HADRON 2015
September 13-18, 2015
Newport News, Virginia USA

UNI GRAZ

Hadron spectrum and structure from the lettuce

Christian B. Lang
University of Graz, Austria

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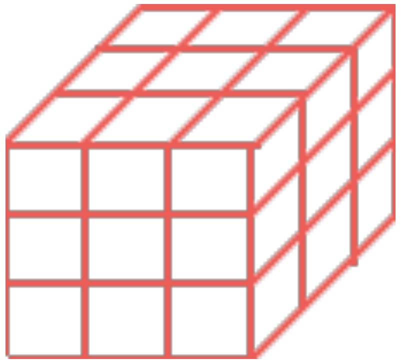
Hadron spectrum and structure from the lattice



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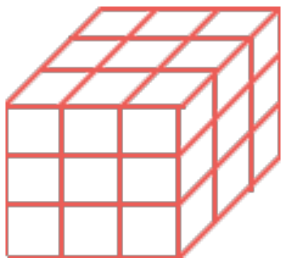
The lattice method

Wilson (1974)

Minkowski continuum:

formal QFT needs regularisation $\longrightarrow$ Euclidean lattice box



continuum QFT in a box

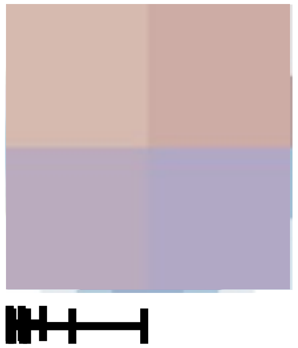
discrete x,y,z,t

discrete p_x, p_y, p_z, p_t

discrete energy spectrum

lattice spacing $a \rightarrow 0$

for $a\ n_L = L > 6$ fm



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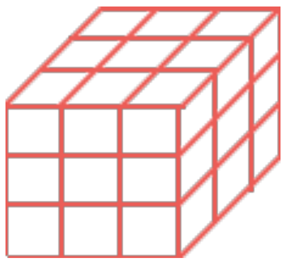

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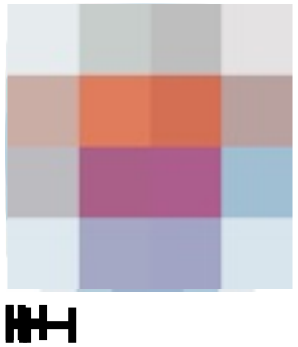

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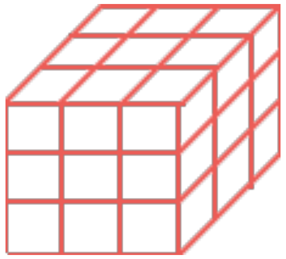

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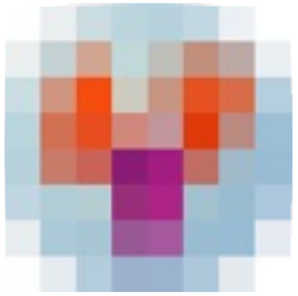

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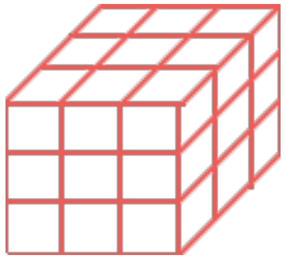

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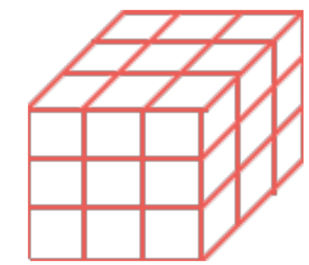

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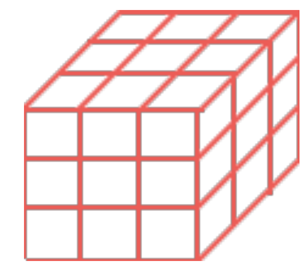

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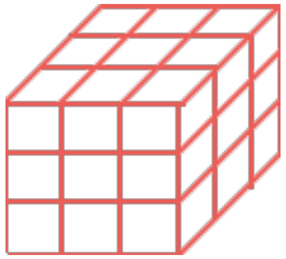

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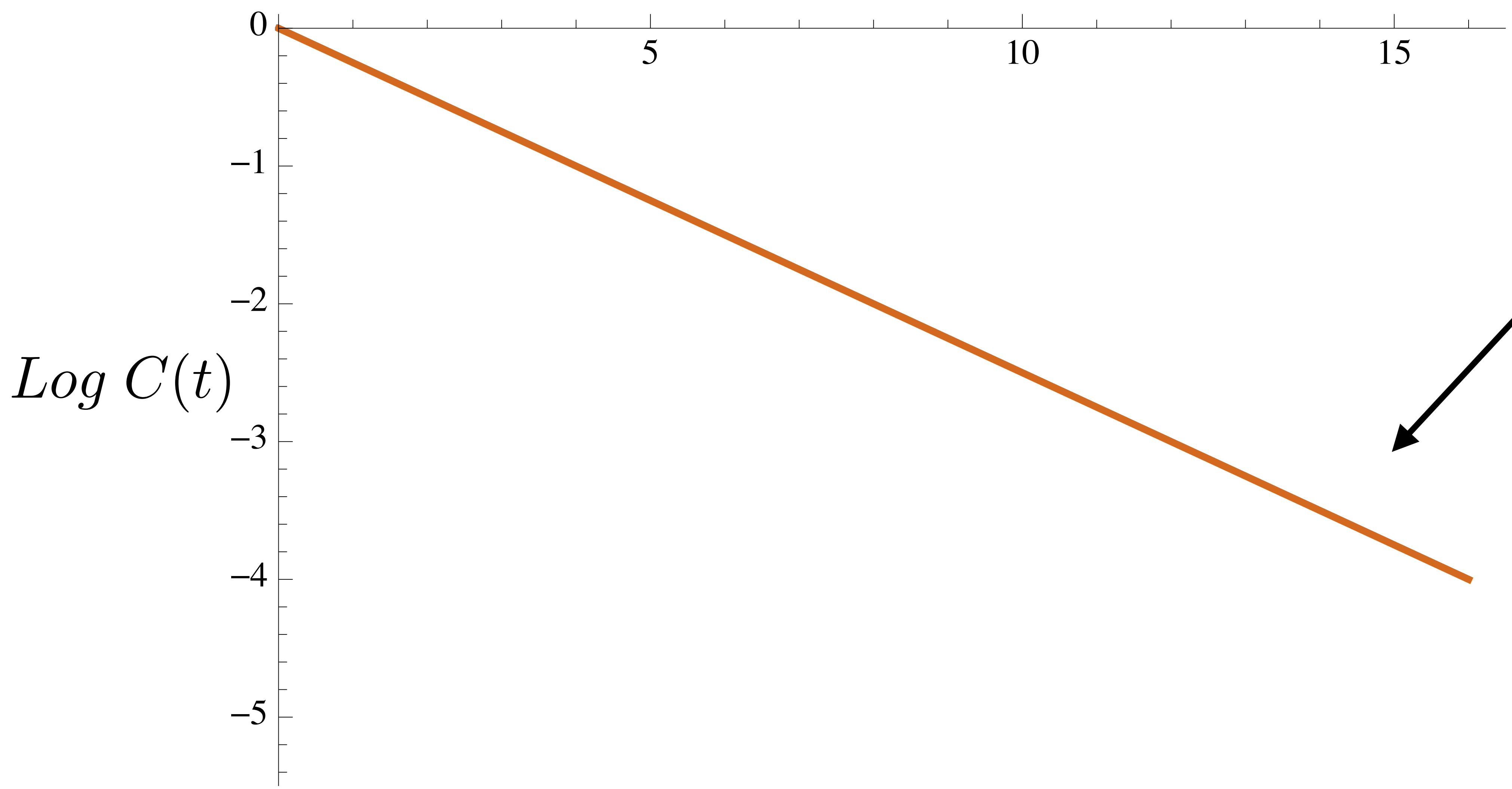
Lattice tools: correlations

$$C_{ij}(t) \equiv \langle X_i(t)\overline{X}_j(0)\rangle = \sum_n \langle X_i|n\rangle\, \mathrm{e}^{-t\,E_n}\, \langle n|\overline{X}_j\rangle$$

X_i lattice operator
n "physical" eigenstate

$$C(t) = e^{-0.25\ t}$$



slope=
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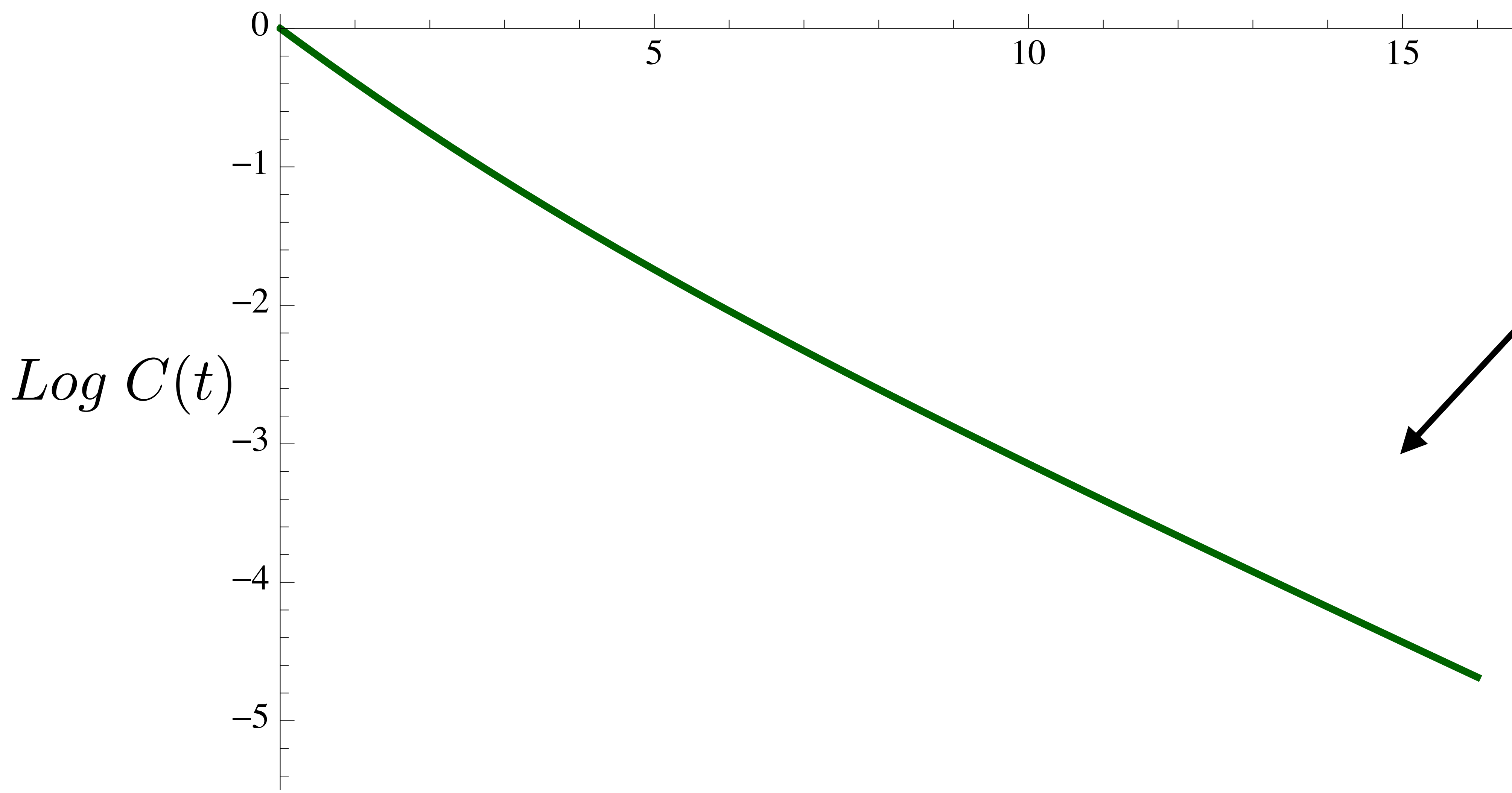
$$C(t) = e^{-0.25\,t} + e^{-0.55\,t}$$



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X_i lattice operator

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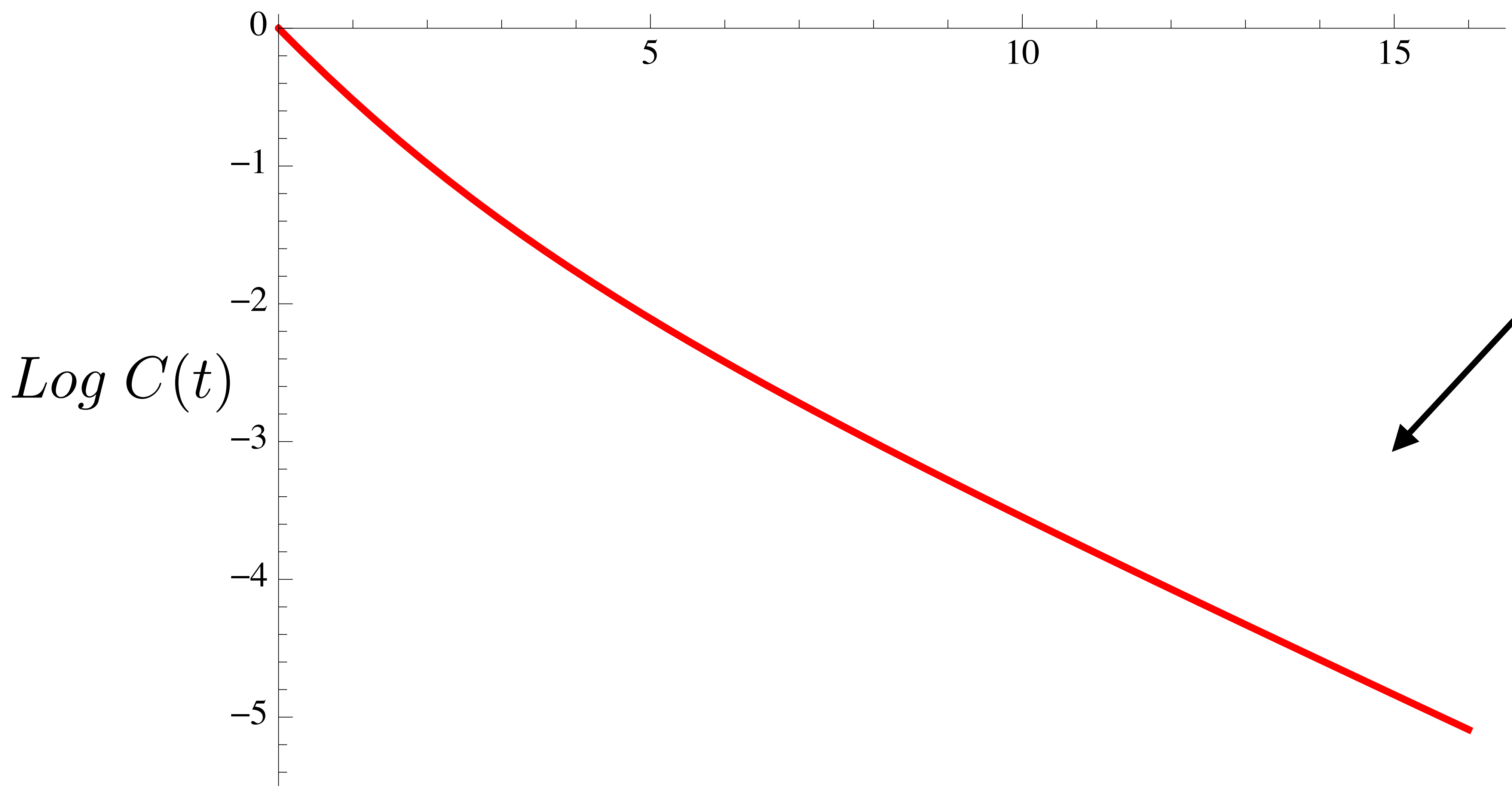
$$C(t) = e^{-0.25\,t} + e^{-0.55\,t} + e^{-0.85\,t}$$



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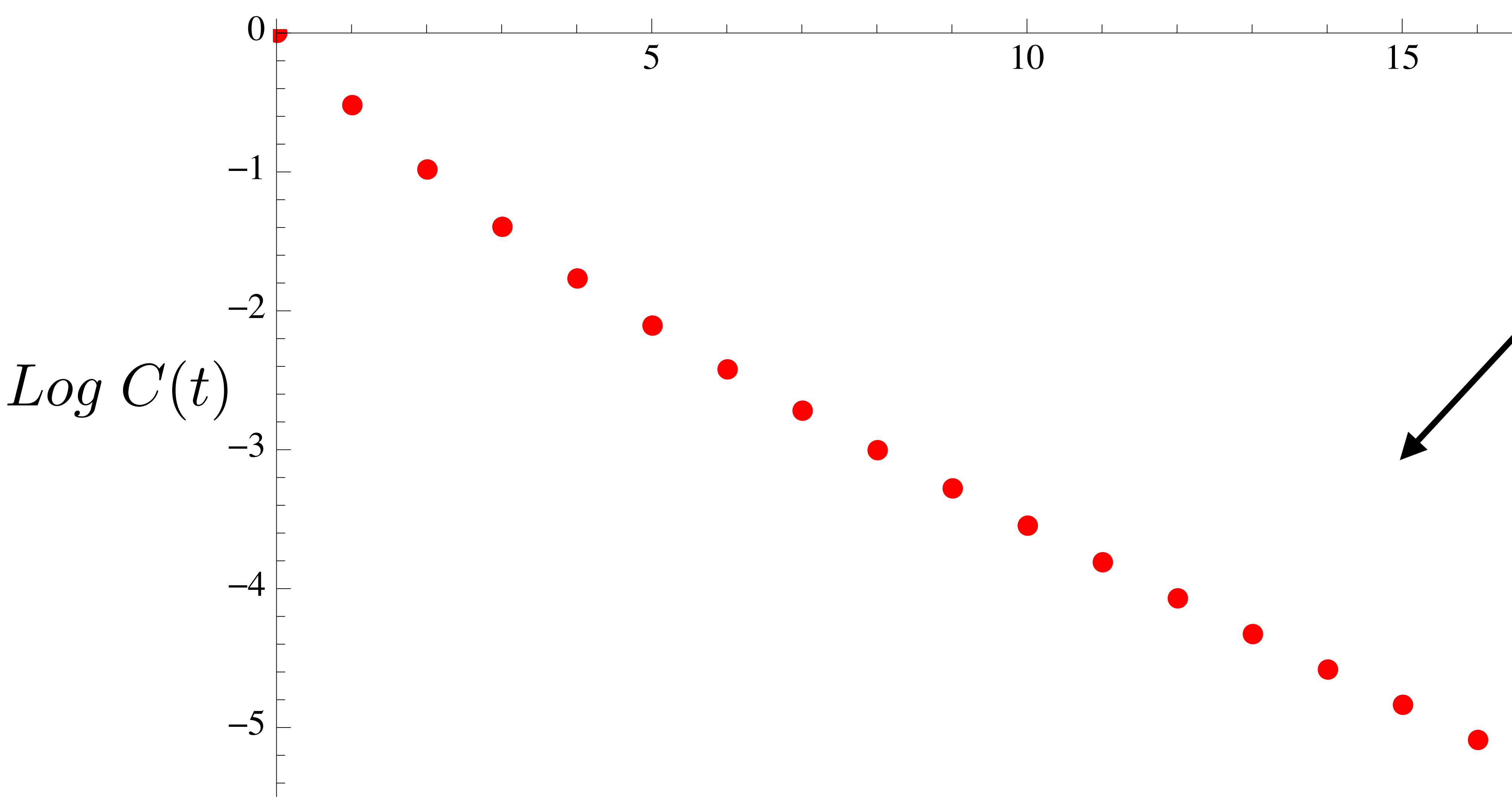

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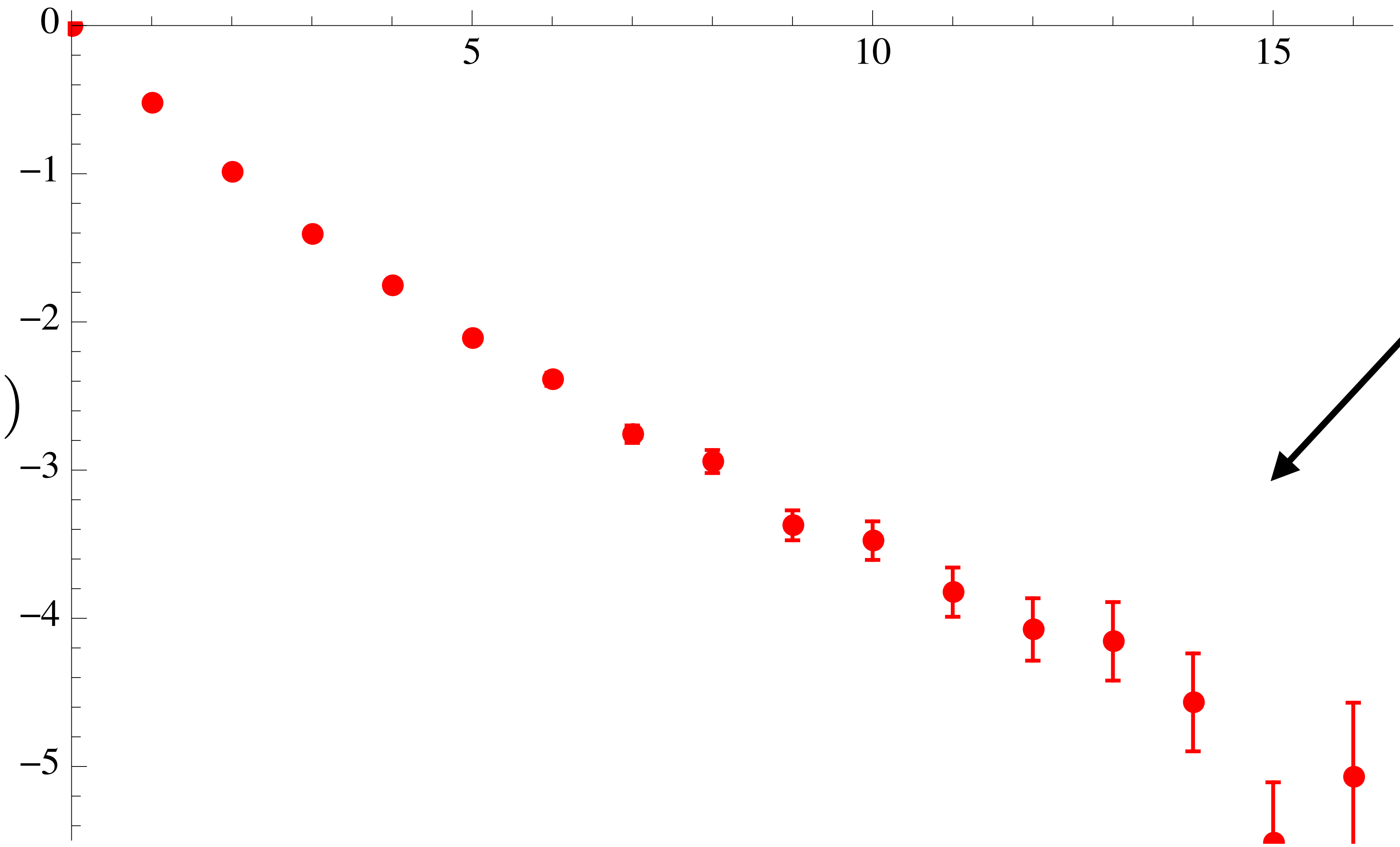
X_i lattice operator

n "physical" eigenstate

$$C(t_n) = e^{-0.25\,t} + e^{-0.55\,t} + e^{-0.95\,t} + \text{noise}$$

$Log\ C(t)$



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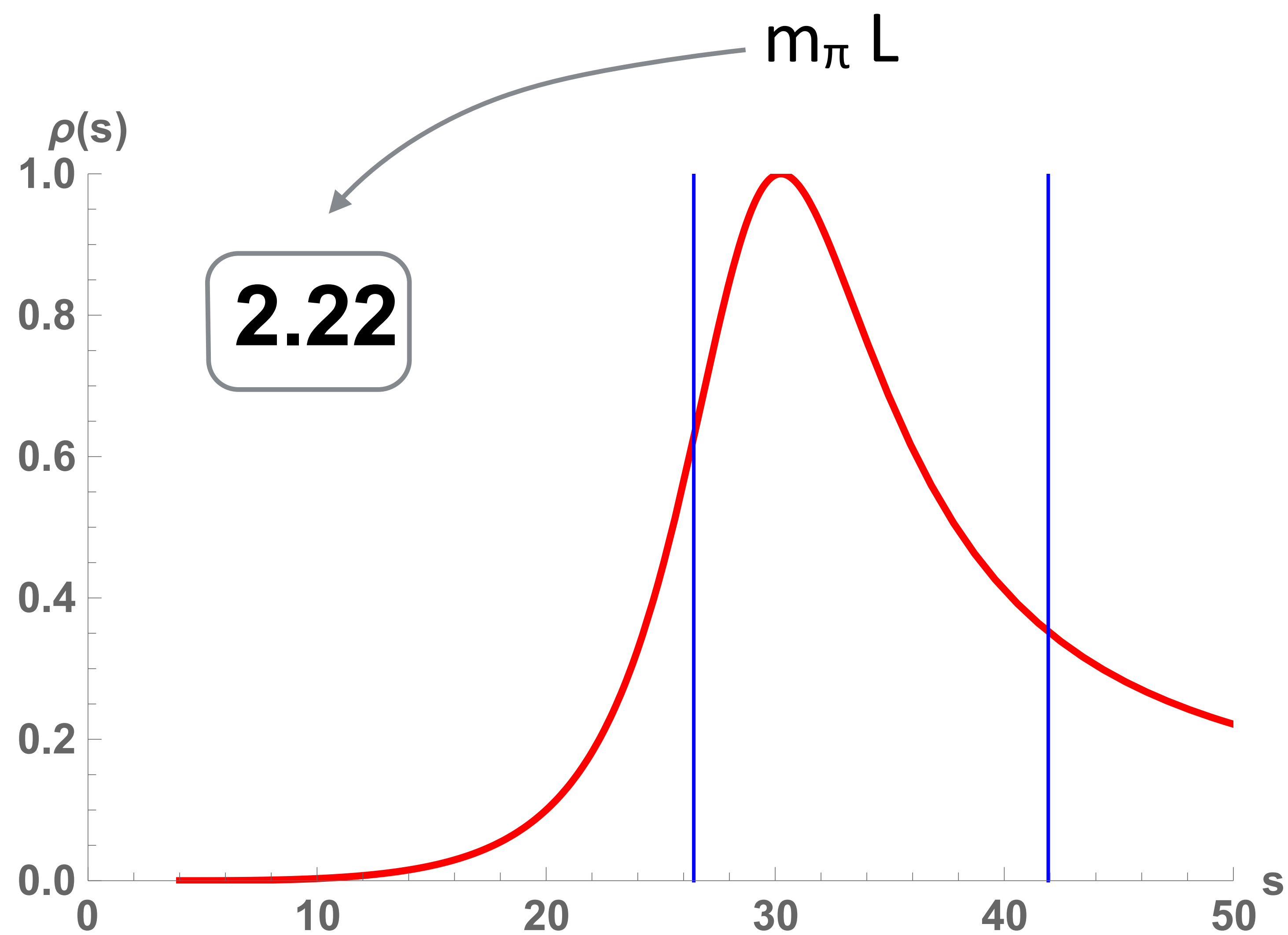
Continuum vs. lattice

Correlation functions have discrete energy levels!

