The $g_2$ Spin Structure Function

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On Behalf of the E08-027 Collaboration
**Electron Scattering**

- Inclusive **unpolarized** cross section:

\[
\frac{d^2\sigma}{d\Omega dE'} = \sigma_{\text{Mott}} \left[ \frac{1}{\nu} F_2(x, Q^2) + \frac{2}{M} F_1(x, Q^2) \tan^2 \frac{\theta}{2} \right]
\]

Structure Function which indicates the parton distribution

- At Bjorken Limit \( Q^2 \to \infty \):

\[
F_1 = \frac{1}{2} \sum_i e_i^2 q_i(x) \quad F_2 = 2x F_1
\]
Electron Scattering

- If the beam and target are polarized, the asymmetric part of the lepton and hadron tensor will not vanish, which leads to 2 additional structure functions $g_1$ and $g_2$

$$
\frac{d^2 \sigma}{d\Omega dE'} = \sigma_{\text{Mott}} \left[ \frac{1}{\nu} F_2(x, Q^2) + \frac{2}{M} F_1(x, Q^2) \tan^2 \frac{\theta}{2} + \gamma g_1(x, Q^2) + \delta g_2(x, Q^2) \right]
$$

2 additional structure functions which are related to the polarized parton distributions
Structure Function

- At Bjorken limit, $g_1$ related to the polarized parton distribution functions

$$g_1 = \frac{1}{2} \sum_i e_i^2 \Delta q_i(x) \quad \Delta q_i(x) = q_i^\uparrow(x) - q_i^\downarrow(x)$$

- However $g_2$ does no show a simple relation with parton distribution functions at Bjorken limit

- $g_{2}^{WW}$ is the leading twist part of the $g_2$:

$$g_2(x, Q^2) = g_{2}^{WW}(x, Q^2) + \bar{g}_2(x, Q^2)$$

which can be calculated from $g_1$ with the Wandzura-Wilczek relation

$$g_{2}^{WW} = -g_1(x, Q^2) + \int_x^1 \frac{dy}{y} g_1(y, Q^2)$$
Structure Function

• Higher twist components can be expressed as:

\[ \bar{g}_2(x, Q^2) = - \int_x^1 \frac{\partial}{\partial y} \left[ \frac{m_q}{M} h_T(y, Q^2) + \zeta(y, Q^2) \right] \frac{dy}{y} \]

  - quark transverse momentum contribution
  - twist-3 part which arises from quark-gluon interactions

• Will get information about higher twist effect when measuring \( g_2 \)
Measurements of $g_2$ and its Moments

- Measurements of $g_2$ need transversely polarized targets, more difficult experimentally.

- 0th moment (no $x$-weighting): Burkhardt-Cottingham (BC) Sum Rule

$$\int_{0}^{1} g_2(x, Q^2)\,dx = 0$$

- Valid at all $Q^2$.

- 2nd moment ($x^2$ weighting):

  - High $Q^2$ – $d_2$, twist-3 color polarizability, test of lattice QCD.
  - Low $Q^2$ – spin polarizabilities, test of Chiral Perturbation Theory (χPT).
Measurements of $g_2$ and its Moments

- High-intensity electron accelerator
- $E_{\text{max}} = 6$ GeV
- $I_{\text{max}} = 200$ uA
- $\text{Pol}_{\text{max}} = 90\%$
- Upgrading to 12 GeV
Measurements of $g_2$ and its Moments

- SLAC E155x: Only dedicated measurement before JLab, not high precision, wider range of $Q^2$ for moment

- $g_2$ Measurements on the neutron at JLab:
  - E97-103: $W>2$ GeV, $Q^2 \approx 1\text{GeV}^2$, $x \approx 0.2$, study higher twist (published)
  - E99-117: $W>2$ GeV, high $Q^2$ (3–5 GeV$^2$) (published)
  - E94–010: moments at low $Q^2$ (0.1–1 GeV$^2$) (published)
  - E97-110: moments at very low $Q^2$ (0.02–0.3 GeV$^2$) (analysis)
  - E01–012: moments at intermediate $Q^2$ (1–4 GeV$^2$) (submitted)
  - E06–014: moments at high $Q^2$ (2–6 GeV$^2$) (published)

