Chiral Effective Field Theories in the meson sector: Status and Perspectives

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Outline

1. Introduction and Motivation

2. Success of ChPT: $\pi\pi$ scattering

3. $\eta \rightarrow 3\pi$ and light quark masses

4. Conclusion and Challenges for the future

1. Introduction and Motivation

1.1 Hadronic Physics

- New era of precision experiments

  ⇒ Build amplitudes to look for exotics, hybrid mesons

- Require building blocks:
  - $\pi\pi$
  - $K\pi$

  ⇒ ChPT + dispersion relations

- Precise tests of the Standard Model

- Look for physics beyond the Standard Model: High precision at low energy as a key to new physics?

1.2 Chiral Symmetry

- Limit $m_k \to 0$

$$\mathcal{L}_{QCD} \to \boxed{\mathcal{L}^0_{QCD} = -\frac{1}{4} G_{\mu\nu} G^{\mu\nu} + \bar{q}_L i\gamma^\mu D_\mu q_L + \bar{q}_R i\gamma^\mu D_\mu q_R}, \quad q = \begin{pmatrix} u \\ d \\ s \end{pmatrix}$$

with $q_{L/R} \equiv \frac{1}{2}(1 \mp \gamma_5) q$

Symmetry: $\boxed{G \equiv SU(3)_L \otimes SU(3)_R \to SU(3)_V}$

- Chiral Perturbation Theory: dynamics of the Goldstone bosons (kaons, pions, eta)

- Goldstone bosons interact weakly at low energy and $m_u, m_d \ll m_s < \Lambda_{QCD}$
Expansion organized in external momenta and quark masses

Weinberg's power counting rule

$$\boxed{\mathcal{L}_{eff} = \sum_{d \geq 2} \mathcal{L}_d \;,\; \mathcal{L}_d = \mathcal{O}\left(p^d\right) \;,\; p \equiv \left\{q, m_q\right\}}$$

$$\boxed{\mathrm{p} << \Lambda_H = 4\pi F_\pi \sim 1\ \mathrm{GeV}}$$

1.3 Chiral expansion

- $$\mathcal{L}_{ChPT} = \underbrace{\mathcal{L}_2}_{\text{LO}:\ \mathcal{O}(p^2)} + \underbrace{\mathcal{L}_4}_{\text{NLO}:\ \mathcal{O}(p^4)} + \underbrace{\mathcal{L}_6}_{\text{NNLO}:\ \mathcal{O}(p^6)} + \ldots.$$

- The structure of the lagrangian is fixed by chiral symmetry but not the coupling constants → LECs appearing at each order

- The method has been rigorously established and can be formulated as a set of calculational rules:

LO : tree level diagrams with $\mathcal{L}_2$

$$\mathcal{L}_2 : F_0, B_0$$

NLO: tree level diagrams with $\mathcal{L}_4$
1-loop diagrams with $\mathcal{L}_2$

$$\mathcal{L}_4 = \sum_{i=1}^{10} L_i \, O_4^i,$$

NNLO: tree level diagrams with $\mathcal{L}_6$
2-loop diagrams with $\mathcal{L}_2$
1-loop diagrams with one vertex from $\mathcal{L}_4$

$$\mathcal{L}_6 = \sum_{i=1}^{90} C_i \, O_6^i$$

- Renormalizable and unitary order by order in the expansion

1.5 ChPT in the meson sector: precision calculations

- Today's standard in the meson sector: 2-loop calculations

- Main obstacle to reaching high precision: determination of the LECs: $\mathcal{O}(p^6)$ LECs proliferation makes the program to pin down/ estimate all of them prohibitive

- In a specific process, only a limited number of LECs appear

- The LECs calculable if QCD solvable, instead
  - Determined from experimental measurement
  - Estimated with models: Resonances, large N_C
  - Computed on the lattice

2. Success: $\pi\pi$ scattering

2.1 $\pi\pi$ scattering lengths

- $\pi\pi$ scattering computed early on, one of the first applications of SU(2) x SU(2) ⇨ converge better
  ⇨ the scattering lengths

Gasser & Leutwyler'83

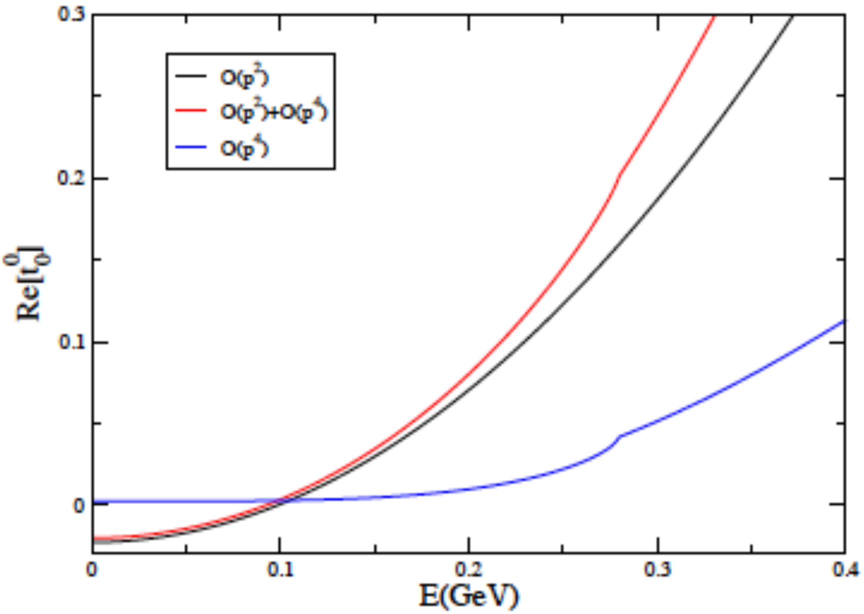
- At NLO: $$a_0^0 = \frac{7M_\pi^2}{32\pi F_\pi^2}\left[1 + \frac{M_\pi^2}{3}\langle r^2\rangle_S^\pi + \frac{200\pi F_\pi^2 M_\pi^2}{7}(a_2^0 + 2a_2^2) - \frac{M_\pi^2}{672\pi^2 F_\pi^2}(15\bar{\ell}_3 - 353)\right]$$
  $$= 0.16 \cdot 1.25 = 0.20$$

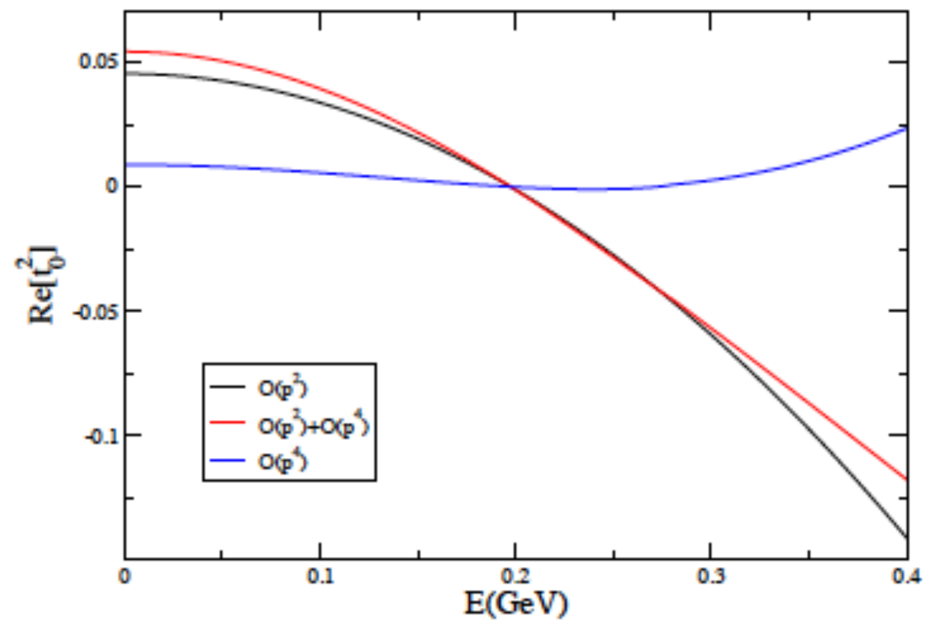
$$2a_0^0 - 5a_0^2 = \frac{3M_\pi^2}{4\pi F_\pi^2}\left[1 + \frac{M_\pi^2}{3}\langle r^2\rangle_S^\pi + \frac{41M_\pi^2}{192\pi^2 F_\pi^2}\right] = 0.547 \cdot 1.14 = 0.624$$

- Higher order corrections are suppressed by $\mathcal{O}$(m/Λ) , Λ = $\mathcal{O}$(1GeV)
  ⇨ expected to be a few percent

2.1 ππ scattering lengths

G. Colangelo





- This momentum dependence is reflected in a chiral log in a^I_0

$$a_0^0 = \frac{7M_\pi^2}{32\pi F_\pi^2}\left[1+\frac{9}{2}\ell_\chi+\ldots\right] \qquad a_0^2 = -\frac{M_\pi^2}{16\pi F_\pi^2}\left[1-\frac{3}{2}\ell_\chi+\ldots\right]$$

$$\ell_\chi = \frac{M_\pi^2}{16\pi^2 F_\pi^2}\ln\frac{\mu^2}{M_\pi^2}$$

How large are yet higher orders?
Is it at all possible to make a precise prediction?

2.2 Roy equations

- Unitarity effects can be calculated *exactly* using dispersive methods

- Unitarity, analyticity and crossing symmetry ≡ *Roy equations*

- Input: imaginary parts above 0.8 GeV
  two subtraction constants, e.g. a_0^0 and a_2^0

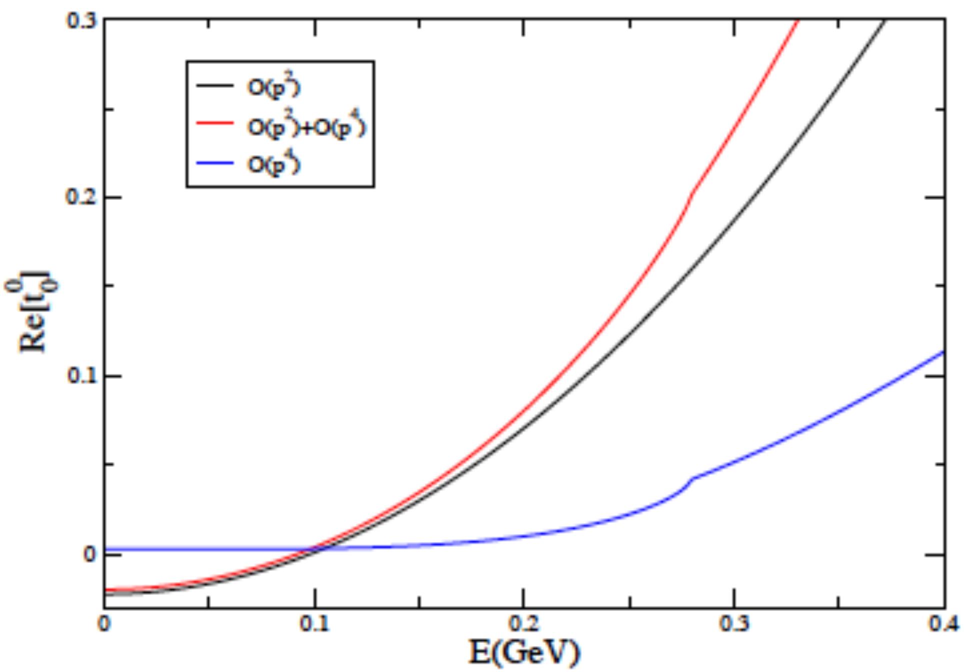
- Output: the full ππ scattering amplitude below 0.8 GeV
  ⇨ extended recently up to 1.15 GeV