Example:

Spectral density of a simple resonance in continuum and the discrete energies for a lattice volume



One cannot arbitrarily fix the energies: they are eigenvalues depending on the control parameters (volume, couplings,...).

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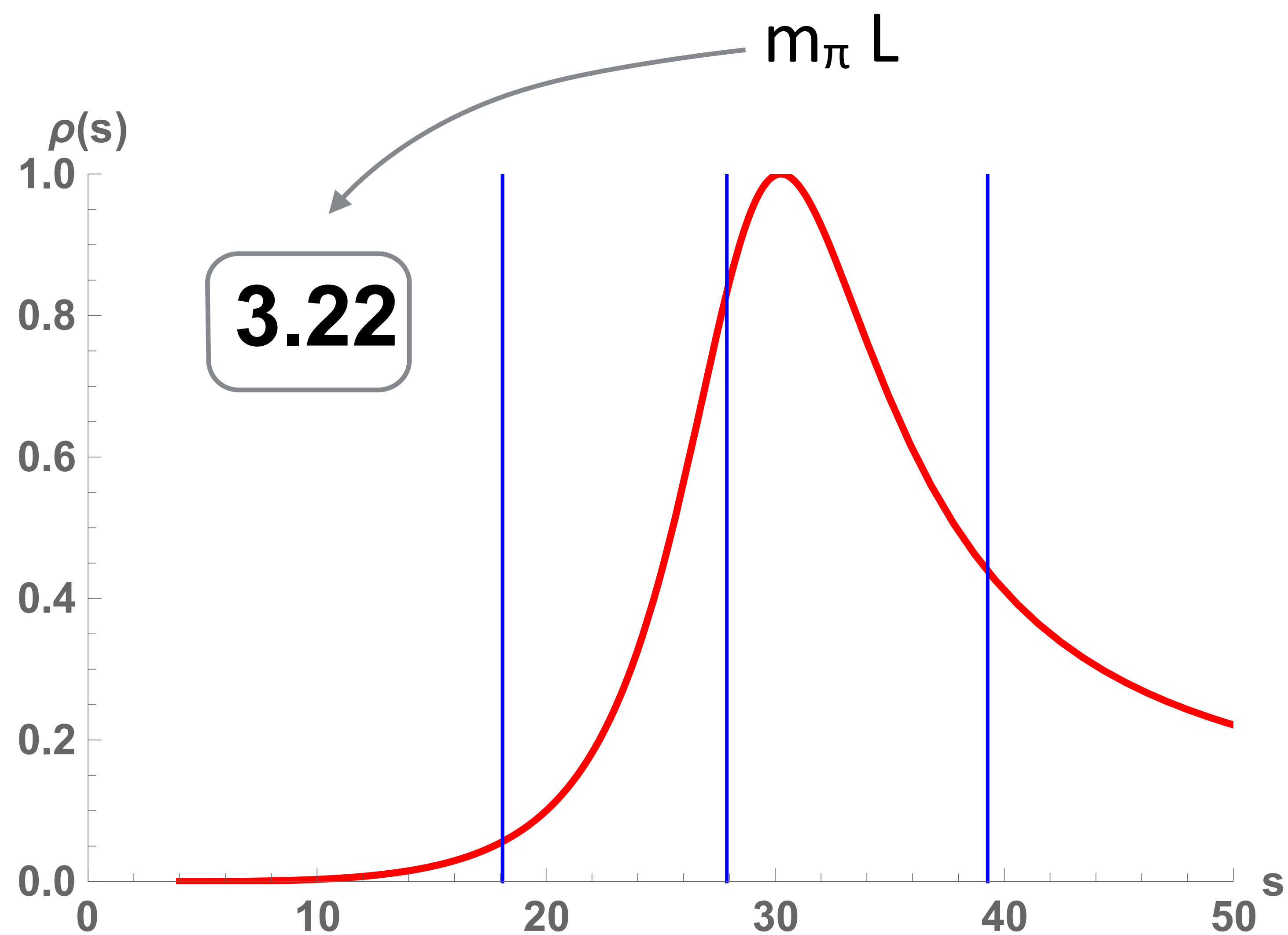

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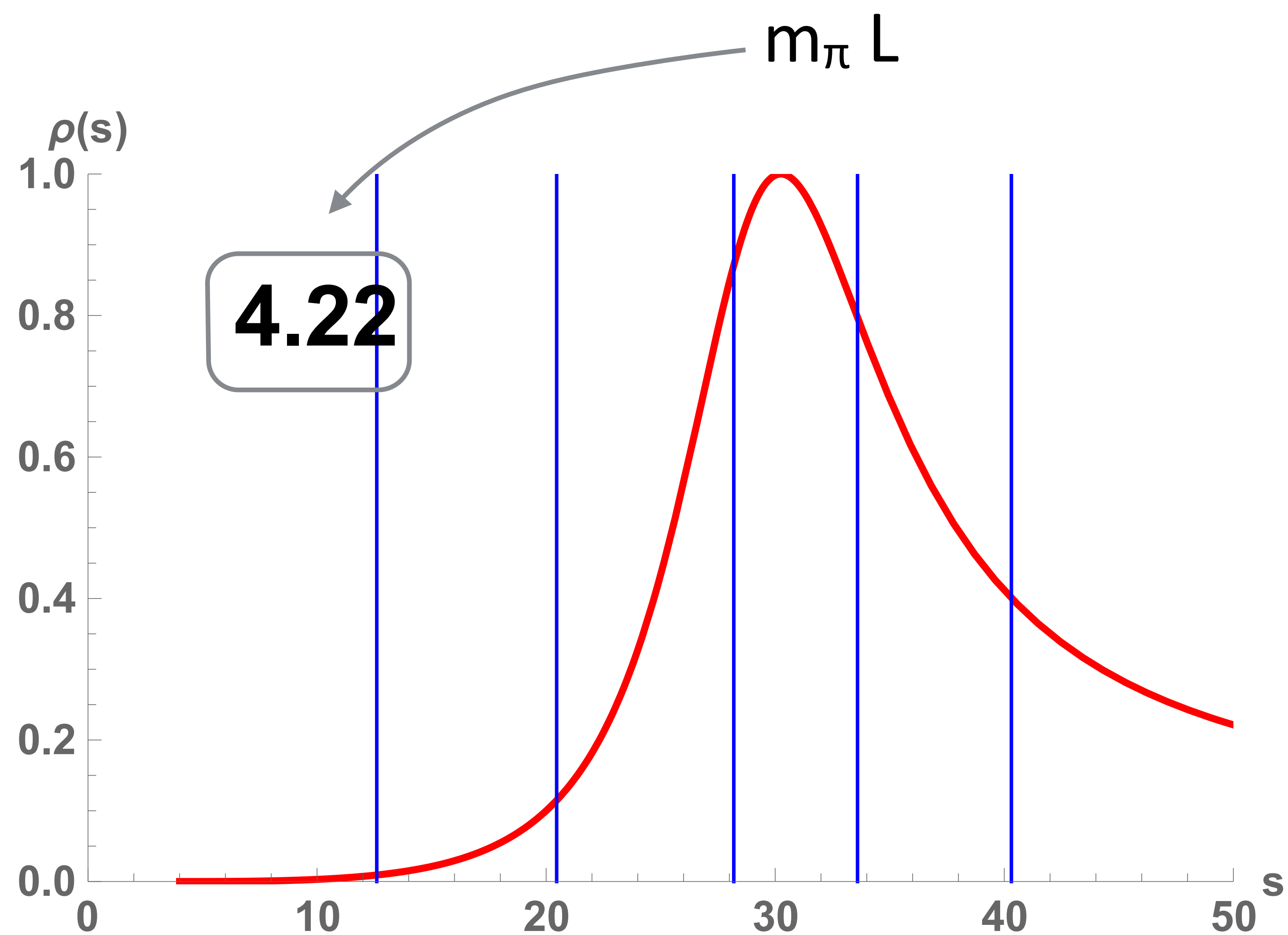

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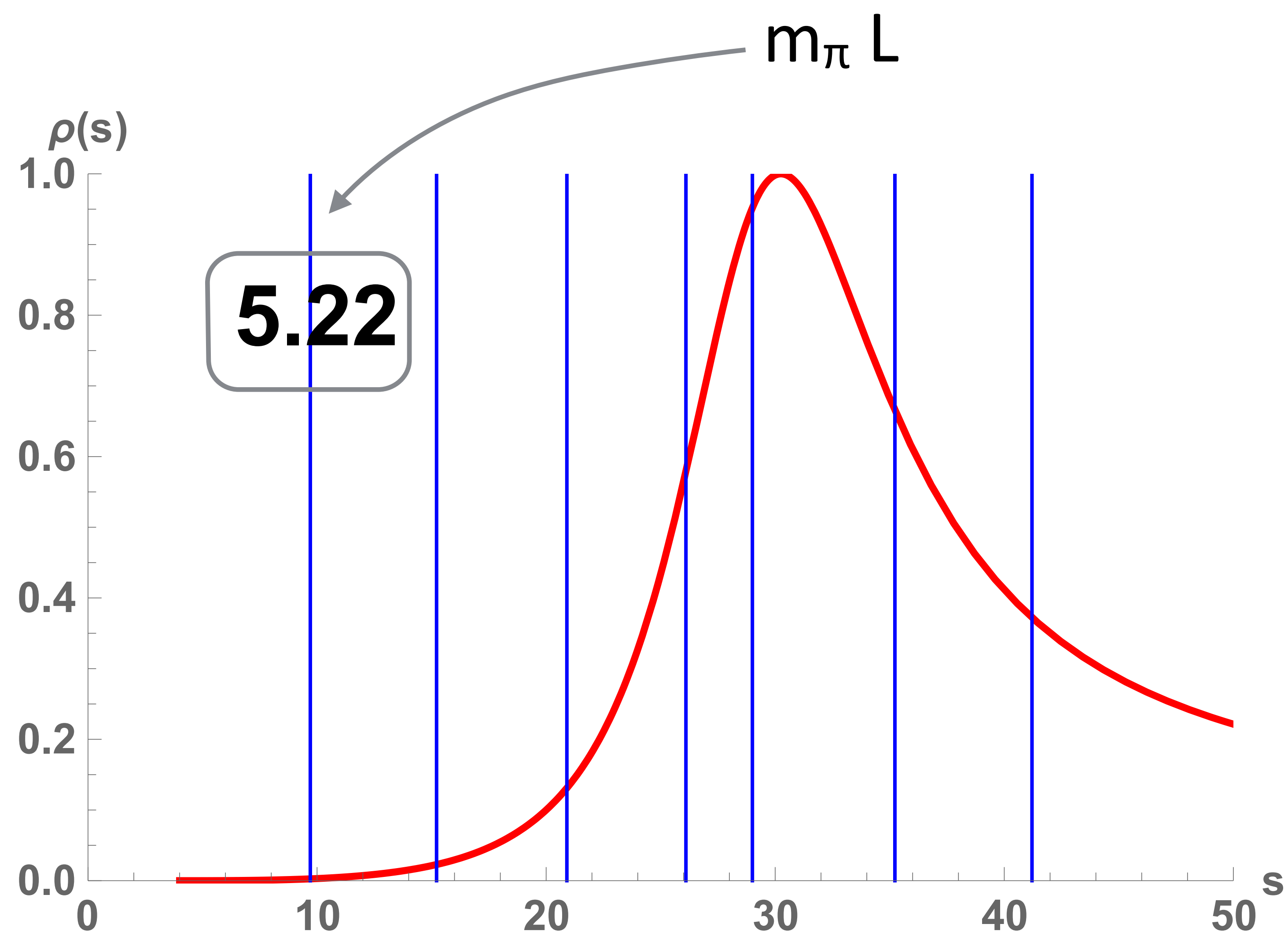

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How to get the energy levels?

- *Lüscher,Wolff: NPB339(90)222*
- *Michael, NPB259(85)58*
- *See also Blossier et al., JHEP0904(09)094*

- Compute all cross-correlations for several lattice operators

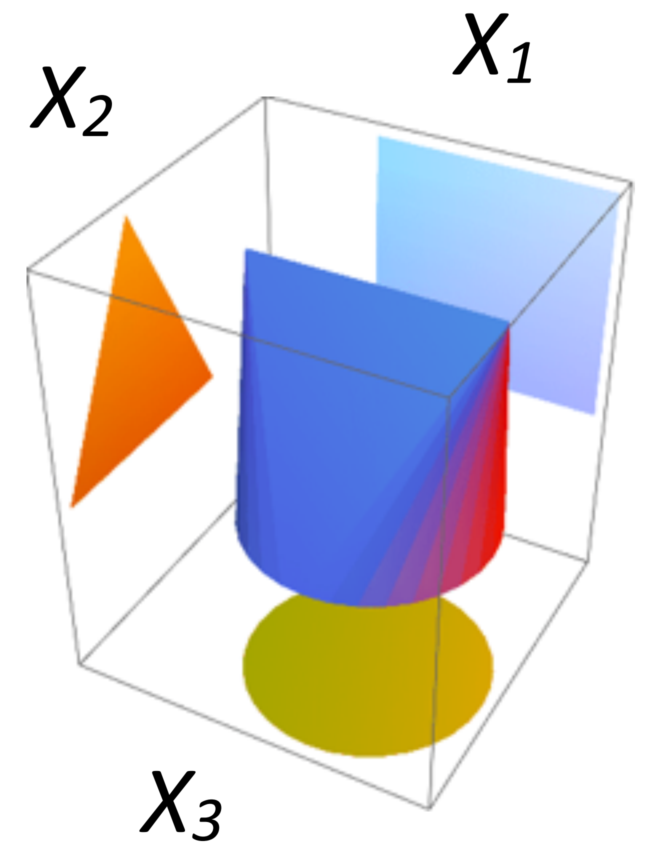
$$C_{ij}(t) \equiv \langle X_i(t)\overline{X}_j(0)\rangle = \sum_n \langle X_i|n\rangle\, \mathrm{e}^{-t\,E_n}\, \langle n|\overline{X}_j\rangle$$

- Solve the eigenvalue problem. The eigenvalues give the energy levels (masses):

$$\lambda^{(n)}(t) \propto e^{-t\,E_n}\left(1+\mathcal{O}(e^{-t\Delta E_n})\right)$$

- The eigenvectors are "fingerprints" of the state and allow to identify the "composition" of the state

X_i : lattice operators with the right quantum numbers, "complete set"!?



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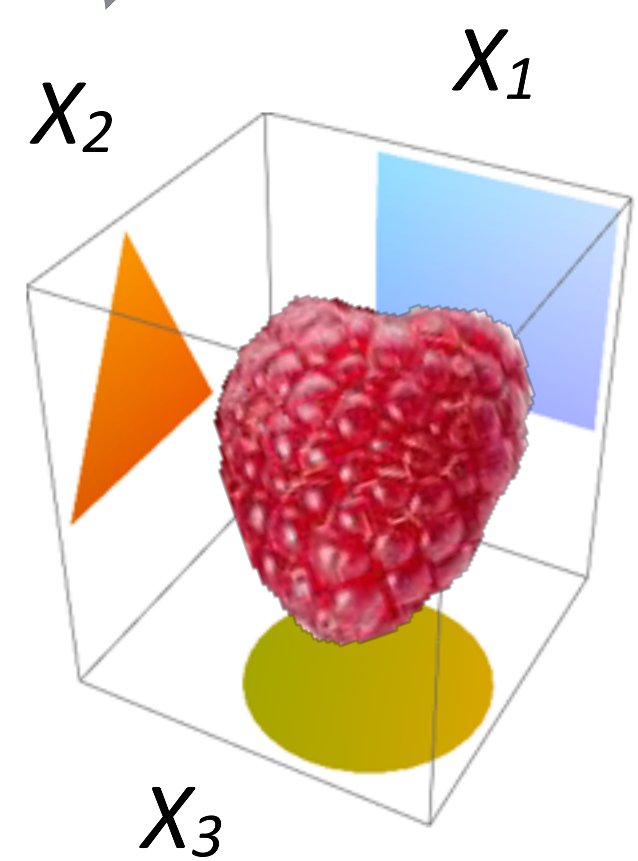
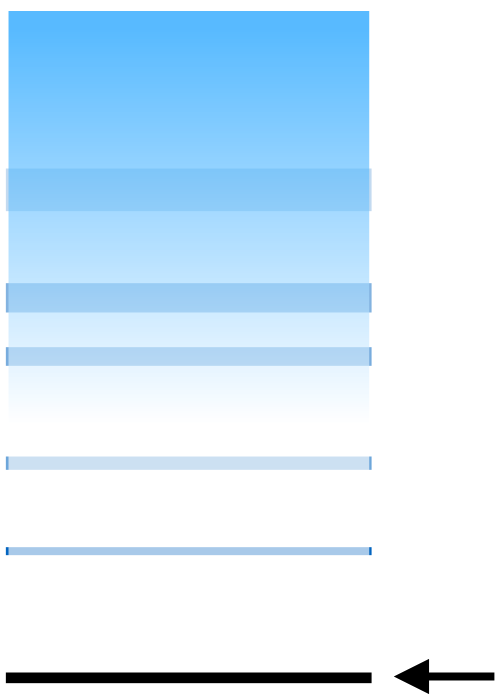


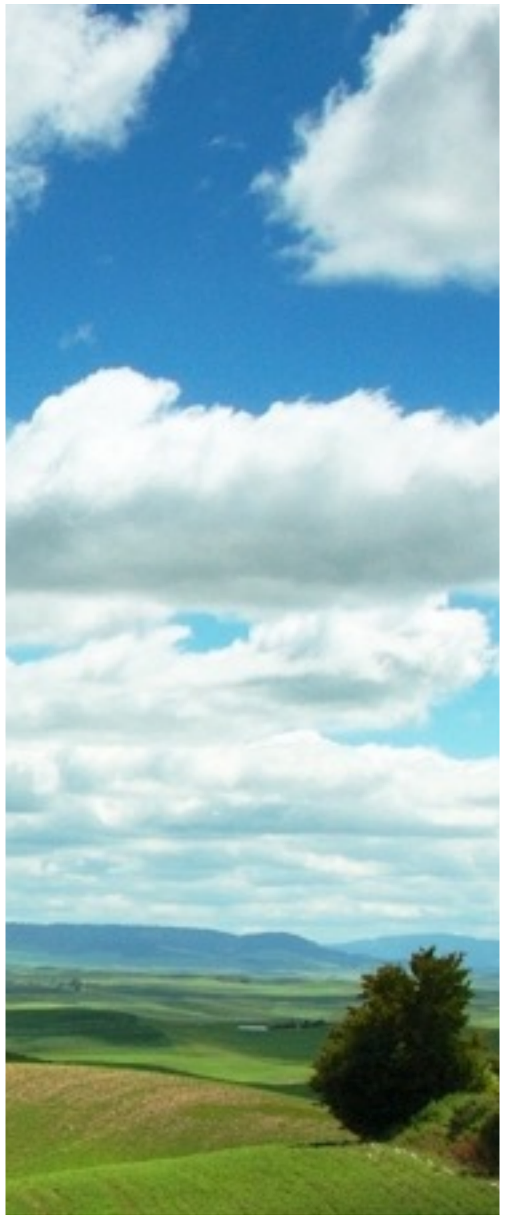
Lattice: Energy levels



Ground state

"Ground state dwellers"

p,n, π, K, D, B,..., form factors, 3-point functions: excited states are a "contamination"



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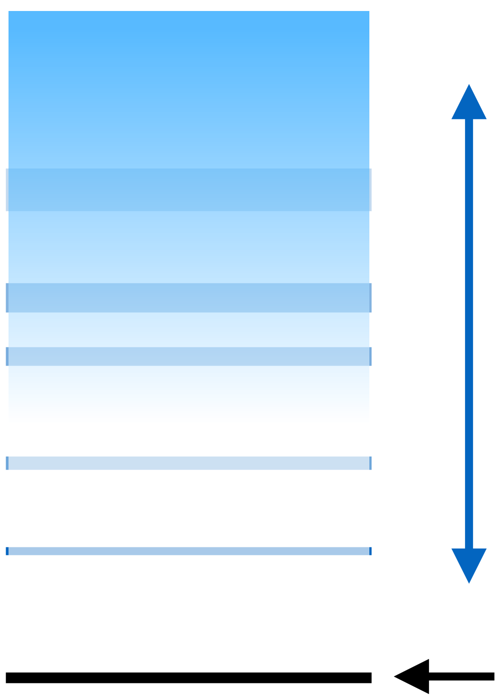


Excited states

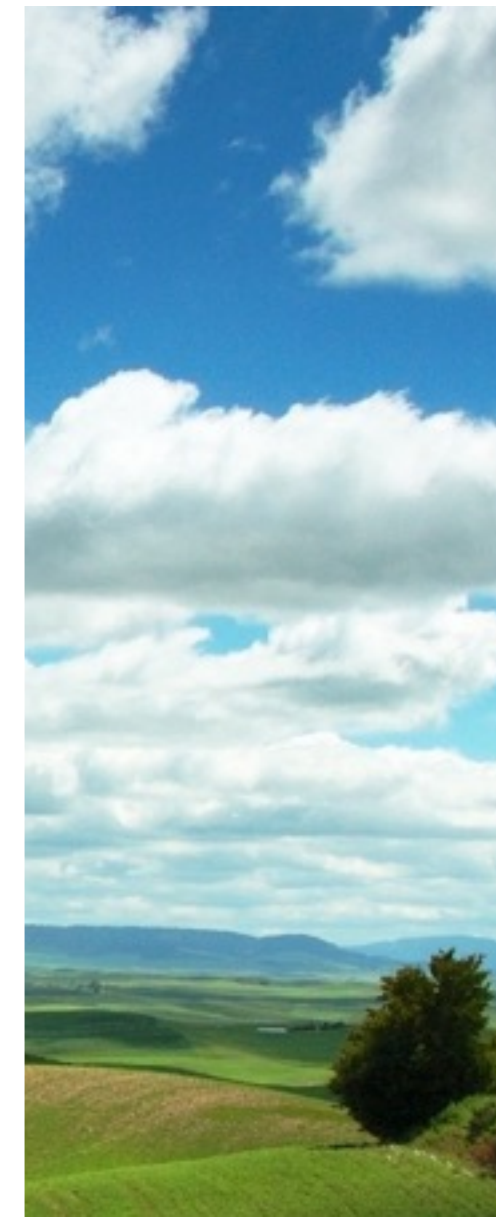
Ground state

"Excited state lovers"

Resonances, transitions, decay, scattering: excited state levels are a "must have"

"Ground state dwellers"

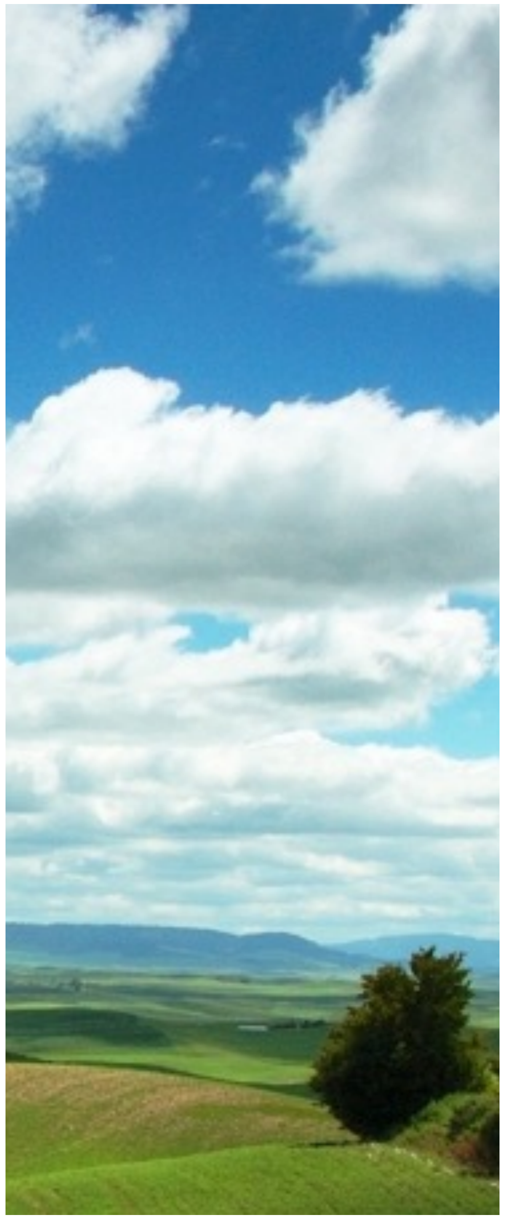
p,n, π, K, D, B,..., form factors, 3-point functions: excited states are a "contamination"



Hadron structure

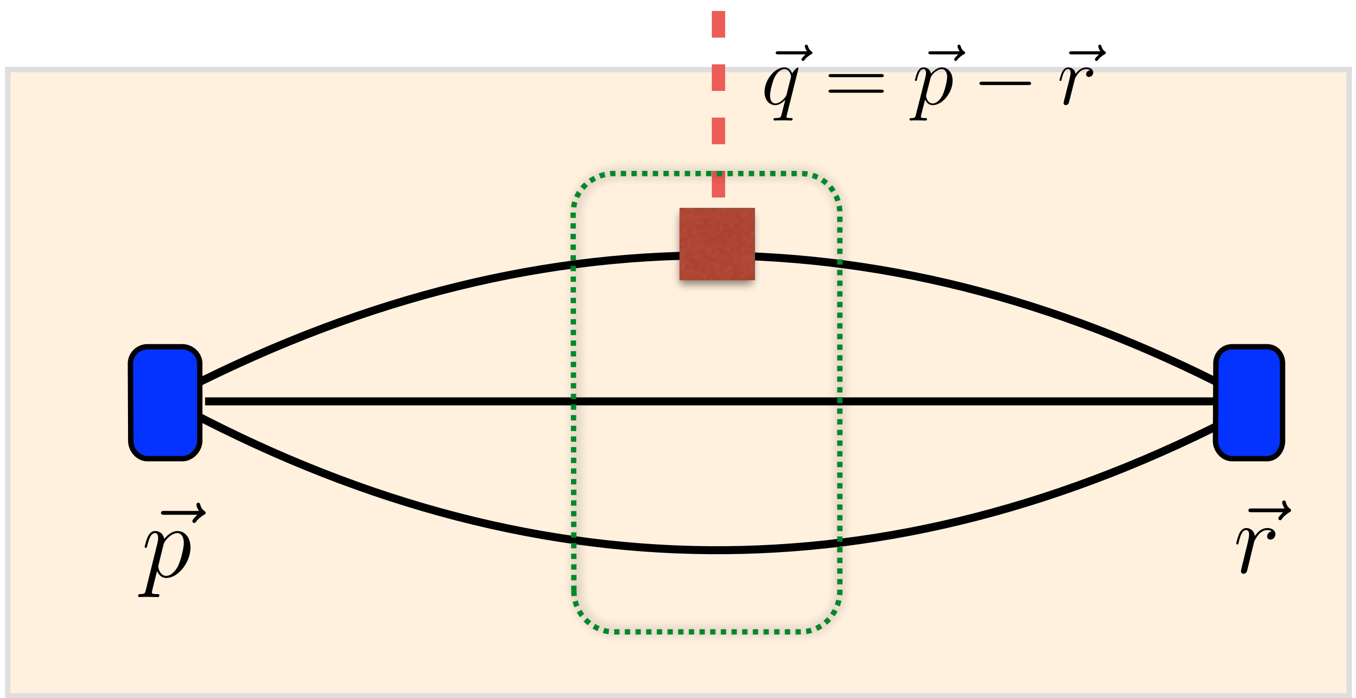
Here a few selected topics:

Benchmark: axial charge g_A
E.m. form factor and charge radii
Proton spin
Radiative transitions



...for ground state dwellers

3-point functions



$$G_3 = \langle 0|Op(t,\vec{p})|\Gamma(\tau)|\overline{O}p(0,\vec{r})0\rangle$$

$$\langle 0|Op(t,\vec{p})|H(\vec{p})\rangle \frac{e^{-E_p(t-\tau)}}{2E_p} \langle H(\vec{p})|\Gamma(\tau)|H(\vec{r})\rangle \frac{e^{-E_r\tau}}{2E_r} \langle H(\vec{r})|\overline{O}p(0,\vec{r})|0\rangle$$

C.B. Lang (2015)

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3-point functions

Op(0,$\vec{r}$)

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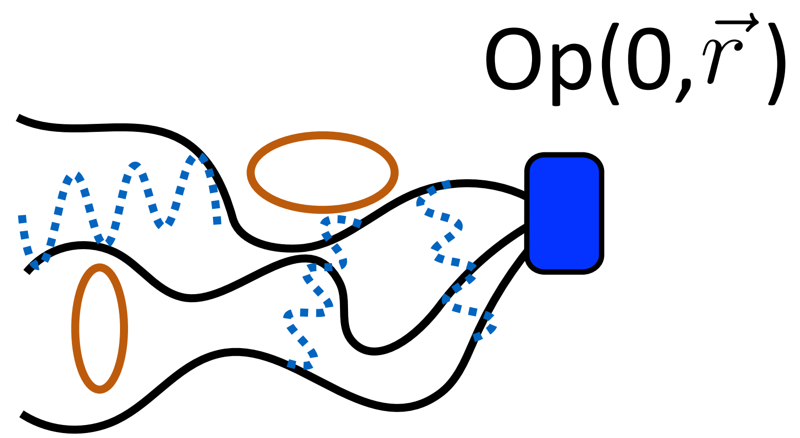

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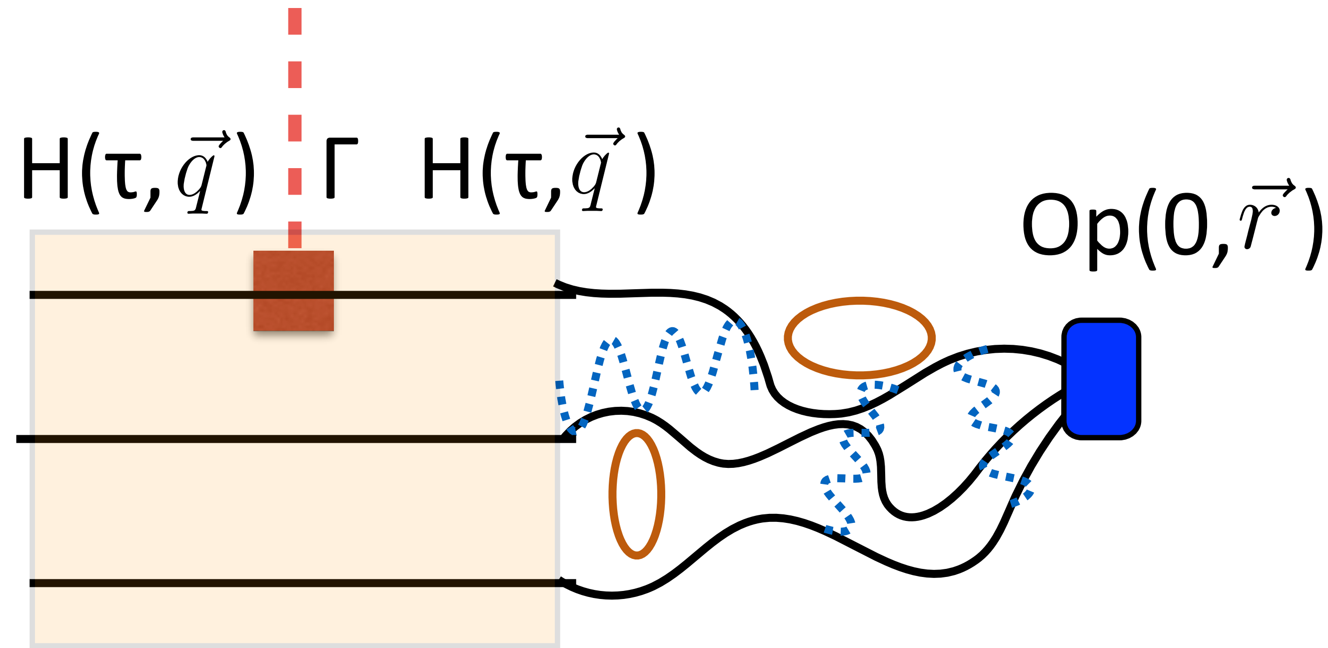

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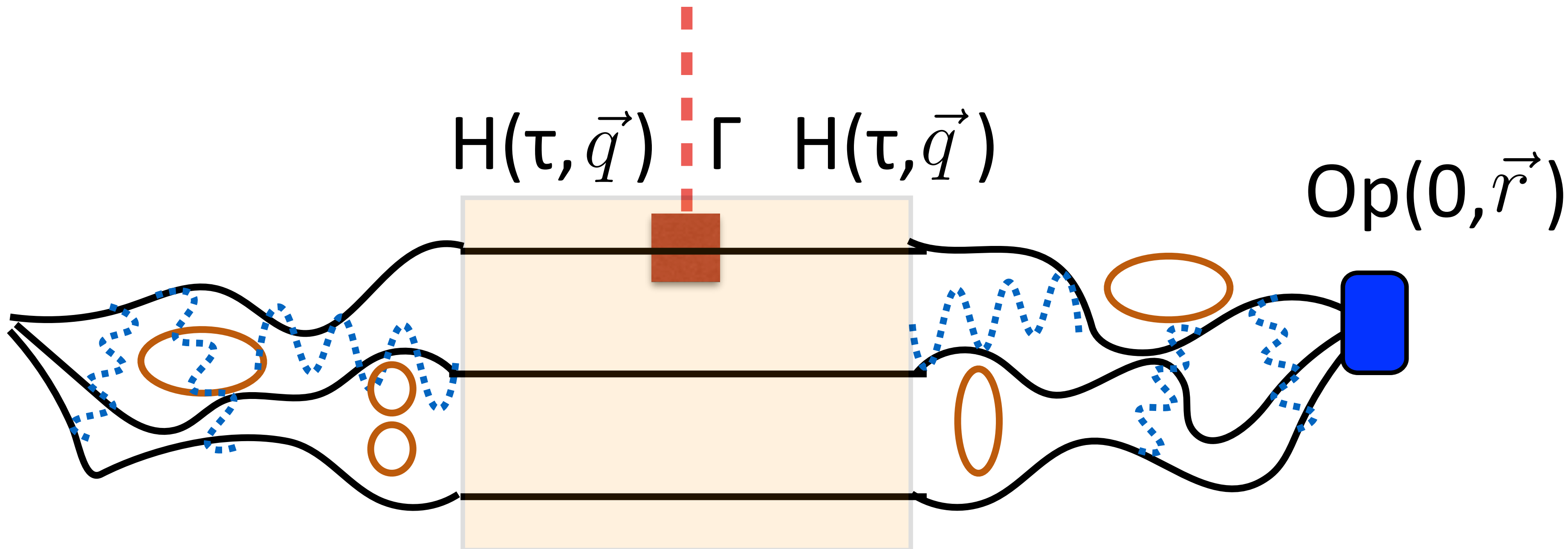

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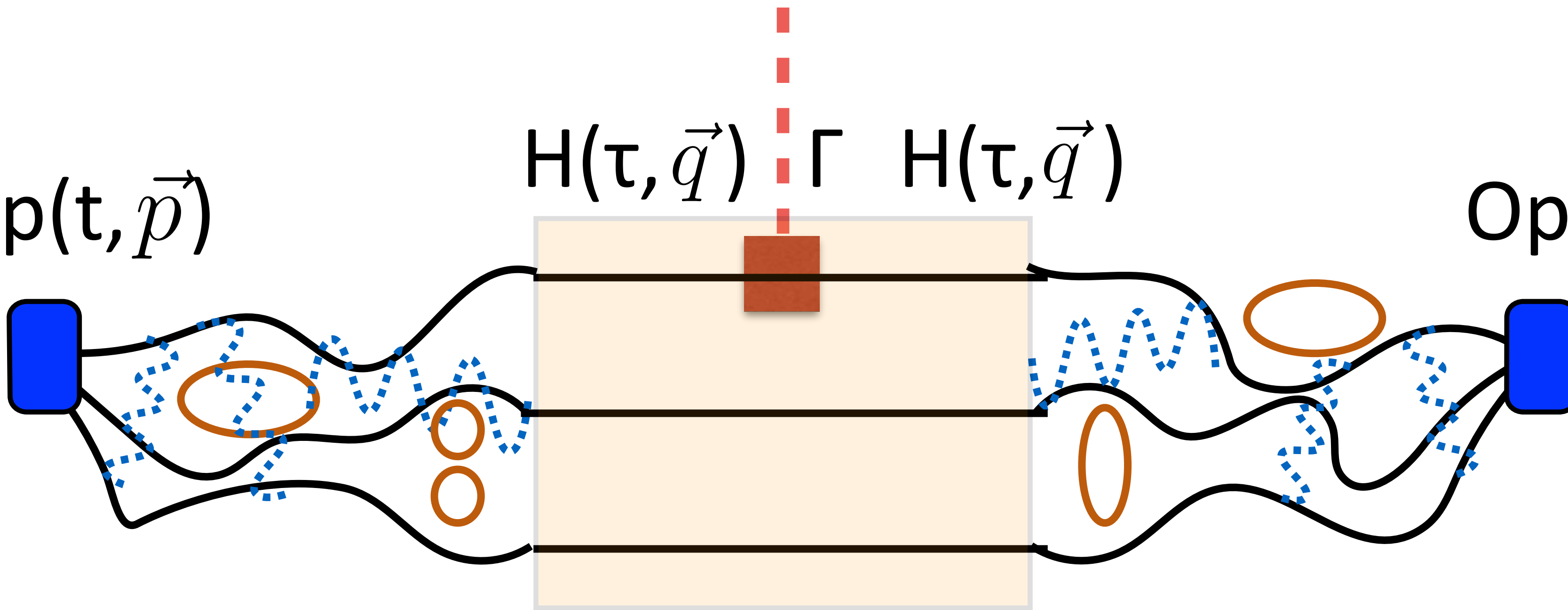

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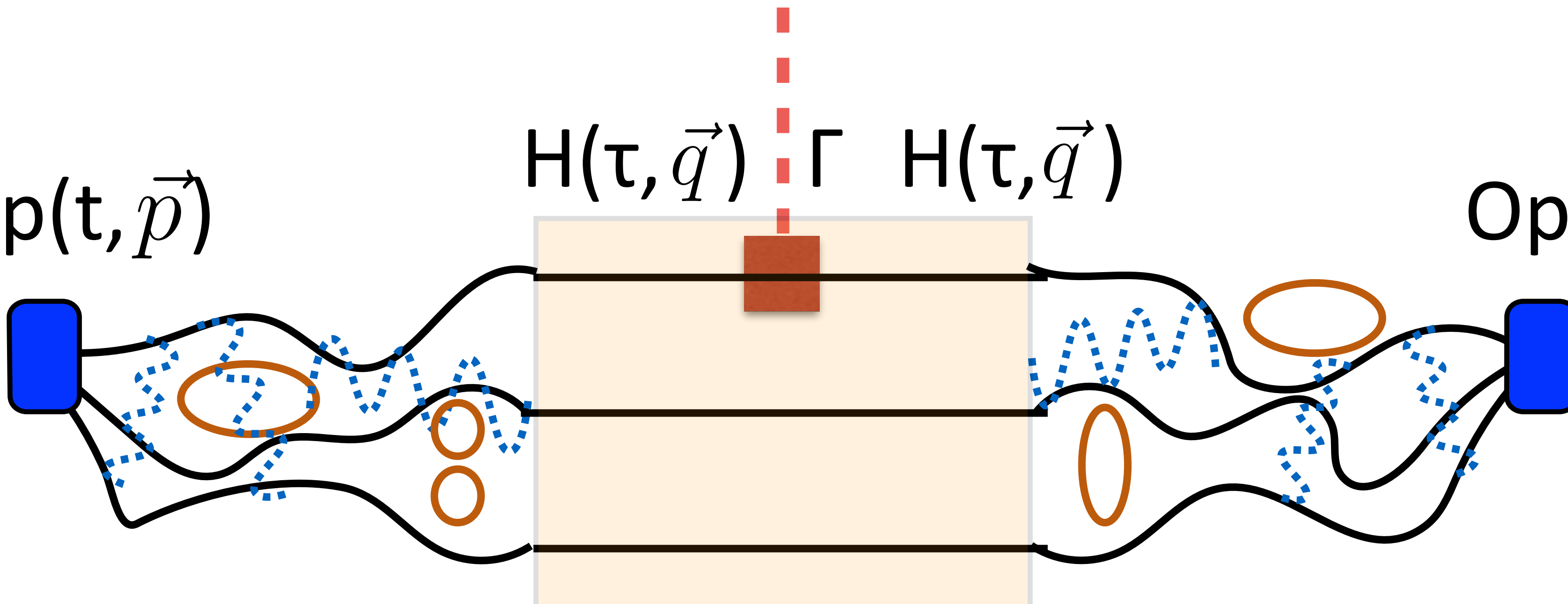

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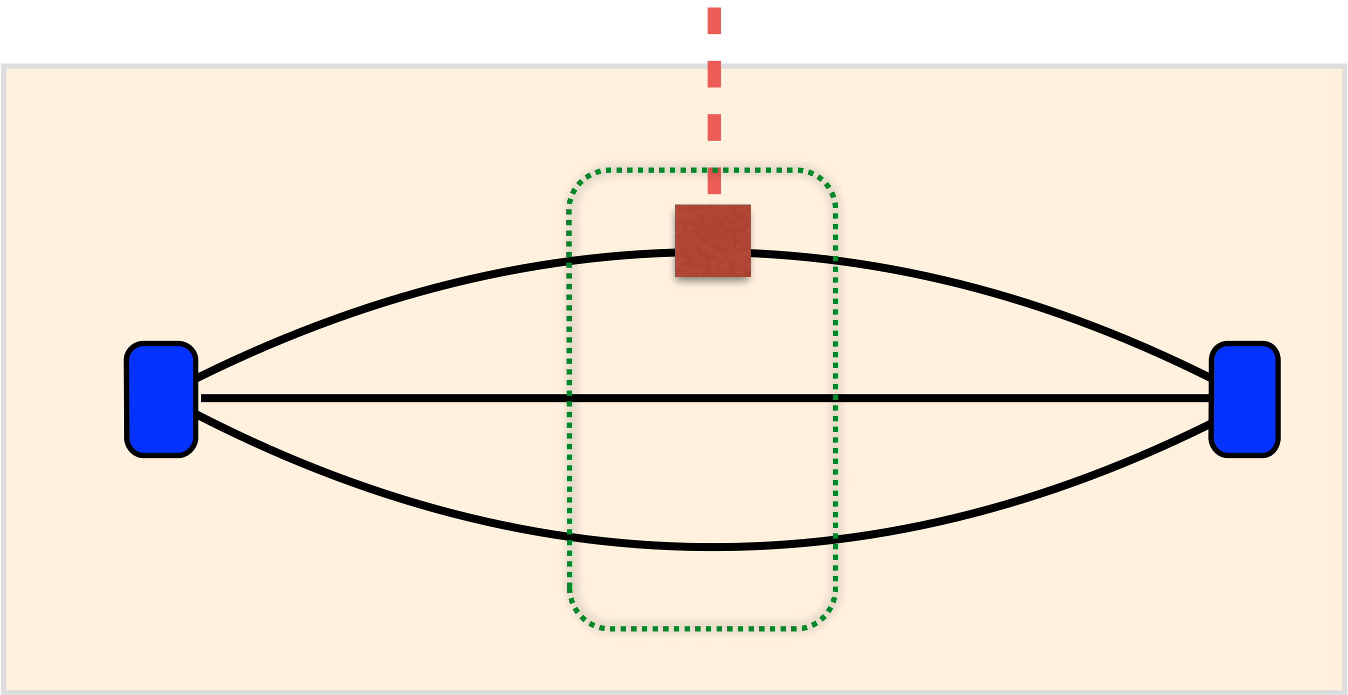
$$G_2 = \langle 0|Op(t,\vec{q})\overline{O}p(0,\vec{p})|0\rangle$$

$$\langle 0|Op(t,\vec{p})|H(\vec{p})\rangle \frac{e^{-E_pt}}{2E_p} \langle H(\vec{p})|\overline{O}p(0,\vec{r})|0\rangle$$

Ratio method



$$R_\Gamma(t,\tau,\vec q) = C_3/C_2\times \ldots \quad \to \langle H(\vec p)|\Gamma(\tau)|H(\vec r)\rangle$$

e.g. $\langle p|\Gamma|n\rangle$ for

$$\Gamma = \quad \overline{u} d \quad \to g_S(q^2)$$

$$\Gamma = \quad \overline{u}\gamma_5 d \quad \to g_P(q^2)$$

$$\Gamma = \quad \overline{u}\gamma_\mu d \quad \to g_V(q^2), \tilde g_T(q^2)$$

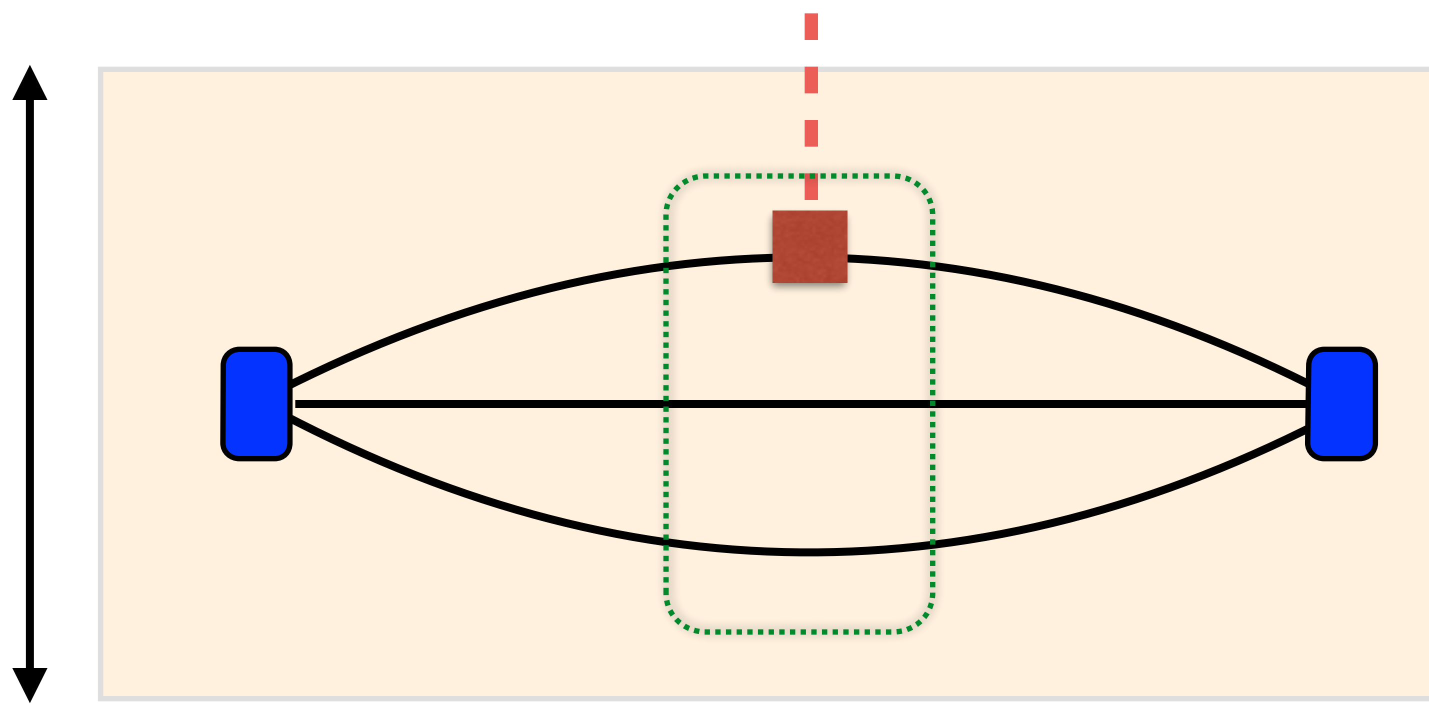
$$\boxed{\Gamma = \quad \overline{u}\gamma_\mu\gamma_5 d \quad \to g_A(q^2)}, \tilde g_P(q^2)$$

$$\Gamma = \quad \overline{u}\sigma_{\mu\nu} d \quad \to g_T(q^2)$$

“Round up the usual suspects”

Volume?

$m_\pi L$



Renormalization factors?

State “contamination”?

Operator?

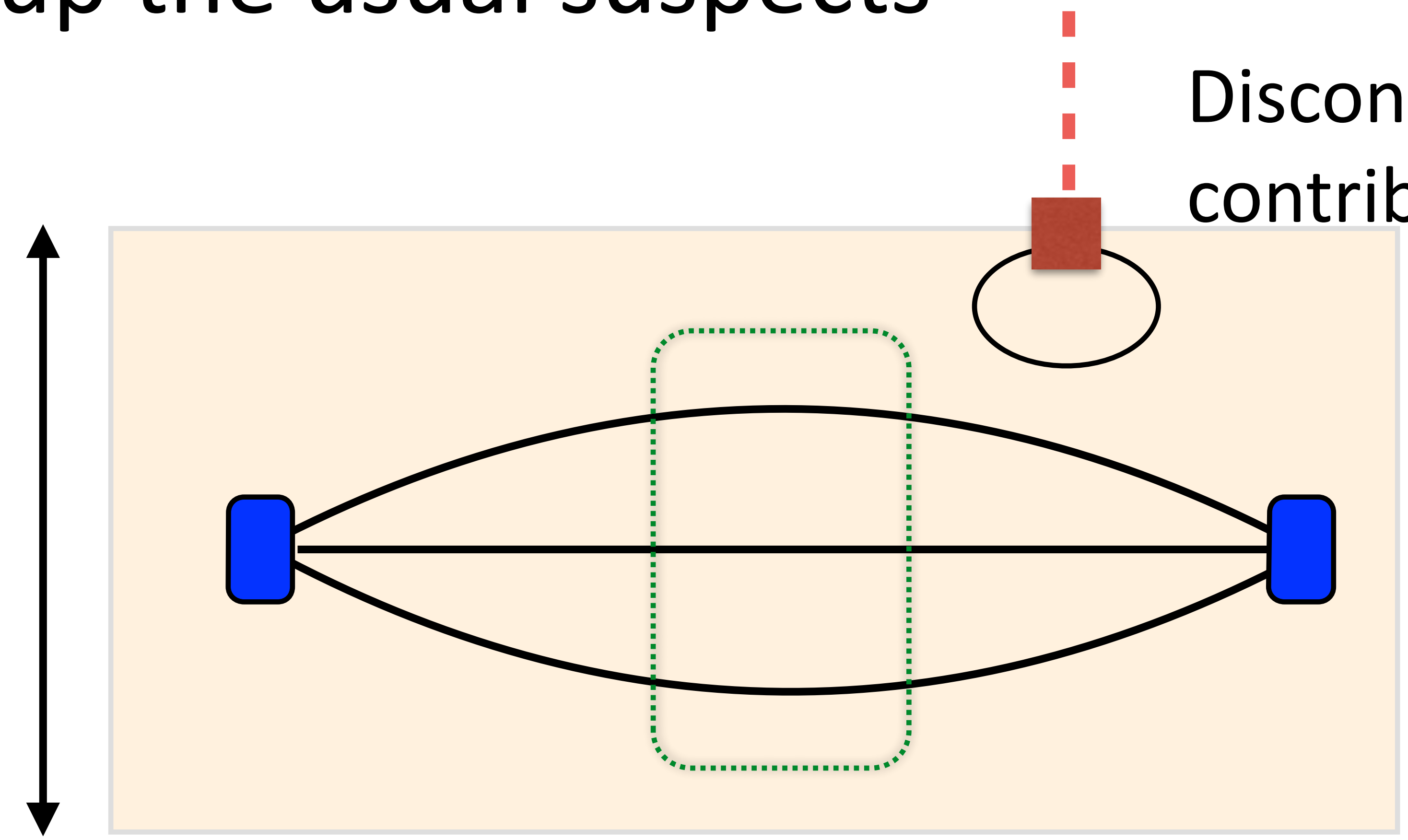
Plateau?

Number of dynamical quarks(2, 2+1, 2+1+1)?

Is m_π close to its physical value?

Is the lattice spacing a small enough?

“Round up the usual suspects”



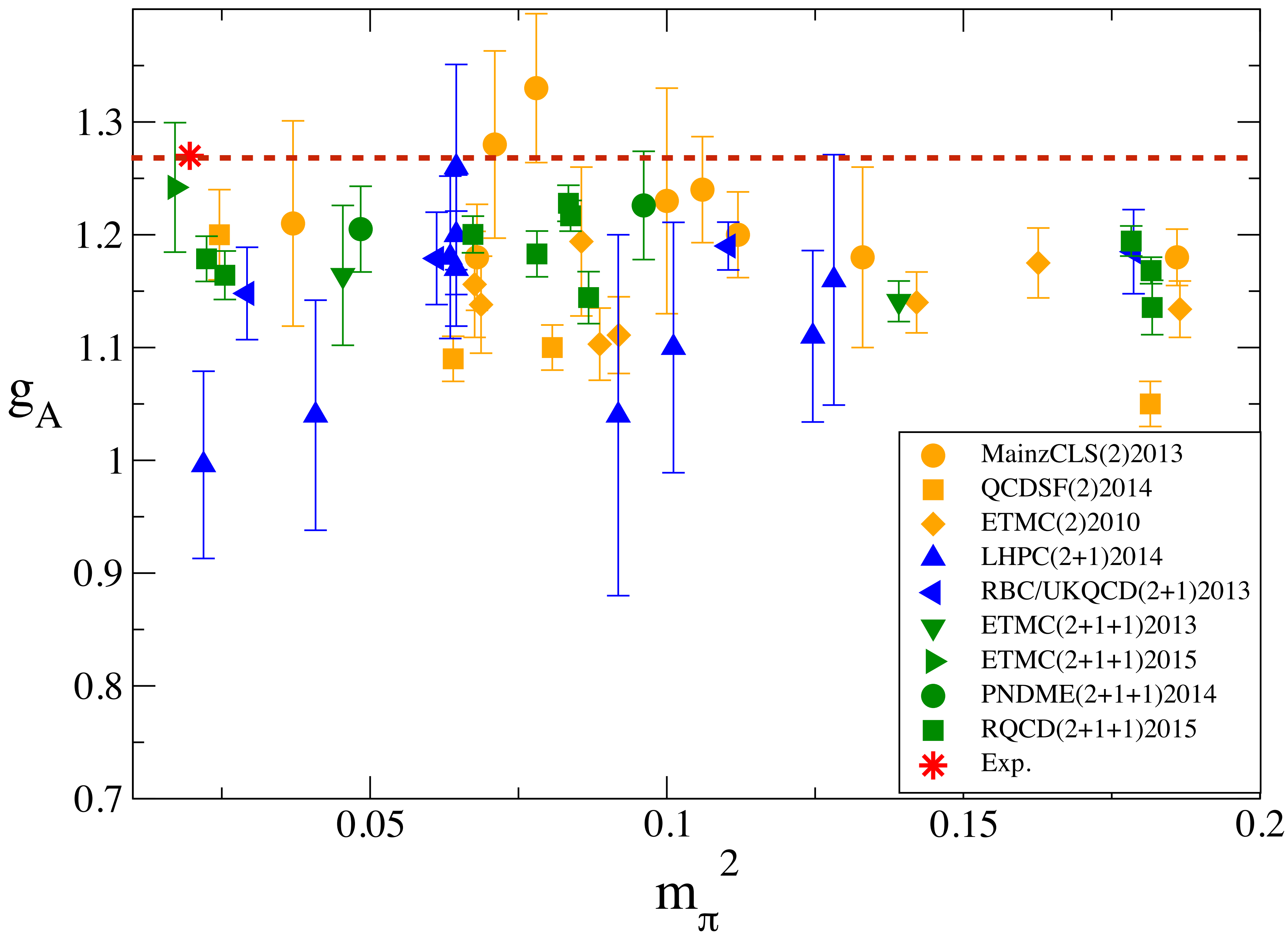
Number of dynamical quarks(2, 2+1, 2+1+1)?

