 2+1$ mixed action, $M_\pi \sim 350–760$ MeV
    Ph. Hagler et. al, Phys. Rev. D77, 094502 (2008).

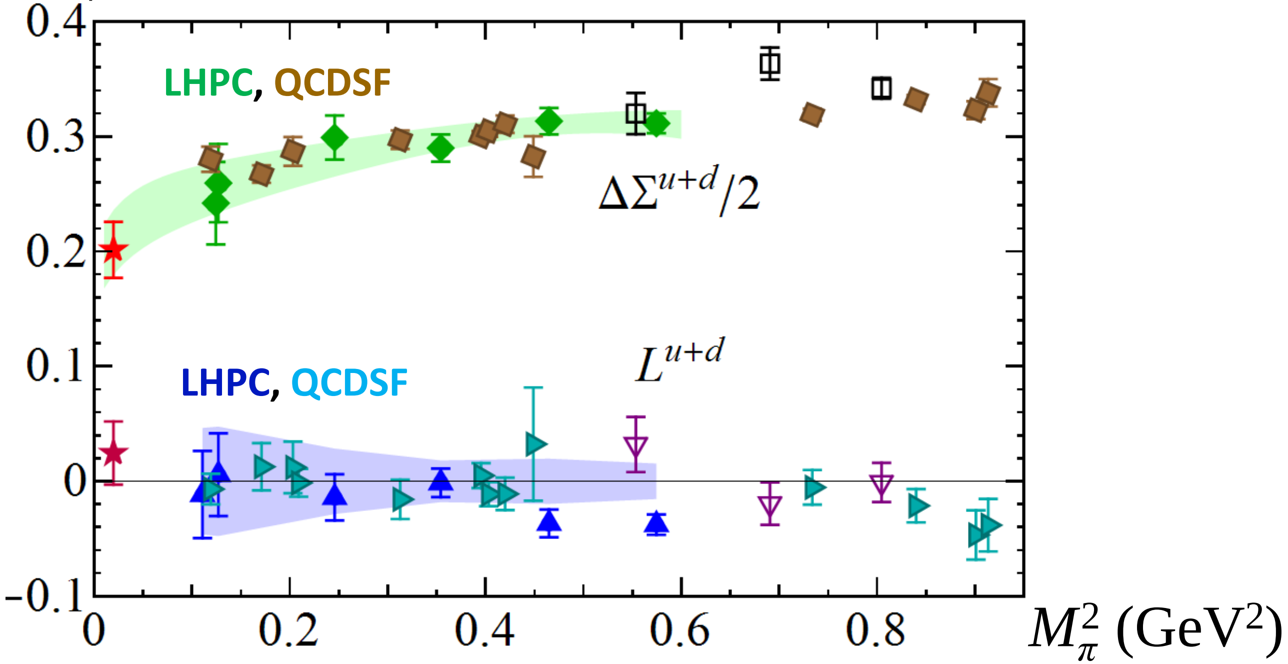


$L^{u+d} \approx 0$

$J^d \approx 0$

Nucleon Spin

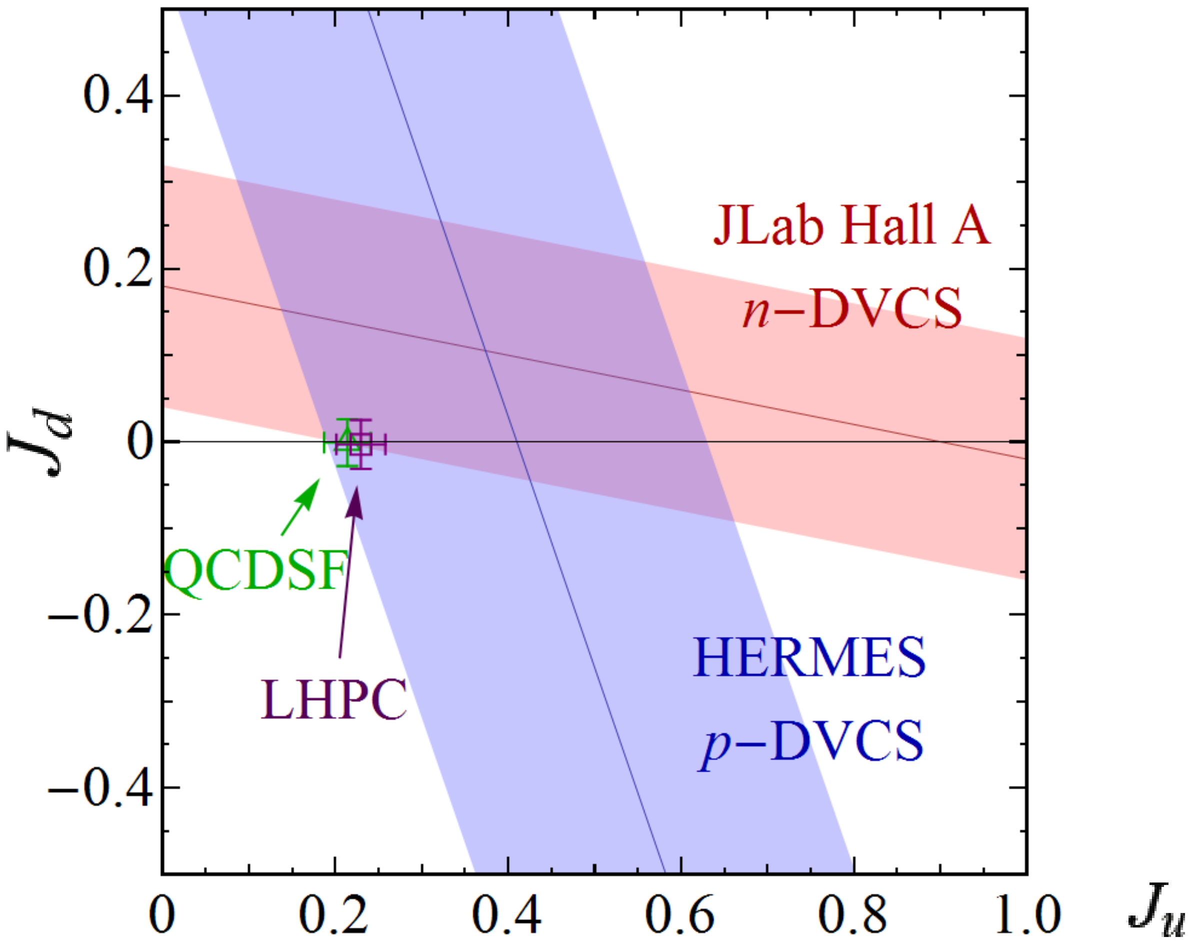
- Using GFF and Ji's sum rule — Xiang-Dong Ji, Phys. Rev. Lett., 78:610 (1997)
- Decomposition according to spin and orbital angular momentum:
  - LHPC: $N_f = 2+1$ mixed action, $M_\pi \sim 350–760$ MeV
  - QCDSF: $N_f = 2$ clover action, $M_\pi \sim 340–950$ MeV



Ph. Hagler et. al, Phys. Rev. D77, 094502 (2008); M. Ohtani et al, PoS (Lat2007) 158