- Numerical solutions of the Roy equations
  Pennington-Protopopescu, Basdevant-Froggatt-Petersen (70s)
  Bern group: Ananthanarayan, Colangelo, Gasser and Leutwyler'00
  Caprini, Colangelo, Leutwyler'11
  Orsay group: Descotes-Genon, Fuchs, Girlanda and Stern'01
  Madrid-Cracow group: Garcia-Martin, Kamisnki Pelaez, Ruiz de Elvira, Yndurain'11

2.3 Combining ChPT and dispersion relations: A happy marriage

G. Colangelo

- In ChPT the two subtraction constants are predicted
- Subtracting the amplitude at threshold (a_0^0 , a_2^0) is not mandatory
- The freedom in the choice of the subtraction point can be exploited to use the chiral expansion where it converges best, i.e. below threshold



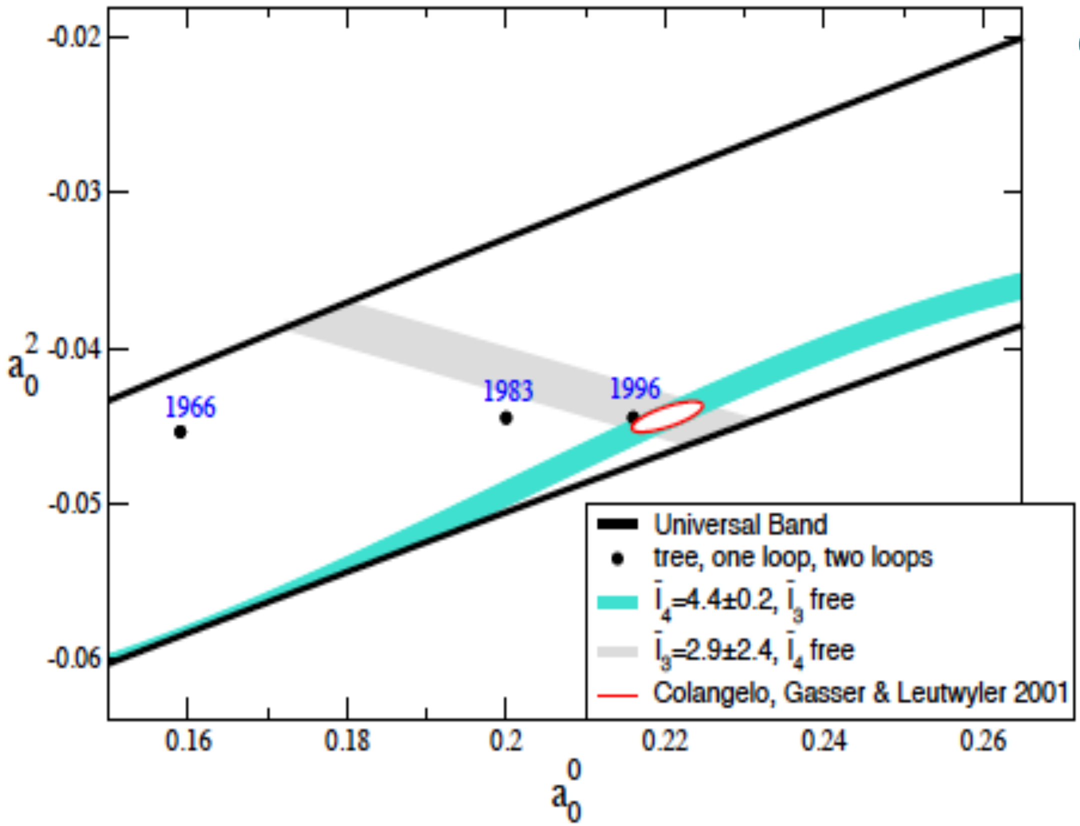
- The convergence of the series at threshold is greatly improved
  - ChPT at threshold:

$$\begin{array}{rcl} a_0^0 &=& 0.159 \to \ 0.200 \to \ 0.216 \\ 10 \cdot a_0^2 &=& -0.454 \to -0.445 \to -0.445 \\ && \quad p^2 \qquad\quad p^4 \qquad\quad p^6 \end{array}$$

  - ChPT below threshold + Roy

$$\boxed{\begin{array}{rcl} a_0^0 &=& 0.197 \to \ 0.2195 \to \ 0.220 \\ 10 \cdot a_0^2 &=& -0.402 \to -0.446 \ \to -0.444 \end{array}}$$

2.4 Chiral Predictions for a_0^0 and a_0^2

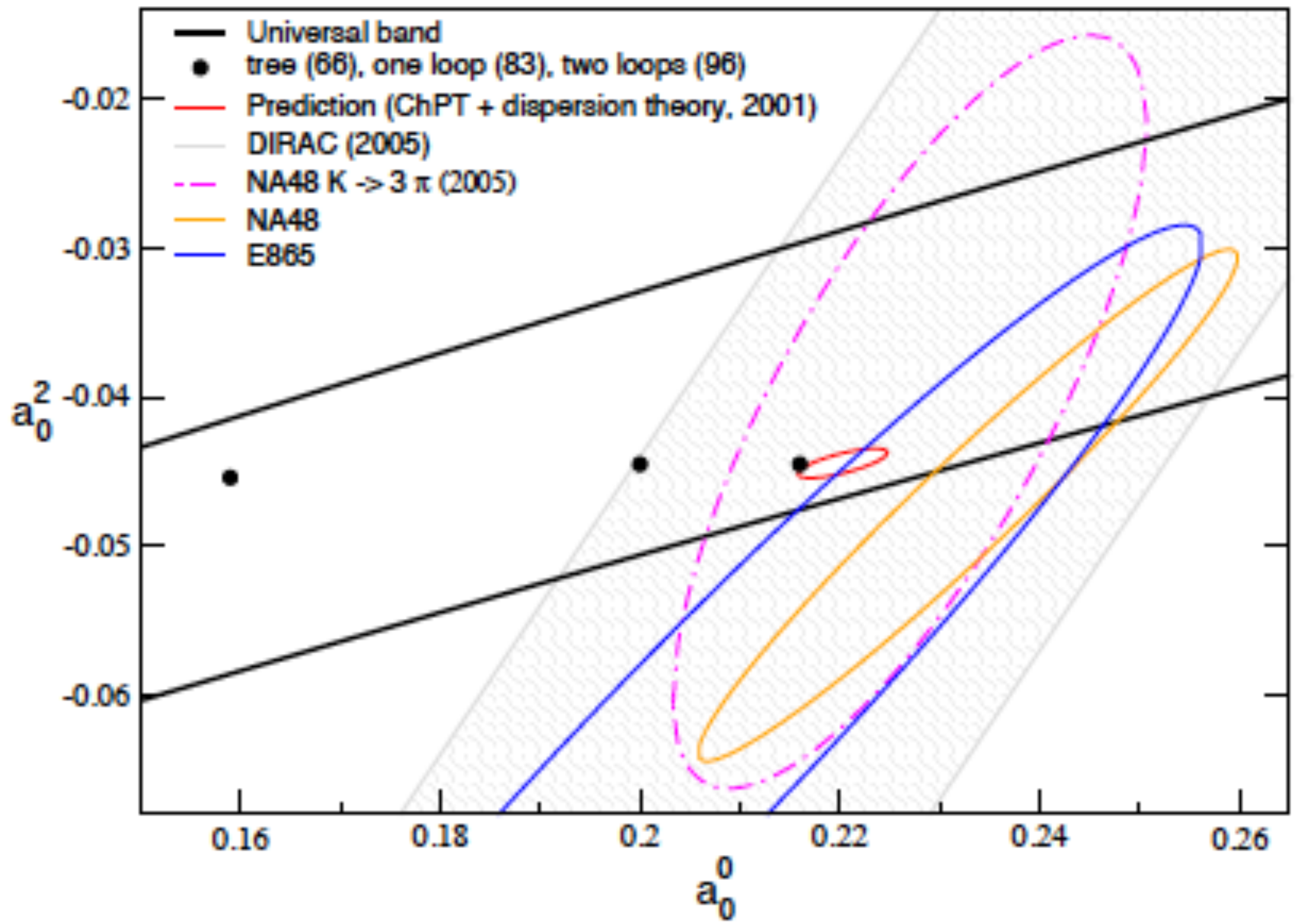


Colangelo, Gasser & Leutwyler'01

Where can we test these predictions?

- Production experiments πN → ππN, ψ → ππω, B → Dππ, . . .
- Extraction of ππ scattering amplitude is not simple
- Best accuracy in Kl4 data, K → 3π, ππ atoms

