Chiral Effective Field Theories in the meson sector: Status and Perspectives

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Outline

1. Introduction and Motivation
2. Success of ChPT: $\pi\pi$ scattering
3. $\eta \rightarrow 3\pi$ and light quark masses
4. Conclusion and Challenges for the future
1. Introduction and Motivation
1.1 Hadronic Physics

- New era of precision experiments
  
  Build amplitudes to look for exotics, hybrid mesons

- Require building blocks:
  - $\pi\pi$
  - $K\pi$

  \[ \text{ChPT} + \text{dispersion relations} \]

- Precise tests of the Standard Model

- Look for physics beyond the Standard Model: High precision at low energy as a key to new physics?
1.2 Chiral Symmetry

- Limit $m_k \to 0$

\[
\mathcal{L}^0_{\text{QCD}} = -\frac{1}{4} G_{\mu\nu} G^{\mu\nu} + \bar{q}_L i \gamma^\mu D_\mu q_L + \bar{q}_R i \gamma^\mu D_\mu q_R, \quad q = \begin{pmatrix} u \\ d \\ s \end{pmatrix}
\]

with $q_{L/R} \equiv \frac{1}{2} \left(1 \mp \gamma_5\right) q$

Symmetry: $G \equiv SU(3)_L \otimes SU(3)_R \to SU(3)_V$

- Chiral Perturbation Theory: dynamics of the Goldstone bosons (kaons, pions, eta)

- Goldstone bosons interact weakly at low energy and $m_u, m_d \ll m_s < \Lambda_{\text{QCD}}$

Expansion organized in external momenta and quark masses

Weinberg’s power counting rule

\[
\mathcal{L}_{\text{eff}} = \sum_{d \geq 2} \mathcal{L}_d, \quad \mathcal{L}_d = \mathcal{O}\left(p^d\right), \quad p \equiv \{q, m_q\}
\]

$p \ll \Lambda_H = 4\pi F_\pi \sim 1 \text{ GeV}$
1.3 Chiral expansion

- \[ \mathcal{L}_{\text{ChPT}} = \mathcal{L}_2 + \mathcal{L}_4 + \mathcal{L}_6 + \ldots \]
  - \( \text{LO} : \mathcal{O}(p^2) \)
  - \( \text{NLO} : \mathcal{O}(p^4) \)
  - \( \text{NNLO} : \mathcal{O}(p^6) \)

- The structure of the lagrangian is fixed by chiral symmetry but not the coupling constants → LECs appearing at each order

- The method has been rigorously established and can be formulated as a set of calculational rules:
  - \( \text{LO} : \) tree level diagrams with \( \mathcal{L}_2 \) \( \mathcal{L}_2 : F_0, B_0 \)
  - \( \text{NLO} : \) tree level diagrams with \( \mathcal{L}_4 \)
  - 1-loop diagrams with \( \mathcal{L}_2 \)
  - \( \mathcal{L}_4 = \sum_{i=1}^{10} L_i O_i^4 \)
  - \( \text{NNLO} : \) tree level diagrams with \( \mathcal{L}_6 \)
  - 2-loop diagrams with \( \mathcal{L}_2 \)
  - 1-loop diagrams with one vertex from \( \mathcal{L}_4 \)
  - \( \mathcal{L}_6 = \sum_{i=1}^{90} C_i O_i^6 \)

- Renormalizable and unitary order by order in the expansion
1.5 ChPT in the meson sector: precision calculations

• Today’s standard in the meson sector: 2-loop calculations

• Main obstacle to reaching high precision: determination of the LECs: $O(p^6)$ LECs proliferation makes the program to pin down/estimate all of them prohibitive

• In a specific process, only a limited number of LECs appear

• The LECs calculable if QCD solvable, instead
  – Determined from experimental measurement
  – Estimated with models: Resonances, large $N_C$
  – Computed on the lattice
2. Success: $\pi\pi$ scattering
2.1 \( \pi\pi \) scattering lengths

- \( \pi\pi \) scattering computed early on, one of the first applications of \( SU(2) \times SU(2) \) converge better the scattering lengths

\[ a_0^0 = \frac{7M^2_\pi}{32\pi F^2_\pi} \left[ 1 + \frac{M^2_\pi}{3} \langle r^2 \rangle^\pi_S + \frac{200\pi F^2_\pi M^2_\pi}{7} (a_2^0 + 2a_2^2) \right] - \frac{M^2_\pi}{672\pi^2 F^2_\pi} (15\bar{\ell}_3 - 353) \]

\[ = 0.16 \cdot 1.25 = 0.20 \]

\[ 2a_0^0 - 5a_2^0 = \frac{3M^2_\pi}{4\pi F^2_\pi} \left[ 1 + \frac{M^2_\pi}{3} \langle r^2 \rangle^\pi_S + \frac{41M^2_\pi}{192\pi^2 F^2_\pi} \right] = 0.547 \cdot 1.14 = 0.624 \]