Nucleon Spin

- J_u-J_d plot with experimental bands

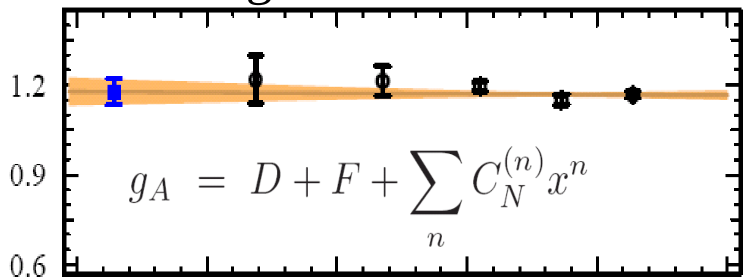


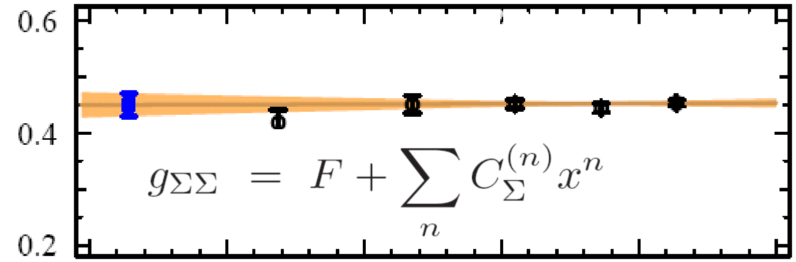
Octet Axial Couplings

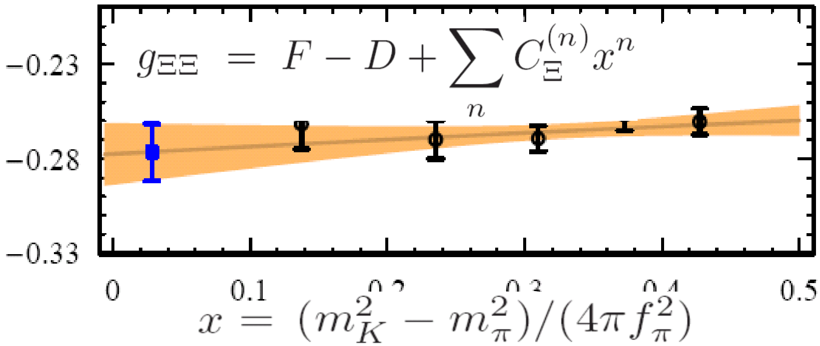
- Full-QCD with pion mass: 350–750 MeV

HWL, K. Orginos, Phys.Rev.D79:034507 (2009)

  - Including first lattice calculation of $g_{\Xi\Xi}$ and $g_{\Sigma\Sigma}$.







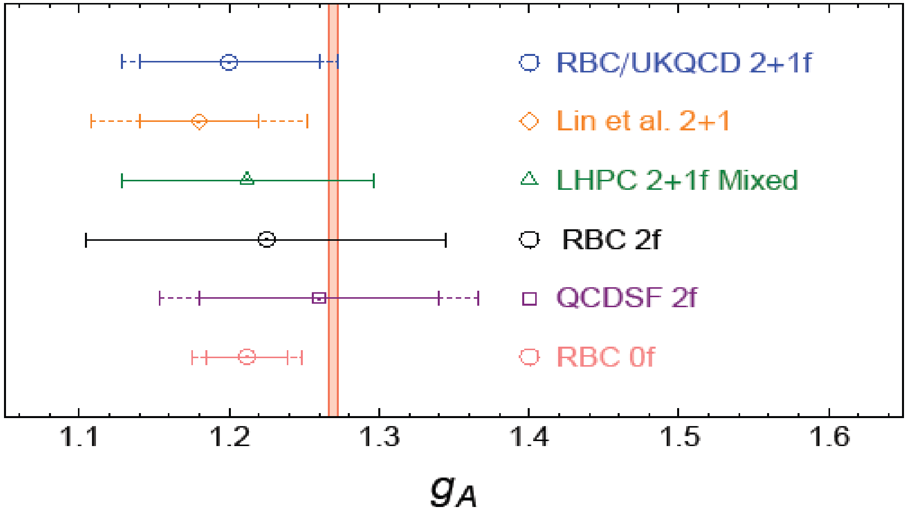
- Systematic errors: finite volume + finite *a*

$$g_A = 1.18(4)_{\rm stat}(6)_{\rm syst}$$
$$g_{\Sigma\Sigma} = 0.450(21)_{\rm stat}(27)_{\rm syst}$$
$$g_{\Xi\Xi} = -0.277(15)_{\rm stat}(19)_{\rm syst}$$

  - XPT: $0.35 < g_{\Sigma\Sigma} < 0.55$; $0.18 < -g_{\Xi\Xi} < 0.36$
  - Large-N_c: $0.30 < g_{\Sigma\Sigma} < 0.36$; $0.26 < -g_{\Xi\Xi} < 0.30$
- Global coupling constants: $D = 0.715(06)(29)$, $F = 0.453(05)(19)$
- Constrained fit for g_A

