A Unified Treatment of the Nucleon, Delta and Roper Resonance

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Understanding hadron structure (& QCD) means charting and computing the distribution of matter and energy within hadrons and nuclei.

- mapping correlations and exposing their influence are the hallmark of nuclear physics
- but a priori have no idea what QCD can produce

Solving QCD will have profound implications.

- it will explain how massless gluons and light quarks bind together to form hadrons & thereby explain the origin of $\sim 98\%$ of the mass in the visible universe
- however given QCD’s complexity, the best promise for progress is a strong interplay between experiment and theory

A key pathway is to exploit opportunities provided by new data on nucleon elastic and transition form factors

- help chart the infrared evolution of QCD’s coupling and dressed-masses
- reveal correlations that are key to nucleon structure e.g. diquarks
Discover the meaning of confinement and its relation to dynamical chiral symmetry breaking

– origin of visible mass –
Nucleon Electromagnetic Form Factors

Nucleon electromagnetic current

\[ \langle J^\mu \rangle = \bar{u}(p') \left[ \gamma^\mu F_1(Q^2) + \frac{i\sigma^{\mu\nu}q_\nu}{2M} F_2(Q^2) \right] u(p) \]

- **Dirac**
- **Pauli**

Provide vital information on the distribution of charge and magnetization within the most basic element of nuclear physics

- form factors also directly probe confinement at all energy scales

Today accurate form factor measurements are creating a paradigm shift in our understanding of nucleon structure:

- proton radius puzzle
- \( \mu_p G_{Ep}/G_{Mp} \) ratio and a possible zero-crossing
- flavour decomposition and evidence for diquark correlations
- meson-cloud effects
- seeking verification of perturbative QCD scaling predictions & scaling violations
**Nucleon Sachs Form Factors**

- Experiment gives Sachs form factors:

\[ G_E = F_1 - \frac{Q^2}{4M^2} F_2 \]
\[ G_M = F_1 + F_2 \]

- Until the late 90s Rosenbluth separation experiments found that the \( \mu_p G_{Ep}/G_{Mp} \) ratio was flat

- Polarization transfer experiments completely altered our picture of nucleon structure

  - distribution of charge and magnetization are not the same

- Proton charge radius puzzle [7σ]

\[ r_{Ep} = 0.84087 \pm 0.00039 \text{ fm} \]

- muonic hydrogen [Pohl et al. (2010)]

\[ \langle r_E^2 \rangle = -6 \frac{\partial}{\partial Q^2} G_E(Q^2) \bigg|_{Q^2=0} \]

\[ r_{Ep} = 0.8775 \pm 0.0051 \text{ fm} \]

CODATA: \( ep + e \)-hydrogen

- one of the most interesting puzzles in hadron physics

- so far defies explanation
Form Factors in Conformal Limit ($Q^2 \to \infty$)

- At asymptotic energies hadron form factors factorize into *parton distribution amplitudes* & a hard scattering kernel [Farrar, Jackson; Lepage, Brodsky]

- only the valence Fock state ($\bar{q}q$ or $qqq$) can contribute as $Q^2 \to \infty$

- both confinement and asymptotic freedom in QCD are important in this limit

- Most is known about $\bar{q}q$ bound states, e.g., for the pion:

\[
Q^2 F_\pi(Q^2) \to 16\pi f_\pi^2 \alpha_s(Q^2)
\]

- For the nucleon, normalization is not known

\[
G_{E,M}(Q^2 \to \infty) \propto \alpha_s^2(Q^2)/Q^4
\]

- orbital angular momentum effects approach

**Gluons play a critical role – formalism must reflex this!**
The equations of motion of QCD \iff QCD’s Dyson–Schwinger equations:
- an infinite tower of coupled integral equations
- tractability \implies must implement a symmetry preserving truncation

The most important DSE is QCD’s gap equation \implies quark propagator

\[ S(p) = \frac{Z(p^2)}{i\not{p} + M(p^2)} \]

- \( S(p) \) has correct perturbative limit
- mass function, \( M(p^2) \), exhibits dynamical mass generation
- complex conjugate poles
  - no real mass shell \implies confinement

A robust description of the nucleon as a bound state of 3 dressed-quarks can only be obtained within an approach that respects Poincaré covariance.

Such a framework is provided by the Poincaré covariant Faddeev equation:

\[ \sum \text{all possible interactions between three dressed-quarks} \]

\[ \text{much of 3-body interaction can be absorbed into renormalized 2-body interactions} \]

\[ \text{Faddeev eq. has solutions at discrete values of } p^2 \ (= M^2) \implies \text{baryon spectrum} \]

A prediction of these approaches is that owing to DCSB in QCD – strong diquark correlations exist within baryons:

\[ \text{any interaction that describes colour-singlet mesons also generates non-pointlike diquark correlations in the colour-} \bar{3} \text{ channel} \]

\[ \text{where scalar } (0^+) \text{ & axial-vector } (1^+) \text{ diquarks most important for the nucleon} \]
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Such a framework is provided by the Poincaré covariant Faddeev equation, which sums all possible interactions between three dressed-quarks. Much of 3-body interaction can be absorbed into renormalized 2-body interactions. The Faddeev equation has solutions at discrete values of $p^2 = M^2$, which correspond to the baryon spectrum.