- $g_2$ Measurements on the proton at JLab:
  - RSS: moments at intermediate $Q^2$ (1–2 GeV$^2$) (published)
  - SANE: moments at high $Q^2$ (2–6 GeV$^2$) (analysis)
  - E08–027 (g2p): moments at very low $Q^2$ (0.02–0.2 GeV$^2$) (analysis)
Measurements of $g_2$ and its Moments

- $g_2$ Measurements on the proton:
  - SLAC: $1 \sim 10 \text{ GeV}^2$
  - SANE: $2 \sim 6 \text{ GeV}^2$
  - RSS: $1 \sim 2 \text{ GeV}^2$
BC Sum Rule: 0th Moment

- **BC Sum Rule:**
  \[ \int_0^1 g_2(x, Q^2)dx = 0 \]

- Violation suggested for proton at large \( Q^2 \)

- **BC Sum = Meas + Low x + Elastic**
  - “Meas”: measured \( x \) range (open circle)
  - “Low x”: unmeasured low-\( x \) part of the integral – assume leading twist behavior
  - “Elastic”: from well known Form Factors (<5%)

- Mostly unmeasured

- P
  - g2p projected
  - Mostly unmeasured

- N

- Q^2 (GeV^2)
  - SLAC E155x
  - Hall C RSS
  - Hall A E94-010
  - Hall A E97-110 (preliminary)
  - Hall A E01-012 (preliminary)
Spin Polarizability: 2nd Moment

- Generalized spin polarizabilities $\gamma_0$ and $\delta_{LT}$ are a benchmark test of $\chi$PT
- One difficulty is how to include the nucleon resonance contributions
  - $\gamma_0$ is sensitive to resonances, $\delta_{LT}$ is not

\[ \gamma_0(Q^2) = \frac{16\alpha M^2}{Q^6} \int_0^{x_0} x^2 [g_1 - \frac{4M^2}{Q^2} x^2 g_2] dx \]

\[ \delta_{LT}(Q^2) = \frac{16\alpha M^2}{Q^6} \int_0^{x_0} x^2 [g_1 + g_2] dx \]
Spin Polarizability: 2nd Moment

- $\delta_{LT}$ is seen as a more suitable testing ground of $\chi$PT – insensitive to $\Delta$ resonance
- Significant disagreement between data and both $\chi$PT calculations
- No proton data yet

Plots by V. Sulkosky
**d₂ and Higher Twist**

\[ d_2(Q^2) = \int_0^1 x^2 [2g_1(x, Q^2) + 3g_2(x, Q^2)] dx \]

\[ = 3 \int_0^1 x^2 [g_2(x, Q^2) - g^{WW}_2(x, Q^2)] dx \]

- Clean access of higher twist (twist-3) effect
- Only contributions from measured region
- Elastic not included, only important for \( Q^2 < 2\text{GeV}^2 \)
- Contributions from unmeasured low \( x \) region usually not significant due to \( x^2 \) weighting.
- A benchmark test of Lattice QCD predictions at high \( Q^2 \)

![Plot](image-url)

Plots by K. Slifer
g2p Experiment at JLab

- First Measurement of the proton structure function $g_2$ in the low $Q^2$ region (0.02–0.2 GeV$^2$)

- Extract spin polarizability $\delta_{LT}$ as a test of $\chi$PT calculations

- Test BC Sum Rule

- Finite size effects:
  - Hydrogen hyperfine splitting: proton structure contributes to uncertainty
  - Proton charge radius: proton polarizability contributes to uncertainty