2.5 Experimental tests



H. Leutwyler

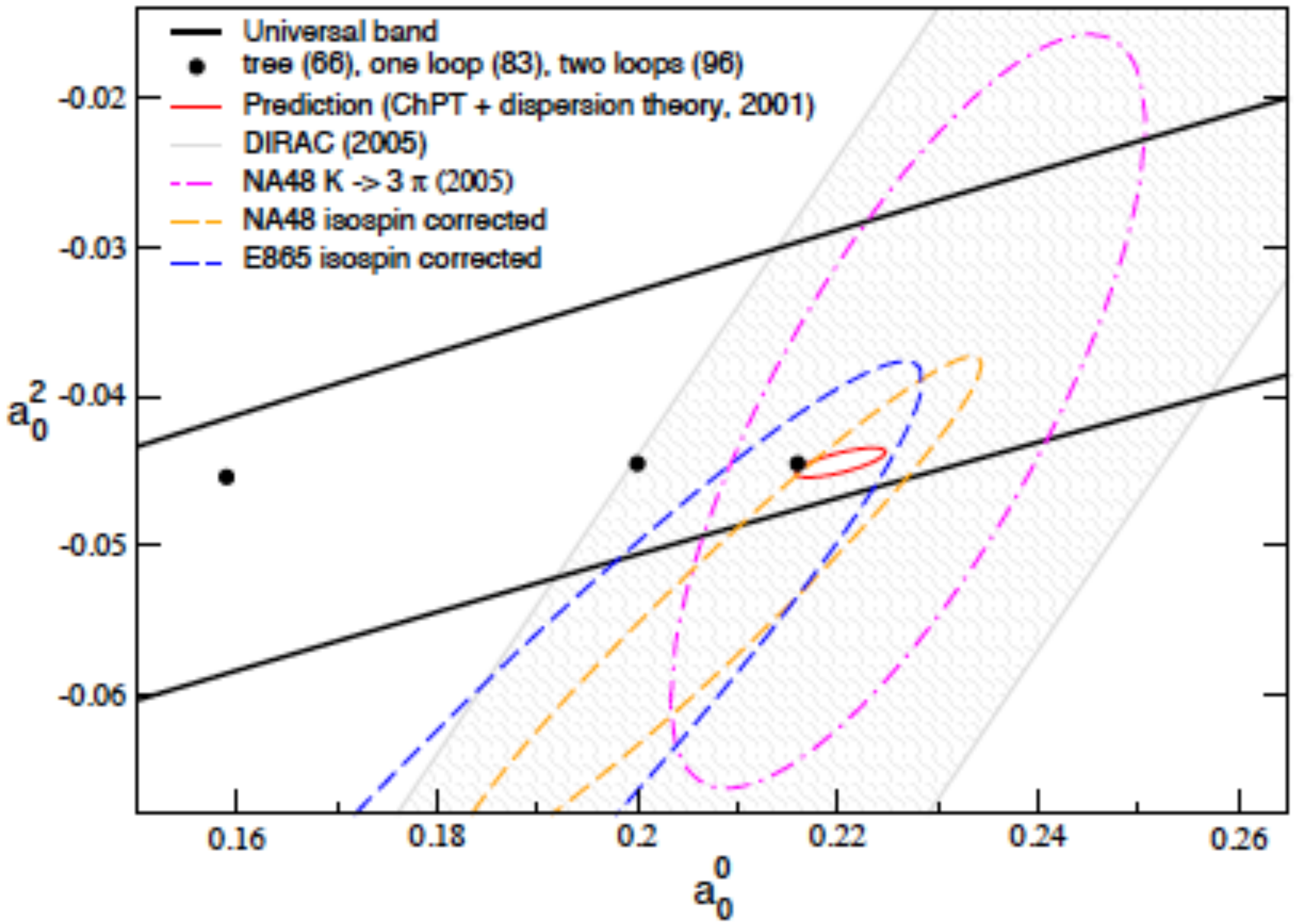
- With the new much more precise NA48 data it seemed that there was a disagreement ⇒ isospin breaking corrections

2.6 On the importance of isospin breaking corrections

- Isospin breaking computed recently
  ⇨ Perfect agreement!

Colangelo, Gasser, Rusetsky'09
Bernard, Descotes-Genon, Knecht '13

H. Leutwyler



2.7 $\pi\pi$ as a building block

- Extremely precise extraction of $\pi\pi$ scattering using ChPT and dispersion relations

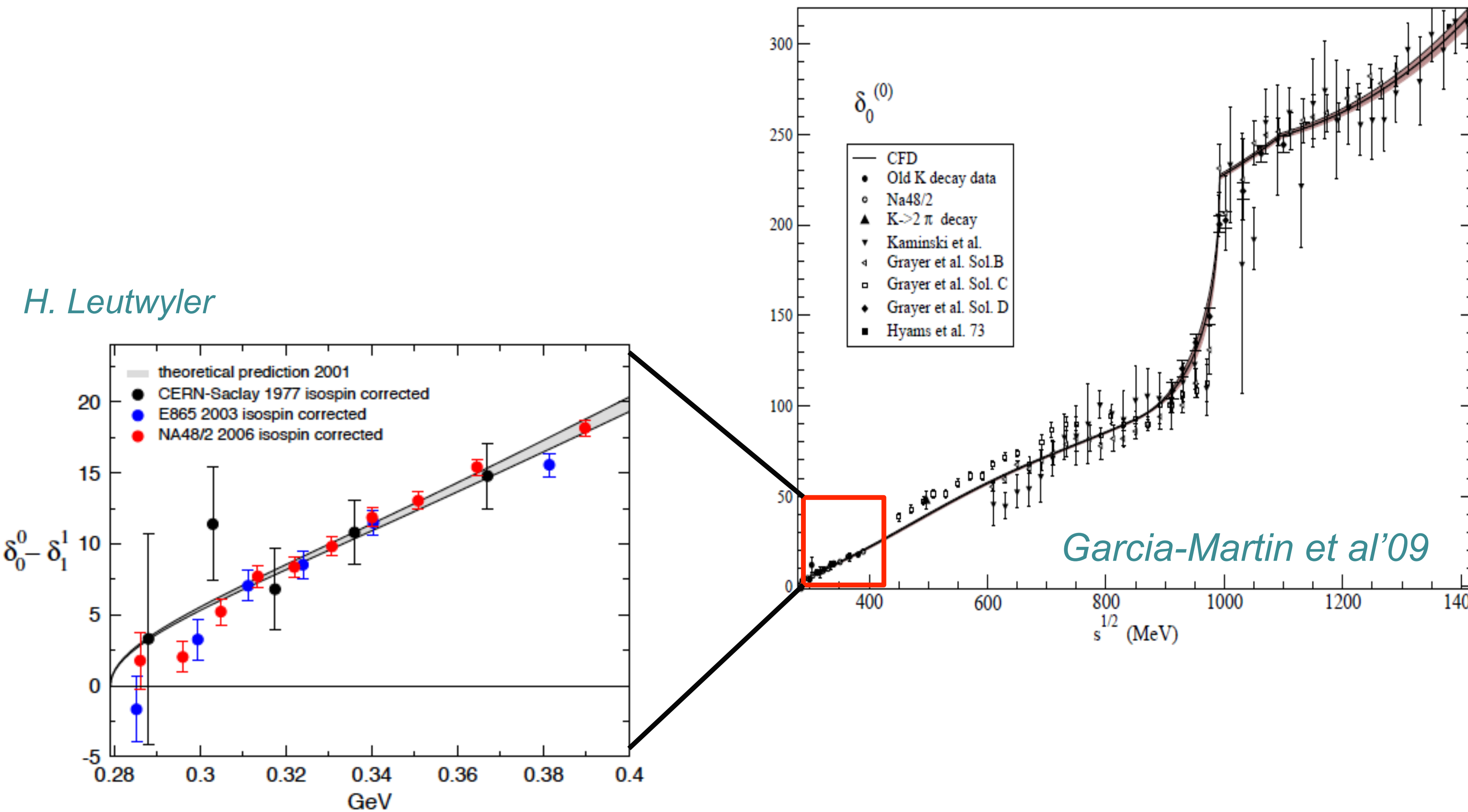
- Similar works done solving Roy-Steiner equations for
  - $K\pi$: *Buettiker, Descotes-Genon, Moussallam'07*
  - πN : *Hoferichter, Ruiz de Elvira, Kubis, Meißner'15*

- Compare to lattice results ⇒ see Talk by *J. Dudek*

- Use these as building blocks for phenomenology:
  - $\pi\pi$ rescattering: e.g., π form factors, $e^+e^- \to \pi\pi$, $\gamma\gamma \to \pi\pi$, $\omega/\varphi/\eta \to 3\pi$, $\tau \to 3\pi\nu_\tau$, $J/\Psi \to \gamma\pi^0\pi^0$. $B \to 3\pi$, $B \to J/\Psi\pi\pi$, etc.
  - $K\pi$ rescattering: e.g., $K\pi$ form factors, $K \to \pi\pi e\nu_e$, $\tau \to K\pi\nu_\tau$, $\tau \to K\pi\pi\nu_\tau$, $D \to K\pi\pi$, $B \to K\pi$

2.7 ππ as a building block

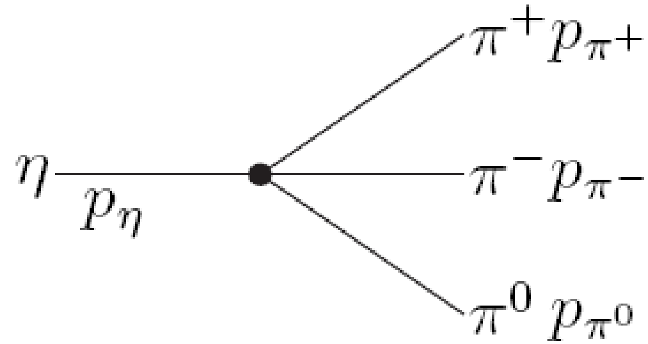
H. Leutwyler



Garcia-Martin et al'09

3. $\eta \to 3\pi$ and light quark masses

3.1 $\eta \to \pi^+ \pi^- \pi^0$



- Decay forbidden by isospin symmetry

  ⇨ $\boxed{A = (m_u - m_d) A_1 + \alpha_{em} A_2}$

- α_{em} effects are small

  Sutherland'66, Bell & Sutherland'68
  Baur, Kambor, Wyler'96, Ditsche, Kubis, Meissner'09

- Decay rate measures the size of isospin breaking ($m_u - m_d$) in the SM:

  $$L_{QCD} \to \boxed{L_{IB} = -\frac{m_u - m_d}{2}\left(\bar{u}u - \bar{d}d\right)}$$

  ⇨ Clean access to ($m_u - m_d$)

- $\boxed{\Gamma_{\eta \to 3\pi} \propto \int \left| A(s,t,u) \right|^2 \propto Q^{-4}}$ ⇨ $\boxed{Q^2 \equiv \frac{m_s^2 - \hat{m}^2}{m_d^2 - m_u^2}}$ $\left[\hat{m} \equiv \frac{m_d + m_u}{2}\right]$

3.2 ChPT

- Slow convergence of the chiral series (SU(3) ChPT)

$$\boxed{\Gamma_{\eta\to 3\pi} = \big(\underset{\text{LO}}{66} + \underset{\text{NLO}}{94} + \underset{\text{NNLO}}{100} + \ldots\big)\,\text{eV} = \big(300 \pm 12\big)\,\text{eV}}$$

PDG'14

LO: *Osborn, Wallace'70*

NLO: *Gasser & Leutwyler'85*

NNLO: *Bijnens & Ghorbani'07*

- CHPT amplitudes have problems with measured Dalitz plot distributions

- Main deficiency: strong $\pi\pi$ rescattering included only perturbatively

- Large $\pi\pi$ final state interactions

  ⇨ call for a dispersive treatment :
  - analyticity, unitarity and crossing symmetry
  - Take into account all the rescattering effects

- Match to CHPT amplitude to obtain Q from rates

3.3 Dispersive method

G. Colangelo, S. Lanz,
H. Leutwyler , E.P., *in progress*

- Decomposition of the amplitude as a function of ππ isospin states

$$M(s,t,u) = M_0(s) + (s-u)M_1(t) + (s-t)M_1(u) + M_2(t) + M_2(u) - \frac{2}{3}M_2(s)$$

Fuchs, Sazdjian & Stern'93

Anisovich & Leutwyler'96

  - ➢ M_I isospin *I* rescattering in two particles
  - ➢ Amplitude in terms of S and P waves ⇨ exact up to NNLO ($\mathcal{O}(p^6)$)
  - ➢ Main two body rescattering corrections inside M_I