A prediction of these approaches is that owing to DCSB in QCD, strong diquark correlations exist within baryons. Any interaction that describes colour-singlet mesons also generates non-pointlike diquark correlations in the colour-$\bar{3}$ channel. The most important for the nucleon are the scalar ($0^+$) and axial-vector ($1^+$) diquarks.
Diquarks

- Diquarks are dynamically generated correlations between quarks inside baryons.
- Typically diquark sizes are similar to analogous mesons: \( r_{0+} \sim r_\pi, \quad r_{1+} \sim r_\rho \).
- These dynamic \( qq \) correlations are not the static diquarks of old.
  - All quarks participate in all diquark correlations.
  - In a given baryon, the Faddeev equation predicts a probability for each diquark cluster.
  - For the nucleon: scalar \((0^+) \sim 70\%\), axial-vector \((1^+) \sim 30\%\).

**Faddeev equation spectrum has significant overlap with constituent quark model and limited relation to Lichtenberg’s quark+diquark model.**

Mounting evidence from hadron structure (e.g. PDFs, form factors) and lattice.
A robust description of form factors is only possible if electromagnetic gauge invariance is respected; equivalently all relevant Ward-Takahashi identities (WTIs) must be satisfied.

For quark-photon vertex WTI implies:

\[ q_\mu \Gamma_{\gamma qq}(p', p) = \hat{Q}_q \left[ S^{-1}_q(p') - S^{-1}_q(p) \right] \]

transverse structure unconstrained

Diagrams needed for a gauge invariant nucleon EM current in DSEs

Feedback with experiment can shed light on elements of QCD via DSEs
Include "anomalous chromomagnetic" term in quark-gluon vertex

\[
\frac{1}{4\pi} g^2 D_{\mu\nu}(\ell) \Gamma_\nu(p', p) \rightarrow \alpha_{\text{eff}}(\ell) D_{\mu\nu}^{\text{free}}(\ell) [\gamma_\nu + i\sigma^{\mu\nu} q_\nu \tau_5(p', p) + \ldots]
\]

In chiral limit anomalous chromomagnetic term can only appear through DCSB – since operator flips quark helicity

EM properties of a spin-\(\frac{1}{2}\) point particle are characterized by two quantities:
- charge: \(e\)
- magnetic moment: \(\mu\)

Expect strong gluon dressing to produce non-trivial electromagnetic structure for a dressed quark
- recall dressing produces – from massless quark – a \(M \sim 400\) MeV dressed quark

Large anomalous chromomagnetic moment in the quark-gluon vertex – produces a large quark anomalous electromagnetic moment
- dressed quarks are not point particles!

[L. Chang, Y.-X. Liu, C. D. Roberts, PRL 106, 072001 (2011)]
**Proton \( \frac{G_E}{G_M} \) Ratio**

Quark anomalous magnetic moment required for good agreement with data

- important for low to moderate \( Q^2 \)
- power law suppressed at large \( Q^2 \)

Illustrates how feedback with EM form factor measurements can help constrain the quark–photon vertex and therefore the quark–gluon vertex within the DSE framework

- knowledge of quark–gluon vertex provides \( \alpha_s(Q^2) \) within DSEs \( \Leftrightarrow \) confinement

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Find that slight changes in $M(p^2)$ on the domain $1 \lesssim p \lesssim 3$ GeV have a striking effect on the $G_E/G_M$ proton form factor ratio

strong indication that position of a zero is very sensitive to underlying dynamics and the nature of the transition from nonperturbative to perturbative QCD

Zero in $G_E = F_1 - \frac{Q^2}{4M_N^2} F_2$ largely determined by evolution of $Q^2 F_2$

$F_2$ is sensitive to DCSB through the dynamically generated quark anomalous electromagnetic moment – vanishes in perturbative limit

the quicker the perturbative regime is reached the quicker $F_2 \to 0$
Prima facie, these experimental results are remarkable

- $u$ and $d$ quark sector form factors have very different scaling behaviour

However, when viewed in context of diquark correlations results are straightforward to understand

- in proton ($uud$) the $d$ quark is “bound” inside a scalar diquark [$ud$] 70% of the time; $u[ud]$ diquark $\Rightarrow 1/Q^2$

Zero in $F_{1p}^d$ a result of interference between scalar and axial-vector diquarks

- location of zero indicates relative strengths – correlated with $d/u$ ratio as $x \rightarrow 1$
Given the challenges posed by non-perturbative QCD it is insufficient to study hadron ground-states alone.