Octet Axial Couplings

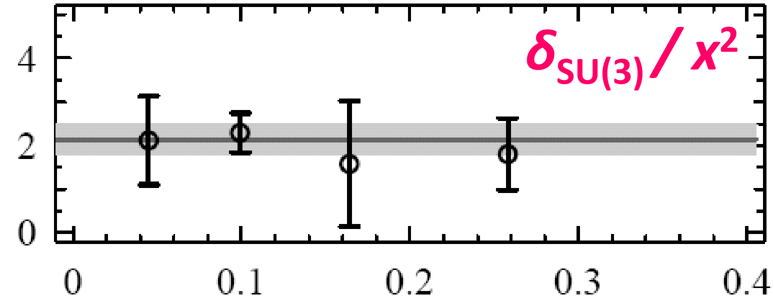
- Most lattice calculations performed on g_A



- SU(3) constrained fit gives Lin et al. smaller extrapolated statistical error than LHPC
- Lighter m_π, finer a, multiple V essential for precise calculation

- SU(3) symmetry breaking

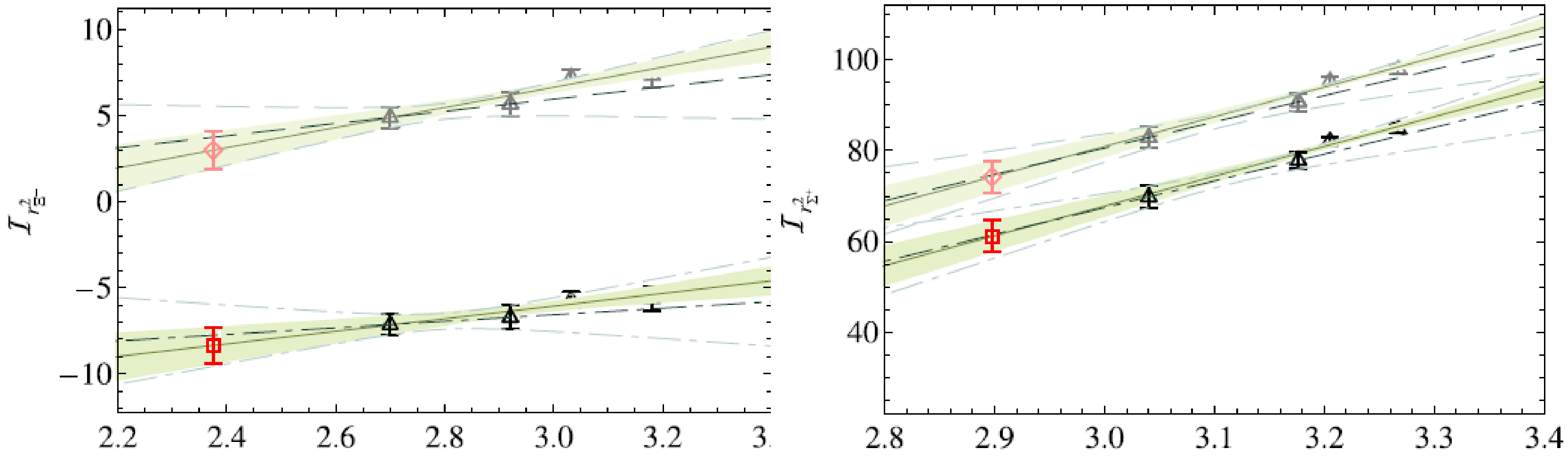
$$\delta_{\mathrm{SU(3)}} = g_{\mathrm{A}} - 2.0 \times g_{\Sigma\Sigma} + g_{\Xi\Xi} = \sum_n c_n x^n$$



- Quadratic behaviour is observed
- Not predicted by any theorem nor chiral perturbation theory ⟶ coincidence?
- 20% breaking at physical point

Hyperon Properties

- Hyperon projects: including
  - Form factors, charge radii, magnetic moments (HWL, K. Orginos, Phys.Rev.D79:074507 (2009))
  - semi-leptonic decays
- Mixed action with pion mass 350–750 MeV
- Predict Ξ^- and Σ^+ charge radii to be 0.67(5) and 0.306(15) fm^2