- Data were taken in Jefferson Lab Hall A in 2012

- Analysis is currently underway
g2p Collaboration

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How to get $g_2$

\[ \Delta \sigma_\parallel = \begin{array}{c} \text{e}^- \rightarrow \bullet \rightarrow - \text{e}^- \rightarrow \bullet \rightarrow \\ \end{array} \]

\[ = \frac{d^2 \sigma_{\uparrow \uparrow}}{d\Omega dE'} - \frac{d^2 \sigma_{\downarrow \uparrow}}{d\Omega dE'} \]

\[ = \frac{4\alpha^2 E'}{M_\nu Q^2 E} \left[ (E + E' \cos \theta) g_1 - 2M x g_2 \right] \]

JLab Hall B experiment EG4 measured this quantity

\[ \Delta \sigma_\perp = \begin{array}{c} \text{e}^- \rightarrow \bullet \uparrow \rightarrow - \text{e}^- \rightarrow \bullet \uparrow \rightarrow \\ \end{array} \]

\[ = \frac{d^2 \sigma_{\uparrow \Rightarrow}}{d\Omega dE'} - \frac{d^2 \sigma_{\downarrow \Rightarrow}}{d\Omega dE'} \]

\[ = \frac{4\alpha^2 E'^2}{M_\nu Q^2 E} \sin \theta \left[ g_1 + \frac{2E}{\nu} g_2 \right] \]

g_2^p experiment will measure this, combining the EG4 data to get $g_2^p$ at low $Q^2$
Experiment Setup

- Major New Installation in Hall A
  - Polarized NH₃ Target with 2.5/5T magnetic field
  - Low current (<100nA) beam line diagnostics
  - Septa magnets

Beam diagnostics:

- Polarized NH₃ Target
- Septa
- Local Dump
- BPM
- Chicane
- Rasters
- BCM

Hall A High Resolution Spectrometer (HRS)
Experiment Setup

- Polarized NH$_3$ Target
- Dynamic nuclear polarization
- Target polarization measured via NMR

Target Polarization Results for 5T Field Setting

- Average Polarization:
  - 2.5 T: ~ 15%
  - 5.0 T: ~ 70%
Experiment Setup

- HRS Detector package
- Vertical Drift Chamber (VDC)
- Particle identification (PID) Detectors
  - High Efficiency (>99%) for gas Cherenkov and lead glass calorimeters

Gas Cherenkov Efficiency

Legend:
- 2.2 GeV, 2.5T, 90deg
- 1.7 GeV, 2.5T, 90deg
- 1.2 GeV, 2.5T, 90deg
- 1.2 GeV, 2.5T, 90deg (short cell)
- 2.2 GeV, 5.0T, 90deg
- 2.2 GeV, 5.0T, 0deg
- 3.3 GeV, 5.0T, 90deg
Kinematics Coverage

\[ M_p < W < 2 \text{ GeV} \]
\[ 0.02 < Q^2 < 0.2 \text{ GeV}^2 \]
\[ \delta_{LT}(Q^2) = \frac{16\alpha M^2}{Q^6} \int_0^{x_0} x^2 [g_1 + g_2] dx \]
\[ \int_0^1 g_2(x, Q^2) dx = 0 \]
Analysis Status

• Completed
  • Run Database
  • HRS Optics
    • Field measurement analysis
    • VDC $t_0$ calibration
    • Simulation package
    • Optics with target field (LHRS)
  • Detector Calibrations/Efficiency Studies
    • Gas Cherenkov
    • Lead Glass Calorimeters
    • Scintillator trigger efficiencies
  • Scalers

• BCM calibration
• Helicity decoding
• Dead time calculations
• Target Polarization Analysis
• BPM Calibrations

• In Progress
  • Raster Size Calibrations
  • Packing Fraction/Dilution Analysis
  • Elastic Analysis
  • Yields/Radiative Corrections
Preliminary Results