- At NLO:

- Higher order corrections are suppressed by \( \mathcal{O}(m/\Lambda) \), \( \Lambda = \mathcal{O}(1\text{GeV}) \) expected to be a few percent

\[ Gasser \ & \ Leutwyler’83 \]
2.1 ππ scattering lengths

- This momentum dependence is reflected in a chiral log in $a_0^1$

\[
a_0^0 = \frac{7M^2_\pi}{32\pi F^2_\pi} \left[ 1 + \frac{9}{2} \ell_\chi + \ldots \right]
\]

\[
a_0^2 = -\frac{M^2_\pi}{16\pi F^2_\pi} \left[ 1 - \frac{3}{2} \ell_\chi + \ldots \right]
\]

\[
\ell_\chi = \frac{M^2_\pi}{16\pi^2 F^2_\pi} \ln \frac{\mu^2}{M^2_\pi}
\]

How large are yet higher orders? Is it at all possible to make a precise prediction?
2.2 Roy equations

- Unitarity effects can be calculated exactly using dispersive methods.

- Unitarity, analyticity and crossing symmetry \( \equiv \text{Roy equations} \)

- **Input:** imaginary parts above 0.8 GeV
  - two subtraction constants, e.g. \( a_0^0 \) and \( a_2^0 \)

- **Output:** the full \( \pi \pi \) scattering amplitude below 0.8 GeV
  - extended recently up to 1.15 GeV

- Numerical solutions of the Roy equations
  - Pennington-Protopopescu, Basdevant-Froggatt-Petersen (70s)
  - Bern group: Ananthanarayan, Colangelo, Gasser and Leutwyler’00
    - Caprini, Colangelo, Leutwyler’11
  - Orsay group: Descotes-Genon, Fuchs, Girlanda and Stern’01
  - Madrid-Cracow group: Garcia-Martín, Kamisnki Pelaez, Ruiz de Elvira, Yndurain’11
2.3 Combining ChPT and dispersion relations: A happy marriage

- In ChPT the two subtraction constants are predicted
- Subtracting the amplitude at threshold \((a_0^0, a_2^0)\) is not mandatory
- The freedom in the choice of the subtraction point can be exploited to use the chiral expansion where it converges best, i.e. below threshold

- The convergence of the series at threshold is greatly improved
  - ChPT at threshold:
    \[
    a_0^0 = 0.159 \rightarrow 0.200 \rightarrow 0.216 \\
    10 \cdot a_2^0 = -0.454 \rightarrow -0.445 \rightarrow -0.445
    \]
  - ChPT below threshold + Roy
    \[
    a_0^0 = 0.197 \rightarrow 0.2195 \rightarrow 0.220 \\
    10 \cdot a_2^0 = -0.402 \rightarrow -0.446 \rightarrow -0.444
    \]
2.4 Chiral Predictions for $a_0^0$ and $a_0^2$

Where can we test these predictions?

- Production experiments $\pi N \rightarrow \pi\pi N$, $\psi \rightarrow \pi\pi\omega$, $B \rightarrow D\pi\pi$, ... 
- Extraction of $\pi\pi$ scattering amplitude is not simple 
- Best accuracy in Kl4 data, $K \rightarrow 3\pi$, $\pi\pi$ atoms

Colangelo, Gasser & Leutwyler’01
With the new much more precise NA48 data it seemed that there was a disagreement with isospin breaking corrections.
2.6 On the importance of isospin breaking corrections

- Isospin breaking computed recently
  Perfect agreement!

Colangelo, Gasser, Rusetsky’09
Bernard, Descotes-Genon, Knecht ‘13

H. Leutwyler
2.7 $\pi\pi$ as a building block

- Extremely precise extraction of $\pi\pi$ scattering using ChPT and dispersion relations

- Similar works done solving Roy-Steiner equations for
  - $K\pi$: Buettiker, Descotes-Genon, Moussallam’07
  - $\pi N$: Hoferichter, Ruiz de Elvira, Kubis, Meißner’15

- Compare to lattice results → see Talk by J. Dudek

- Use these as building blocks for phenomenology:
  - $\pi\pi$ rescattering: e.g., $\pi$ form factors, $e^+e^- \rightarrow \pi\pi$, $\gamma\gamma \rightarrow \pi\pi$, $\omega/\varphi/\eta \rightarrow 3\pi$, $\tau \rightarrow 3\pi\nu_\tau$, $J/\Psi \rightarrow \gamma\pi^0\pi^0$. $B \rightarrow 3\pi$, $B \rightarrow J/\Psi\pi\pi\pi$, etc.