Is m_π close to its physical value?

Is the lattice spacing a small enough?

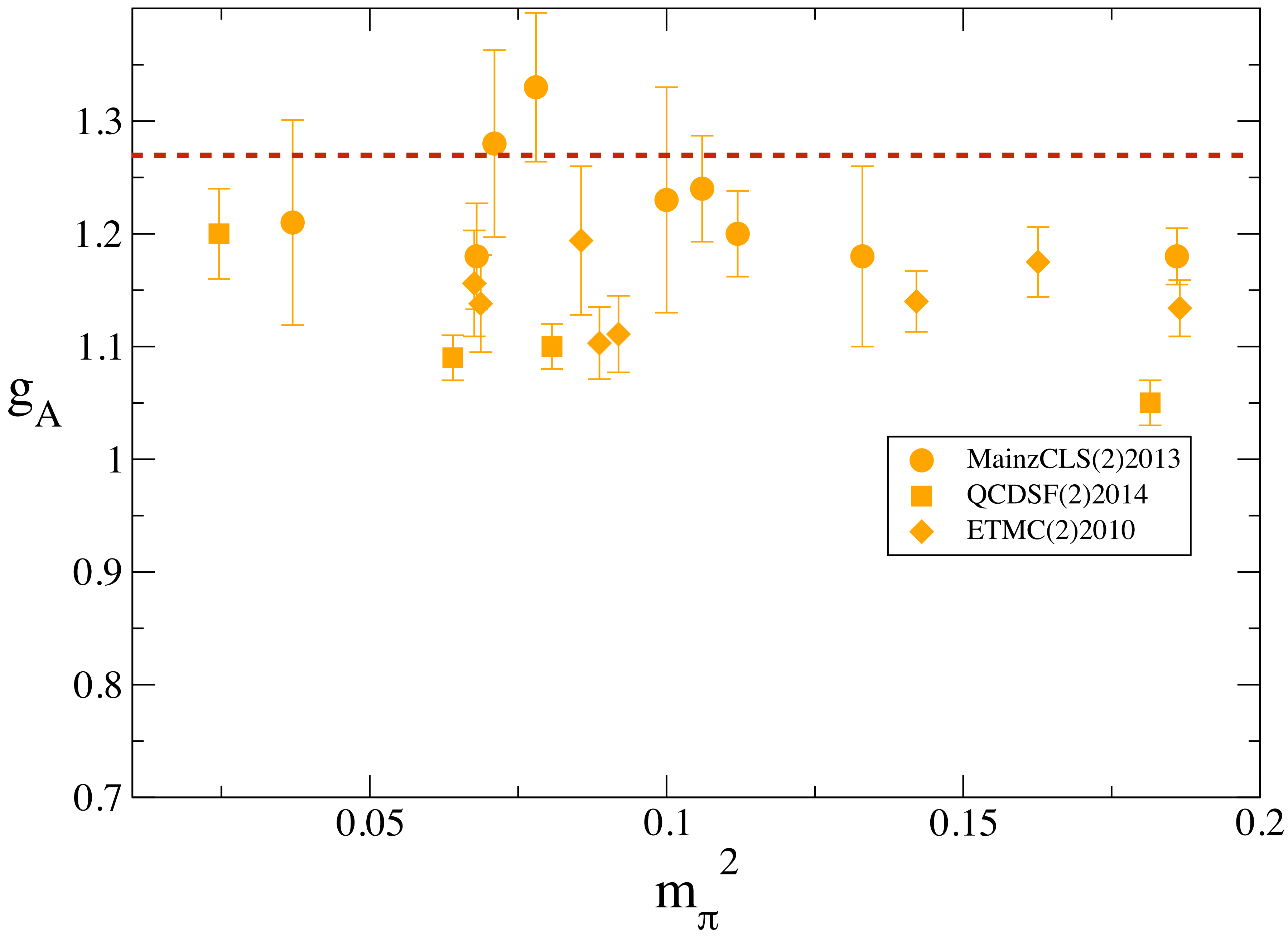
$g_A(0)=g_A$ - Lattice results

(Exp.: 1.2723(23) g_V)

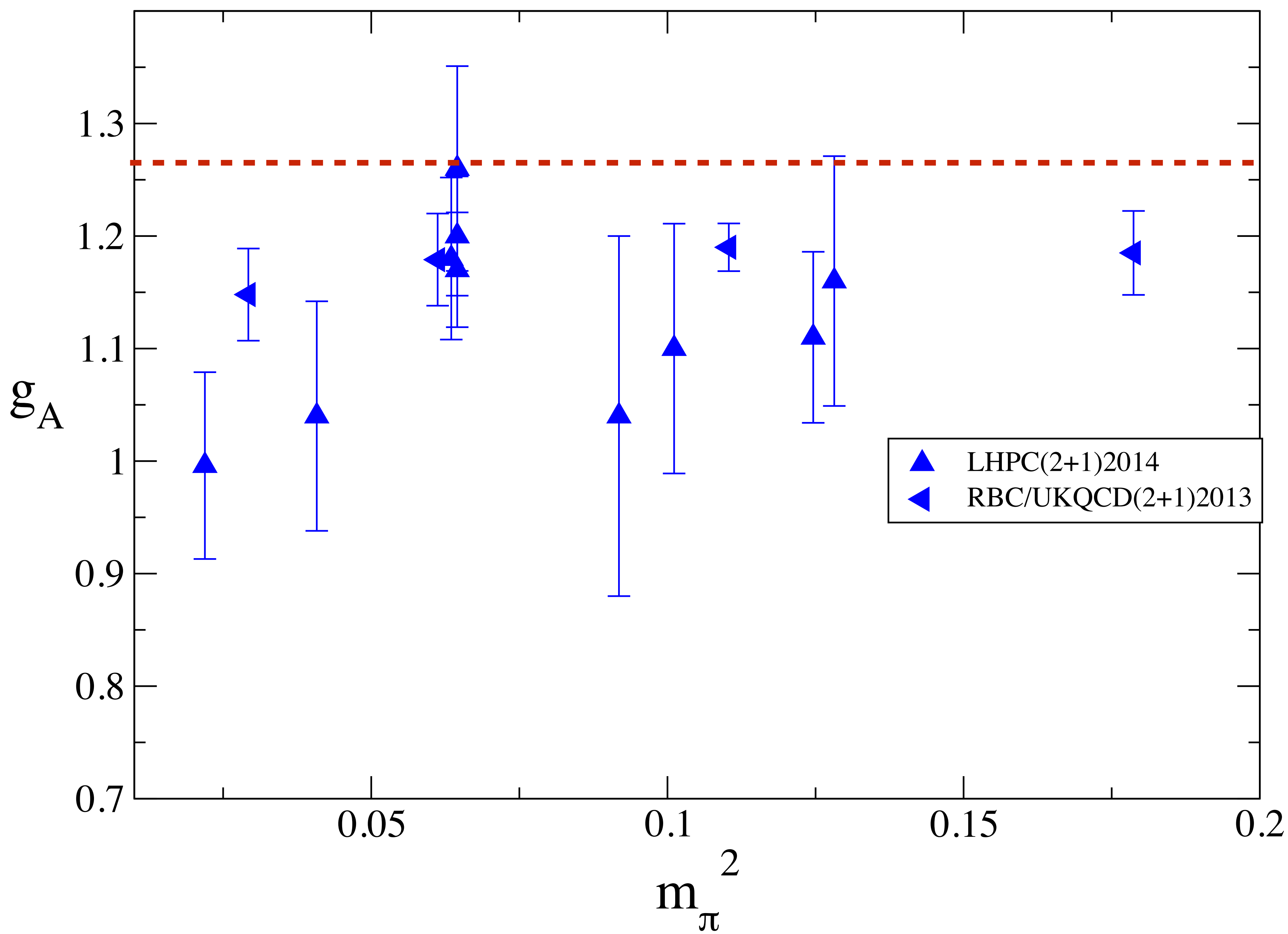


Thanks to Martha Constantinou and Sara Collins for help

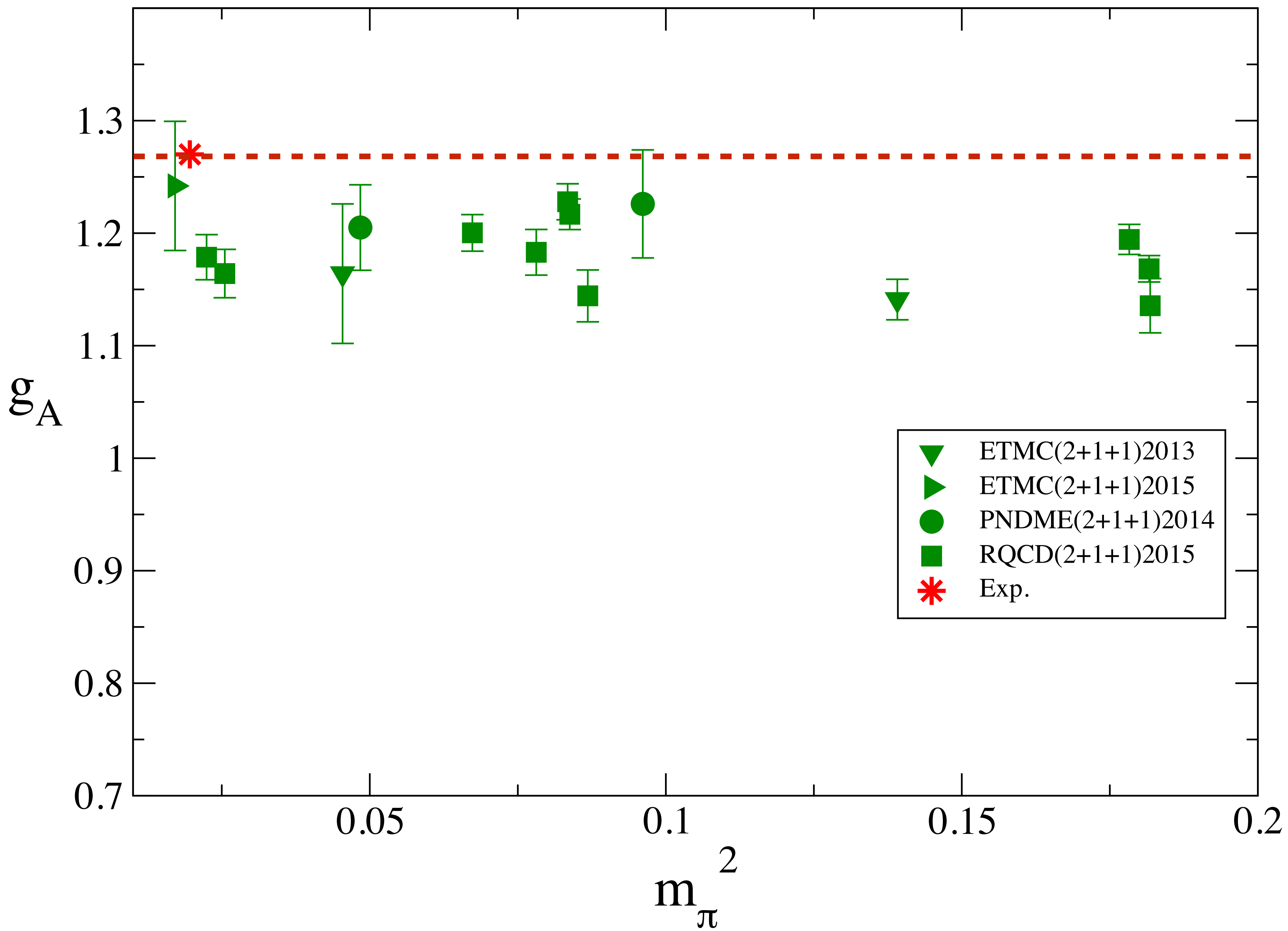
Number of dynamical quarks: 2



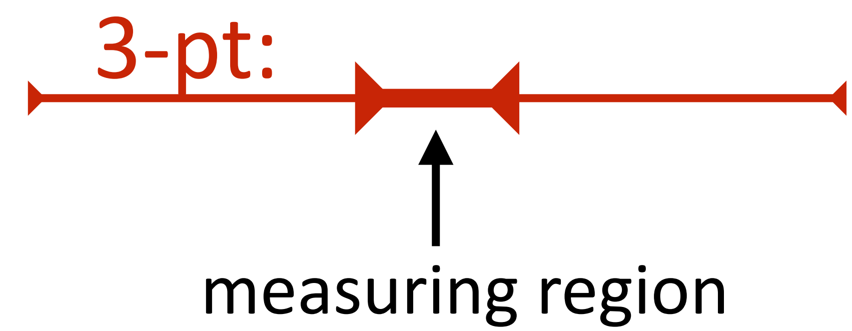
Number of dynamical quarks: 2+1



Number of dynamical quarks: 2+1+1



Plateau?

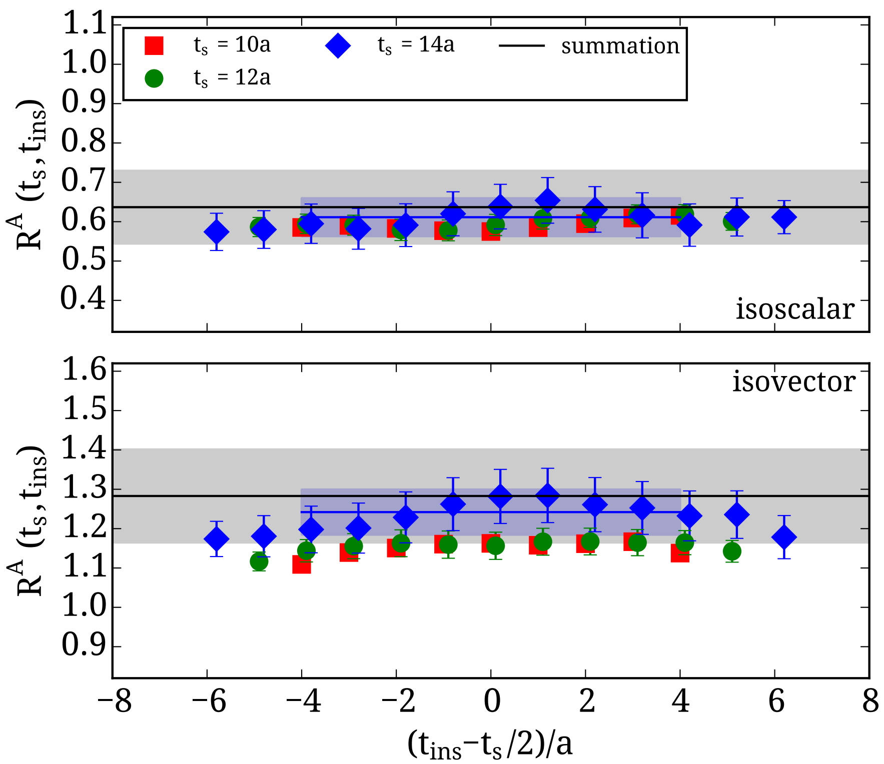


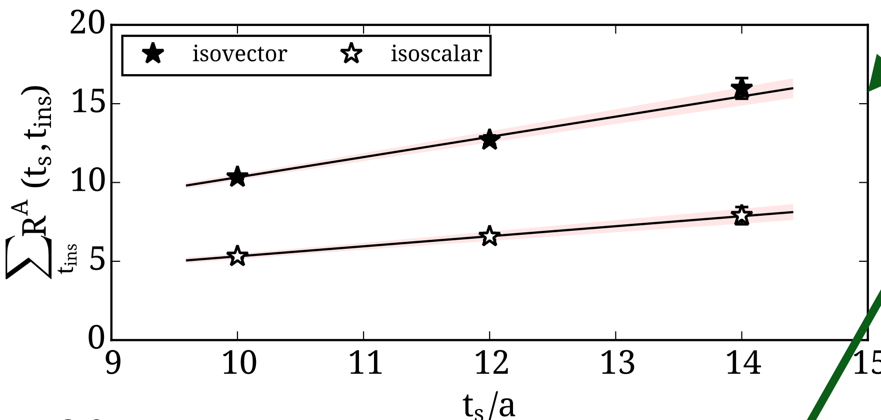
Various methods:

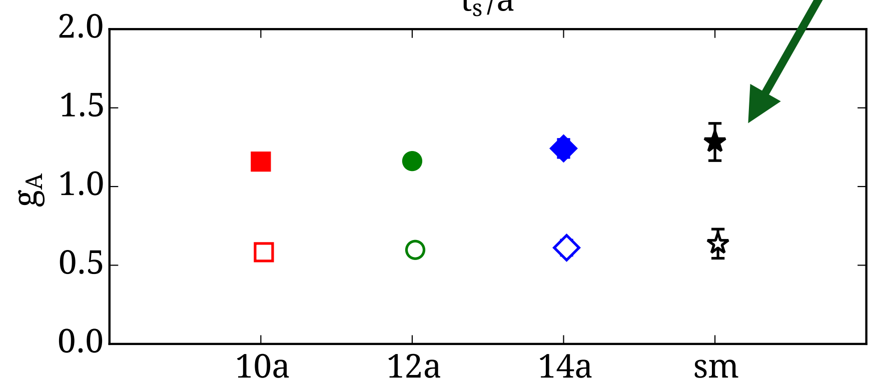
plateau

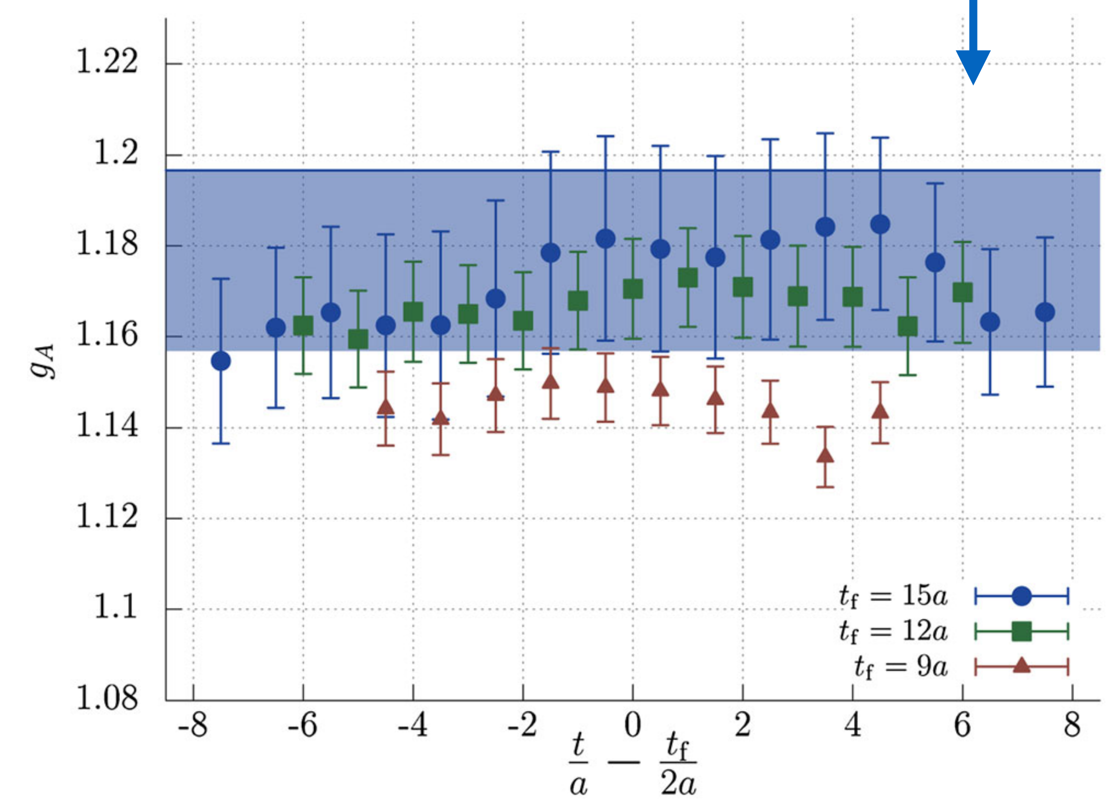
summation

fit including 1st excitation





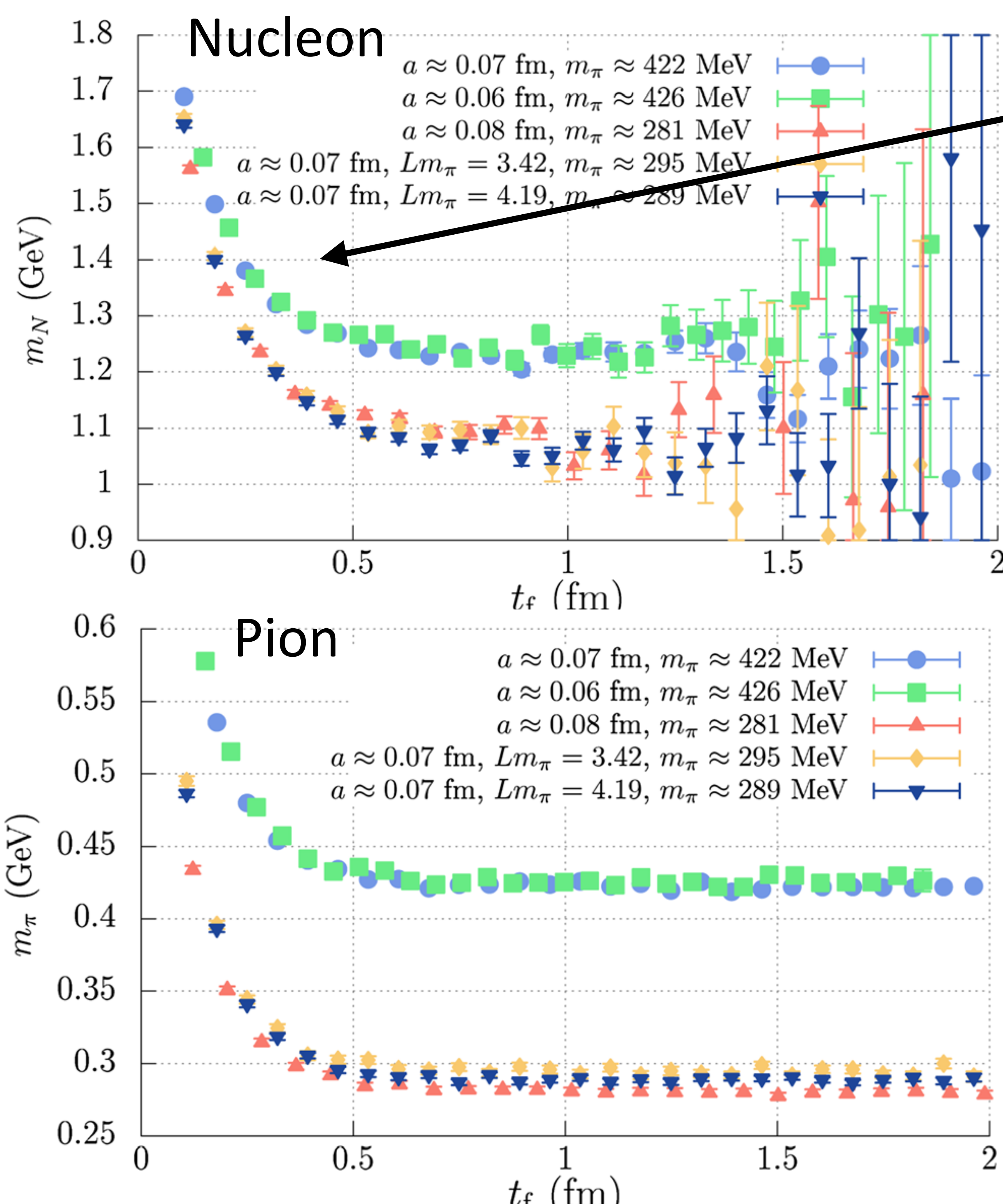




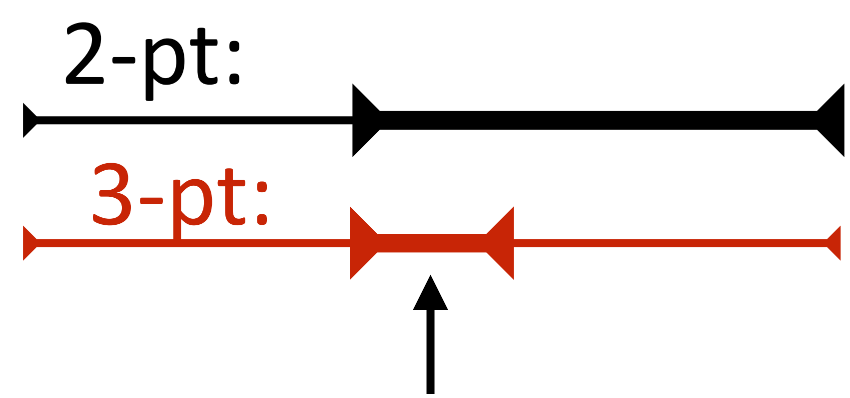
Bali et al, RQCD,
PR D 91, 054501 (2015)

Abdel-Rehim et al., ETMC, [arXiv:1507.04936]

Contamination by excited states?