Asymmetry

\[ \Delta \sigma_\perp = \sigma_{\text{total}} \cdot A_\perp \]
Conclusion of g2p

- g2p experiment will provide first measurement of the proton structure function $g_2$ in the low $Q^2$ region (0.02–0.2 GeV$^2$)

- The result will provide insight on several outstanding physics puzzles:
  - Spin polarizability $\delta_{LT}$ discrepancy seen for neutron data
  - BC Sum Rule violation suggested for proton at large $Q^2$
  - Contribute to the uncertainty of some finite size effects like hydrogen hyperfine splitting and proton charge radius puzzle
Future Experiments

- JLab at 12 GeV
  - Hall A
    - E12–06–122: A1n in valence quark region (8.8 and 6.6 GeV)
  - Hall B
    - E12–06–109: longitudinal spin structure of the nucleon
  - Hall C
    - E12–06–110: A1n in valence quark region (11 GeV)
    - E12–06–121: g2n and d2n at high $Q^2$

![Graph showing $d^2_n$ with SHMS/HMS](image)

![Graph showing $d^2$](image)

Figure 13: $x^2g^2_n$ vs. $x$, presenting the statistical errors expected from the proposal measurement (colored circles). Existing world data are also shown.

Note: The points associated with the present measurement are distributed along different horizontal lines, each representing a common $<Q^2>$ value. This is in marked contrast to the existing world data for $g^2_n$ for $Q^2 > 1$ GeV$^2$ which were measured over $Q^2$ values ranging from 1—15 GeV$^2$ and were "evolved" to a common $Q^2$ prior to computing $d^2_n$.

Figure 14: $\bar{d}^2_n(Q^2)$ without the nucleon elastic contribution are presented with the estimated statistical errors for the proposed measurement. The SLAC E155 [38] neutron result is also shown here (open square). The solid line is the MAID calculation [55] while the dashed line is a HB χPT calculation [56] valid only at very low $Q^2$. The lattice prediction [51] at $Q^2 = 5$ GeV$^2$ for the neutron $d^2_n$ reduced matrix element is negative but consistent with zero. We note that all models shown in Fig. 3 predict a negative value or zero at large $Q^2$ where the elastic contribution is negligible. At moderate $Q^2$ the data show a positive $\bar{d}^2_n$, indicating a low decrease with $Q^2$. The combined SLAC + JLab data shows a positive $d^2_n$ value but with still a large error bar.
Thanks
Backups
Electron Scattering

- Important kinematics variables:
  - $\nu = E - E'$
  - $Q^2$: Momentum transfer squared
  - $W$: Invariant mass of residual hadronic system
  - $x = \frac{Q^2}{2M\nu}$: Bjorken variable: fraction momentum of struck quark
Structure Function

• “twist” in Operator Production Expansion

\[ T_{\mu\nu}(P, q) = i \int d^4 z \exp(iq \cdot z) \langle N(P) | T(j_\mu(z)j_\nu(0)) | N(P) \rangle \]
\[ = \sum_{n=\text{even}} \langle N(P) | O_n^{\mu_1} \cdots \mu_n | N(P) \rangle \frac{2^n}{(Q^2)^n} \left( P_{\mu\nu}^{(L)} C_n^{(L)}(Q^2) q_{\mu_1} \cdots q_{\mu_n} \right) \]
\[ + \left[ -q^2 g_{\mu_1 \mu_2} g_{\mu_2 \nu} + [g_{\mu_1 q_{\mu_2} q_{\nu} + g_{\mu_2 q_{\nu} q_{\mu_1} q_{\mu_2}}] - g_{\mu\nu} q_{\mu_1} q_{\mu_2} \right] \times C_n^{(2)}(Q^2) q_{\mu_3} \cdots q_{\mu_n} \]
\[ (5.125) \]