Nucleon transition form factors provide a critical extension to elastic form factors – providing more windows into and different perspectives on quark-gluon dynamics.

e.g. nucleon resonances are more sensitive to long-range effects in QCD than the properties of ground states . . . analogous to exotic and hybrid mesons

Important example is $N \rightarrow \Delta$ transition – parametrized by three form factors:

- $G^*_E(Q^2)$, $G^*_M(Q^2)$, $G^*_C(Q^2)$
- if both $N$ and $\Delta$ were purely $S$-wave then $G^*_E(Q^2) = 0 = G^*_C(Q^2)$

When analyzing the $N \rightarrow \Delta$ transition it is common to construct the ratios:

\[ R_{EM} = - \frac{G^*_E}{G^*_M}, \quad R_{SM} = - \frac{|q|}{2 M \Delta} \frac{G^*_C}{G^*_M} \]
$N \rightarrow \Delta$ form factors from the DSEs

For $R_{SM} = -\frac{|q|}{2 M_\Delta} \frac{G^*_C}{G^*_M}$ DSEs reproduces rapid fall off with $Q^2$

Find that $R_{EM} = -\frac{G^*_E}{G^*_M}$ is a particular sensitive measure of quark orbital angular momentum within the nucleon and $\Delta$

At large $Q^2$ helicity conservation demands: $R_{SM} \rightarrow \text{constant}$, $R_{EM} \rightarrow 1$

however these asymptotic results are not reached until incredibly large $Q^2$ – which will not be accessible at any present or foreseeable facility

Comparison with Argonne-Osaka results for $N \rightarrow \Delta$ suggest that the pion cloud is masking expected zero-crossing in $R_{EM}$
Results are indistinguishable from data for $Q^2 \gtrsim 0.7 \text{GeV}^2$.

With same set of inputs provide a unified description of nucleon, Delta and $N \to \Delta$ form factors.

For example, same
- quark propagators
- diquark masses and amplitudes
- Faddeev kernel
- electromagnetic current operator

Image courtesy of Victor Mokeev
Three poles, each seeded by a single dressed quark core:

Two poles associated with Roper resonance and the third with the next higher $P_{11}$ resonance


The Excited Baryon Analysis Center (EBAC), resolved a fifty-year puzzle by demonstrating that the Roper resonance is the proton’s first radial excitation

- its lower-than-expected mass owes to a dressed-quark core shielded by a dense cloud of pions and other mesons

[Decadal Report on Nuclear Physics: Exploring the Heart of Matter]
The Faddeev equation that produces the nucleon also gives its excited states amplitudes for the lightest excited state typically possess a zero therefore lightest nucleon excited state is a radial excitation ⇐⇒ Roper resonance

“quark core” mass: $M_R = 1.73$ GeV; c.f. Argonne-Osaka group $M_R = 1.76$ GeV

Now have a unified description of the nucleon, Delta and Roper baryons

Find e.g. that the Roper charge radius is 80% larger than the nucleon’s
Results agree well with data for $Q^2 \gtrsim 2 m_N^2$ & at the real photon point.

However contemporary kernels just produce a hadron’s *dressed-quark core*.

- Pion cloud contributions are absent from our calculation, however these are inferred from the deviation with data.
- On domain $0 < Q^2 \lesssim 2 m_N^2$ pion cloud contributions should be negative and deplete the transition form factors.
Looking to the Future

- The DSEs are primarily a hadron structure tool
  - a detailed study of the hadron spectrum is (currently) beyond the scope of the approach
  - need much more sophisticated kernels with explicit resonance contributions

- However, studies are underway to determine the “quark core” contributions to other low-lying nucleon to resonance transitions:
  - e.g., $N(1535) - J^P = \frac{1}{2}^-$ & $N(1520) - J^P = \frac{3}{2}^-$

- More broadly we are beginning a detailed study of nucleon generalized and transverse momentum dependent PDFs
  - an aim is to predict a large amount of hadron structure data and relate it to a single universal quark-gluon vertex
  - constraints from experiment & lattice will result in significantly improved kernels

- DSEs can then be used to study domains not explored by experiment and not reachable by lattice

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Conclusion

- QCD will only be solved by deploying a diverse array of experimental and theoretical methods
- must define and solve the problems of confinement and its relationship with DCSB

- These are two of the most important challenges in fundamental Science

- Nucleon elastic and transition form factors provide an important avenue with which to address these critical questions

- We have provided a unified treatment of the nucleon, Delta and Roper elastic and transition form factors
  - demonstrating e.g. that the location of zero’s in form factors – e.g. $G_{Ep}$, $F_{1p}^d$ – provide tight constraints on QCD dynamics

- Continuum-QCD approaches are essential; are at the forefront of guiding experiment & provide rapid feedback; building intuition & understanding
Backup Slides
quark aem term has important influence on Pauli form factors at low $Q^2$
Quark anomalous chromomagnetic moment – which drives the large anomalous electromagnetic moment – has only a minor impact on neutron Sachs form factor ratio

- Predict a zero-crossing in $G_{En}/G_{Mn}$ at $Q^2 \sim 11\text{ GeV}^2$

- DSE predictions were confirmed on domain $1.5 \lesssim Q^2 \lesssim 3.5\text{ GeV}^2$
Proton $G_E$ form factor and DCSB

Recall: $G_E = F_1 - \frac{Q^2}{4M_N^2} F_2$

Only $G_E$ is sensitive to these small changes in the mass function

Accurate determination of zero crossing would put important constraints on quark-gluon dynamics within DSE framework