contributions from excited states

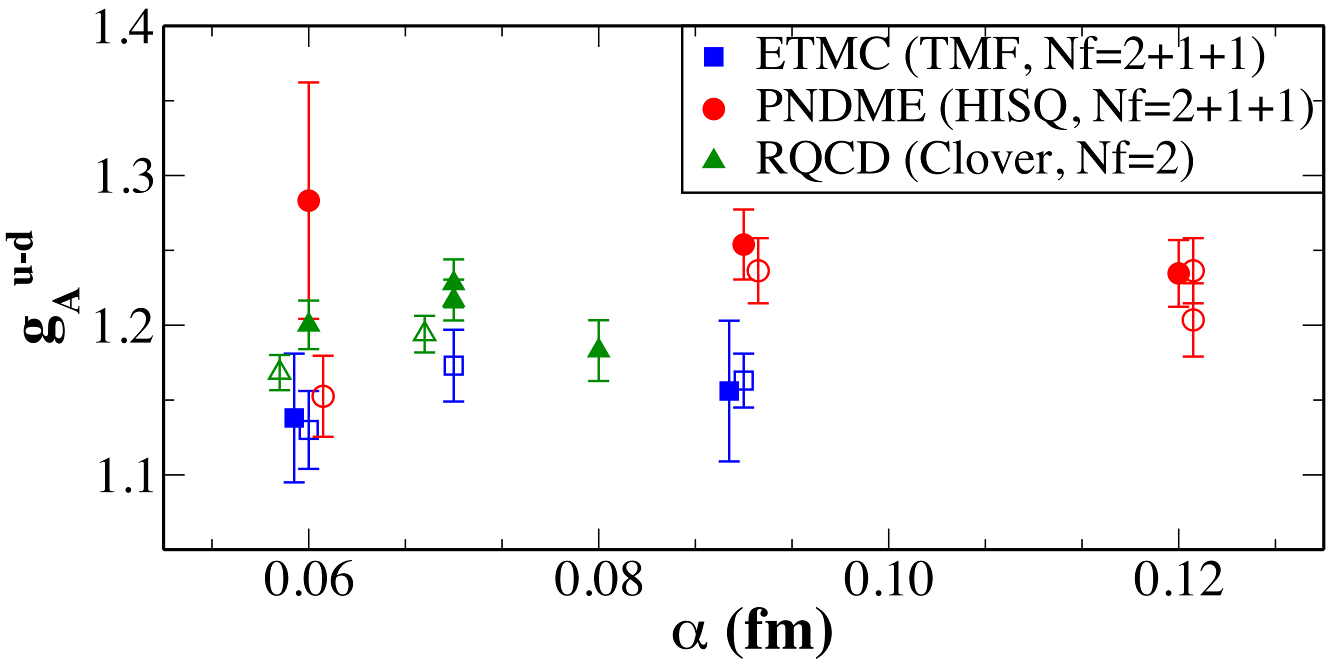


measuring region

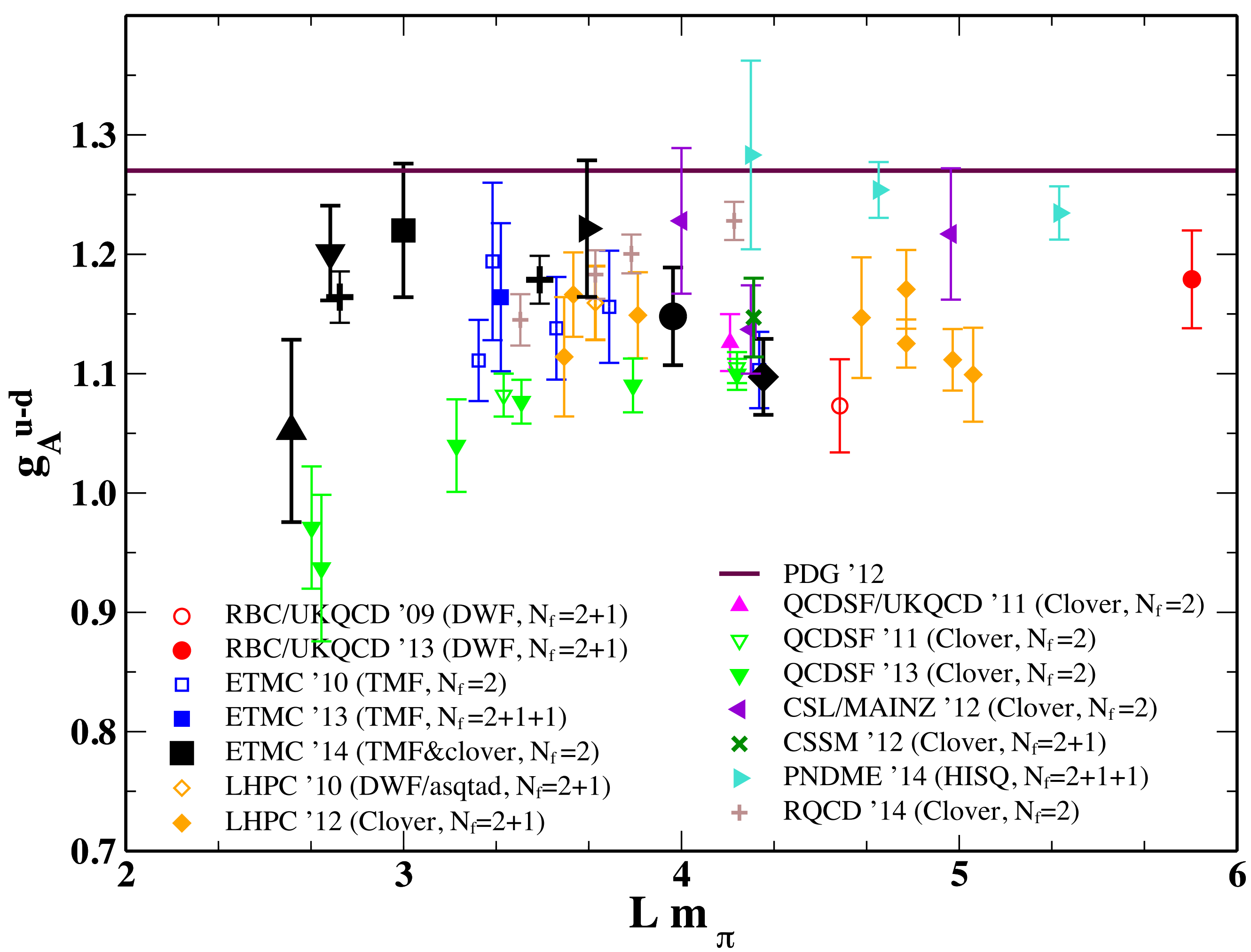
Bali et al, RQCD,

PR D 91, 054501 (2015)

Lattice spacing and volume



Dependence on the lattice spacing a: no obvious systematic trend



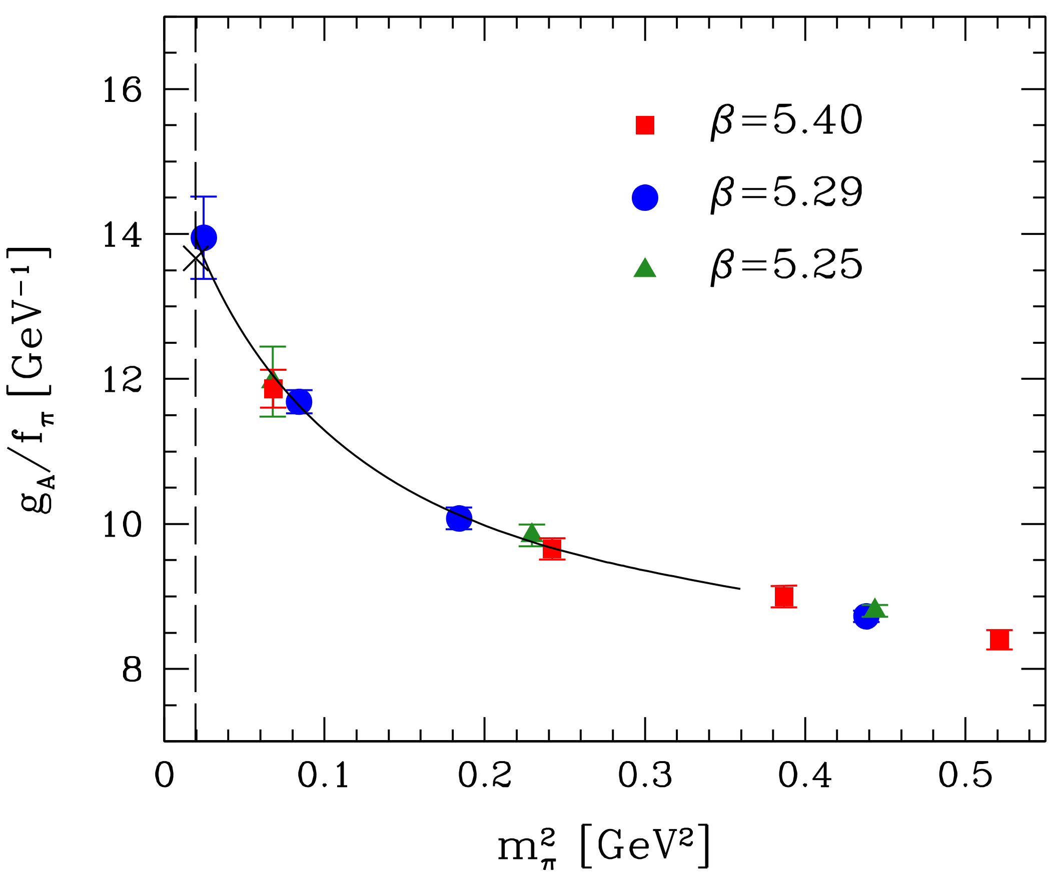
Dependence on the spatial volume: no obvious systematic trend

Both figures from

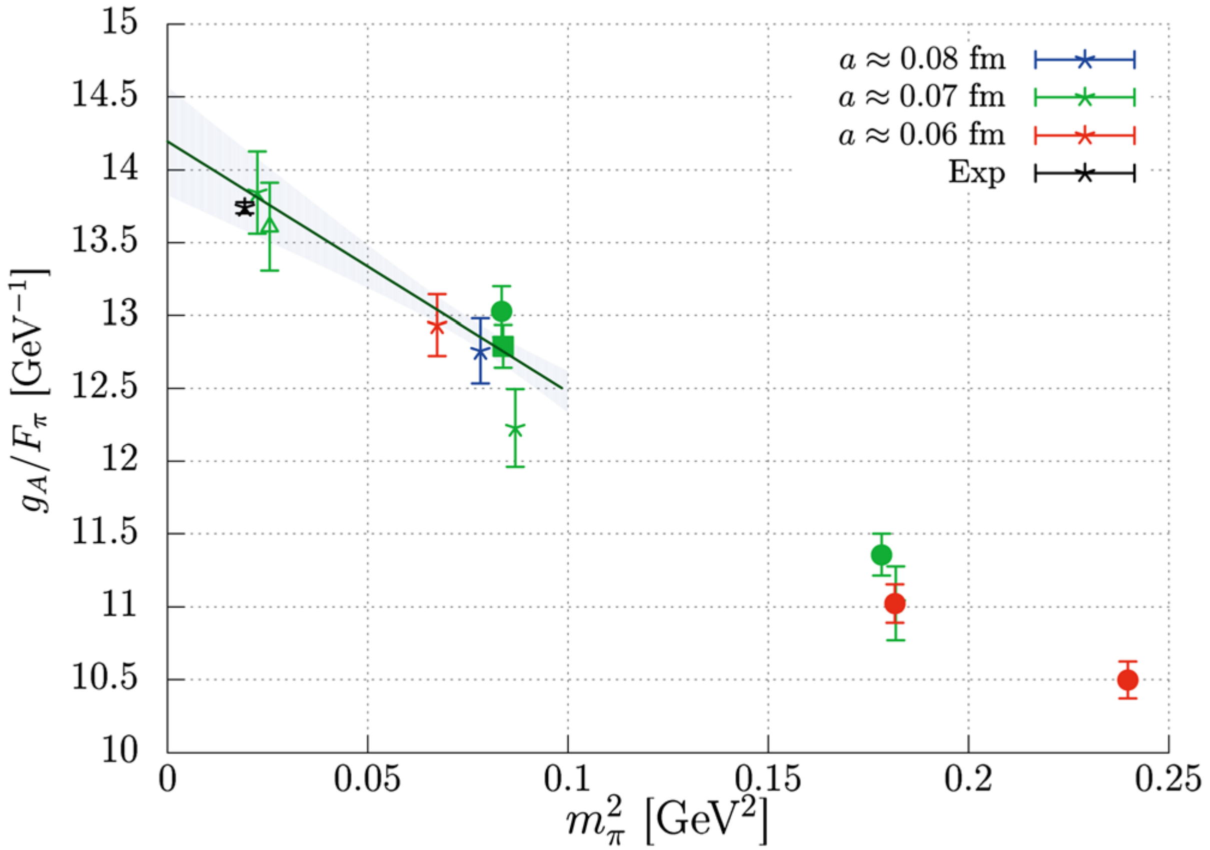
M. Constantinou, POS LATTICE 2014 [arXiv:1411.0078]

Extrapolation to physical m_π

In g_A/F_π the finite volume effect cancel partially, ratio smooth in ChPT:

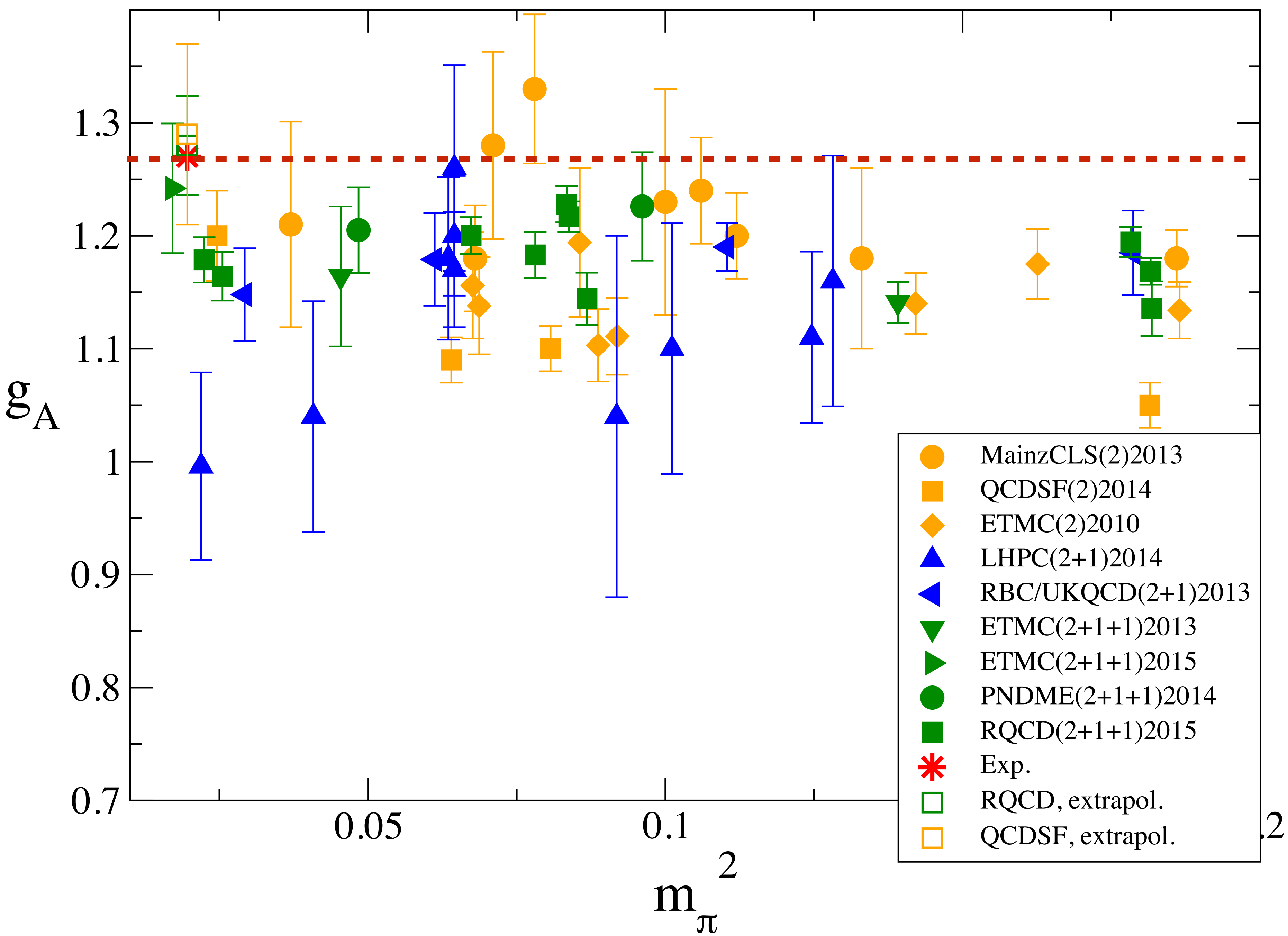


R. Horsley et al., QCDSF,
Phys. Lett. B 732, 41 (2014).



Bali et al, RQCD,
PR D 91, 054501 (2015) [arXiv:1412.7336|

g_A (Exp.: 1.2723(23) g_V)



Thanks to Martha Constantinou and Sara Collins for help

Summary g_A

Medium happy…

Further recent results:

- Disconnected contrib. to isoscalar (S and A) are O(7%)

  Abdel-Rehim et al., Phys.Rev. D89, 034501 (2014)

- Isovector g_S (excited states contributions need large $\tau \approx 1.5$ fm!) and g_T

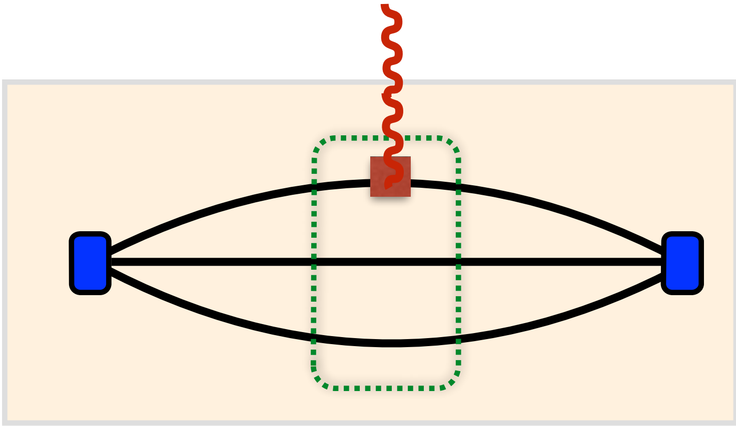
  Bali et al, RQCD, PR D 91, 054501 (2015)
  Abdel-Rehim et al., ETMC, [arXiv:1507.04936]

- ChPT study: nucleon-pion-state contributions in the determination of the nucleon axial charge are few percent;

  Oliver Bär, [arXiv 1508.01021]

Electromagnetic form factors

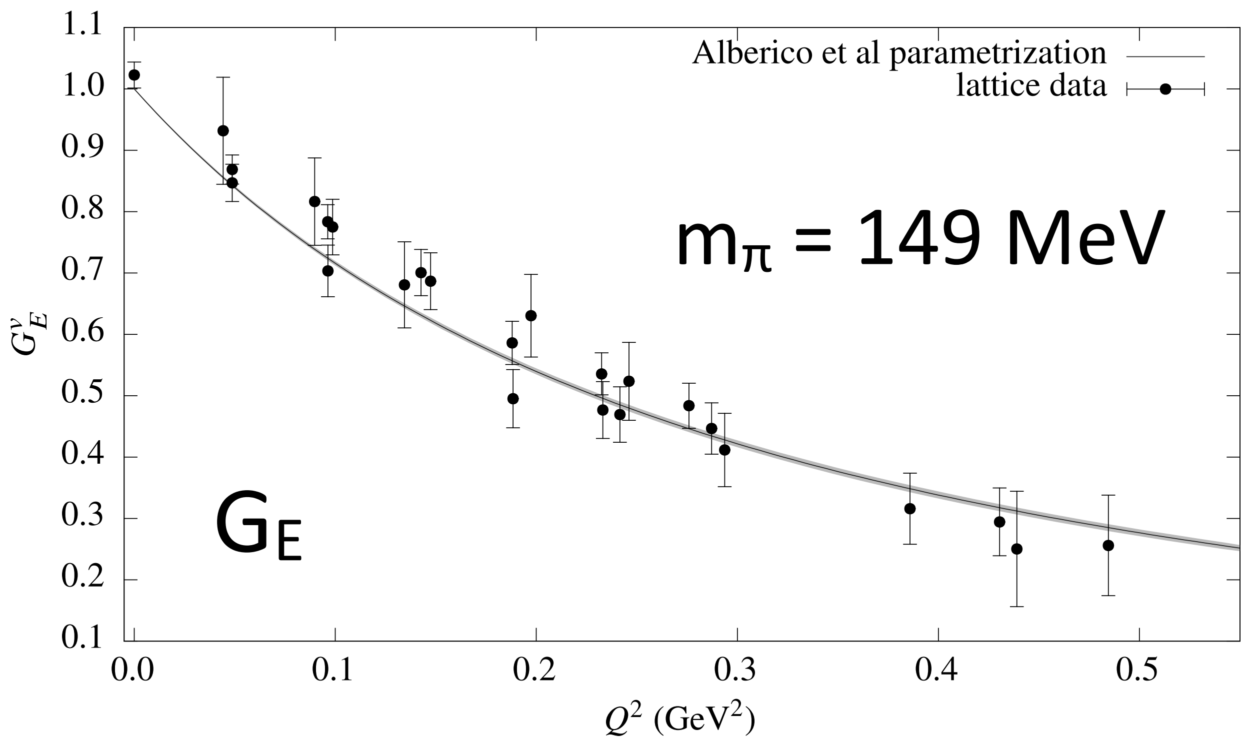
What's new:

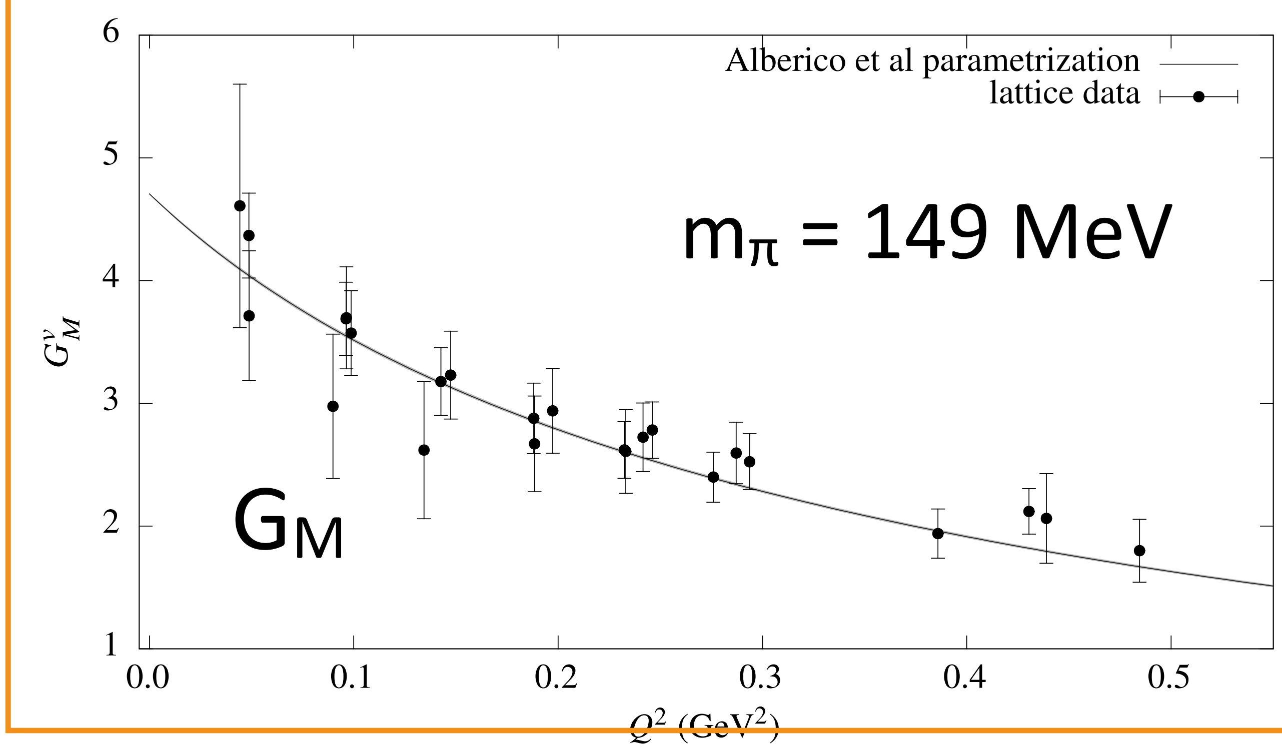


- small m_π (close to physical)
- systematic studies of analysis methods (plateau, summation, 2 state fits)
- large lattices

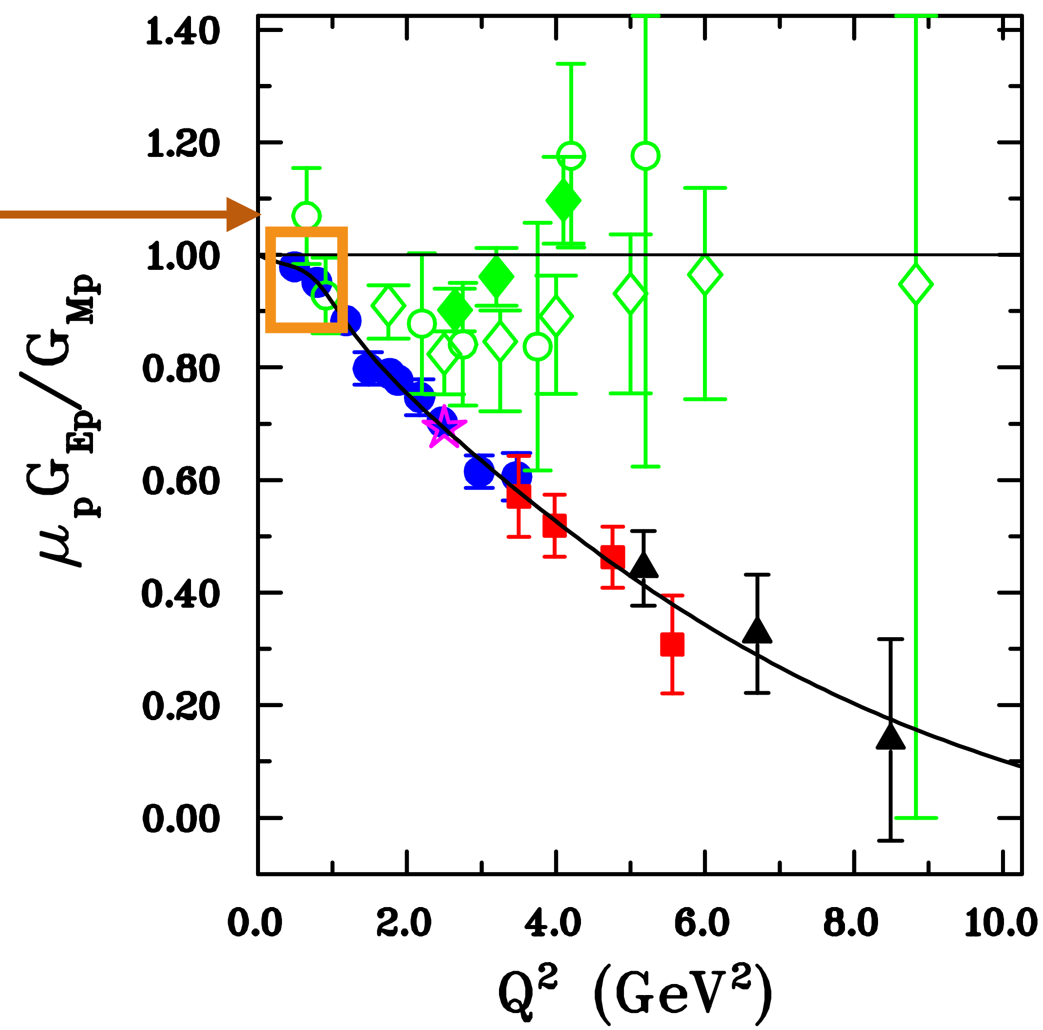
- LHPC, Green et al., Phys. Rev. D 90, 074507 (2014) [arXiv:1404.4029].
  m_π= 149 MeV, nf=2+1
- MainzCLS, Capitani et al., [arXiv:1504.04628]
  m_π=193 MeV, nf=2
- ETMC, Abdel-Rehim et al.,PoS(LATTICE2014)148 [arXiv:1501.01480]
  m_π=135 MeV, nf=2+1+1
- PACS-CS, prelim. results presented by Yamazaki at LATTICE2015
  m_π=145 MeV, nf=2+1

Electromagnetic form factors





typical
lattice
range



Punjabi et al.,
[arXiv;1503.01425]

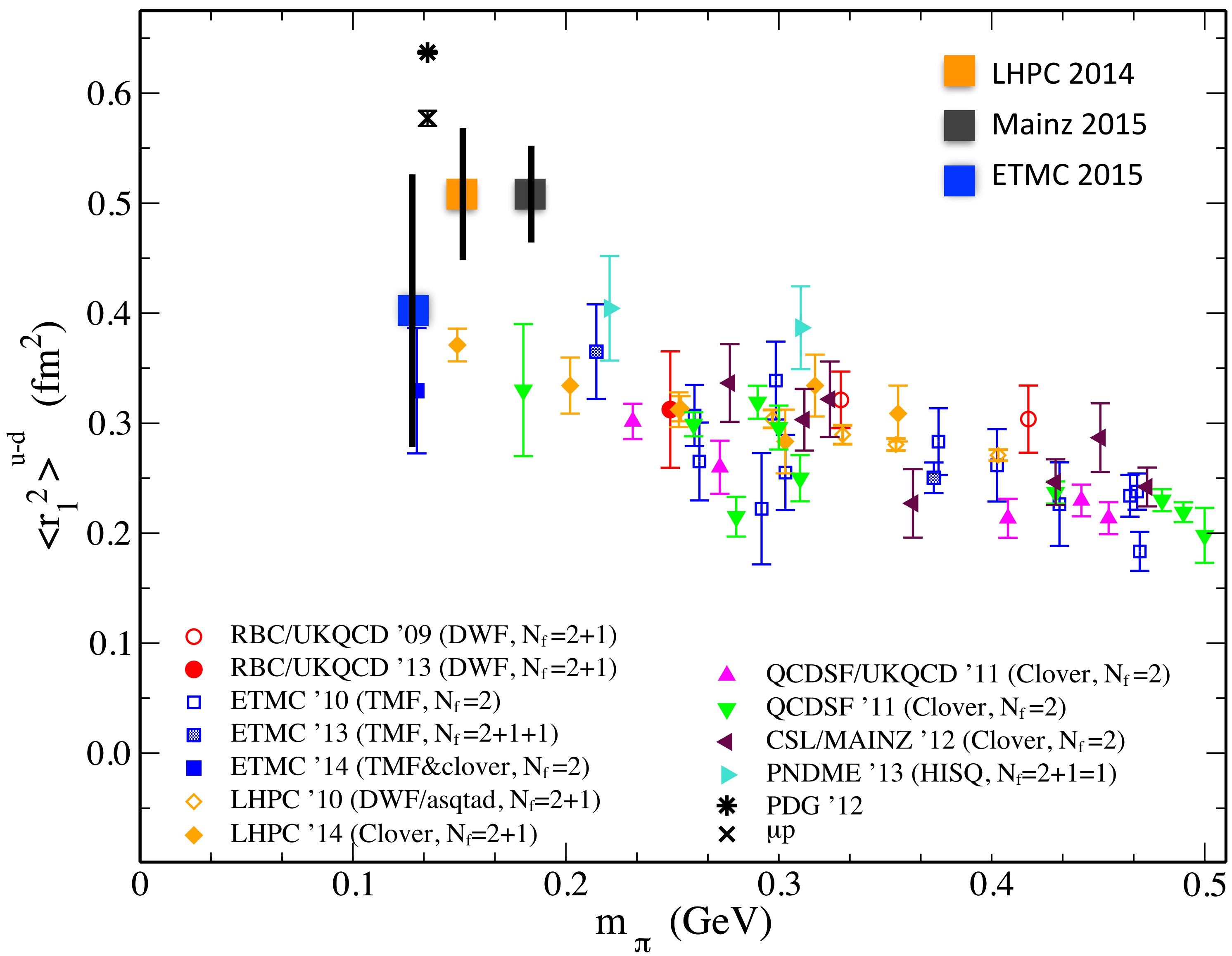
LHPC, Green et al.
Phys. Rev. D 90, 074507 (2014)
[arXiv:1404.4029].