Structure of Nucleon, eq 5.125

• quark–quark and quark–gluon correlation
Proton Polarizability

- Proton electric and magnetic polarizabilities: response to low-frequency, long-wavelength electromagnetic fields
- From the dispersion relation of the real Compton scattering (RCS) amplitude, one could derive electric and magnetic polarizability and forward spin spin polarizability

\[
\alpha + \beta = \frac{1}{2\pi^2} \int_{\nu_0}^{\infty} \frac{\sigma_T}{\nu'} d\nu' \\
\gamma_0 = -\frac{1}{4\pi^2} \int_{\nu_0}^{\infty} \frac{\sigma_{TT}}{\nu'^3} d\nu'
\]

Electric and magnetic polarizability
\[
\sigma_T = \frac{1}{2}(\sigma_{1/2} + \sigma_{3/2})
\]

Forward spin polarizability
\[
\sigma_{TT} = \frac{1}{2}(\sigma_{1/2} - \sigma_{3/2})
\]
Generalized Longitudinal-Transverse Polarizability

• Start from forward spin-flip doubly-virtual Compton scattering (VVCS) amplitude $g_{TT}$ and $g_{LT}$

$$\text{Re}[g_{TT}^{\text{non-pole}}(\nu, Q^2)] = \frac{\nu}{2\pi^2} \mathcal{P} \int_{\nu}^{\infty} \frac{d\nu' K}{\nu'^2 - \nu^2} \sigma_{TT}(\nu', Q^2)$$

$$\text{Re}[g_{LT}^{\text{non-pole}}(\nu, Q^2)] = \frac{1}{2\pi^2} \mathcal{P} \int_{\nu}^{\infty} \frac{d\nu' \nu' K}{\nu'^2 - \nu^2} \sigma_{LT}(\nu', Q^2)$$

• $g_{TT}$ and $g_{LT}$ can be expanded in power series of $\nu$

$O(\nu^3)$ term of $g_{TT}$ leads to the generalized forward spin polarizability $\gamma_0$:

$$\gamma_0(Q^2) = \frac{1}{2\pi^2} \int_{\nu}^{\infty} \frac{K(\nu, Q^2)}{\nu} \frac{\sigma_{TT}(\nu, Q^2)}{\nu^3} d\nu$$

$$= \frac{16\alpha M^2}{Q^6} \int_0^{x_0} x^2 [g_1 - \frac{4M^2}{Q^2} x^2 g_2] dx$$

$O(\nu^2)$ term of $g_{LT}$ leads to the generalized longitudinal-transverse polarizability $\delta_{LT}$:

$$\delta_{LT}(Q^2) = \frac{1}{2\pi^2} \int_{\nu}^{\infty} \frac{K(\nu, Q^2)}{\nu} \frac{\sigma_{LT}(\nu, Q^2)}{Q\nu^2} d\nu$$

$$= \frac{16\alpha M^2}{Q^6} \int_0^{x_0} x^2 [g_1 + g_2] dx$$
δ_{LT} puzzle

• At low $Q^2$, the generalized polarizabilities have been evaluated with NLO $\chi$PT calculations:


• One issue in the calculation is how to properly include the nucleon resonance contributions, especially the $\Delta$ resonance

  • $\gamma_0$ is sensitive to resonances

  • $\delta_{LT}$ is insensitive to the $\Delta$ resonance

• $\delta_{LT}$ should be more suitable than $\gamma_0$ to serve as a testing ground for the chiral dynamics of QCD
δ_{LT} puzzle

Kochelev’s new calculation result:
- Include the axial-anomaly $a_1(1260)$ meson contribution
- Improves agreement with neutron

FIG. 3. (a) The generalized longitudinal-transverse polarizability of the neutron $\delta_{LT}^n$ and (b) of the proton $\delta_{LT}^p$. Same as Fig. 2 but with the result of Lorentz-invariant $\chi$PT of Ref. [2].