  - $K\pi$ rescattering: e.g., $K\pi$ form factors, $K \rightarrow \pi\pi\nu_e$, $\tau \rightarrow K\pi\nu_\tau$, $\tau \rightarrow K\pi\pi\nu_\tau$, $D \rightarrow K\pi\pi\pi$, $B \rightarrow K\pi$
2.7 $\pi\pi$ as a building block

H. Leutwyler

Garcia-Martin et al’09

Emilie Passemard
3. $\eta \rightarrow 3\pi$ and light quark masses
3.1 $\eta \to \pi^+ \pi^- \pi^0$

- Decay forbidden by isospin symmetry
  \[ A = \left( m_u - m_d \right) A_1 + \alpha_{em} A_2 \]

- $\alpha_{em}$ effects are small: Sutherland’66, Bell & Sutherland’68
  Baur, Kambor, Wyler’96, Ditsche, Kubis, Meissner’09

- Decay rate measures the size of isospin breaking $(m_u - m_d)$ in the SM:
  \[ L_{QCD} \to L_{IB} = -\frac{m_u - m_d}{2} \bar{u}u - \bar{d}d \]

- Clean access to $(m_u - m_d)$

- $\Gamma_{\eta \to 3\pi} \propto \int |A(s, t, u)|^2 \propto Q^{-4}$
  \[ Q^2 \equiv \frac{m_s^2 - \hat{m}^2}{m_d^2 - m_u^2} \]
  \[ \hat{m} \equiv \frac{m_d + m_u}{2} \]
3.2 ChPT

- Slow convergence of the chiral series (SU(3) ChPT)

\[ \Gamma_{\eta \rightarrow 3\pi} = (66 + 94 + 100 + \ldots) \text{eV} = (300 \pm 12) \text{eV} \]

- CHPT amplitudes have problems with measured Dalitz plot distributions

- Main deficiency: strong \( \pi\pi \) rescattering included only perturbatively

- Large \( \pi\pi \) final state interactions call for a dispersive treatment:
  - analyticity, unitarity and crossing symmetry
  - Take into account all the rescattering effects

- Match to CHPT amplitude to obtain Q from rates
3.3 Dispersive method

- **Decomposition** of the amplitude as a function of $\pi\pi$ isospin states

\[
M(s,t,u) = M_0(s) + (s-u)M_1(t) + (s-t)M_1(u) + M_2(t) + M_2(u) - \frac{2}{3}M_2(s)
\]

- $M_I$ isospin $I$ rescattering in two particles
- Amplitude in terms of S and P waves exact up to NNLO ($O(p^6)$)
- Main two body rescattering corrections inside $M_I$

- **Unitary relation** for $M_I(s)$:

\[
disc M_I(s) = 2i \left( M_I(s) + \hat{M}_I(s) \right) \sin \delta_I(s) e^{-i\delta_I(s)} \theta(s - 4M^2_{\pi})
\]

G. Colangelo, S. Lanz, H. Leutwyler, E.P., in progress

Fuchs, Sazdjian & Stern’93
Anisovich & Leutwyler’96

Emilie Passemear
3.3 Dispersive method

- **Unitary relation for** $M_I(s)$:

\[
\text{disc } M_I(s) = 2i \left( M_I(s) + \hat{M}_I(s) \right) \sin \delta_I(s) e^{-i\delta_I(s)} \theta \left( s - 4M^2_\pi \right)
\]

- **Dispersive relation for** the $M_I$’s

\[
M_I(s) = \Omega_I(s) \left( P_I(s) + \frac{s''}{\pi} \int_{4M^2_\pi}^{\infty} ds' \frac{\sin \delta_I(s')}{s'' |\Omega_I(s')| (s' - s - i\epsilon)} \hat{M}_I(s') \right)
\]

- $\hat{M}_I(s)$: singularities in the t and u channels, depend on the other $M_I(s)$

\[
\hat{M}_I(s) = \text{inhomogeneities at } M(s,t,u)
\]

- **Angular averages of the other functions**

\[
\text{Coupled equations}
\]

- **Omnès function**
3.4 Combining ChPT and dispersion relations

- As for $\pi\pi$, combine dispersion relations with ChPT where it works the best.
- Use representation holding up to and including NNLO $\pi\pi$ partial-wave discontinuities for $l = 0, 1$ only and $l=0,1,2$.
- Interesting matching point: Adler zero. The real part of the amplitude along the line $s=u$ has a zero. Chiral SU(2) prediction small higher order corrections.