- Unitary relation for $M_I(s)$:

$$disc\ M_I(s) = 2i\ \left(M_I(s) + \hat{M}_I(s)\right)\ \sin\delta_I(s) e^{-i\delta_I(s)} \theta\left(s - 4M_\pi^2\right)$$

right-hand cut   left-hand cut

3.3 Dispersive method

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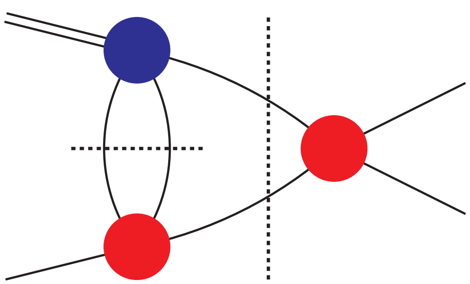


- Dispersion relation for the M_I's

$$\boxed{M_I(s) = \Omega_I(s)\left(P_I(s) + \frac{s^n}{\pi}\int_{4M_\pi^2}^{\infty}\frac{ds'}{s'^n}\frac{\sin\delta_I(s')\,\hat{M}_I(s')}{|\Omega_I(s')|\left(s'-s-i\varepsilon\right)}\right)}$$

$$\left[\Omega_I(s) = \exp\left(\frac{s}{\pi}\int_{4M_\pi^2}^{\infty} ds'\frac{\delta_I(s')}{s'(s'-s-i\varepsilon)}\right)\right]$$

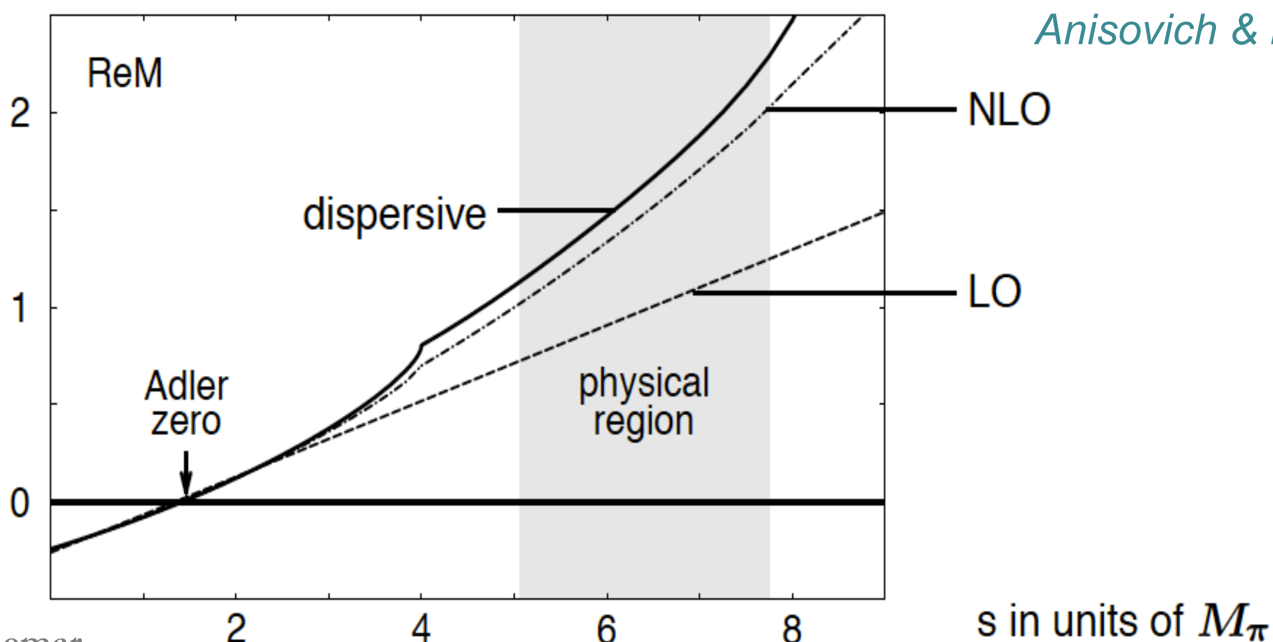
Omnès function

- $\hat{M}_I(s)$: singularities in the t and u channels, depend on the other $M_I(s)$
  subtract $M_I(s)$ from the partial wave projection of $M(s,t,u)$
  ⇨ Angular averages of the other functions ⇨ Coupled equations

3.4 Combining ChPT and dispersion relations

- As for $\pi\pi$, combine dispersion relations with ChPT where it works the best
- Use representation holding up to and including NNLO
  $\pi\pi$ partial-wave discontinuities for I = 0,1 only and I=0,1,2
- Interesting matching point: Adler zero
  The real part of the amplitude along the line s=u has a zero
  Chiral SU(2) prediction ⇨ small higher order corrections

Anisovich & Leutwyler'96



3.5 Different recent analyses

1. *Schneider, Kubis, Ditsche 2011*: 2-loop NREFT approach
   Allows investigation of isospin-violating corrections

2. *Kampf, Knecht, Novotny, Zdrahal 2011*: Analytic dispersive approach
   Match to absorptive part of NNLO chiral amplitude where differences between NLO and NNLO are small $\Rightarrow$ R (Q)
   Problem: do not reproduce the Adler's zero

3. *Guo et al. 2015: JPAC* analysis, Khuri-Treiman equations solved numerically using Pasquier inversion techniques
   - Madrid/Cracow ππ phase shifts, 3 subtraction constants
   - Match to NLO ChPT near Adler zero $\Rightarrow$ Q

   See talk by *V. Mathieu*

4. *Colangelo, Lanz, Leutwyler, E.P. in progress:* dispersive approach following *Anisovich, Leutwyler*
   - Electromagnetic effects to NLO fully taken into account (*Ditsche, Kubis, Meißner'09*)
   - Matching to one loop ChPT : Taylor expand the partial wave around s=0

3.6 Dalitz plot parameters: Charged channel

- Dalitz plot measurement of $\eta \to \pi^+ \pi^- \pi^0$

  Amplitude expanded in X and Y around X=Y=0

$$\boxed{\left|A_c(s,t,u)\right|^2 = N\left(1 + aY + bY^2 + dX^2 + fY^3\right)}$$

$$Q_c \equiv M_\eta - 2M_{\pi^+} - M_{\pi^0}$$

$$\boxed{X = \frac{\sqrt{3}}{2M_\eta Q_c}(u-t)} \qquad \boxed{Y = \frac{3}{2M_\eta Q_c}\left(\left(M_\eta - M_{\pi^0}\right)^2 - s\right) - 1}$$

C. Fernandez-Ramirez

	$-a$	b	d
KLOE 2015	1.095(4)	0.145(6)	0.081(7)
BESIII 2015	1.128(17)	0.153(17)	0.085(18)
WASA/COSY 2014	1.144(18)	0.219(51)	0.086(23)
NNLO CHPT	1.271(75)	0.394(102)	0.055(57)
NREFT	1.213(14)	0.308(23)	0.050(3)
JPAC	1.116(32)	0.188(12)	0.063(4)

See talks by *Daniel Lersch (WASA), Liqing Qin (BESSIII)*

3.7 Comparison of results for α: neutral decay

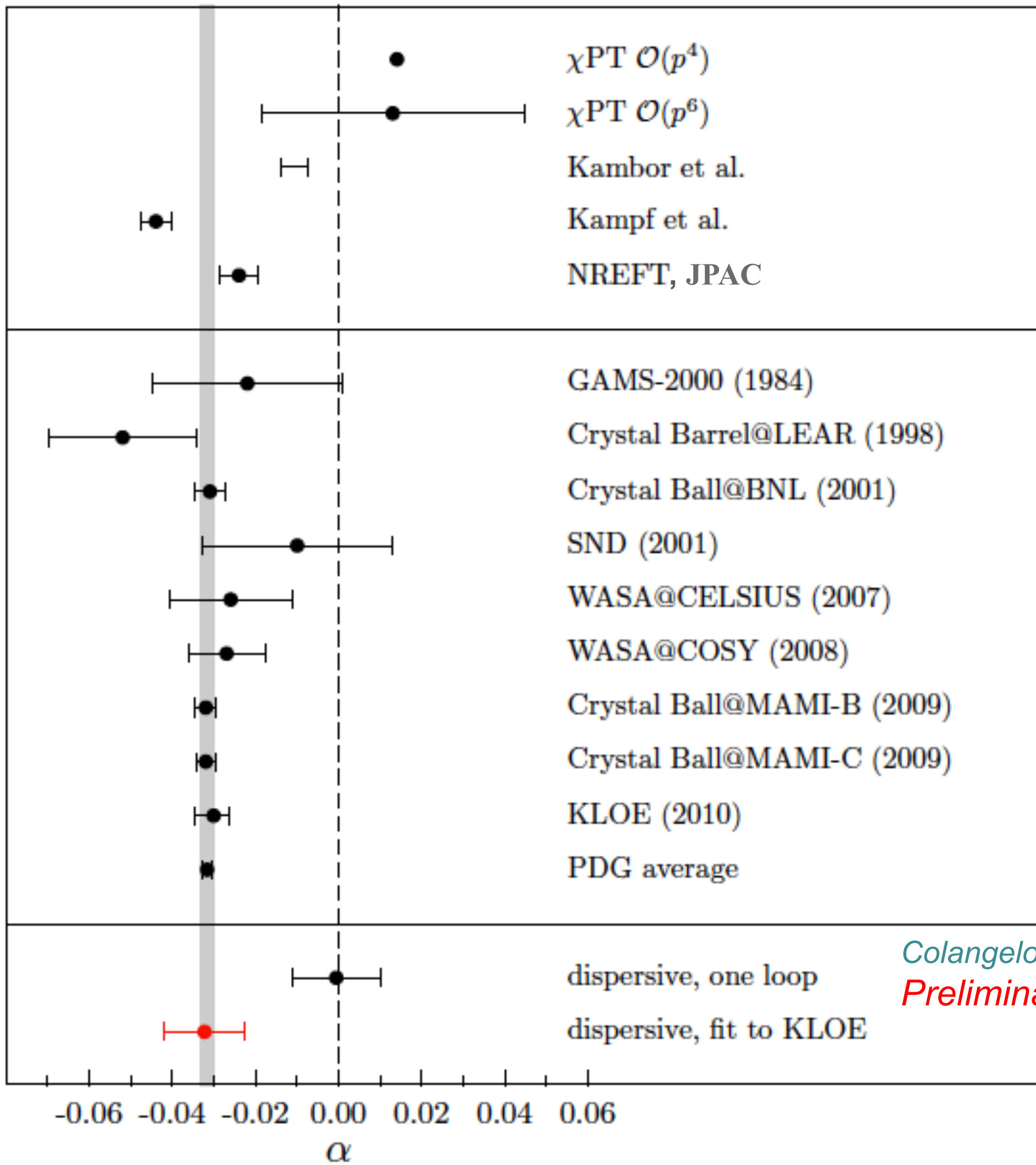
- Dalitz plot measurement of $\eta \to 3\pi^0$