Kochelev & Oh. arXiv:1103.4892
Hydrogen Hyperfine Structure

- Hydrogen hyperfine splitting in the ground state has been measured to a relative high accuracy of 10^\(-13\).

\[ \Delta E = 1420.4057517667(9) \text{MHz} \]

\[ = (1 + \delta)E_F \]

\[ \delta = (\delta_{\text{QED}} + \delta_R + \delta_{\text{small}}) + \Delta_S \]

- \( \Delta_S \) is the proton structure correction and has the largest uncertainty

\[ \Delta S = \Delta_Z + \Delta_{\text{pol}} \]

- \( \Delta_Z \) can be determined from elastic scattering, which is \(-41.0 \pm 0.5 \times 10^2\).

- \( \Delta_{\text{pol}} \) involves contributions of the inelastic part (excited state), and can be extracted to 2 terms corresponding to 2 different spin-dependent structure function of proton.
Hydrogen Hyperfine Structure

\[ \Delta_{\text{pol}} = \frac{\alpha m_e}{\pi g_p m_p} (\Delta_1 + \Delta_2) \]

\[ \Delta_2 = -24m_p^2 \int_0^\infty \frac{dQ^2}{Q^4} B_2(Q^2) \]

\[ B_2(Q^2) = \int_0^{x_{th}} dx \beta_2(\tau) g_2(x, Q^2) \]

\[ \beta_2(\tau) = 1 + 2\tau - 2\sqrt{\tau(\tau + 1)} \]

- \( B_2 \) is dominated by low Q2 part
- \( g_2^p \) is unknown in this region, so there may be huge error when calculating \( \Delta_2 \)
- This experiment will provide a constraint

Nazaryan, Carlson, Griffieon, PRL, 96(2006)163001
• The finite size of the nucleus plays a small but significant role in atomic energy levels
• Simplest: proton
• 2 ways to measure:
  • energy splitting of the $2S_{1/2}-2P_{1/2}$ level (Lamb shift)
  • scattering experiment
• The result do not match when using muonic hydrogen
  • $<R_p> = 0.84184\pm0.00067$ fm by Lamb shift in muonic hydrogen
  • $<R_p> = 0.87680\pm0.0069$ fm CODATA world average

Experiment Setup

- Chicane and Local Dump
  - Outgoing beam will be tilted by the large target field
  - Use Chicane to provide an incident angle
  - Use local dump to stop non-straight beam
Experiment Setup

- Septa magnets

- Detector package has a minimum angle limit at 12.5°

- Use septa magnets to bend 5.6° scattered electrons to 12.5° to allow access to the lowest possible $Q^2$
Experiment Setup

- Hall A High Resolution Spectrometer
  - High momentum resolution: $10^{-4}$ level over a range of 0.8–4.0 GeV/c
  - High momentum acceptance: $|\delta p/p| < 4.5\%$
  - Wide range of angular settings: 12.5°–150° for left arm, 12.5°–130° for right arm
  - Angular acceptance: ±30 mrad (Horizontal) and ±60 mrad (Vertical)
Analysis Status

Raw Data
- Run Info DB
- Detector Calibration
- Scaler Info
- Target Info
  - Optics
  - Beam Info
Cooked Data
- PID Cuts
- Efficiencies
- Deadtime
Selected Events
- Acceptance Cuts
Raw Results
- Radiative Correction
- Dilution Factors
Physic Results
- Target Polarization
- Acceptance
HRS Optics: Overview

- HRS has a series of magnets
- 3 quadrupoles to focus and 1 dipole to disperse on momentums
- Optics study will provide a matrix to transform VDC readouts to kinematics variables which represents the effects of these magnets

\[
\begin{pmatrix}
\delta \\
\theta \\
y \\
\phi
\end{pmatrix}_{tg} =
\begin{pmatrix}
\langle \delta | x \rangle & \langle \delta | \theta \rangle \\
\langle \theta | x \rangle & \langle \theta | \theta \rangle \\
\langle y | y \rangle & \langle y | \phi \rangle \\
\langle \phi | y \rangle & \langle \phi | \phi \rangle
\end{pmatrix}
\begin{pmatrix}
x \\
\theta \\
y \\
\phi
\end{pmatrix}
\]
Optics for g2p