Diagram: (Anisovich & Leutwyler’96)
3.5 Different recent analyses

1. **Schneider, Kubis, Ditsche 2011**: 2-loop NREFT approach
   Allows investigation of isospin-violating corrections

2. **Kampf, Knecht, Novotny, Zdrahal 2011**: Analytic dispersive approach
   Match to absorptive part of NNLO chiral amplitude where differences
   between NLO and NNLO are small R (Q)
   Problem: do not reproduce the Adler’s zero

3. **Guo et al. 2015: JPAC** analysis, Khuri-Treiman equations solved
   numerically using Pasquier inversion techniques
   - Madrid/Cracow ππ phase shifts, 3 subtraction constants
   - Match to NLO ChPT near Adler zero Q
   See talk by V. Mathieu

4. **Colangelo, Lanz, Leutwyler, E.P. in progress**: dispersive approach following
   Anisovich, Leutwyler
   - Electromagnetic effects to NLO fully taken into account (Ditsche,
     Kubis, Meißner’09)
   - Matching to one loop ChPT: Taylor expand the partial wave around s=0
3.6 Dalitz plot parameters: Charged channel

- Dalitz plot measurement of $\eta \to \pi^+ \pi^- \pi^0$

Amplitude expanded in X and Y around $X=Y=0$

$$\left| A_c(s,t,u) \right|^2 = N \left( 1 + aY + bY^2 + dX^2 + fY^3 \right)$$

$$Q_c \equiv M_\eta - 2M_{\pi^+} - M_{\pi^0}$$

$$X = \frac{\sqrt{3}}{2M_\eta Q_c} (u-t)$$

$$Y = \frac{3}{2M_\eta Q_c} \left( (M_\eta - M_{\pi^0})^2 - s \right)^{-1}$$

<table>
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<tr>
<th></th>
<th>$-a$</th>
<th>$b$</th>
<th>$d$</th>
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<td>BESIII 2015</td>
<td>1.128(17)</td>
<td>0.153(17)</td>
<td>0.085(18)</td>
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<td>WASA/COSY 2014</td>
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<td>0.219(51)</td>
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<td>NNLO CHPT</td>
<td>1.271(75)</td>
<td>0.394(102)</td>
<td>0.055(57)</td>
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<td>NREFT</td>
<td>1.213(14)</td>
<td>0.308(23)</td>
<td>0.050(3)</td>
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<td>JPAC</td>
<td>1.116(32)</td>
<td>0.188(12)</td>
<td>0.063(4)</td>
</tr>
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</table>

See talks by Daniel Lersch (WASA), Liqing Qin (BESSIII)
3.7 Comparison of results for $\alpha$: neutral decay

- Dalitz plot measurement of $\eta \to 3\pi^0$

$$|A_n(s,t,u)|^2 = N\left(1 + 2\alpha Z\right)$$

$$Z = X^2 + Y^2$$

$$Z = \frac{2}{3} \sum_{i=1}^{3} \left(\frac{3T_i}{Q_n} - 1\right)^2$$

$$Q_n \equiv M_\eta - 3M_{\pi^0}$$

- Colangelo et al., JPAC

- Preliminary

- $\chiPT\,O(p^4)$
- $\chiPT\,O(p^6)$
- Kambor et al.
- Kampf et al.
- NREFT, JPAC

- Crystal Barrel@LEAR (1998)
- Crystal Ball@BNL (2001)
- SND (2001)
- WASA@CELSIUS (2007)
- WASA@COSY (2008)
- Crystal Ball@MAMI-B (2009)
- Crystal Ball@MAMI-C (2009)
- KLOE (2010)
- PDG average

- dispersive, one loop
- dispersive, fit to KLOE
3.8 Quark mass ratio

\[ \Gamma_{\eta \to 3\pi} \propto \int |A(s, t, u)|^2 \propto Q^{-4} \]

\[ Q^2 \equiv \frac{m_s^2 - \hat{m}^2}{m_d^2 - m_u^2}, \quad \hat{m} \equiv \frac{m_d + m_u}{2} \]

\[ A(s, t, u) = \frac{N}{Q^2} M(s, t, u) \]

- \( M(s, t, u) \) determined through the dispersive analysis of the data but for \( N \) one has to rely on ChPT
3.9 $\eta \rightarrow 3\pi$ and light quark masses

![Graph showing the relationship between $m_s/m_d$ and $m_u/m_d$ with annotations for lattice, intersection, and $\eta$ decay (preliminary).]
4. Conclusion and outlook
4.1 Conclusion