$$\boxed{\left|A_n(s,t,u)\right|^2 = N(1 + 2\alpha Z)}$$

$$\boxed{Z = X^2 + Y^2}$$

$$Z = \frac{2}{3}\sum_{i=1}^{3}\left(\frac{3T_i}{Q_n} - 1\right)^2$$

$$Q_n \equiv M_\eta - 3M_{\pi^0}$$



Colangelo et al.
Preliminary

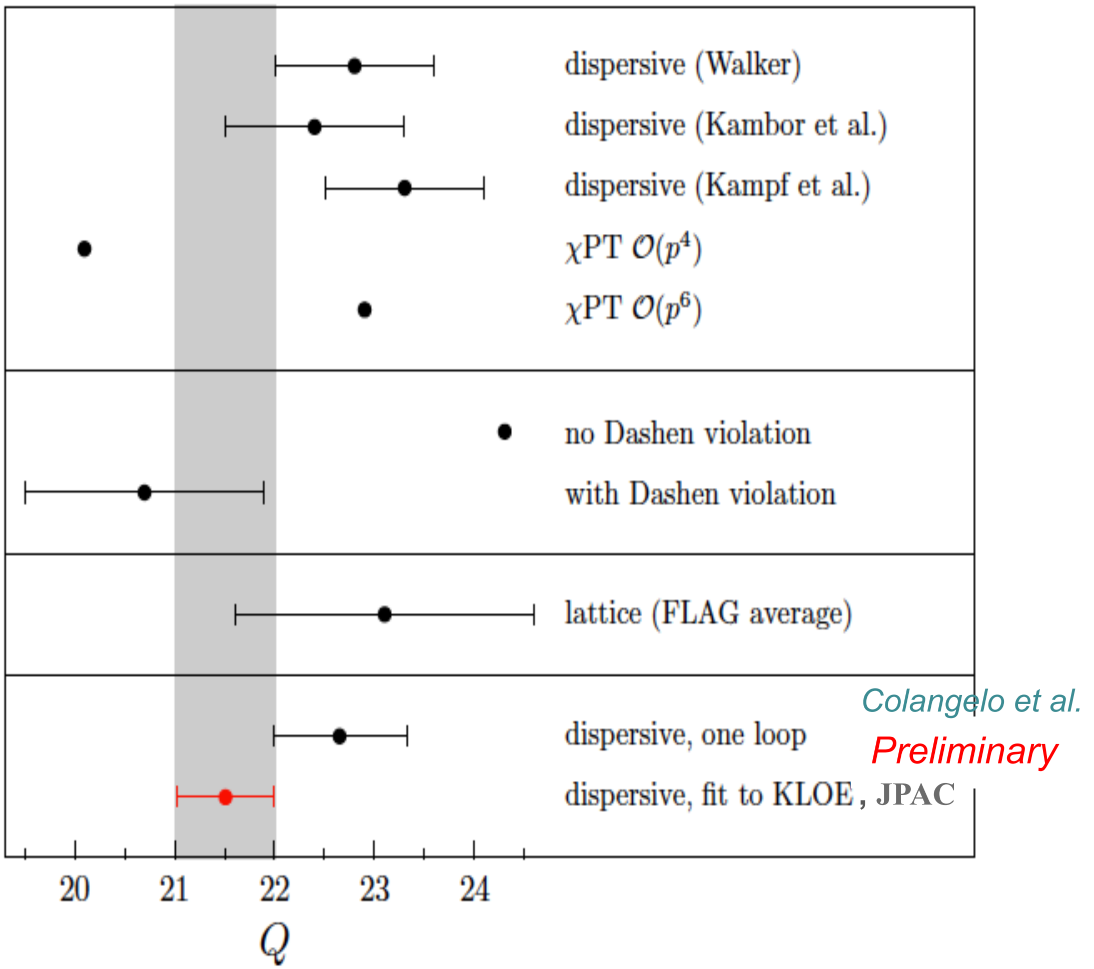
3.8 Quark mass ratio

$$\Gamma_{\eta\to 3\pi} \propto \int \left| A(s,t,u) \right|^2 \propto Q^{-4}$$

$$Q^2 \equiv \frac{m_s^2 - \hat{m}^2}{m_d^2 - m_u^2} \qquad \left[\hat{m} \equiv \frac{m_d + m_u}{2} \right]$$

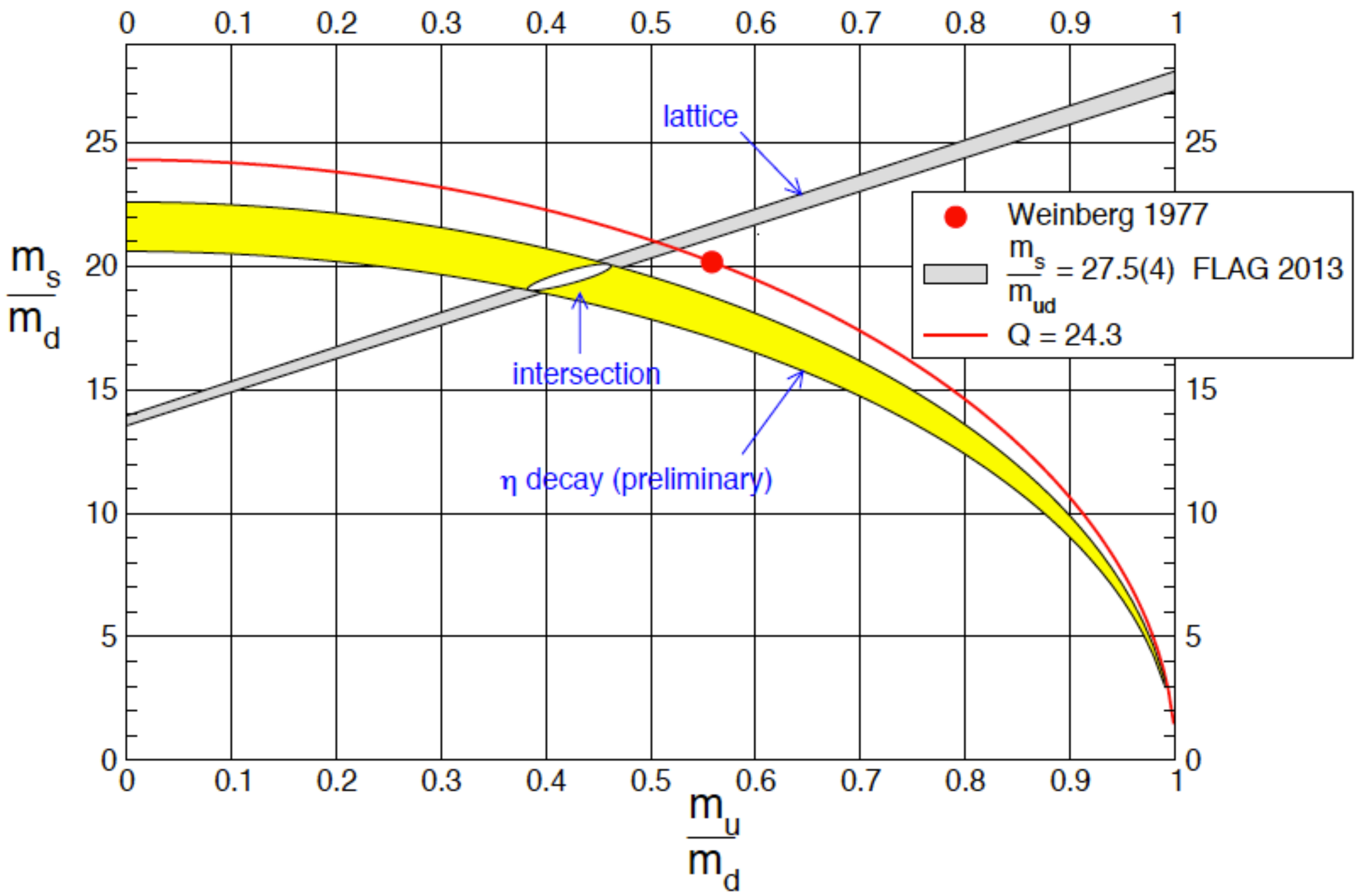
$$A(s,t,u) = \frac{N}{Q^2} M(s,t,u)$$

- M(s,t,u) determined through the dispersive analysis of the data but for N one has to rely on ChPT



3.9 $\eta \to 3\pi$ and light quark masses

H. Leutwyler

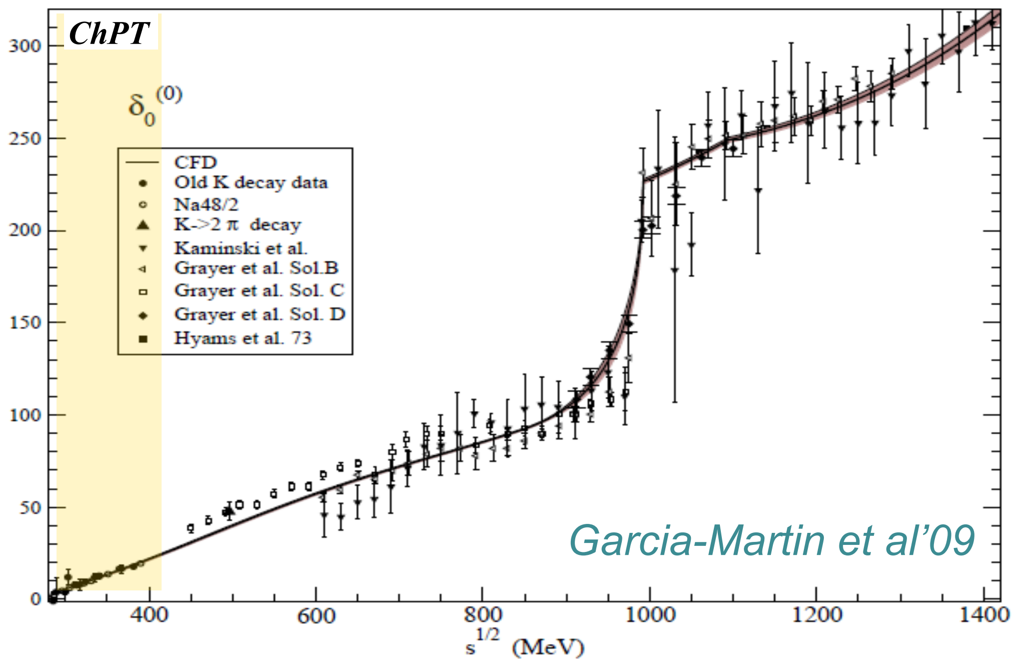


4. Conclusion and outlook

4.1 Conclusion

- ChPT is a very interesting tool at low energy
  - Model independent
  - Build amplitude using a power counting scheme

