Compositeness of near-threshold quasi-bound states

14 September 2015 @ Hadron 2015
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Contents

- Introduction ~compositeness of bound state~
- Extension to the quasi-bound state
- Applications to exotic hadrons ~ $\Lambda(1405), a_0(980), f_0(980)$ ~
- Conclusions
Exotic hadrons

Hadrons which do not coincide with the predictions of the quark model. More complicated internal structure can be expected.

- tetra quark, penta quark
- hadron molecule …

It is important to reveal the internal structure of exotics.

\[ \Lambda(1405) \]

\[ \Lambda \text{ state}(uds) \]

\[ \bar{K}N \text{ bound state} \]

Compositeness of bound state

Previous work
Introduced to study deuteron by Weinberg.
S. Weinberg, Phys. Rev. 137, B672 (1965)

Condition
• weakly bound
• stable state
• s-wave

Output
• \( X \); weight of composite state \((0 < X < 1)\)
• \( Z \); wave function renormalization \((0 < Z < 1)\)
• \( a_0 \); scattering length
• \( B \); binding energy

\[
R = \frac{1}{\sqrt{2\mu B}}
\]
\((\mu \); reduced mass of scat. state)
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Extension to the quasi-bound state.

**Setting**

- The state is weakly bound from channel 1.
- The state decays to channel 2.
- We consider the compositeness of channel 1; $X$.
- We write probability to find channel 2 or bare state as $Z$.

**Problem**

- How does the existence of decaying channel affect the compositeness relation?
- When can we use the relation?

$$a_0 = R \left\{ \frac{2X}{1+X} + \mathcal{O} \left( \frac{R_{typ}}{R} \right) \right\}$$
Extension to the quasi-bound state.

**Extension of the system**

To estimate the contribution of decaying mode, we use EFT which contains scattering and discrete states.

**Bound state**

\[ |p\rangle \text{ scattering channel} \]

\[ |B_0\rangle \text{ discrete channel} \]

**Quasi-bound state**

\[ |p\rangle \]

\[ |p'\rangle \text{ decaying channel} \]

\[ \nu \]

\[ \mathcal{H}_{\text{int}} = \]

\[ H|QB\rangle = E_{QB}|QB\rangle \]

\[ E_{QB} = -B - i\frac{\Gamma}{2} \text{ ; complex} \]


\[ H|B\rangle = E_B|B\rangle \]

\[ E_B = -B \text{ ; real} \]
Extension to the quasi-bound state.

Definition of compositeness

Bound state

The probabilistic interpretation is valid for $X$ and $Z$.

Quasi-bound state

To normalize unstable state, we introduce Gamow state $|QB\rangle$.

Normalization condition becomes

$$\langle QB|QB \rangle = \langle QB^*|QB \rangle = 1.$$ 


The expectation value of the any operator becomes complex number.

- $X + Z = 1$
- $0 < X, Z < 1$, $X, Z \in \mathbb{C}$

The probabilistic interpretation is not valid!
Extension to the quasi-bound state.

Assuming $|E_h|$ is small, we expand $a_0$ with respect to $1/R$.

$$a_0 = R \left\{ \frac{2X}{1+X} + \mathcal{O} (|R_{\text{typ}}/R|) + \sqrt{\frac{\mu^3}{\mu^3}} \mathcal{O} (|l/R|^3) \right\}$$

original

new

If $|R_{\text{typ}}/R|$ and $|l/R|^3$ are sufficiently smaller than 1, we can extract $X$ from $a_0$ and $E_{QB}$.

Notice

- $a_0$, $E_h$, $X$ are all complex numbers, then above relation is established among them.
- If the the contribution of decaying mode is neglected, the compositeness relation is same to the one for bound state.
- The same argument is valid for the case with $\text{Re} \ E_h > 0$. 

Interpretation of $X$

$X$ is a complex number.

- close to bound state case
  \[
  \begin{cases} 
  X = 0.8 - 0.1i \\
  Z = 0.2 + 0.1i 
  \end{cases}
  \]
  The probabilistic interpretation is seemed to be possible.

- When real part is not in $[0,1]$
  \[
  \begin{cases} 
  X = 1.9 - 0.2i \\
  Z = -0.9 + 0.2i 
  \end{cases}
  \]

- When imaginary part is large.
  \[
  \begin{cases} 
  X = 0.9 - 0.8i \\
  Z = 0.1 + 0.8i 
  \end{cases}
  \]
  It is difficult to interpret $X$ as a probability.

Is there any good prescription to interpret the complex value?
Interpretation of $X$

$X$ is a complex number.

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  \begin{cases}
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  small cancellation in $X+Z$

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  \]
  large cancellation in $X+Z$

Is there any good prescription to interpret the complex value?
Our proposal

For probabilistic interpretation we define the following real quantities.