- ChPT is a very interesting tool at low energy
  - Model independent
  - Build amplitude using a power counting scheme
- precise predictions in the meson sector

Garcia-Martin et al’09
4.1 Conclusion

- ChPT is a very interesting tool at low energy
  - Model independent
  - Build amplitude using a power counting scheme
  - Precise predictions in the meson sector

- But when one wants to go to higher energy or more precise prediction
  Nicely complement by dispersion relation: analyticity, unitarity, crossing
  Ex: $\pi\pi$ scattering, $\eta \rightarrow 3\pi$

Garcia-Martin et al'09
4.2 Outlook: Challenges for the future

- ChPT is a very interesting tool at low energy
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![Graph showing ChPT and Dispersion Relations](image-url)
4.2 Outlook: Challenges for the future

- What can we do if one wants to explore the intermediate energy region: resonance appear not taken into account by ChPT
  Include coupled channels:
    - Analytically: e.g. dispersive approach, unitary coupled channel (Ed Berger talk)
      Ex: $\pi\pi$ scalar form factors ($\pi\pi$ and KK), Donoghue, Gasser, Leutwyler’90, Moussallam’99, Daub et al’13, Celis, Cirigliano, E.P.’14
      see Christoph Hanhart’s talk

\[
\langle \pi^+\pi^- | m_u\bar{u}u + m_d\bar{d}d | 0 \rangle \equiv \Gamma_\pi(s)
\]

\[
\langle \pi^+\pi^- | m_s\bar{s}s | 0 \rangle \equiv \Delta_\pi(s)
\]

Celis, Cirigliano, E.P.’14
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      see Christoph Hanhart’s talk)
    - Lattice QCD: see Jo Dudek’s talk
      Ex: \( \pi\pi \), KK scattering, P wave Wilson, Briceno, Dudek, Edwards, Thomas’15
      \( K\pi \) and \( K\eta \) scattering Hadron spectrum, Wilson, Dudek, Edwards, Thomas’14
      Include resonances: see talk by Ed Berger
      - RChPT, Unitarized ChPT e.g., Pelaez, Oller, Oset’99
      - Lattice with Unitarized ChPT Bolton, Briceno, Wilson’15
  - If one wants to explore the full Dalitz plot in B, D decays N/D
    see talk by A. Jackura

Oller and Oset’98
5. Back-up
4.1 Precision Physics at intermediate energies

- ChPT is a very interesting tool at low energy
  - Model independent
  - Rely on unitarity, analyticity
  - Build amplitude using a power counting scheme
  - Precise predictions in the meson sector but unknowns LECs to be determined

- But when one wants to go to higher energy or more precise prediction
  - Nicely complement by dispersion relation
  
  **Ex:** \( \pi \pi \) scattering, \( \eta \to 3\pi \)

- Still if one wants to explore the intermediate energy region: resonance appear not taken into account by ChPT
  - For \( \pi \pi \): \( l=1 \): \( \rho(770), \rho(1450), \rho(1700), \ldots \), \( l=0 \): “\( \sigma(\sim 500) \)”, \( f_0(980), \ldots \)
  - For \( K\pi \): \( l=1 \): \( K^*(892), K^*(1410), K^*(1680), \ldots \), \( l=0 \): “\( \kappa(\sim 800) \)”, …
1.4 Construction of an effective theory: ChPT

- Degrees of freedom: **Goldstone bosons** (GB)
  
  Symmetry group: \[ G \equiv SU(3)_L \otimes SU(3)_R \]

- Build all the corresponding invariant operators including explicit symmetry breaking parameters

  \[ \mathbf{L}_{\text{ChPT}} \equiv \mathbf{L}(U, \chi) \]

  **GB’s**  **Masses \sim m_q**

- Goldstone bosons interact weakly at low energy and \( m_u, m_d = m_s < \Lambda_{QCD} \)
  
  expansion organized in **external momenta** and **quark masses**

  **Weinberg’s power counting rule**

\[
\mathcal{L}_{\text{eff}} = \sum_{d \geq 2} \mathcal{L}_d, \quad \mathcal{L}_d = \mathcal{O}(p^d), \quad p \equiv \{q, m_q\}
\]

\[ p \ll \Lambda_H = 4\pi F_\pi \sim 1 \text{ GeV} \]
2.1 Low energy constants

- Recent fit by Ecker and Bijnens of NLO LECs $L_i (i = 1,\ldots,10)$ and NNLO LECs $C_i (i = 1,\ldots,90)$
  Update and extension of Bijnens, Jemos 2012