Charge radii

	m_π (MeV)	$(r_1^2)^v$ (fm^2)	κ^v	$\kappa^v(r_2^2)^v$ (fm^2)
LHPC	149	0.498(55)	3.76(38)	2.68(62)
Mainz	193	0.501(42)	3.33(35)	2.61(9)
ETMC	135	0.398(126)	3.21(35)	2.52(63)
Exp.		0.640(9) or 0.578(2)	3.706	2.47(8) or 2.96(21)

ep μp

Obtained from extrapolating suitable fits to t=0



LHPC, Green et al., Phys. Rev. D 90, 074507 (2014) [arXiv:1404.4029]

Mainz CLS, Capitani et al., [arXiv: 1504.04628]

ETMC, Abdel-Rehim et al., PoS(LATTICE2014)148 [arXiv:1501.01480

Consistent lattice results but maybe not small enough Q^2

Electromagnetic form factors

Lattice: problems to reach small and large values of q^2

$q^2 \approx \vec{k}^2 (2\pi/L_s)^2$

small (non-vanishing) q^2 needs larger volumes

large q^2 : noise increases with momentum transfer k

(k^2=6 corresponds to q^2 =0.96 GeV^2 for L_s=3 fm)

New approach working at larger q^2:

Feynman-Hellmann relation between <H|*O*|H> and the derivative of a 2-pt function

(Insert operator *O* as extra term in the action, cf. σ_N)

CSSM/QCDSF/UKQCD, Chambers et al., Phys. Rev. D 90, 014510 (2014), Young at LATTICE2015

Proton spin

Jaffe & Manohar, Nucl. Phys. B337, 509 (1990)

Ji, Phys. Rev. Lett. 78, 610 (1997)

Quark contributions to spin

$$\frac{1}{2} = \boxed{\sum_q J_q} + J_G \qquad J_q = L_q + \frac{1}{2}\Delta\Sigma^q$$

$$\langle x \rangle^q \,, \langle p' | T^{\mu\nu} | p \rangle$$

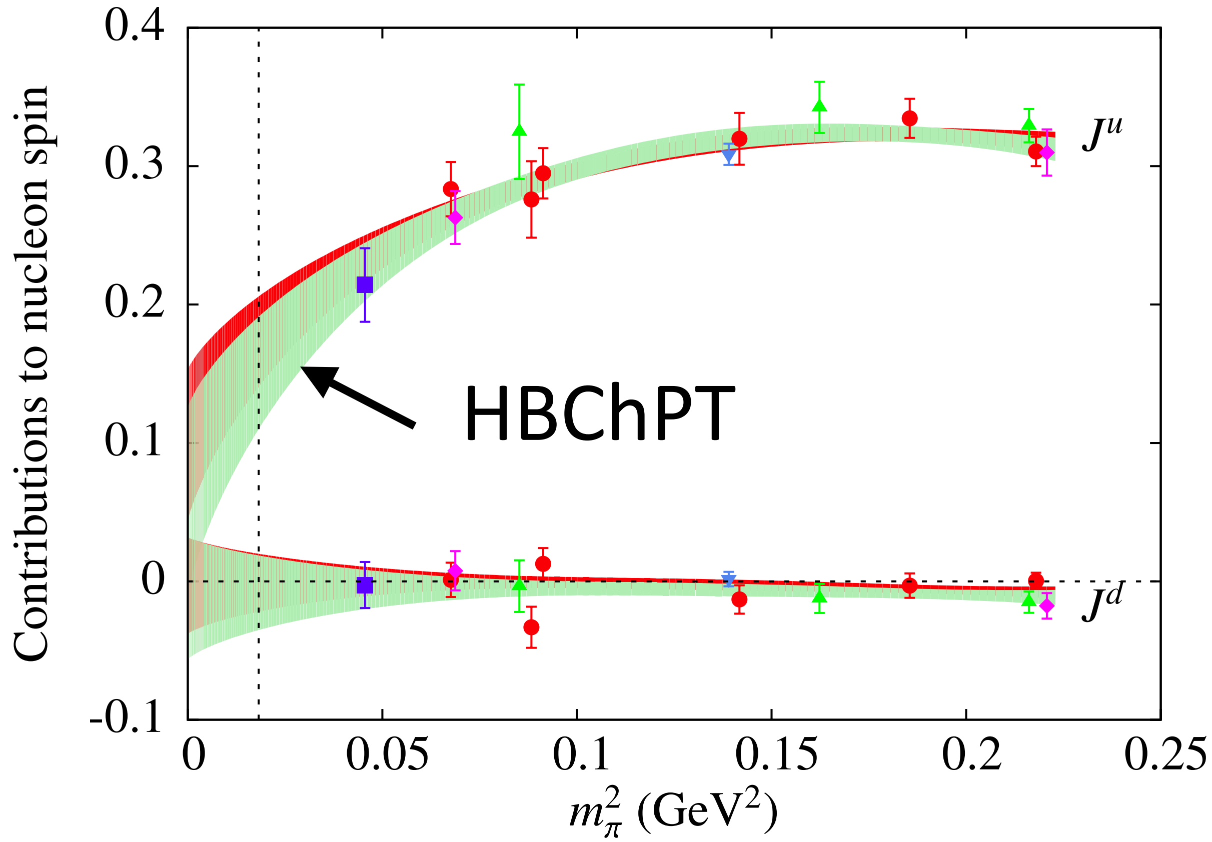
needs matrix elements with derivatives

Results: isovector and isoscalar contributions

For individual $\Delta\Sigma^q$ one needs isoscalar (disconnected) contributions $\longrightarrow$ stochastic source methods

Proton spin

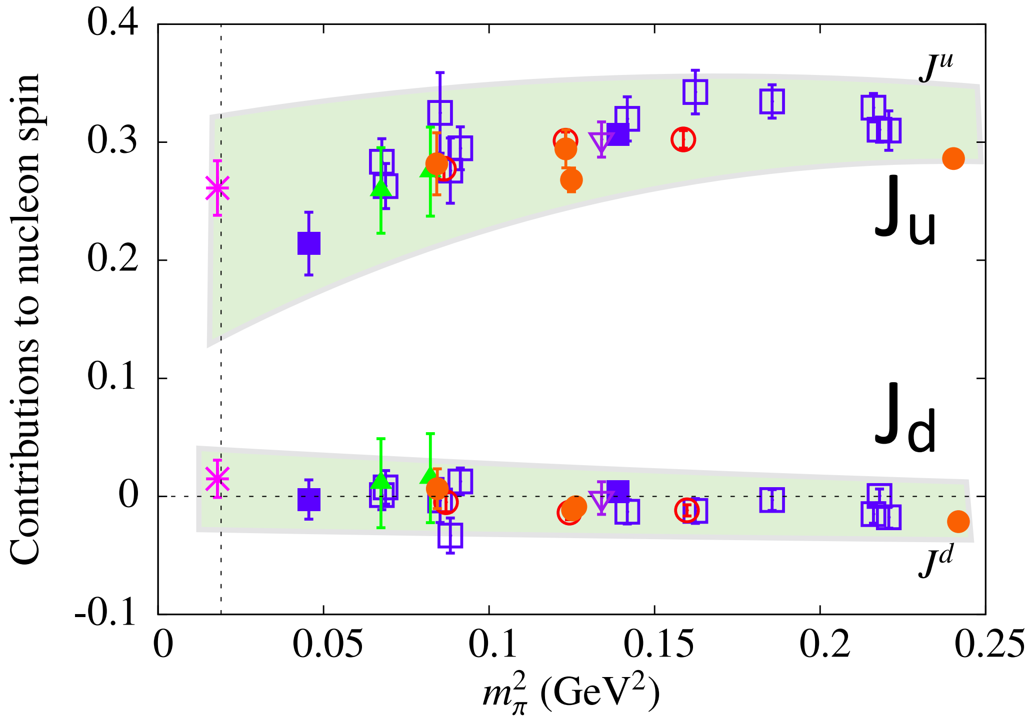
comparison: LHPC, QCDSF/UKQCD, ETMC

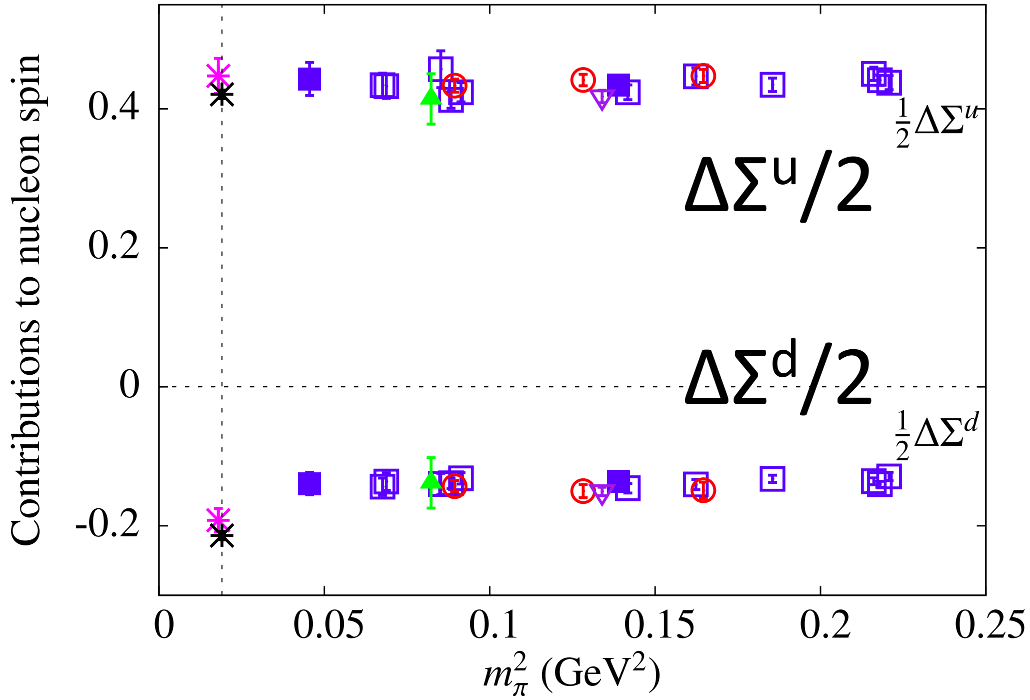


Alexandrou et al., Phys.Rev. D 88 (2013) 014509

$$\Delta u + \Delta d = 0.35(6)$$

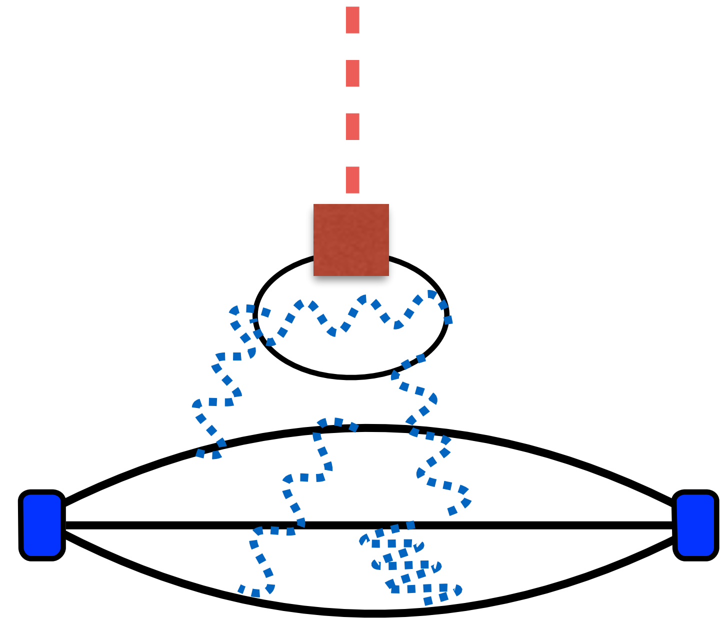
see also Yang et al. (χQCD collaboration) PoS(LATTICE2014)138 [arXiv:1504.04052]





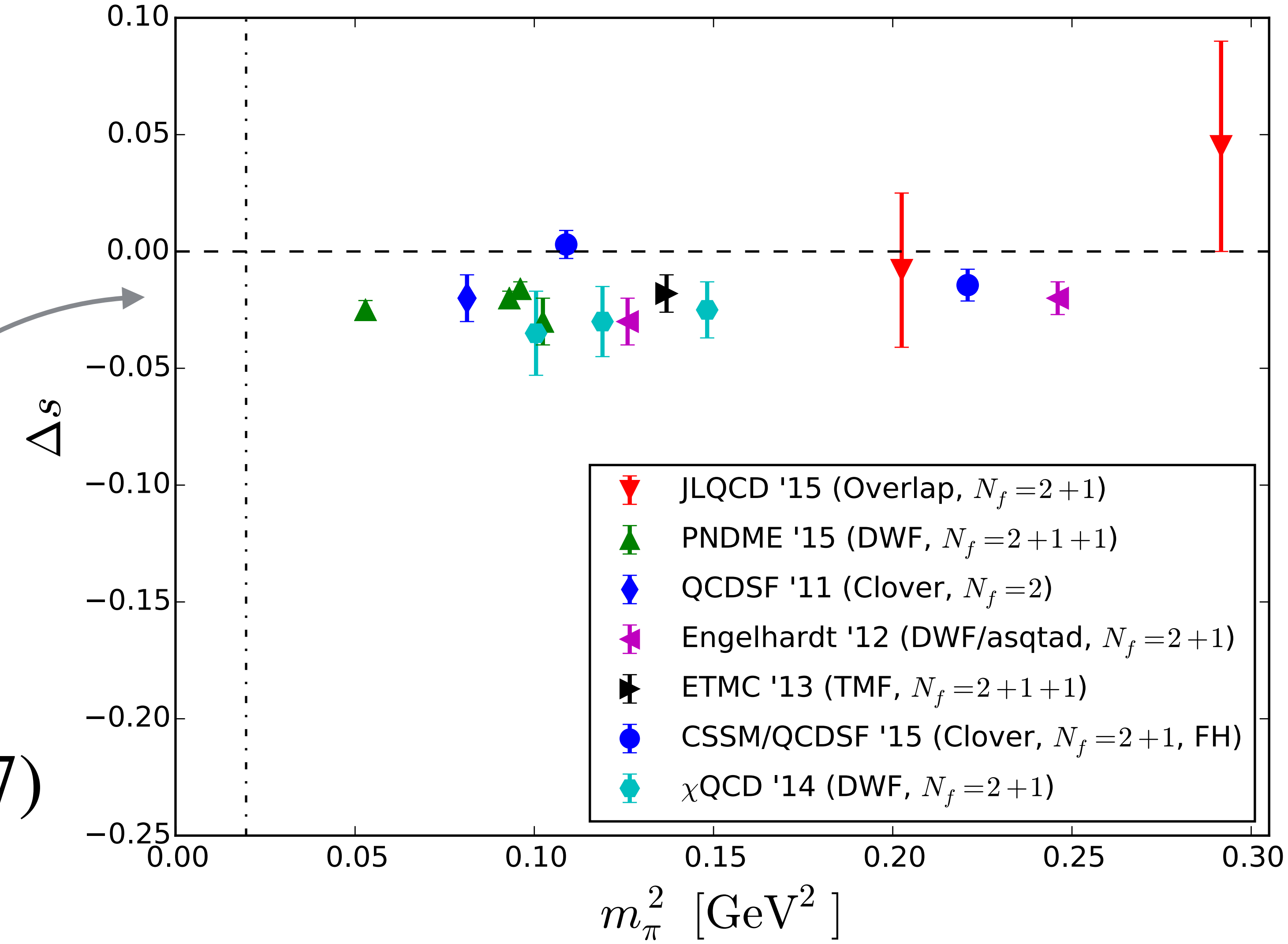
M. Constantinou, POS LATTICE 2014 hep-lat 1411.0078

Disconnected contributions



$$\Delta\Sigma^s = \int_0^1 dx\ (\Delta\Sigma^s(x) + \Delta\overline{\Sigma}^s(x))$$

from the vacuum sea,
disconnected
(connected via gluons)



$\Delta s = -0.02(1)$

$$\rightarrow\ \Delta\Sigma = \Delta u + \Delta d + \Delta s = 0.33(7)$$

COMPASS(2007) **0.33**(3)(5)

Zanotti, LATT2015

Recent: Chambers et al.[arXiv:1508.06856]

Proton spin

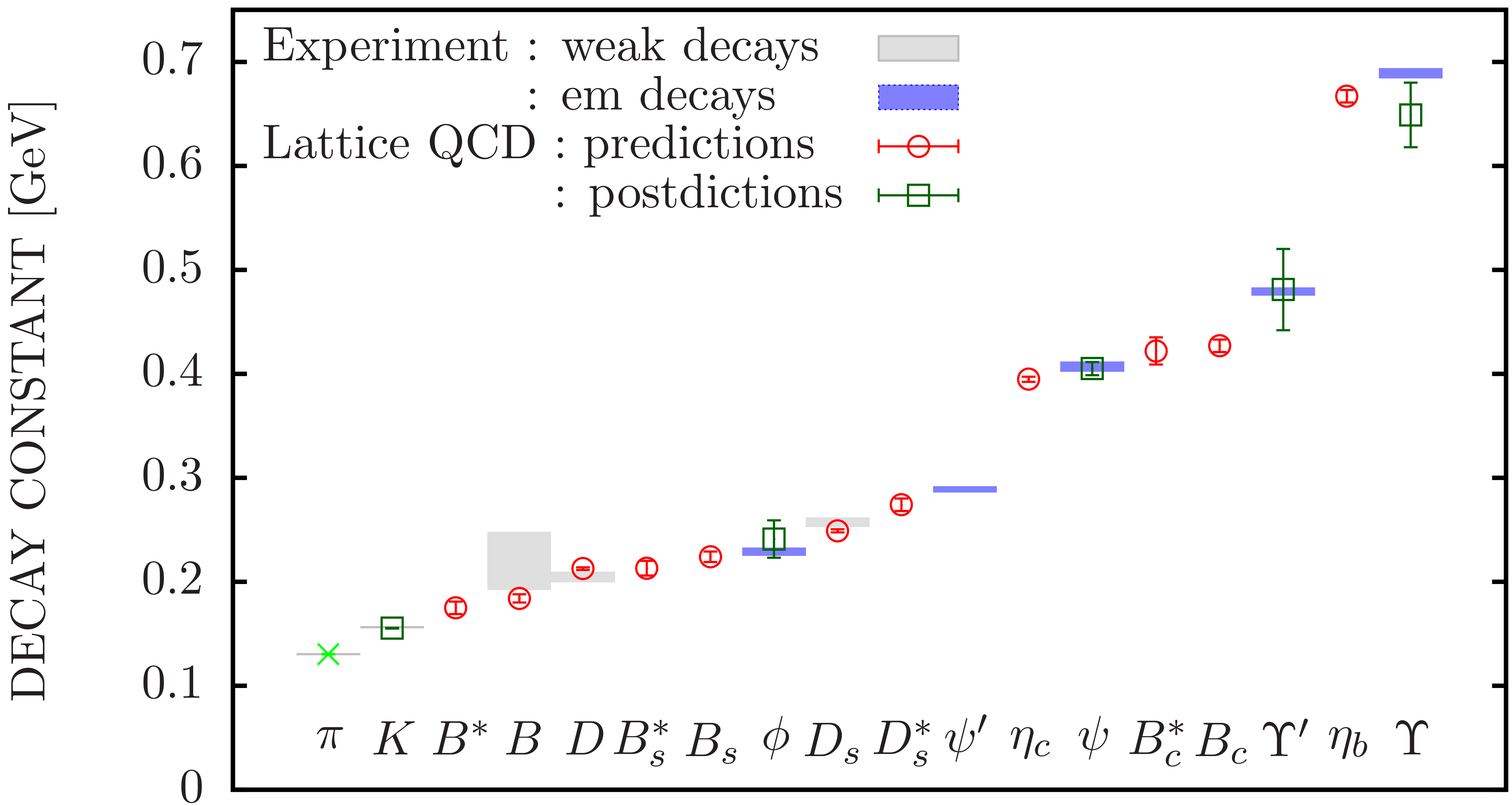
Summary:
lattice results compatible with experiments, but:

- Uncertainties still large
- m_π too large, rely on extrapolation
- Other components? Gluonic contributions?

Yang et al., LATT2015, prelim.
RBC/UKQCD in progress

Review: Liu, at SPIN2014, [arXiv:1504.06601]

Meson decay constants



Colquhoun et al. (HPQCD), Phys. Rev. D 91,114509 (2015)
[arXiv: 1503.05762]

Compilation of low energy parameters

Leptonic and semileptonic decay constants, CKM matrix elements, quark masses, quark condensate, α_S , …

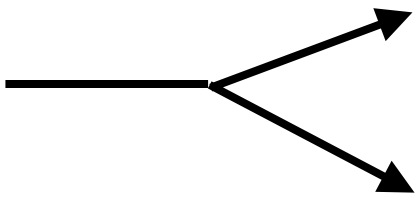
→ Flavor Lattice Averaging Group - FLAG

http://itpwiki.unibe.ch/flag

"Review of lattice results concerning low energy particle physics"

Eur. Phys. J. C (2014) 74:2890 [arXiv: 1310.8555]

Radiative decays (prepare the path for N*→N γ)

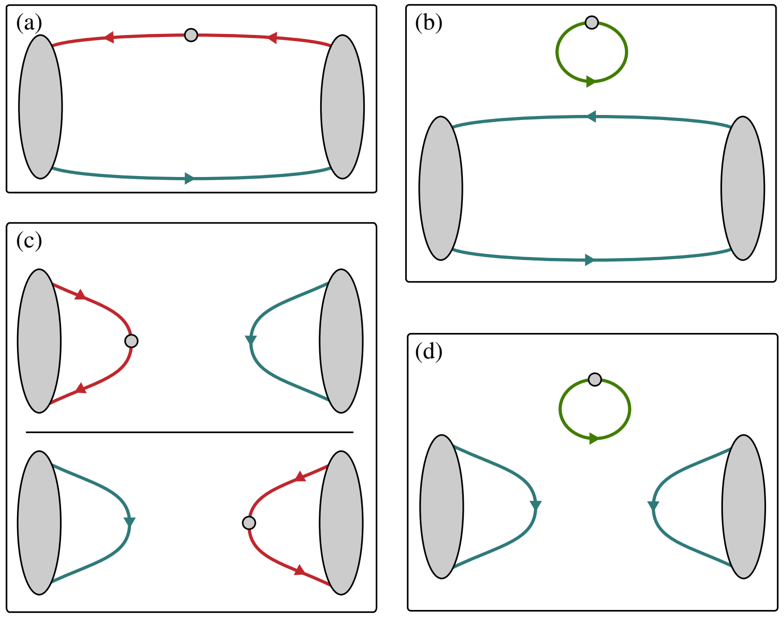


Lellouch & Lüscher Commun.Math.Phys. 219, 31 (2001)

Briceño et al., Phys.Rev. D91, 034501 (2015).