\( \tilde{X} \); probability to find the scattering state in physical state

\( \tilde{Z} \); probability to find the other states

\( U \); degree of uncertainty

conditions:

- \( \tilde{X} + \tilde{Z} = 1 \)
- \( 0 \leq \tilde{X}, \tilde{Z} \leq 1 \)
- When there is no cancelation,
  \( \tilde{X} = X, \tilde{Z} = Z, U = 0 \).
- \( U \) becomes large when the cancelation becomes large.

\[
\tilde{X} \equiv \frac{1 - |Z| + |X|}{2}
\]
\[
\tilde{Z} \equiv \frac{1 - |X| + |Z|}{2}
\]
\[
U \equiv |Z| + |X| - 1
\]

If \( U \) is small, we can interpret \( \tilde{X} \) as the compositeness.


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Compositeness of exotics

\( \Lambda(1405) \) in \( I = 0 \) \( \bar{K}N \) scattering

We use \( E_{QB} \) and \( a_0 \) in the following papers.


<table>
<thead>
<tr>
<th>Ref.</th>
<th>( E_{QB} ) (MeV)</th>
<th>( a_0 ) (fm)</th>
<th>( X )</th>
<th>( \tilde{X} )</th>
<th>( U )</th>
<th>( \frac{R_{\text{typ}}}{R} )</th>
<th>( \frac{l}{R} )</th>
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<td>1.39-i0.85</td>
<td>1.3+i0.1</td>
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<td>0.5</td>
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<td>(3)</td>
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<tr>
<td>(4)-2</td>
<td>-3-i12</td>
<td>1.52-i1.85</td>
<td>1.0+i0.5</td>
<td>0.8</td>
<td>0.6</td>
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</tr>
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- \( |R_{\text{typ}}/R|, |l/R|^3 \) and \( U \) are small enough. \( a_0 = R \left\{ \frac{2X}{1+X} + \mathcal{O}(|R_{\text{typ}}/R|) + \sqrt{\frac{\mu^3}{\mu_3^3}}\mathcal{O}(|l/R|^3) \right\} \)
- \( R_{\text{typ}} \) is estimated from \( \rho \) exchange interaction.
- \( \tilde{X} \) is close to 1.

\( \Lambda(1405) : \bar{K}N \) composite dominance
Compositeness of exotics

$s a_0(980)$ in $K\overline{K}$ scattering

We determine $E_{QB}$ and $a_0$ from Flatte parameters which are obtained experimental analysis.


(1) G. S. Adams et al. [CLEO Collaboration], Phys. Rev. D 84, 112009 (2011)

| Set | $E_{QB}$ (MeV) | $a_0$ (fm) | $X$ | $\tilde{X}$ | $U$ | $\left|\frac{R_{typ}}{R}\right|$ | $\left|\frac{l}{R}\right|^3$ |
|-----|----------------|-------------|-----|-------------|-----|----------------|----------------|
| (1) | 31-i70         | -0.03-i0.53 | 0.2-i0.2 | 0.3         | 0.1 | 0.3            | 0.1            |
| (2) | 3-i25          | 0.17-i0.77  | 0.2-i0.2 | 0.2         | 0.1 | 0.1            | 0.0            |
| (3) | 9-i36          | 0.05-i0.63  | 0.2-i0.2 | 0.2         | 0.1 | 0.2            | 0.0            |
| (4) | 14-i5          | -0.13-i2.19 | 0.8-i0.4 | 0.7         | 0.3 | 0.1            | 0.0            |
| (5) | 15-i29         | -0.13-i0.52 | 0.1-i0.4 | 0.1         | 0.1 | 0.2            | 0.0            |

• $|R_{typ}/R|$, $|l/R|^3$ and $U$ are small enough.
• $R_{typ}$ is estimated from $\rho$ exchange interaction.
• $\tilde{X}$ is close to 1.

$a_0(980)$ : small $K\overline{K}$ fraction
Compositeness of exotics

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- $|R_{typ}/R|$, $|l/R|^3$ and $U$ are small enough. $a_0 = R \left\{ \frac{2X}{1+X} + \mathcal{O}(|R_{typ}/R|) + \sqrt{\frac{\mu^3}{\mu^3}} \mathcal{O}(|l/R|^3) \right\}$
- $R_{typ}$ is estimated from $\rho$ exchange interaction.
- $\tilde{X}$ is close to 1.


(1) G. S. Adams et al. [CLEO Collaboration], Phys. Rev. D 84, 112009 (2011)
Compositeness of exotics

$f_0(980)$ in $K\bar{K}$ scattering

We determine $E_{QB}$ and $a_0$ from Flatte parameters which are obtained experimental analysis.


(1) T. Aaltonen et al. [CDF Collaboration], Phys. Rev. D 84, 052012 (2011)

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- $|R_{typ}/R|$, $l/R^3$ and $U$ are small enough. 
  $a_0 = R \left\{ \frac{2X}{1 + X} + O(|R_{typ}/R|) + \sqrt{\frac{\mu^3}{\mu^3}} O (|l/R|^3) \right\}$

- $R_{typ}$ is estimated from $\rho$ exchange interaction.

- Values of $\tilde{X}$ are not consistent. More precise analysis is needed.
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Conclusions


• We extend the compositeness relation to quasi-bound states.

\[
a_0 = R \left\{ \frac{2X}{1+X} + \mathcal{O} \left( \frac{|R_{typ}/R|}{1} \right) + \mathcal{O} \left( \frac{|l/R|^3}{1} \right) \right\}
\]

If the gap of threshold energy is sufficiently large, we can ignore the effect of decaying channel to calculate compositeness.