- New ingredients:
  - relations $l_i (L_j ; C_j) (j = 1,\ldots,4)$ Gasser, Haefeli, Ivanov, Schmid 2007
    altogether 17 input data
  - penalize bad convergence of meson masses
  - intelligent guesses (priors) for 34 (combinations of the $C_i$
  - renormalization scale $\mu = 0.77$ GeV

- Fitting procedure:
  - minimization/random walk in restricted $C_i$ –space
  - iterate after possible modification of Ci –space
  - normal $\chi^2$ fit for $L_i$ for (fixed) “best” values of the $C_i$
    “best” values for $L_i$
2.1 Low energy constants

<table>
<thead>
<tr>
<th>(10^3 L'_A)</th>
<th>NNLO free fit</th>
<th>NNLO BE14</th>
<th>NLO 2014</th>
<th>GL 1985</th>
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<td>0.68(11)</td>
<td>0.24(11)</td>
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<td>0.64(06)</td>
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<td>0.76(18)</td>
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\[F_0 \text{ [MeV]}\]

<table>
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<tr>
<th>NNLO free fit</th>
<th>64</th>
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<tbody>
<tr>
<td>NNLO BE14</td>
<td>71</td>
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</table>

- Strong sensitivity to (large-Nc) suppressed \(L_4\)
  -\(10^3 L'_4 = 0.3\); \(NLO: 0.3 \leq 10^3 L'_4 \leq 0.3\) fixed \(\rightarrow L_A = 2 L_1 - L_2\) and \(L_6\) automatically suppressed

- NNLO only makes sense with certain set of \(C_r\)

- Except for last column: no estimate of higher-order uncertainties
2.1 Low energy constants

<table>
<thead>
<tr>
<th>$10^3 L_A'$</th>
<th>NNLO free fit</th>
<th>NNLO BE14</th>
<th>NLO 2014</th>
<th>GL 1985</th>
</tr>
</thead>
<tbody>
<tr>
<td>$10^3 L_1'$</td>
<td>0.68(11)</td>
<td>0.24(11)</td>
<td>0.4(2)</td>
<td>0.7(3)</td>
</tr>
<tr>
<td>$10^3 L_2'$</td>
<td>0.64(06)</td>
<td>0.53(06)</td>
<td>1.0(1)</td>
<td>1.3(7)</td>
</tr>
<tr>
<td>$10^3 L_3'$</td>
<td>0.59(04)</td>
<td>0.81(04)</td>
<td>1.6(2)</td>
<td></td>
</tr>
<tr>
<td>$10^3 L_4'$</td>
<td>-2.80(20)</td>
<td>-3.07(20)</td>
<td>-3.8(3)</td>
<td>-4.4(2.5)</td>
</tr>
<tr>
<td>$10^3 L_5'$</td>
<td>0.76(18)</td>
<td>0.3</td>
<td>0.0(3)</td>
<td>-0.3(5)</td>
</tr>
<tr>
<td>$10^3 L_6'$</td>
<td>0.50(07)</td>
<td>1.01(06)</td>
<td>1.2(1)</td>
<td>1.4(5)</td>
</tr>
<tr>
<td>$10^3 L_7'$</td>
<td>0.49(25)</td>
<td>0.14(05)</td>
<td>0.0(4)</td>
<td>-0.2(3)</td>
</tr>
<tr>
<td>$10^3 L_8'$</td>
<td>-0.19(08)</td>
<td>-0.34(09)</td>
<td>-0.3(2)</td>
<td>-0.4(2)</td>
</tr>
<tr>
<td>$10^3 L_9'$</td>
<td>0.17(11)</td>
<td>0.47(10)</td>
<td>0.5(2)</td>
<td>0.9(3)</td>
</tr>
</tbody>
</table>

| $F_0$ [MeV] | 64 | 71 |

- Reasonable convergence of observables (enforced for masses)
- Qualitative evidence for resonance saturation, even for scalars
- Last 3 columns: good stability
1.4 Construction of an effective theory: ChPT

- Degrees of freedom: Goldstone bosons (GB)
  
  Symmetry group: \( G \equiv SU(3)_L \otimes SU(3)_R \)

- Build all the corresponding invariant operators including explicit symmetry breaking parameters

\[
\mathcal{L}_{\text{ChPT}} \equiv \mathcal{L}^0_{QCD} + \mathcal{L}_m \quad \text{with} \quad \mathcal{L}_m = -\bar{q} \mathcal{M} q, \quad \mathcal{M} = \text{diag}(m_u, m_d, m_s)
\]