⇨ precise predictions in the meson sector



4.1 Conclusion

- ChPT is a very interesting tool at low energy
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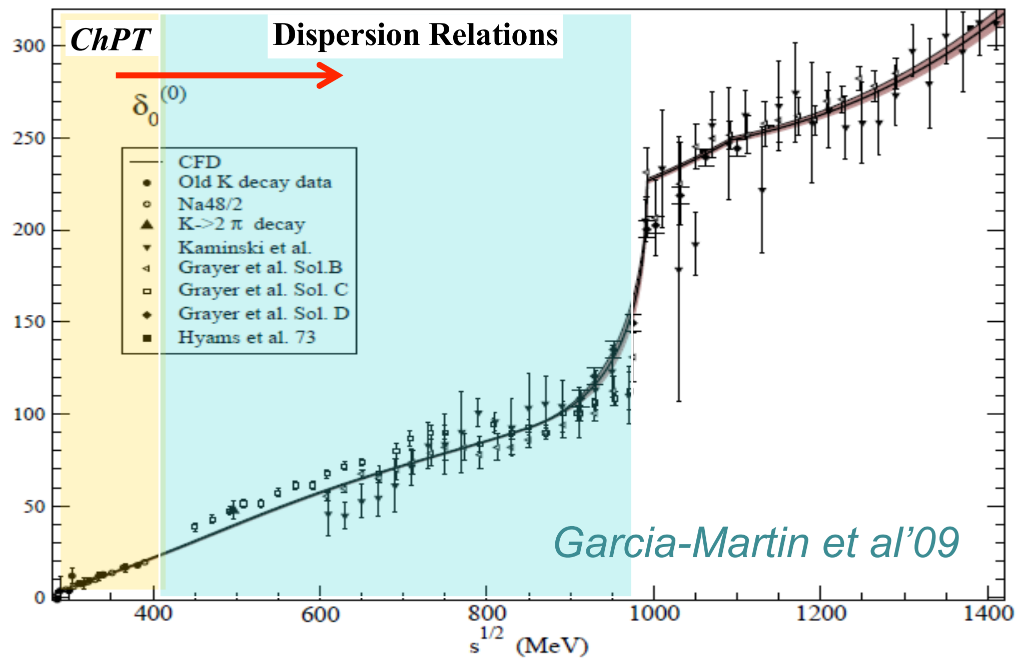
  ⇨ precise predictions in the meson sector

- But when one wants to go to higher energy or more precise prediction
  ⇨ nicely complement by dispersion relation: analyticity, unitarity, crossing Ex: ππ scattering, η → 3π



4.2 Outlook: Challenges for the future

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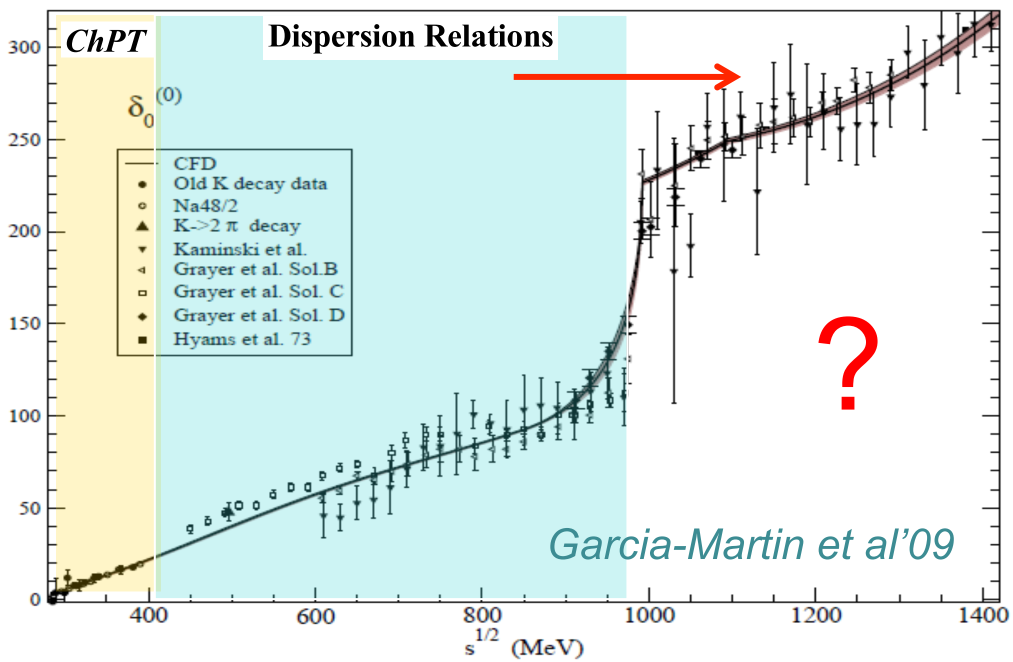
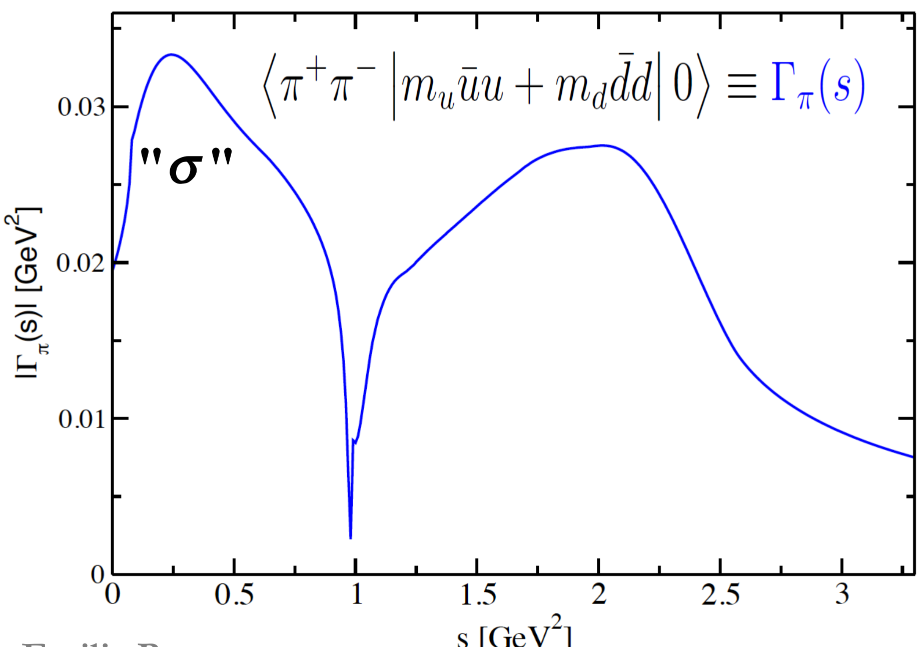


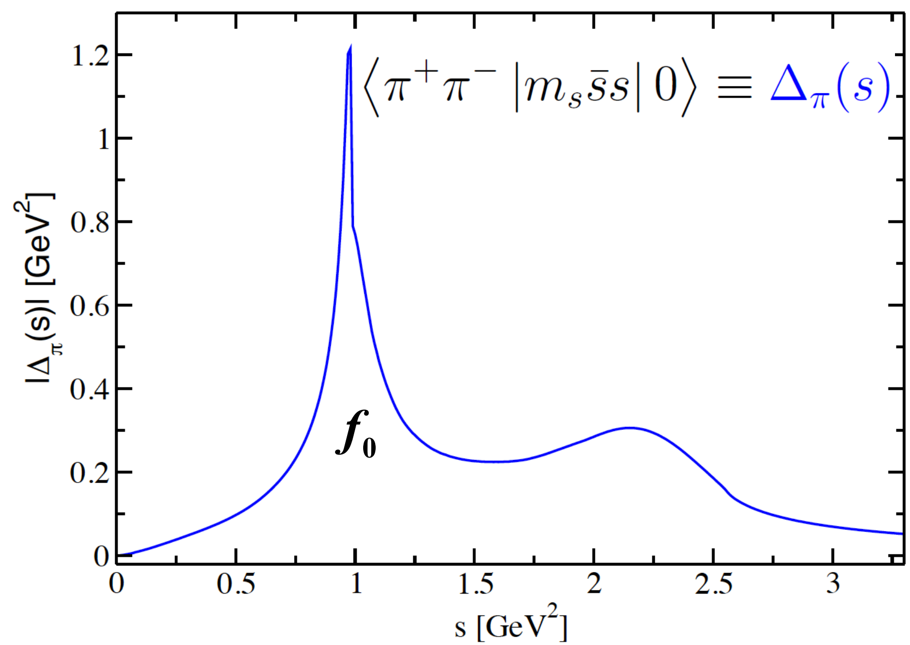
4.2 Outlook: Challenges for the future

- What can we do if one wants to explore the intermediate energy region: resonance appear ⇨ not taken into account by ChPT
  ⇨ Include coupled channels:
  - Analytically: e.g. dispersive approach, unitary coupled channel (*Ed Berger* talk)
    Ex: ππ scalar form factors (ππ and KK), *Donoghue, Gasser, Leutwyler'90, Moussallam'99, Daub et al'13, Celis, Cirigliano, E.P.'14*