• We construct interpretation of complex \( X \).

\[
\tilde{X} \equiv \frac{1 - |Z| + |X|}{2}, \quad U \equiv |X| + |Z| - 1
\]

If the uncertainty \( U \) is small, we interpret \( \tilde{X} \) as the compositeness.

• We apply the method to exotic hadrons and discuss the internal structures.

\[\Lambda(1405) : \bar{K}N \text{ composite dominance}\]

\[a_0(980) : \text{not } K\bar{K} \text{ dominance}\]
Back up slides
Motivation

- A problem of model approaches

  When we set the model parameters,
  
  the contributions from outside of
  the model space is renormalized in
  the model parameters.

  If the model prediction coincide well with experiment,
  the description by the model may not be described
  the physics of phenomena.

To avoid such ambiguity,
we should use model independent approach.
Flatte parametrization

To get \( E_h \) and \( a_0 \) from Flatte parameters

\[
T = \frac{1}{M^2 - s - i (g_1 \rho_{\alpha \pi} + g_2 \rho_K \bar{K})}
\]

for \( a_0(980) : \alpha \) denote \( \eta \)  

\( f_0(980) : \alpha \) denote \( \pi \)

\[
\rho_{\alpha \beta} = 2 p_{\alpha \beta} / \sqrt{s}
\]

\( g_1 \) and \( g_2 \) were determined fitting the scattering amplitude.

\[
E_h
\]

Find pole position of the T matrices

\[
a_0
\]

Normalize Kbar-K amplitude \( f(s) \) so as to \( f(s) \) satisfies

\[
f(s)^{-1} \rightarrow -a_0 - ik + O(k^2)
\]

(\( k \) is a momentum of \( K \) or Kbar)

\[
a_0 = -f(0)
\]
Power counting(1)

Neglecting collection terms, the compositeness relation is rewritten by scattering length $a_0$ and effective range $r_e$.

$$a_0 = R \left\{ \frac{2X}{1+X} + \mathcal{O}(|R_{\text{typ}}/R|) + \mathcal{O}\left(|l/R|^3\right) \right\}$$

$$X(a_0, r_e) = \left(1 - \frac{2r_e}{a_0}\right)^{-1/2}$$

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$s \Lambda(1405)$
Power counting (2)

\[ X(a_0, r_e) = \left(1 - \frac{2r_e}{a_0}\right)^{-1/2} \]

- \( X \sim 0 \quad \rightarrow \quad |r_e/a_0| \sim \infty \)
- \( X \sim 0.5 \quad \rightarrow \quad |r_e/a_0| \sim 1.5 \)
- \( X \sim 1 \quad \rightarrow \quad |r_e/a_0| \sim 0 \)

\[ a_0(980) \]

| Set | \( E_{QB} \) (MeV) | \( a_0 \) (fm) | \( X \) | \( \tilde{X} \) | \( |r_e/a_0| \) |
|-----|------------------|----------------|--------|---------|----------------|
| (1) | 31-i70           | -0.03-i0.53    | 0.2-i0.2 | 0.3     | 4.8            |
| (2) | 3-i25            | 0.17-i0.77     | 0.2-i0.2 | 0.2     | 6.5            |
| (3) | 9-i36            | 0.05-i0.63     | 0.2-i0.2 | 0.2     | 7.2            |
| (4) | 14- i 5          | -0.13-i2.19    | 0.8-i0.4 | 0.7     | 0.5            |
| (5) | 15-i29           | -0.13-i0.52    | 0.1-i0.4 | 0.1     | 13             |
\[ X(a_0, r_e) = \left(1 - \frac{2r_e}{a_0}\right)^{-1/2} \]

- \( X \sim 0 \rightarrow |r_e/a_0| \sim \infty \)
- \( X \sim 0.5 \rightarrow |r_e/a_0| \sim 1.5 \)
- \( X \sim 1 \rightarrow |r_e/a_0| \sim 0 \)

\( f_0(980) \)

| Set  | \( E_{QB} \) (MeV) | \( a_0 \) (fm) | \( X \) | \( \tilde{X} \) | \( |r_e/a_0| \) |
|------|------------------|----------------|-------|---------|----------------|
| (1)  | 19-i30           | 0.02-i0.95     | 0.3-i0.3 | 0.4     | 2.6            |
| (2)  | -6-i10           | 0.84-i0.85     | 0.3-i0.1 | 0.3     | 5.4            |
| (3)  | -8-i28           | 0.64-i0.83     | 0.4-i0.2 | 0.4     | 2.1            |
| (4)  | 10-i18           | 0.51-i1.58     | 0.7-i0.3 | 0.6     | 0.7            |
| (5)  | -10-i29          | 0.49-i0.67     | 0.3-i0.1 | 0.3     | 4.0            |
| (6)  | 10-i7            | 0.52-i2.41     | 0.9-i0.2 | 0.9     | 0.2            |