- Goldstone bosons interact weakly at low energy and \( m_u, m_d = m_s < \Lambda_{QCD} \)

  expansion organized in external momenta and quark masses

  *Weinberg’s power counting rule*

\[
\mathcal{L}_{\text{eff}} = \sum_{d \geq 2} \mathcal{L}_d, \quad \mathcal{L}_d = \mathcal{O}(p^d), \quad p \equiv \{q, m_q\}
\]

\[
p \ll \Lambda_H = 4\pi F_\pi \sim 1 \text{ GeV}
\]
1.4 Chiral expansion

- \( \mathcal{L}_{\text{ChPT}} = \mathcal{L}_2 + \mathcal{L}_4 + \mathcal{L}_6 + \ldots \)
  - **LO**: \( \mathcal{O}(p^2) \)
  - **NLO**: \( \mathcal{O}(p^4) \)
  - **NNLO**: \( \mathcal{O}(p^6) \)

- **Renormalizable** and **unitary** order by order in the expansion

- The structure of the lagrangian is fixed by chiral symmetry but not the coupling constants \( \rightarrow \) **LECs** appearing at each order

  \[
  \mathcal{L}_2 : \quad F_0, B_0, \quad \mathcal{L}_4 = \sum_{i=1}^{10} L_i \, O_4^i, \quad \mathcal{L}_6 = \sum_{i=1}^{90} C_i \, O_6^i
  \]

- **LECs** describe the influence of heavy degrees of freedom not contained in the ChPT lagrangian

- **Naturalness**: LECs of order one
2.2 Pion polarizabilities: success of ChPT at NNLO

- New ingredients:
  - relations $l_i \ (L_i ; C_i \ ) \ (j = 1, \ldots, 4)$ Gasser, Haefeli, Ivanov, Schmid 2007
together 17 input data
  - penalize bad convergence of meson masses
  - intelligent guesses (priors) for 34 (combinations of the ) $C_i$
  - renormalization scale $\mu = 0.77$ GeV

- Fitting procedure:
  - minimization/random walk in restricted $C_i$–space
  - iterate after possible modification of $C_i$–space
  - normal $\chi^2$ fit for $L_i$ for (fixed) “best” values of the $C_i$
  “best” values for $L_i$
1.5 ChPT in the meson sector: precision calculations

\[ \mathcal{L}_{\text{ChPT}} = \mathcal{L}_2 + \mathcal{L}_4 + \mathcal{L}_6 + \ldots \]

- LO: \( \mathcal{O}(p^2) \)
- NLO: \( \mathcal{O}(p^4) \)
- NNLO: \( \mathcal{O}(p^6) \)

- The structure of the lagrangian is fixed by chiral symmetry but not the coupling constants \( \rightarrow \) LECs appearing at each order

\[ \mathcal{L}_2: \ F_0, B_0, \quad \mathcal{L}_4 = \sum_{i=1}^{10} L_i O_4^i, \quad \mathcal{L}_6 = \sum_{i=1}^{90} C_i O_6^i \]

- LECs describe the influence of heavy degrees of freedom not contained in the ChPT lagrangian

- Naturalness: LECs of order one

- Today’s standard in the meson sector: 2-loop calculations

- Main obstacle to reaching high precision: determination of the LECs
2.4 Chiral Predictions for

- ChPT: Calculations at
  - NLO
  - NNLO

- Prediction obtained matching $O(p6)$ $\chi$PT to Roy equations (disp. relations)

\[
\begin{align*}
a_0^0 &= 0.220 \pm 0.001 + 0.009 \Delta \ell_4 - 0.002 \Delta \ell_3 \\
10 \cdot a_0^2 &= -0.444 \pm 0.003 - 0.01 \Delta \ell_4 - 0.004 \Delta \ell_3
\end{align*}
\]

\[
\bar{\ell}_4 = 4.4 + \Delta \ell_4 \quad \bar{\ell}_3 = 2.9 + \Delta \ell_3 \quad [\Delta \ell_4 = 0.2, \Delta \ell_3 = 2.4]
\]

- Adding the uncertainties in quadrature:

\[
\begin{align*}
a_0^0 &= 0.220 \pm 0.005 \\
10 \cdot a_0^2 &= -0.444 \pm 0.01 \\
a_0^0 - a_0^2 &= 0.265 \pm 0.004
\end{align*}
\]
3.3 Different recent analyses

1. **Schneider, Kubis, Ditsche 2011**: 2-loop NREFT approach
   - allows investigation of isospin-violating corrections
   - relations between charged and neutral Dalitz plots