Bernard et al., JHEP 1209, 023 (2012), [arXiv:1205.4642]
Agadjanov et al., Nucl. Phys. B886, 1199 (2014), [arXiv:1405.3476]

$\rho \to \pi\gamma^*$ transition form factor



- *Assuming ρ is stable*
- Variationally optimised operators

Shultz et al. (HSC), Phys. Rev. D91, 114501 (2015)
[arXiv:1501.07457]

HSC, heavy pseudoscalars O(700 MeV)

Owen et al. (CSSM), [arXiv:1505.02876]

PACS-CS 2+1, m_π=157 MeV

Tue. 1C3 Briceño

New: $\pi\,\gamma^* \to \rho \to \pi\pi$

Here ρ is a resonance

Briceño et al. [arXiv:1507.06622]

$$\mathcal{H}^{\mu}_{\pi\pi,\pi\gamma^\star} = \left\langle \text{out}; \pi, \mathrm{P}_\pi \middle| \mathcal{J}^{\mu}_{\mathrm{x}=0} \middle| \text{in}; \pi\pi, \mathrm{P}_{\pi\pi}, \ell = 1 \right\rangle$$

$$\langle \pi_{out}, \Lambda_\pi | J^\mu | \pi\pi_{in}, \Lambda_{\pi\pi} \rangle$$

$$\mathcal{A}_{\pi\pi,\pi\gamma^\star}(E^\star_{\pi\pi}, Q^2)$$

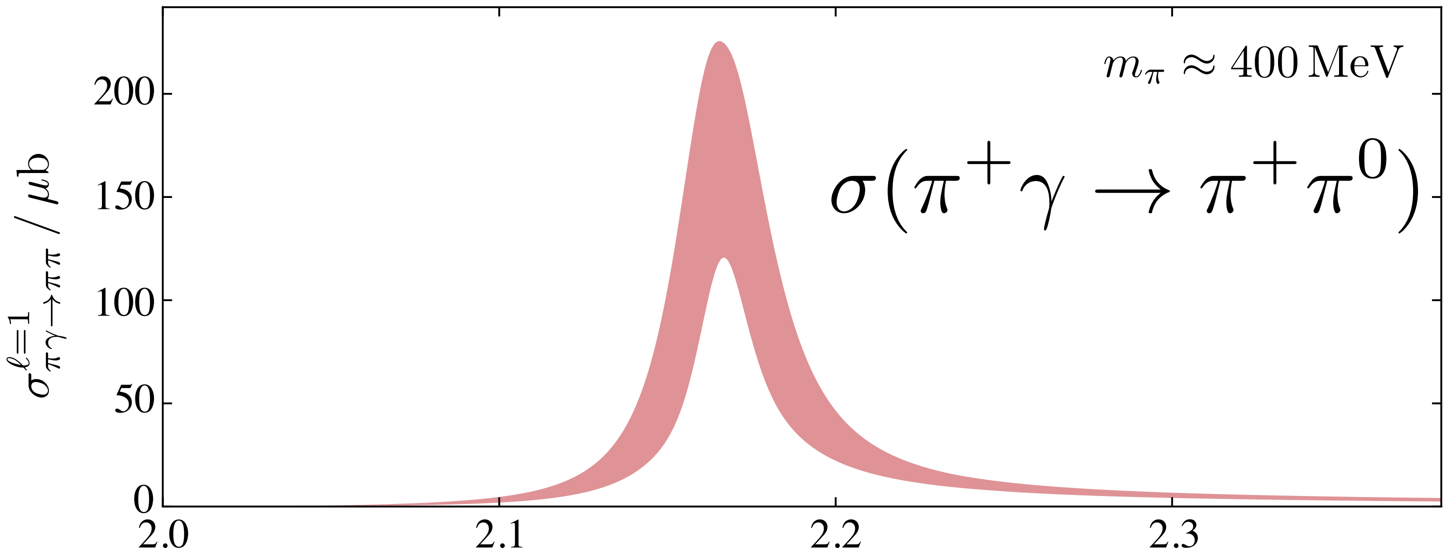
parametrize A and continue analytically to ρ pole

→ Form factor

$$F_{\pi\rho}(E^\star_{\pi\pi}, Q^2)$$

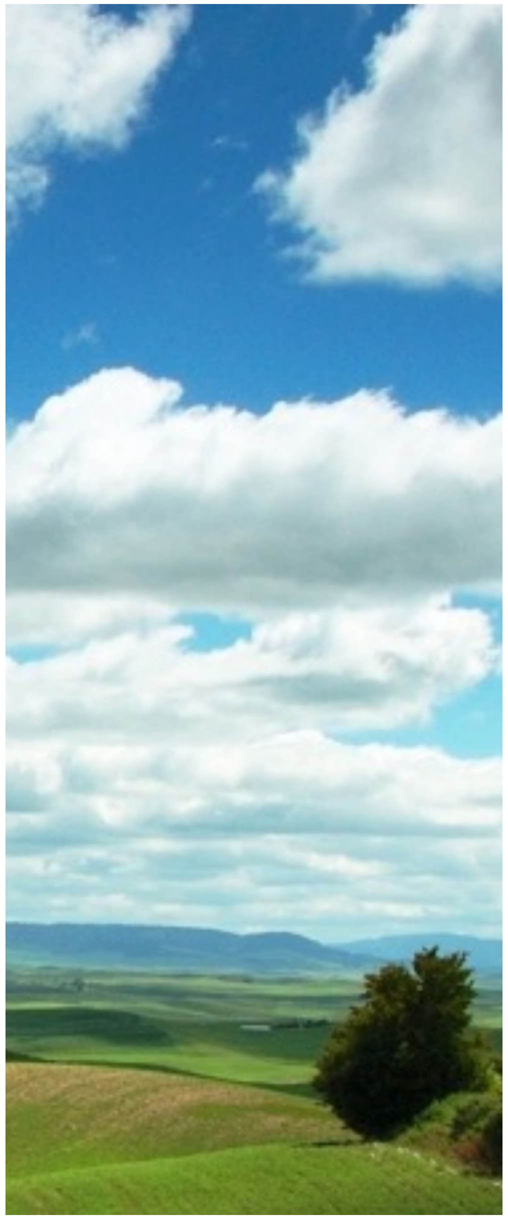
m_π=400 MeV

$-0.4 \leq (Q/\mathrm{GeV})^2 \leq 1$



Spectroscopy

...for excited state lovers



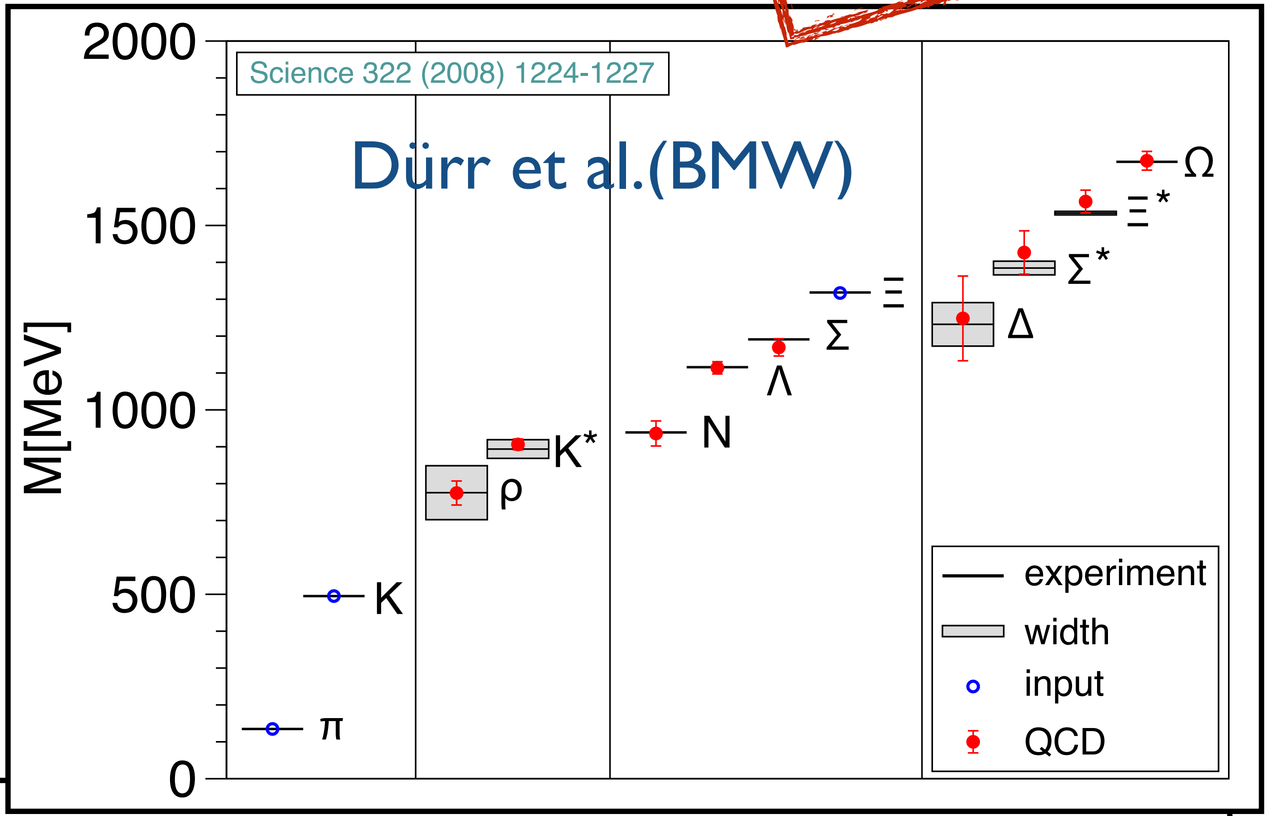
- Single hadron approach
  qqq or $\bar{q}q$
- Multi-hadron approach:
  resonances and scattering
- Heavy quark results

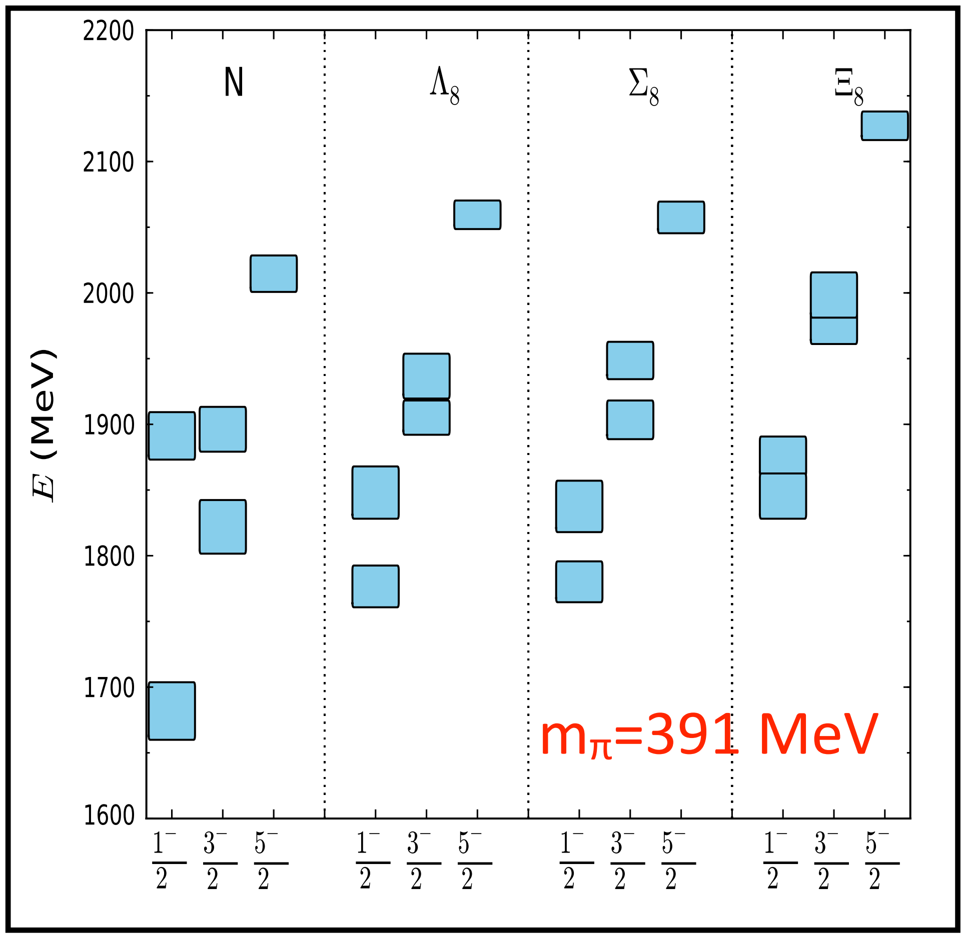
Milestones

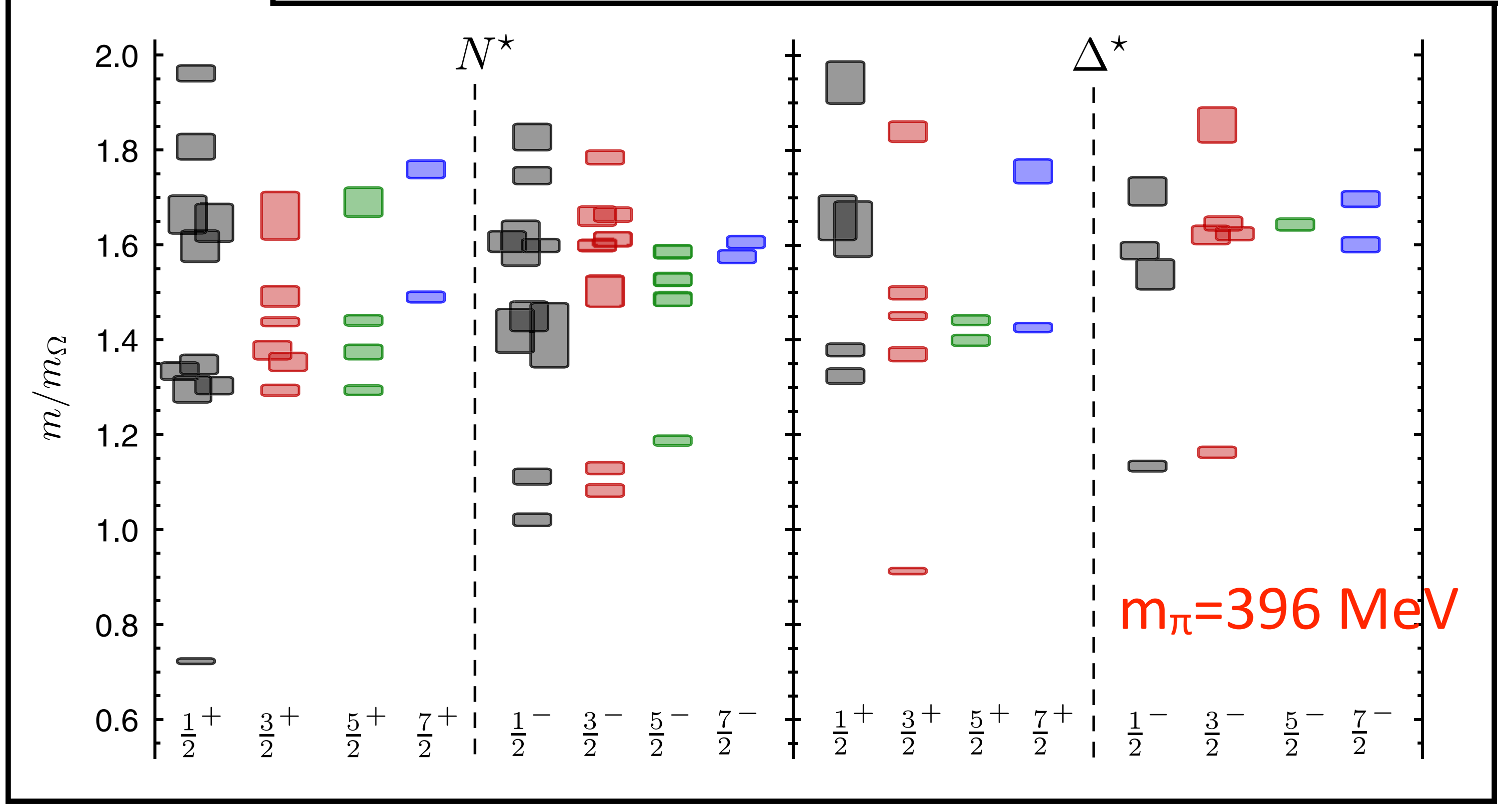
Single hadron approximation

BMW(2008)

HSC(2011, 2013)





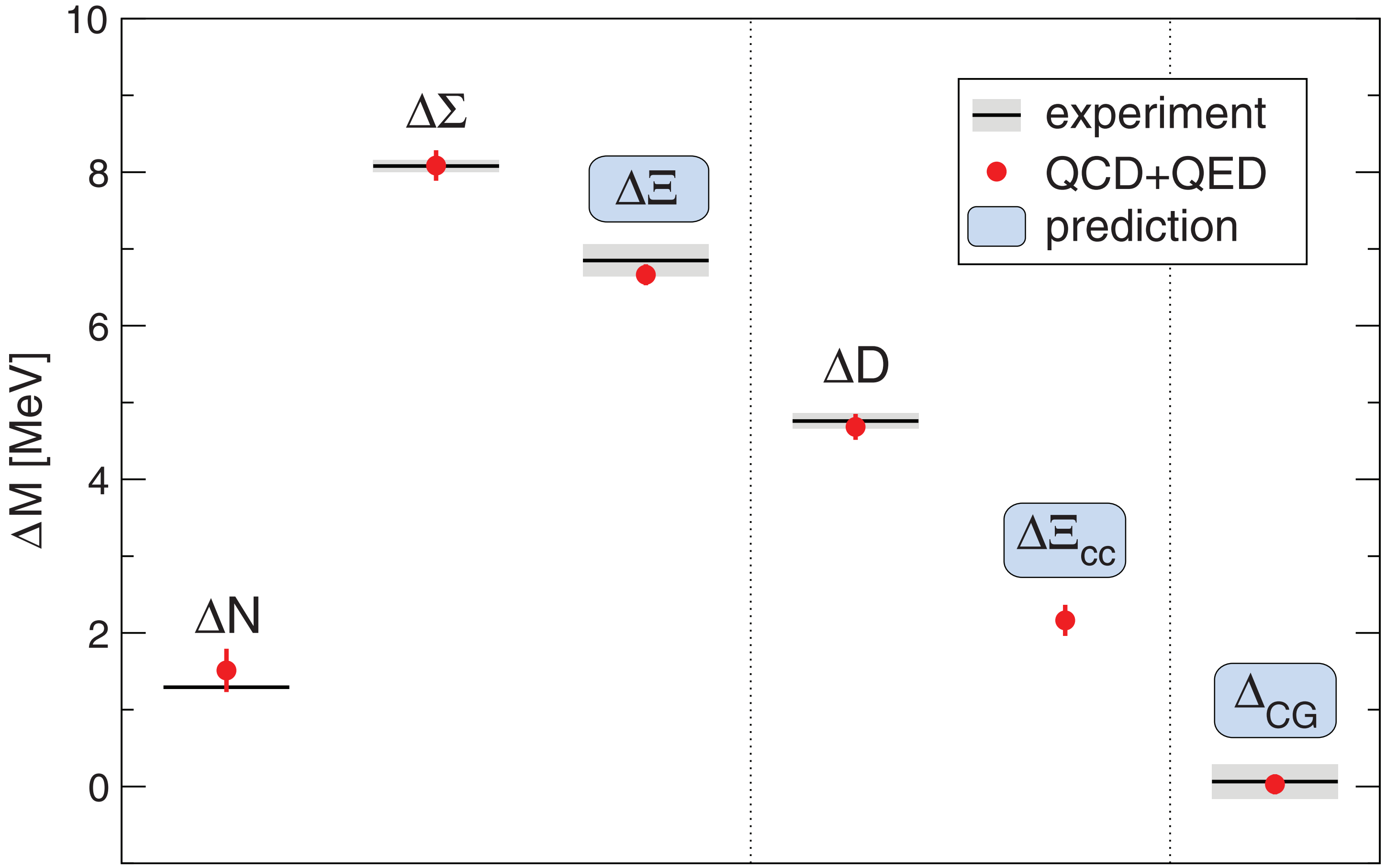


Edwards et al. (HSC) Phys.Rev. D87, 054506 (2013). Edwards et al. (HSC) Phys. Rev. D 84, 074508 (2011)

Neutron-proton mass difference

Single hadron approximation

Borsanyi et al., *Science* **347**, 1452 (2015)



- 1+1+1+1 quark species
- QCD + (non-compact) QED

  QED needs special care: gauge fixing, finite volume corrections O(1/L)*), regularization scheme**)

*) see also Davoudi &Savage, Phys.Rev. D 90, 054503 (2014)

**) see also Endres et al., [arXiv:1507.08916]

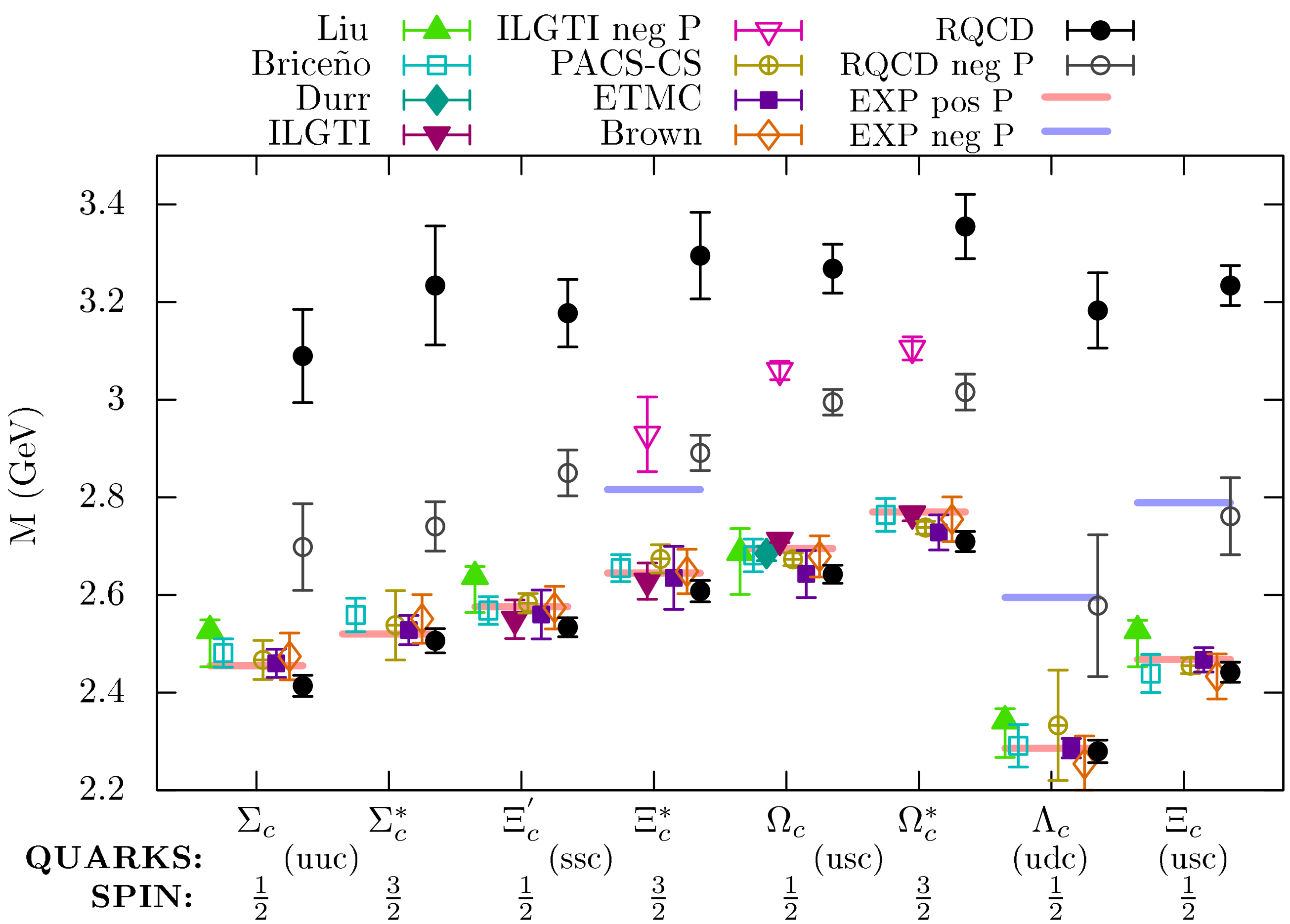
0≤ α ≤0.17 interpolated
197 MeV ≤ m_π ≤ 440 MeV
lattice spacing 0.06-0.1 fm