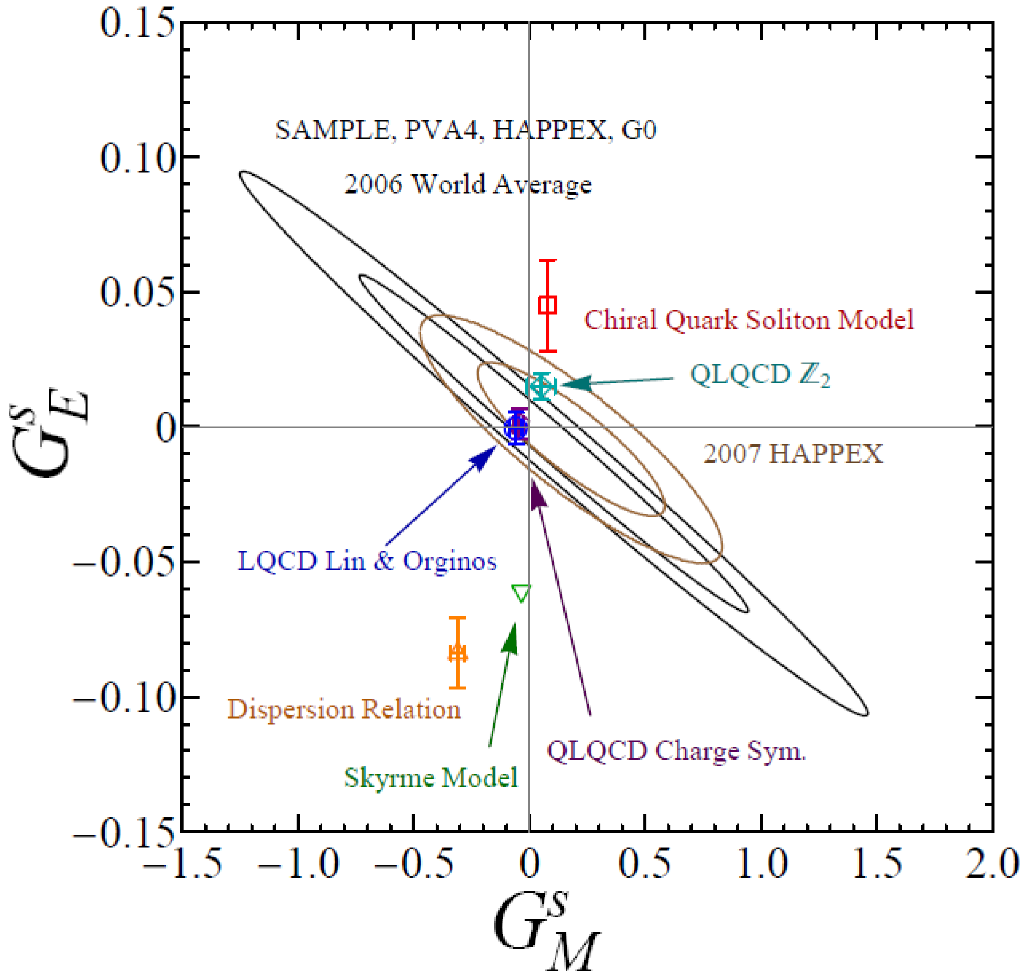


- Future: extending to Λ hyperon and decuplet-octet transition

Strangeness in the Nucleon

§ G_E^s-G_M^s plots

HAPPEX: Phys.Rev.Lett.98:032301 (2007)
SAMPLE, PVA4,HAPPEX,G0: Phys.Rev.Lett. 97, 102002 (2006)



Indirect lattice calculation
D. Leinweber et al., Phys.Rev.Lett.97:022001 (2006); D. Leinweber et al., Phys.Rev.Lett.94:212001 (2005); HWL,0903.4080 [hep-lat] (SPIN 2008)

Recent direct lattice calculation
M. Deka: et al., Phys.Rev.D79:094502 (2009)

{–0.017(25)(07), +0.027(16)(8)} at $Q^2 = 0$

Summary

Exciting era using Lattice QCD for nuclear physics

§ Improvement

- Huge leaps due to increasing computational resources world-wide and improved algorithms

§ Universality

- Different lattice actions/groups with independent calculations provide consistency checks: so far so good…

§ Confidence

- Reproducing well measured experimental values gives us confidence for predicted quantities that haven't/couldn't be measured by experiments

§ Variety

- There are many different aspects of hadron structure; only presented a few examples