2. **Kampf, Knecht, Novotny, Zdrahal 2011**: Analytic dispersive approach
   - Amplitudes involve 6 parameters (subtraction constants)
   - Fit to Dalitz plot distribution (KLOE 2008: $\eta \rightarrow \pi^+\pi^-\pi^0$)
   - Predict Dalitz plot parameter $\alpha$ (neutral decay mode)
   - Match to absorptive part of NNLO chiral amplitude where differences between NLO and NNLO are small

Problem: do not reproduce the Adler’s zero
3.3 Different recent analyses

3. **Guo et al. 2015: JPAC** analysis, Khuri Treiman equations solved numerically using Pasquier inversion techniques
   - Madrid/Cracow ππ phase shifts, 3 subtraction constants
   - Fit experimental Dalitz plot (*WASA/COSY* 2014: $\eta \rightarrow \pi^+\pi^-\pi^0$)
     - predict Dalitz plot parameter $\alpha$
   - Match to NLO ChPT near Adler zero

4. **Colangelo, Lanz, Leutwyler, E.P. in progress**: dispersive approach following Anisovich, Leutwyler
   - Electromagnetic effects to NLO fully taken into account (*Ditsche, Kubis, Meißner’09*)
   - Dispersive amplitudes: Bern ππ phase shifts, 6 subtraction constants
   - Fit simultaneously Charged (*WASA, KLOE*) and Neutral Dalitz plots (*MAMI*)
   - Matching to one loop ChPT: Taylor expand the partial wave around $s=0$
3.3 Different recent analyses

3. **Guo et al. 2015: JPAC** analysis, Khuri Treiman equations solved numerically using Pasquier inversion techniques
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   - Matching to one loop ChPT: Taylor expand the partial wave around $s=0$
3.4 Dalitz plot parameters

- Dalitz plot measurement: Amplitude expanded in X and Y around X=Y=0

\[ |A_c(s,t,u)|^2 = N \left( 1 + aY + bY^2 + dX^2 + fY^3 \right) \]

\[ X = \frac{\sqrt{3}}{2M_{\eta}Q_c} (u-t) \]

\[ Y = \frac{3}{2M_{\eta}Q_c} \left( (M_\eta - M_{\pi^+})^2 - s \right) - 1 \]

\[ Z = X^2 + Y^2 \]

\[ Q_c \equiv M_\eta - 2M_{\pi^+} - M_{\pi^0} \]

<table>
<thead>
<tr>
<th>Source</th>
<th>$-a$</th>
<th>$b$</th>
<th>$d$</th>
<th>$\alpha$</th>
</tr>
</thead>
<tbody>
<tr>
<td>KLOE 2015</td>
<td>1.095(4)</td>
<td>0.145(6)</td>
<td>0.081(7)</td>
<td>−0.055(15)</td>
</tr>
<tr>
<td>BESIII 2015</td>
<td>1.128(17)</td>
<td>0.153(17)</td>
<td>0.085(18)</td>
<td></td>
</tr>
<tr>
<td>WASA/COSY 2014</td>
<td>1.144(18)</td>
<td>0.219(51)</td>
<td>0.086(23)</td>
<td></td>
</tr>
<tr>
<td>NNLO CHPT</td>
<td>1.271(75)</td>
<td>0.394(102)</td>
<td>0.055(57)</td>
<td>0.013(32)</td>
</tr>
<tr>
<td>KKNZ</td>
<td></td>
<td></td>
<td></td>
<td>−0.044(4)</td>
</tr>
<tr>
<td>NREFT</td>
<td>1.213(14)</td>
<td>0.308(23)</td>
<td>0.050(3)</td>
<td>−0.025(5)</td>
</tr>
<tr>
<td>JPAC</td>
<td>1.116(32)</td>
<td>0.188(12)</td>
<td>0.063(4)</td>
<td>−0.022(4)</td>
</tr>
<tr>
<td>PDG 2014</td>
<td></td>
<td></td>
<td></td>
<td>−0.0315(15)</td>
</tr>
</tbody>
</table>
3.3 Qualitative results of our analysis

- Plot of $Q$ versus $\alpha$:

- All the data give consistent results. The preliminary outcome for $Q$ is intermediate between the lattice result and the one of Kastner and Neufeld.

NB: Isospin breaking has not been accounted for.

From kaon mass splitting:

$$Q = 20.7 \pm 1.2$$

*Kastner & Neufeld’08*
3.3 Qualitative results of our analysis

- Plot of $Q$ versus $\alpha$:

All our preliminary results give a negative value for $\alpha$. In particular the result using KLOE data for $\eta \rightarrow \pi^+ \pi^- \pi^0$ is in perfect agreement with the PDG value!

NB: Isospin breaking has not been accounted for.