    ⇨ see *Christoph Hanhart*'s talk

Celis, Cirigliano, E.P.'14





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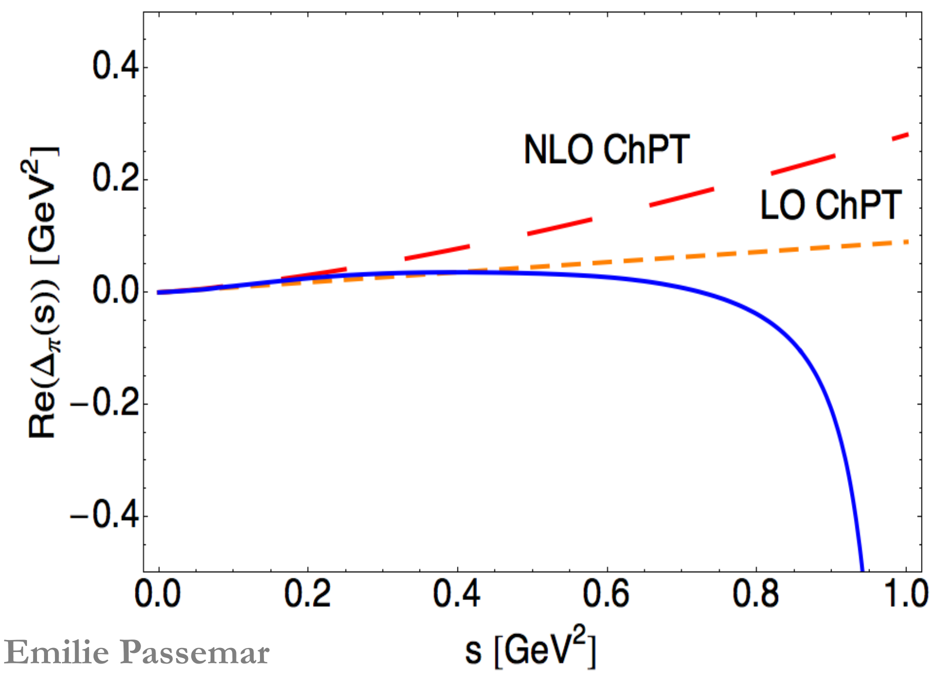
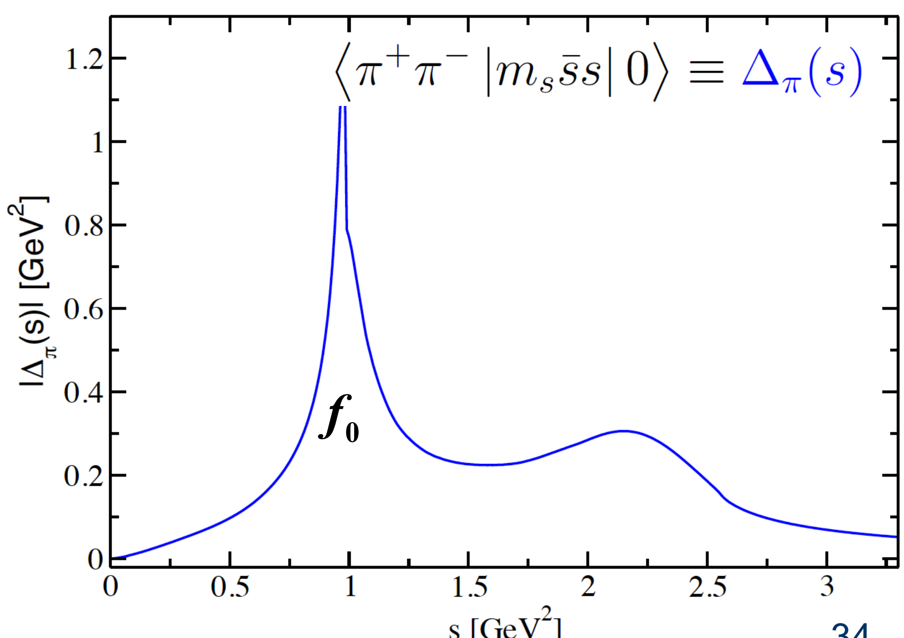
Celis, Cirigliano, E.P.'14





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    ⇨ see Christoph *Hanhart*'s talk)

    γγ → ππ, KK *Dai & Pennington'13,* η → 3π *Albaladejo & Moussallam'15*

  – Lattice QCD: see *Jo Dudek*'s talk

    Ex: ππ, KK scattering, P wave *Wilson, Briceno, Dudek, Edwards, Thomas'15*

    Kπ and Kη scattering *Hadron spectrum, Wilson, Dudek, Edwards, Thomas'14*

  ⇨ Include resonances: see talk by *Ed Berger*

  – RChPT, Unitarized ChPT e.g., *Pelaez, Oller, Oset'99*

  – Lattice with Unitarized ChPT *Bolton, Briceno, Wilson'15*

- If one wants to explore the full Dalitz plot in B, D decays ⇨ N/D *Oller and Oset'98*

see talk by *A. Jackura*

5. Back-up

4.1 Precision Physics at intermediate energies

- ChPT is a very interesting tool at low energy
  - Model independent
  - Rely on unitarity, analyticity
  - Build amplitude using a power counting scheme

  ⇨ precise predictions in the meson sector but unknowns LECs to be determined

- But when one wants to go to higher energy or more precise prediction
  ⇨ nicely complement by dispersion relation

  Ex: $\pi\pi$ scattering, $\eta \to 3\pi$

- Still if one wants to explore the intermediate energy region: resonance appear ⇨ not taken into account by ChPT
  - For $\pi\pi$: *I=1*: $\rho(770)$, $\rho(1450)$, $\rho(1700)$, …, *I=0*: "$\sigma(\sim500)$", $f_0(980)$,…
  - For $K\pi$: *I=1*: $K^*(892)$, $K^*(1410)$, $K^*(1680)$, …, *I=0*: "$\kappa(\sim800)$", …

1.4 Construction of an effective theory: ChPT

- Degrees of freedom: Goldstone bosons (GB)

  Symmetry group: $\boxed{G \equiv SU(3)_L \otimes SU(3)_R}$

- Build all the corresponding invariant operators including explicit symmetry breaking parameters

  ⇨ $\boxed{L_{ChPT} \equiv L\left(U, \chi\right)}$

  GB's   Masses ~ m_q

- Goldstone bosons interact weakly at low energy and m_u, m_d = $m_s < \Lambda_{QCD}$

  ⇨ expansion organized in external momenta and quark masses

Weinberg's power counting rule

$$\boxed{\mathcal{L}_{eff} = \sum_{d \geq 2} \mathcal{L}_d \;, \mathcal{L}_d = \mathcal{O}\left(p^d\right) \;, p \equiv \left\{q, m_q\right\}}$$

$$\boxed{\mathrm{p} \ll \Lambda_H = 4\pi F_\pi \sim 1\ \mathrm{GeV}}$$

2.1 Low energy constants

- Recent fit by Ecker and Bijnens of NLO LECs Li (i = 1,…,10) and NNLO LECs Ci (i = 1,…,90)
  Update and extension of Bijnens, Jemos 2012

- New ingredients:
  - relations li (Li ; Ci) (j = 1,…, 4) Gasser, Haefeli, Ivanov, Schmid 2007
    altogether 17 input data
  - penalize bad convergence of meson masses
  - intelligent guesses (priors) for 34 (combinations of the) Ci
  - renormalization scale μ = 0.77 GeV

- Fitting procedure:
  - minimization/random walk in restricted Ci –space
  - iterate after possible modication of Ci –space
  - normal χ2 fit for Li for (fixed) “best" values of the Ci
    “best" values for Li

2.1 Low energy constants

	NNLO free fit	NNLO BE14	NLO 2014	GL 1985
$10^3 L_A^r$	0.68(11)	0.24(11)	0.4(2)	
$10^3 L_1^r$	0.64(06)	0.53(06)	1.0(1)	0.7(3)
$10^3 L_2^r$	0.59(04)	0.81(04)	1.6(2)	1.3(7)
$10^3 L_3^r$	−2.80(20)	−3.07(20)	−3.8(3)	−4.4(2.5)
$10^3 L_4^r$	0.76(18)	0.3	0.0(3)	−0.3(5)
$10^3 L_5^r$	0.50(07)	1.01(06)	1.2(1)	1.4(5)
$10^3 L_6^r$	0.49(25)	0.14(05)	0.0(4)	−0.2(3)
$10^3 L_7^r$	−0.19(08)	−0.34(09)	−0.3(2)	−0.4(2)
$10^3 L_8^r$	0.17(11)	0.47(10)	0.5(2)	0.9(3)
F_0 [MeV]	64	71		

- Strong sensitivity to (large-Nc) suppressed L_4
  ⇨ enforce small L4 (supported by lattice), $10^3 L^r_4 = 0.3$;
  NLO: $0.3 \leq 10^3 L^r_4 \leq 0.3$ fixed → $L_A = 2 L_1 - L_2$ and L_6 automatically suppressed
- NNLO only makes sense with certain set of Cr
- Except for last column: no estimate of higher-order uncertainties

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F_0 [MeV]	64	71		

- Reasonable convergence of observables (enforced for masses)
- Qualitative evidence for resonance saturation, even for scalars
- Last 3 columns: good stability

1.4 Construction of an effective theory: ChPT

- Degrees of freedom: Goldstone bosons (GB)

  Symmetry group: $\boxed{G \equiv SU(3)_L \otimes SU(3)_R}$

- Build all the corresponding invariant operators including explicit symmetry breaking parameters

⇨ $\boxed{\mathcal{L}_{ChPT} \equiv \mathcal{L}^0_{QCD} + \mathcal{L}_m}$ with $\mathcal{L}_m = -\bar{q}\mathcal{M}q$, $\mathcal{M} = \mathrm{diag}\left(m_u, m_d, m_s\right)$

- Goldstone bosons interact weakly at low energy and $m_u, m_d = m_s < \Lambda_{QCD}$

  ⇨ expansion organized in external momenta and quark masses

Weinberg's power counting rule

$$\boxed{\mathcal{L}_{eff} = \sum_{d\geq 2} \mathcal{L}_d \; , \; \mathcal{L}_d = \mathcal{O}\left(p^d\right) \; , p \equiv \left\{q, m_q\right\}}$$

$$\boxed{\mathrm{p} << \Lambda_H = 4\pi F_\pi \sim 1 \text{ GeV}}$$

1.4 Chiral expansion

- $$\mathcal{L}_{ChPT} = \underbrace{\mathcal{L}_2}_{\text{LO}: \mathcal{O}(p^2)} + \underbrace{\mathcal{L}_4}_{\text{NLO}: \mathcal{O}(p^4)} + \underbrace{\mathcal{L}_6}_{\text{NNLO}: \mathcal{O}(p^6)} + \cdots$$

- Renormalizable and unitary order by order in the expansion

- The structure of the lagrangian is fixed by chiral symmetry but not the coupling constants → LECs appearing at each order

$$\mathcal{L}_2 : \; F_0, B_0, \qquad \mathcal{L}_4 = \sum_{i=1}^{10} L_i \, O_4^i, \qquad \mathcal{L}_6 = \sum_{i=1}^{90} C_i \, O_6^i$$

- LECs describe the influence of heavy degrees of freedom not contained in the ChPT lagrangian

- Naturalness: LECs of order one

2.2 Pion polarizabilities: success of ChPT at NNLO

- New ingredients:
  - relations li (Li ; Ci) (j = 1,…, 4) Gasser, Haefeli, Ivanov, Schmid 2007
    altogether 17 input data
  - penalize bad convergence of meson masses
  - intelligent guesses (priors) for 34 (combinations of the) Ci
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- Fitting procedure:
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    “best" values for Li

1.5 ChPT in the meson sector: precision calculations

- $$\mathcal{L}_{ChPT} = \underbrace{\mathcal{L}_2}_{\text{LO}\,:\,\mathcal{O}(p^2)} + \underbrace{\mathcal{L}_4}_{\text{NLO}\,:\,\mathcal{O}(p^4)} + \underbrace{\mathcal{L}_6}_{\text{NNLO}\,:\,\mathcal{O}(p^6)} + \ldots.$$

- The structure of the lagrangian is fixed by chiral symmetry but not the coupling constants → LECs appearing at each order

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- LECs describe the influence of heavy degrees of freedom not contained in the ChPT lagrangian

- Naturalness: LECs of order one

- Today's standard in the meson sector: 2-loop calculations

- Main obstacle to reaching high precision: determination of the LECs

2.4 Chiral Predictions for

- ChPT : Calculations at
  – NLO
  – NNLO

- Prediction obtained matching O(p6) χPT to Roy equations (disp. relations)

$$\begin{aligned} a_0^0 &= 0.220 \pm 0.001 + 0.009\Delta\ell_4 - 0.002\Delta\ell_3 \\ 10 \cdot a_0^2 &= -0.444 \pm 0.003 - 0.01\Delta\ell_4 - 0.004\Delta\ell_3 \end{aligned}$$

$$\bar{\ell}_4 = 4.4 + \Delta\ell_4 \qquad \bar{\ell}_3 = 2.9 + \Delta\ell_3 \qquad [\Delta\ell_4 = 0.2,\ \Delta\ell_3 = 2.4]$$