- Septa magnet
- Target magnetic field
- Optics matrix will cover septa magnet
- Target magnetic field will break the focusing nature of the spectrometer so more difficult

\[
\begin{pmatrix}
\delta \\
\theta \\
y \\
\phi
\end{pmatrix}_t g = \\
\begin{pmatrix}
\langle \delta | x \rangle & \langle \delta | \theta \rangle \\
\langle \theta | x \rangle & \langle \theta | \theta \rangle \\
\langle y | y \rangle & \langle y | \phi \rangle \\
\langle \phi | y \rangle & \langle \phi | \phi \rangle
\end{pmatrix}
\begin{pmatrix}
x \\
\theta \\
y \\
\phi
\end{pmatrix}
\]
Optics Goal

- The g2p experiment will measure the proton structure function $g_2$ in the low $Q^2$ region (0.02–0.2 GeV$^2$) for the first time
- Goal: 5% systematic uncertainty when measuring cross section
- Optics Goal:
  - <1.0% systematic uncertainty of scattering angle, which will contribute <4.0% to the uncertainty of cross section
    \[
    \sigma \sim \frac{1}{\sin^4(\theta/2)}
    \]
  - Momentum uncertainty is not as sensitive, but it is not hard to reach $10^{-4}$ level
Angle Calibration

- Determine the center scattering angle
- Survey: ~1mrad
- Idea: Use elastic scattering on different target materials
  \[ \Delta E' = \frac{E}{1 + \frac{E}{M_1}(1 - \cos \theta)} - \frac{E}{1 + \frac{E}{M_2}(1 - \cos \theta)} \]
- Data taking: Carbon foil in LHe, or CH\textsubscript{2} foil
- Two elastic peak took at the same time
- The accuracy to determine this difference is <50KeV -> <0.5mrad
Matrix Calibration

• Calibrate the angle and momentum matrix elements:
  • Use carbon foil target and point beam
  • Use sieve slit to get the real scattering angle from geometry
  • Angle: Fit with data which we already know the real scattering angle
  • Momentum: Use the real scattering angle to calculate elastic scattering momentum of carbon target

**Figure A-8:** Sieve Pattern Reconstruction.

Each settings were calculated using magnet field readouts from dipoles.

**Figure A-9** shows the effect of this $dp_{\text{kin}}$ correction in the waterfall target elastic scattering. The hydrogen elastic peak after the correction can finally be clearly identified. Of course, this method is only valid for elastic scattering from known targets.
Matrix Calibration: Angle

LHRS

Before Calibration

After Calibration

Resolution: 1.4 mrad (RMS)
Matrix Calibration: Angle

Before Calibration

After Calibration

Resolution: 1.6 mrad (RMS)
Matrix Calibration: Momentum

LHRS

Before Calibration

After Calibration

RMS: 1.4x10^{-4}
Matrix Calibration: Momentum

Before Calibration

After Calibration

RMS: $1.7 \times 10^{-4}$
Optics Study with Target Field

• To include target field
  • Normal sieve slit method is not useful
• Idea: separate reconstruction process to 2 parts:
  • Use HRS optics matrix to do the reconstruction from VDC to sieve slit
  • Use the target field map to do a ray trace of the scattered particle from sieve slit to target
Optics Study with Target Field

- Use carbon foil target and point beam
- Sieve pattern is decided by both the beam position and the reconstructed angle
- Directly use BPM readout to provide beam position here
Optics Study with Target Field

- Compare reconstructed target theta and phi angle with the calculated result

![Calculated theta and phi](image1)

![Reconstructed theta and phi](image2)