- Adding the uncertainties in quadrature:

$$\boxed{\begin{aligned} a_0^0 &= 0.220 \pm 0.005 \\ 10 \cdot a_0^2 &= -0.444 \pm 0.01 \\ a_0^0 - a_0^2 &= 0.265 \pm 0.004 \end{aligned}}$$

3.3 Different recent analyses

1. *Schneider, Kubis, Ditsche 2011*: 2-loop NREFT approach
   - allows investigation of isospin-violating corrections
   - relations between charged and neutral Dalitz plots

2. *Kampf, Knecht, Novotny, Zdrahal 2011*: Analytic dispersive approach
   - Amplitudes involve 6 parameters (subtraction constants)
   - Fit to Dalitz plot distribution (KLOE 2008: $\eta \to \pi^+\pi^-\pi^0$)
   - Predict Dalitz plot parameter α (neutral decay mode)
   - Match to absorptive part of NNLO chiral amplitude where differences between NLO and NNLO are small $\Rightarrow$ R (Q)

   Problem: do not reproduce the Adler's zero

3.3 Different recent analyses

3. *Guo et al. 2015: JPAC* analysis, Khuri Treiman equations solved numerically using Pasquier inversion techniques
   - Madrid/Cracow ππ phase shifts, 3 subtraction constants
   - Fit experimental Dalitz plot (*WASA/COSY* 2014: $\eta \to \pi^+\pi^-\pi^0$)
     ⇨ predict Dalitz plot parameter α
   - Match to NLO ChPT near Adler zero ⇨ Q

4. *Colangelo, Lanz, Leutwyler, E.P. in progress:* dispersive approach following *Anisovich, Leutwyler*
   - Electromagnetic effects to NLO fully taken into account (*Ditsche, Kubis, Meißner'09*)
   - Dispersive amplitudes: Bern ππ phase shifts, 6 subtraction constants
   - Fit similtanously Charged (*WASA*, *KLOE*) and Neutral Dalitz plots (*MAMI*)
   - Matching to one loop ChPT : Taylor expand the partial wave around s=0

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3.4 Dalitz plot parameters

- Dalitz plot measurement : Amplitude expanded in X and Y around X=Y=0

$$\left|A_c(s,t,u)\right|^2 = N\left(1 + aY + bY^2 + dX^2 + fY^3\right)$$

$$X = \frac{\sqrt{3}}{2M_\eta Q_c}(u-t)$$

$$Y = \frac{3}{2M_\eta Q_c}\left(\left(M_\eta - M_{\pi^0}\right)^2 - s\right) - 1$$

$$\left|A_n(s,t,u)\right|^2 = N\left(1 + 2\alpha Z + 6\beta Y\left(X^2 - \frac{Y^2}{3}\right) + 2\gamma Z^2\right)$$

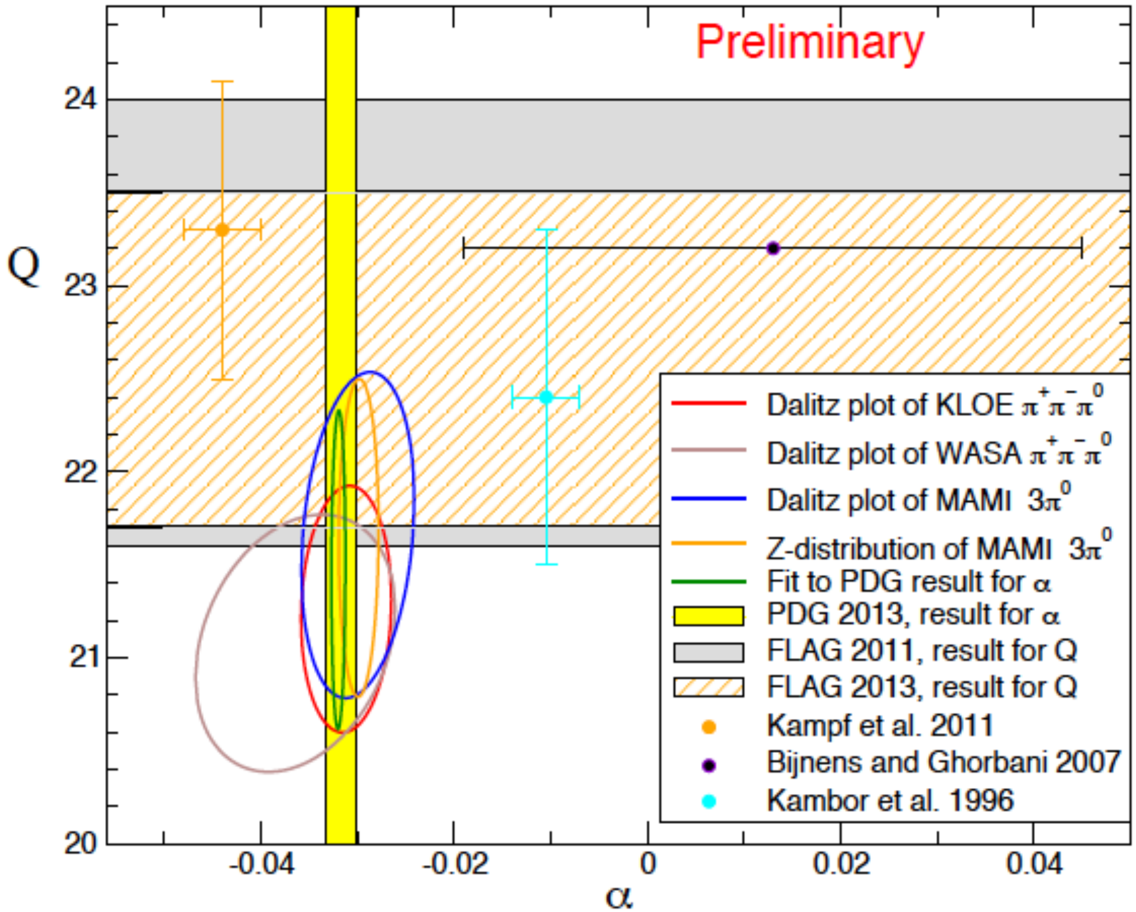
$$Z = X^2 + Y^2$$

$$Q_c \equiv M_\eta - 2M_{\pi^+} - M_{\pi^0}$$

	$-a$	b	d	α
KLOE 2015	1.095(4)	0.145(6)	0.081(7)	
BESIII 2015	1.128(17)	0.153(17)	0.085(18)	−0.055(15)
WASA/COSY 2014	1.144(18)	0.219(51)	0.086(23)	
NNLO CHPT	1.271(75)	0.394(102)	0.055(57)	0.013(32)
KKNZ				−0.044(4)
NREFT	1.213(14)	0.308(23)	0.050(3)	−0.025(5)
JPAC	1.116(32)	0.188(12)	0.063(4)	−0.022(4)
PDG 2014				−0.0315(15)

3.3 Qualitative results of our analysis

- Plot of Q versus α :



NB: Isospin breaking has not been accounted for

From kaon mass spliting :

$$\boxed{Q = 20.7 \pm 1.2}$$

Kastner & Neufeld'08

- All the data give consistent results. The preliminary outcome for Q is intermediate between the lattice result and the one of Kastner and Neufeld.

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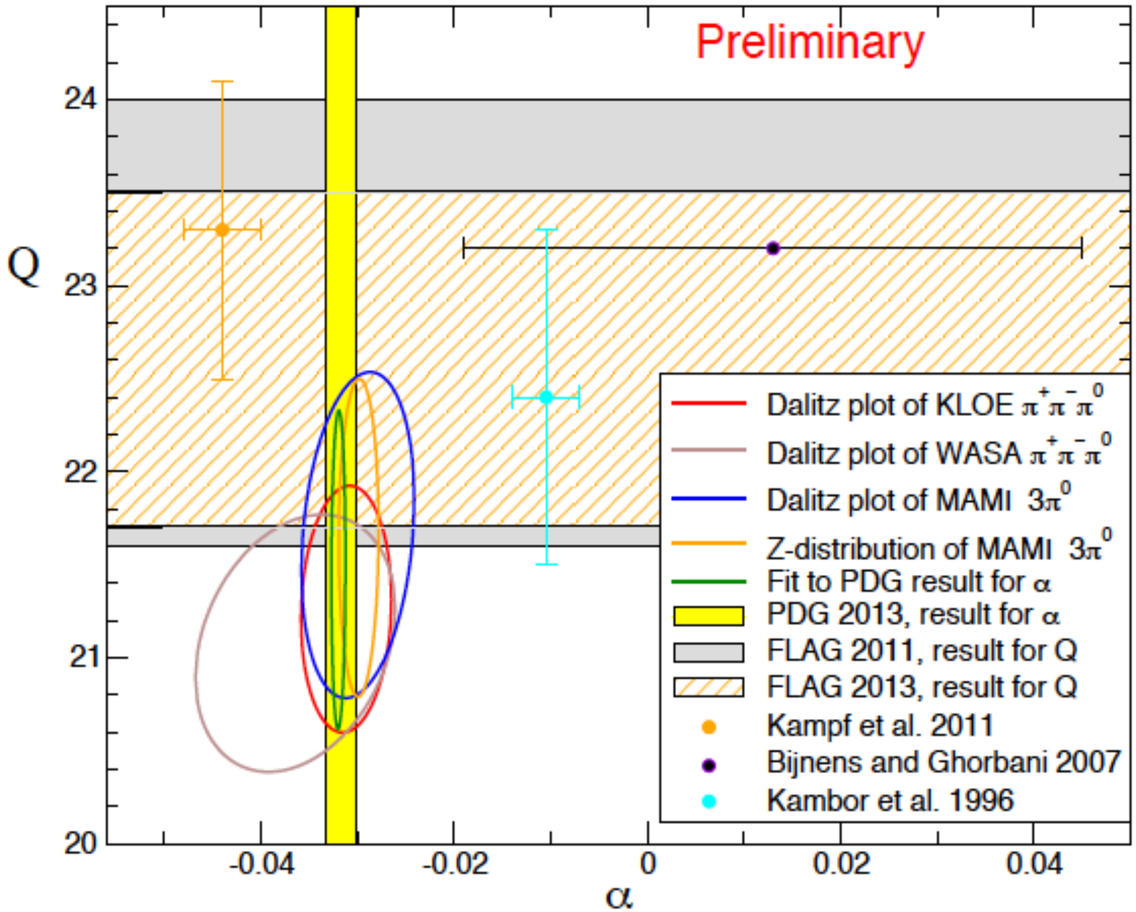

NB: Isospin breaking has not been accounted for

- All our preliminary results give a negative value for α. In particular the result using KLOE data for $\eta \to \pi^+ \pi^- \pi^0$ is in perfect agreement with the PDG value!