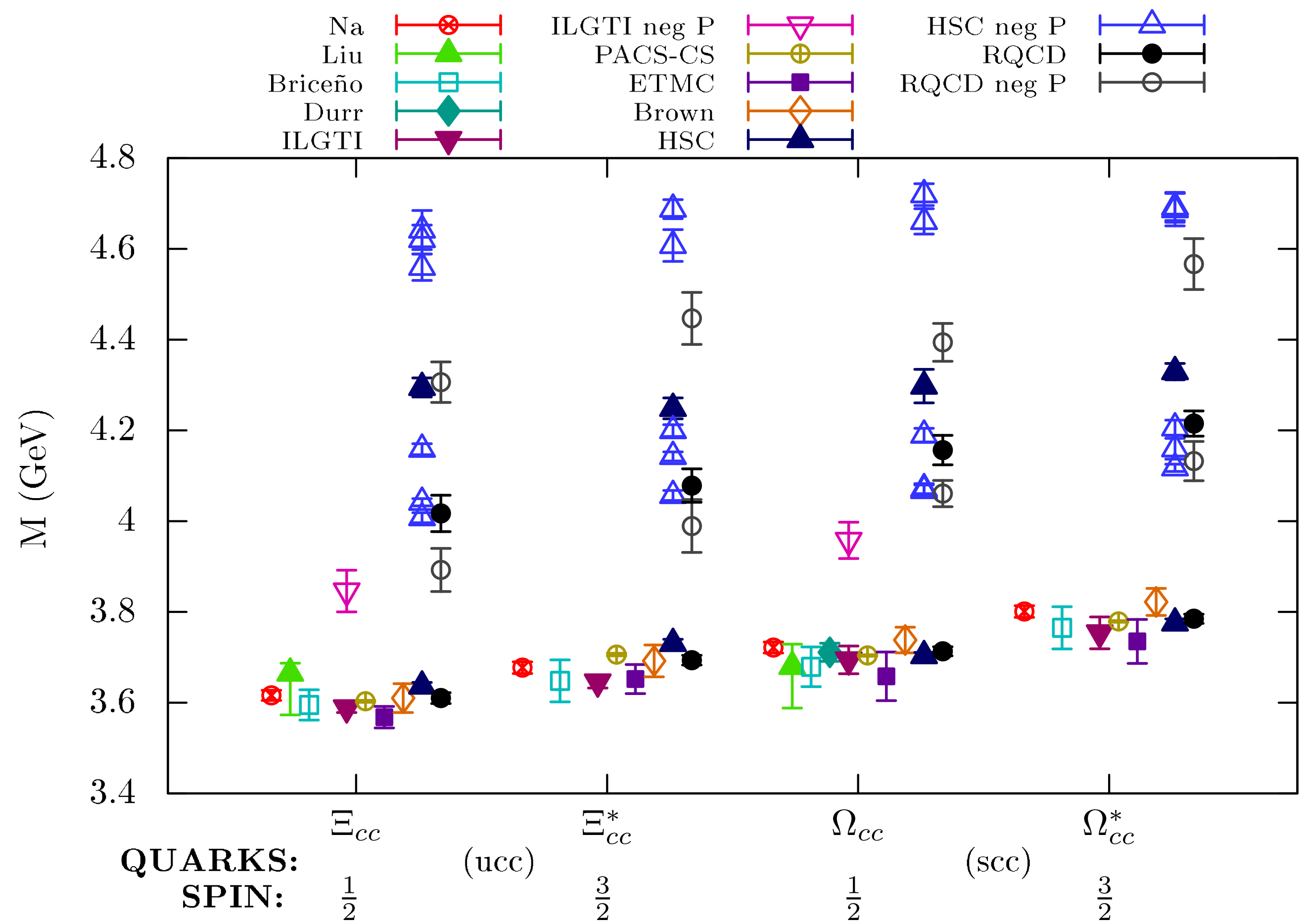
extrapolated to the physical point in continuum

input: $M_{\pi^+}/M_\Omega, M_{K^+}/M_\Omega, M_{K^0}/M_\Omega, M_{D^0}/M_\Omega$

c-Baryons and cc-Baryons

Single hadron approximation





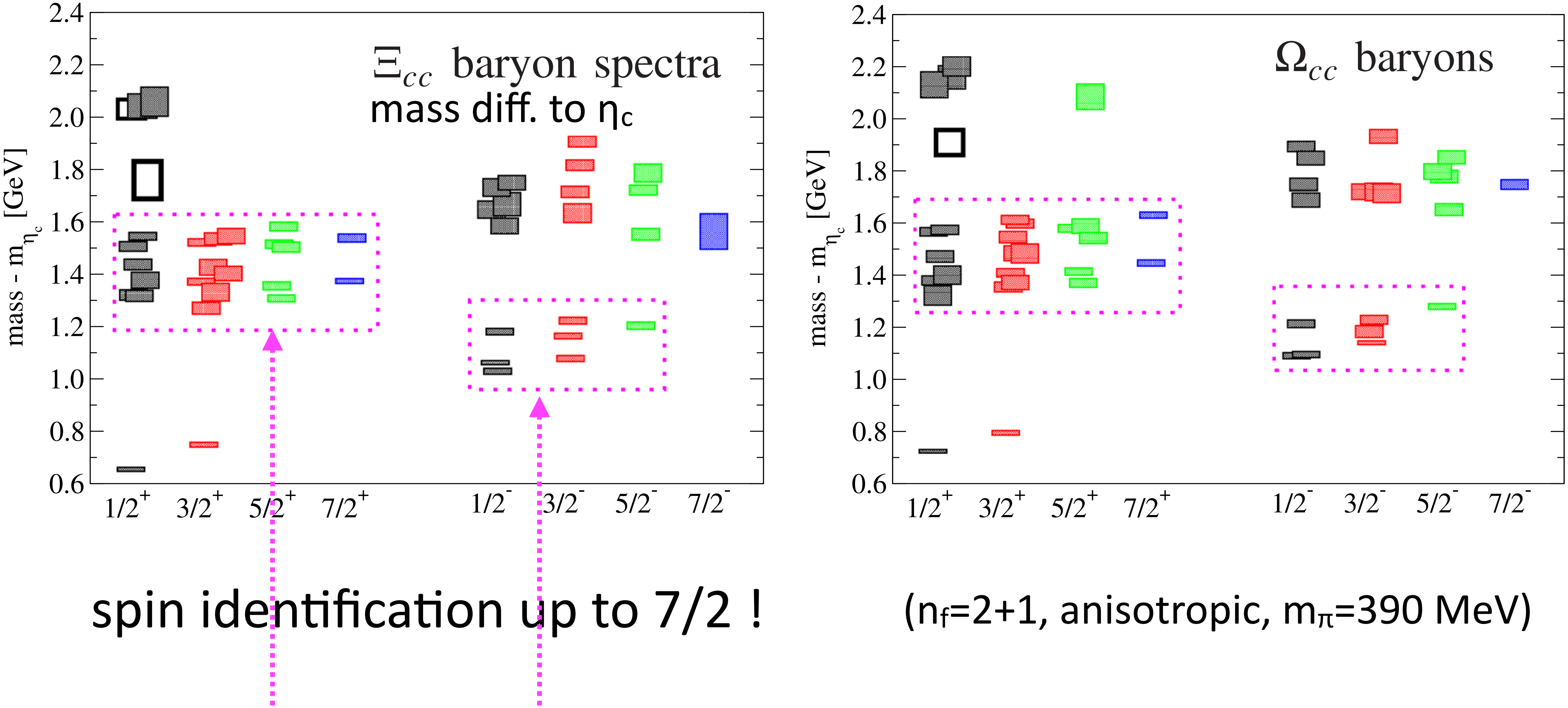
Rubio et al. (RQCD), Phys. Rev. D 92, 034504 (2015) [arXiv: 1503.08440]

+excited levels

cc-Baryons

Single hadron approximation

Padmanath et al. (HSC), Phys. Rev. D 91, 094502 (2015).



spin identification up to 7/2 !

(n_f=2+1, anisotropic, m_π=390 MeV)

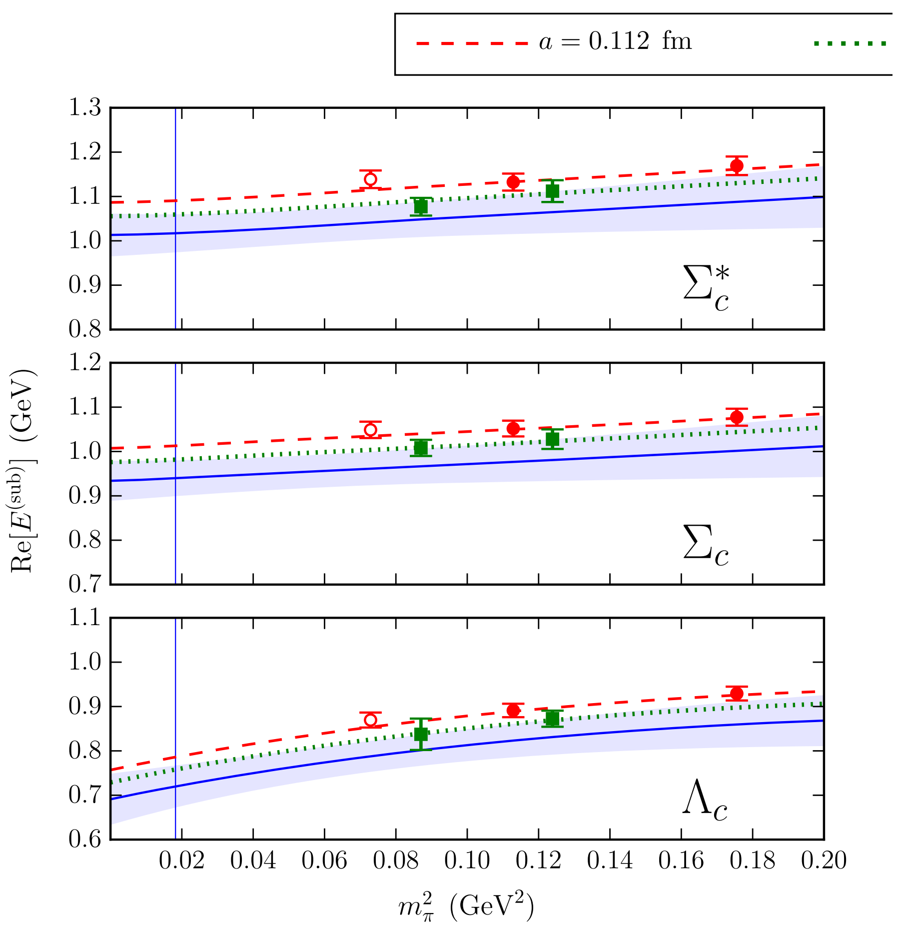
numbers match non.rel. quark spinor model: SU(6)xO(3)

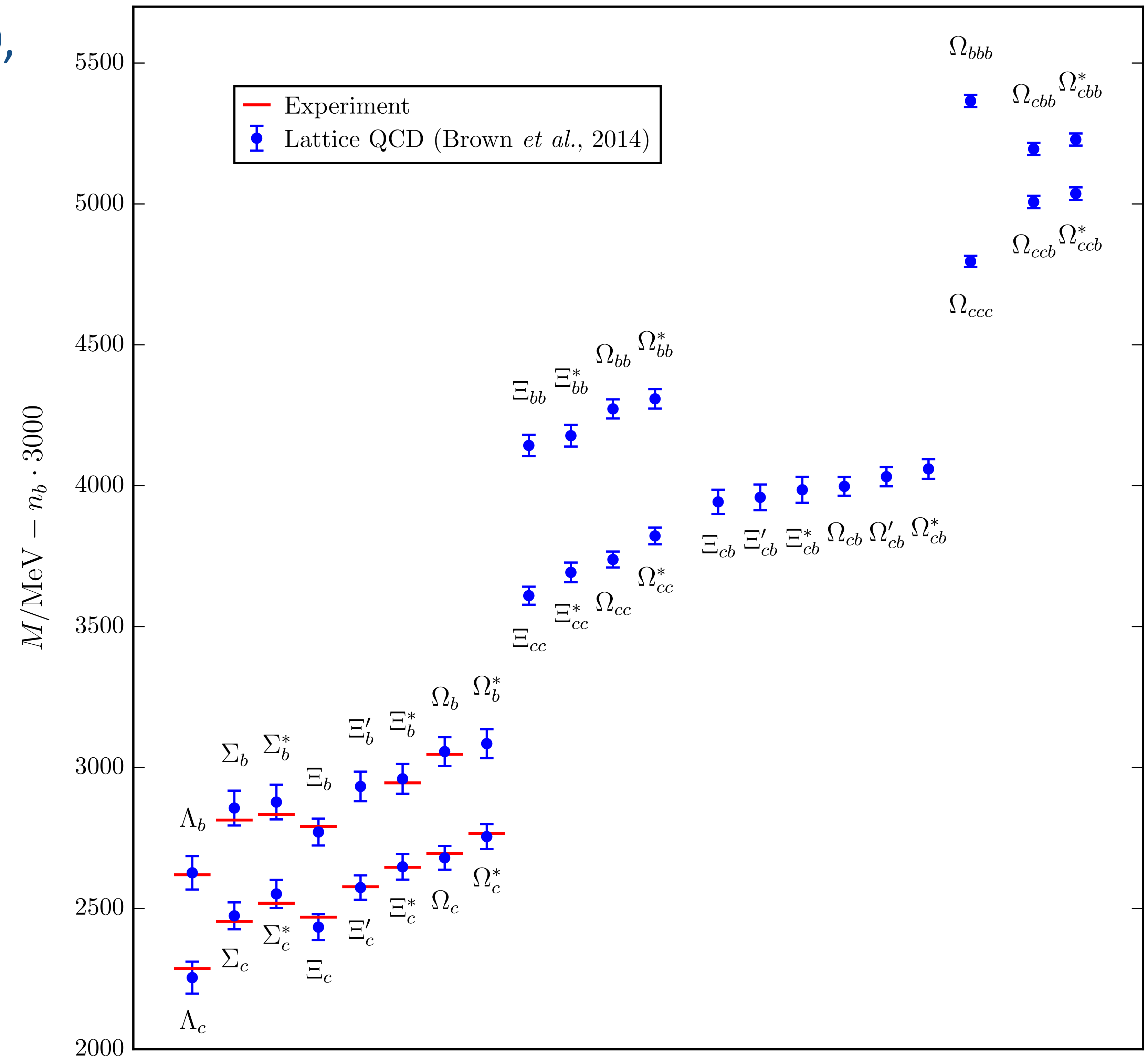
Baryons with 1,2,3 heavy (c, b) quarks

Single hadron approximation

Brown et al. Phys. Rev. D 90, 094507 (2014).

Extrapolation to continuum limit and physical point





References: Charmed baryons and mesons

Single hadron approximation

Baryons

Namekawa et al. (PACS-CS), Phys. Rev. D 87, 094512 (2013) [arXiv:1301-4743].

Alexandrou et al.(ETMC), Phys. Rev. D 90, 074501 (2014).

Brown et al. Phys. Rev. D 90, 094507 (2014).

Padmanath et al. (HSC), Phys. Rev. D 91, 094502 (2015) [arXiv: 1502.01845].

Rubio et al. (RQCD), Phys. Rev. D 92, 034504 (2015) [arXiv: 1503.08440]

Charmonium and charmed mesons

L. Liu et al. (HSC), JHEP 1207, 126 (2012), [1204.5425]

cf Thomas, 1C2,Tue.

Moir et al.(HSC), J. High Energy Phys. 05 (2013) 021 [arXiv:1301.7670].

Galloway et al. (HPQCD) PoS (LATTICE2014) 092 [arXiv:1411.1318]

Rubio et al. (RQCD), Phys. Rev. D 92, 034504 (2015) [arXiv: 1503.08440]

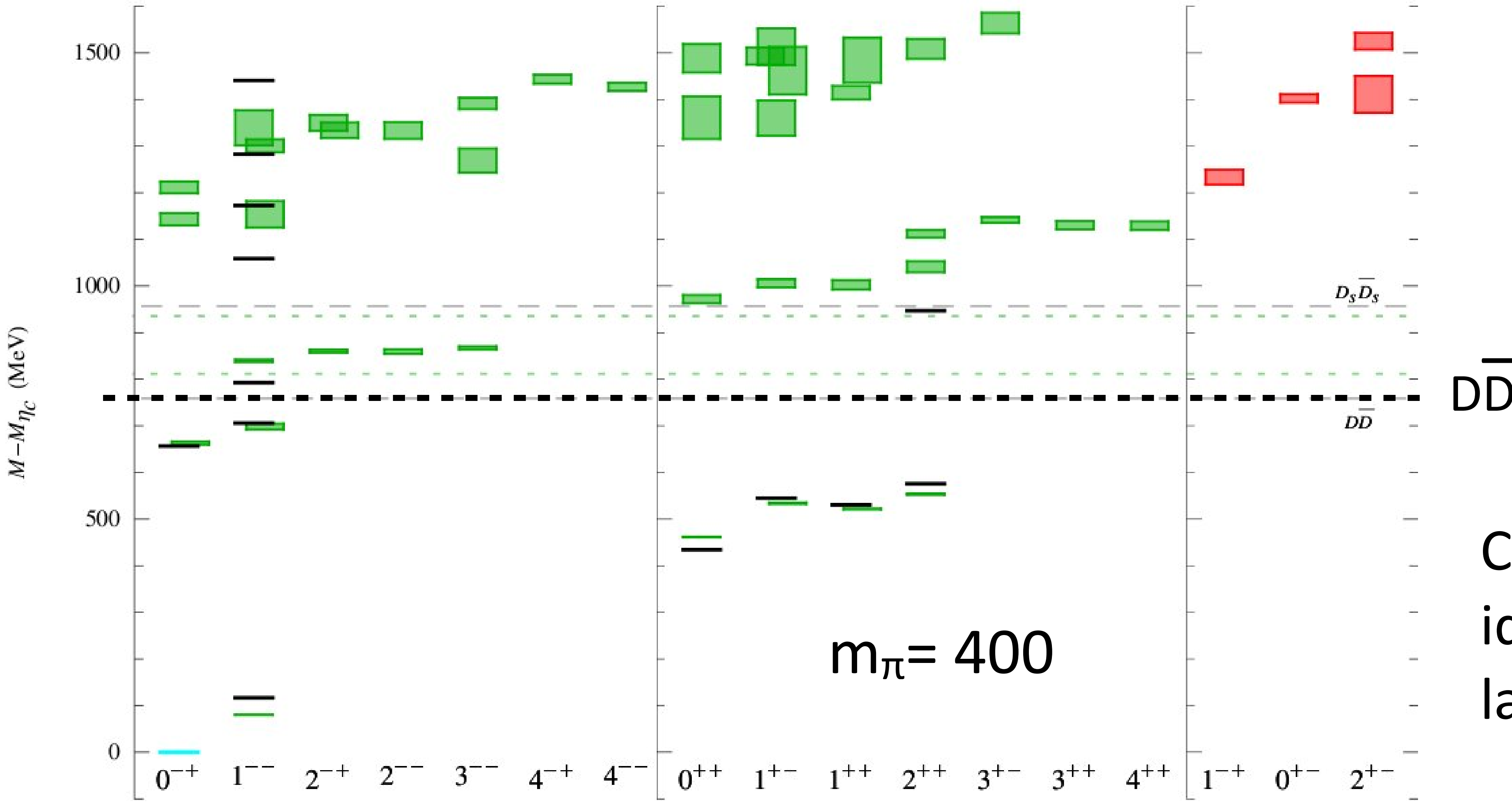
Quarkonium-nucleus bound states

Beane et al. (NPLQCD Collaboration) Phys. Rev. D 91, 114503 [arXiv:1410.7069]

Charmonium

Single hadron approximation

L. Liu et al. (HSC), JHEP 1207, 126 (2012), [1204.5425]



Challenge: Spin identification (from lattice irreps)

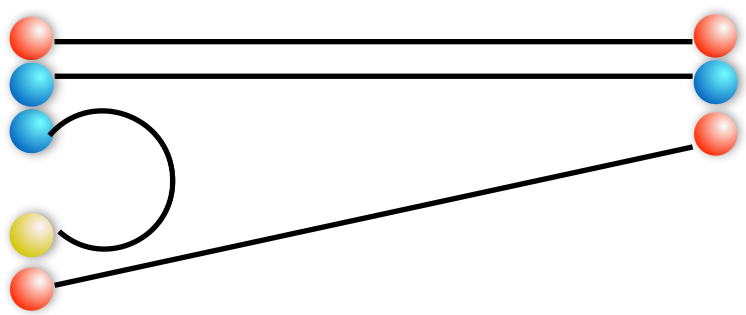
See also: Mohler et al., Phys.Rev. D 87, 034501 (2013) [arXiv:1208.4059]
Galloway et al. (HPQCD) PoS (LATTICE2014) 092 [arXiv:1411.1318]
(extrapolation to physical point)

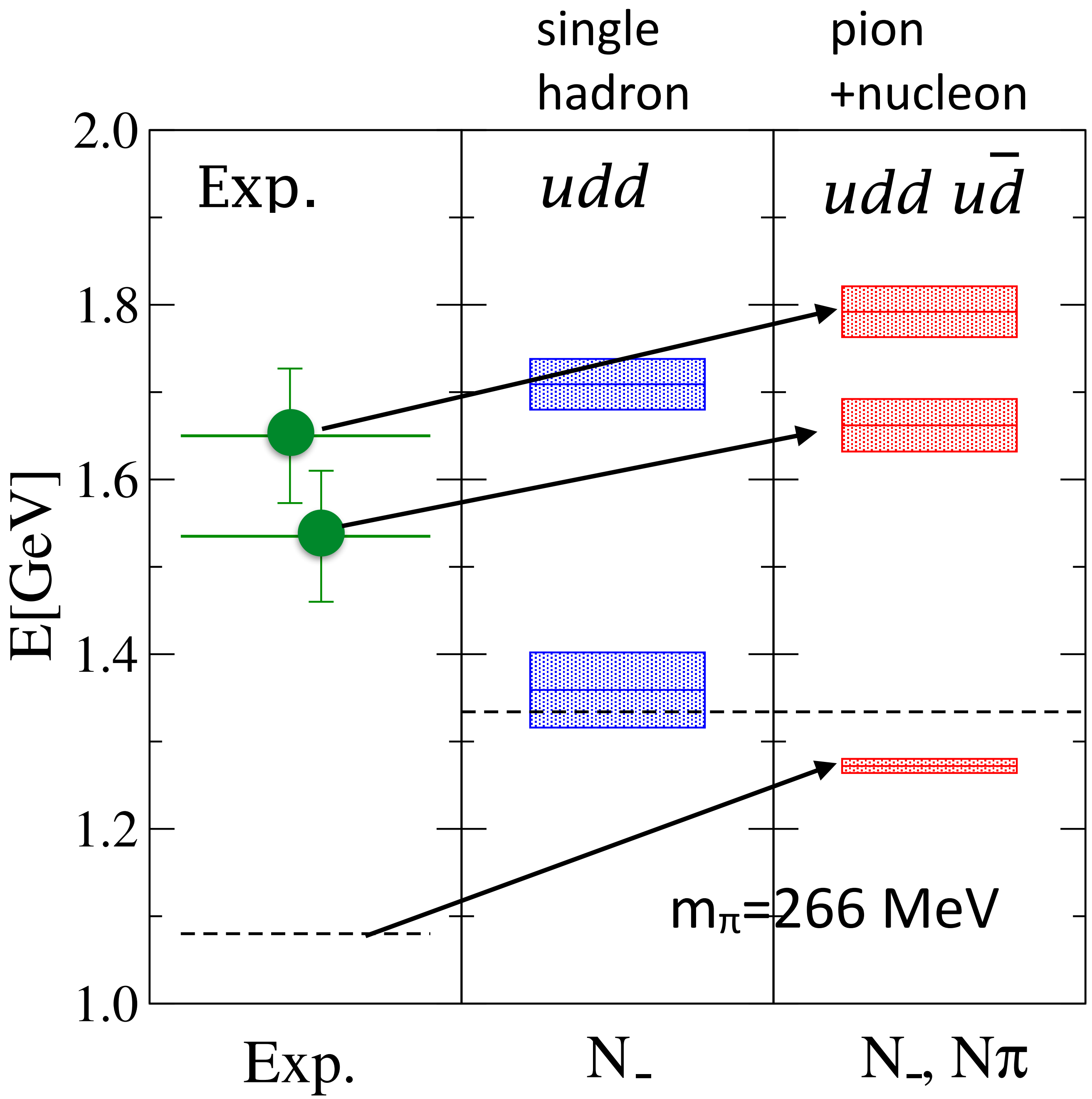
But: Effect of open 2-hadron channel?

N*(1535),N*(1650)
Nπ negative parity

CBL&Verduci, PRD87 (2013) 054502
[arXiv:1212.5055]

needs annihilation terms



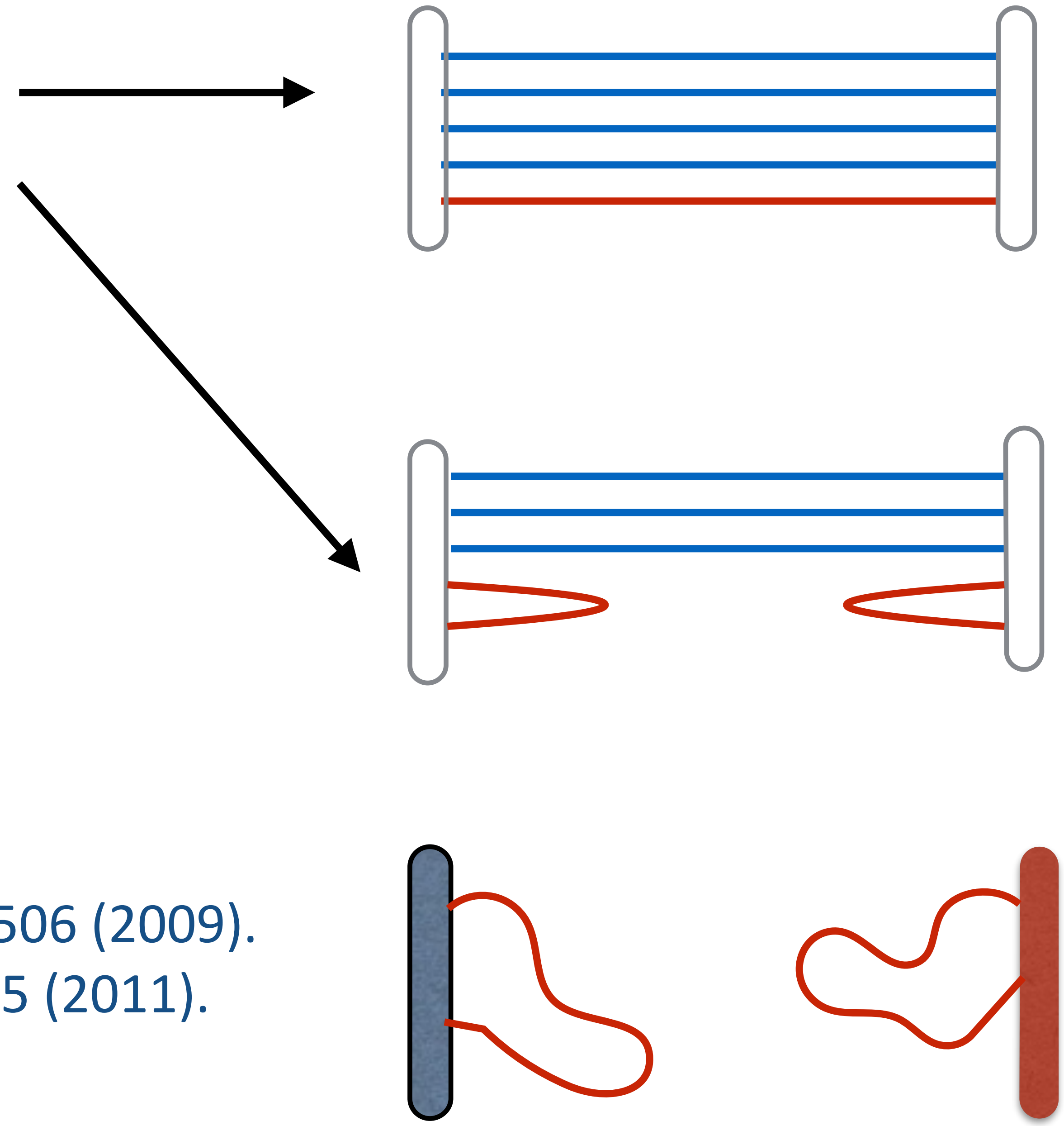


See also Kiratidis et al., Phys. Rev. D 91, 094509 (2015) [arXiv: 1501.07667]

(different operator basis for Nπ: (Nπ)(p=0) instead of N(p=0)π(p=0))

What is the challenge?

More quarks propagators
Backtracking loops are
expensive!



“All-to-all propagators”:
- Stochastic sources
- Distillation

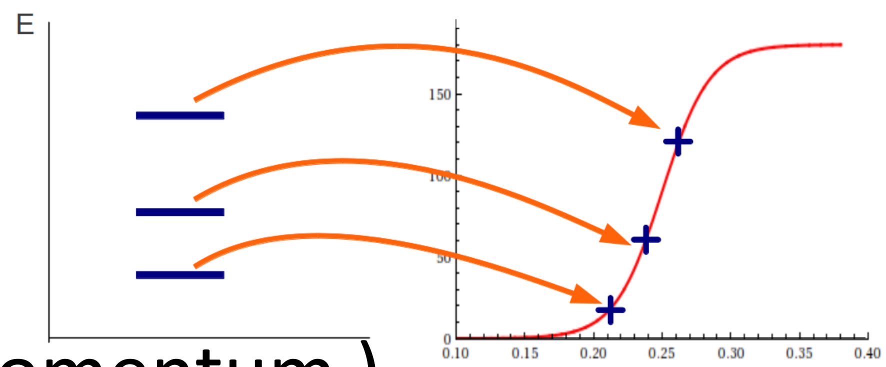
Peardon et al. (HSC), Phys. Rev. D 80, 054506 (2009).
Morningstar et al., Phys. Rev. D 83, 114505 (2011).

Multi hadron approach

The excited states are related to resonances that decay hadronically.
We need to extend the space of operators to multi-hadron operators:
$(q\underline{q})(q\underline{q}),(qqq)(q\underline{q}),(qqq)(\underline{qqq})\ldots$

Lüscher: energy levels give phase shift values in the elastic region

Lüscher, CMP 105(86) 153,
NP B354 (91) 531, NP B 364 (91) 237



Extension to moving frames (operators with momentum)

Rummukainen, Gottlieb: NP B 450(1995) 397
Kim, Sharpe: NP B 727 (2005) 218
Leskovec, Prelovsek, PR D85 (2012) 114507
Göckeler et al., PR D 86, 094513 (2012)
Döring et al., Eur.Phys.J.A48 (2012)114

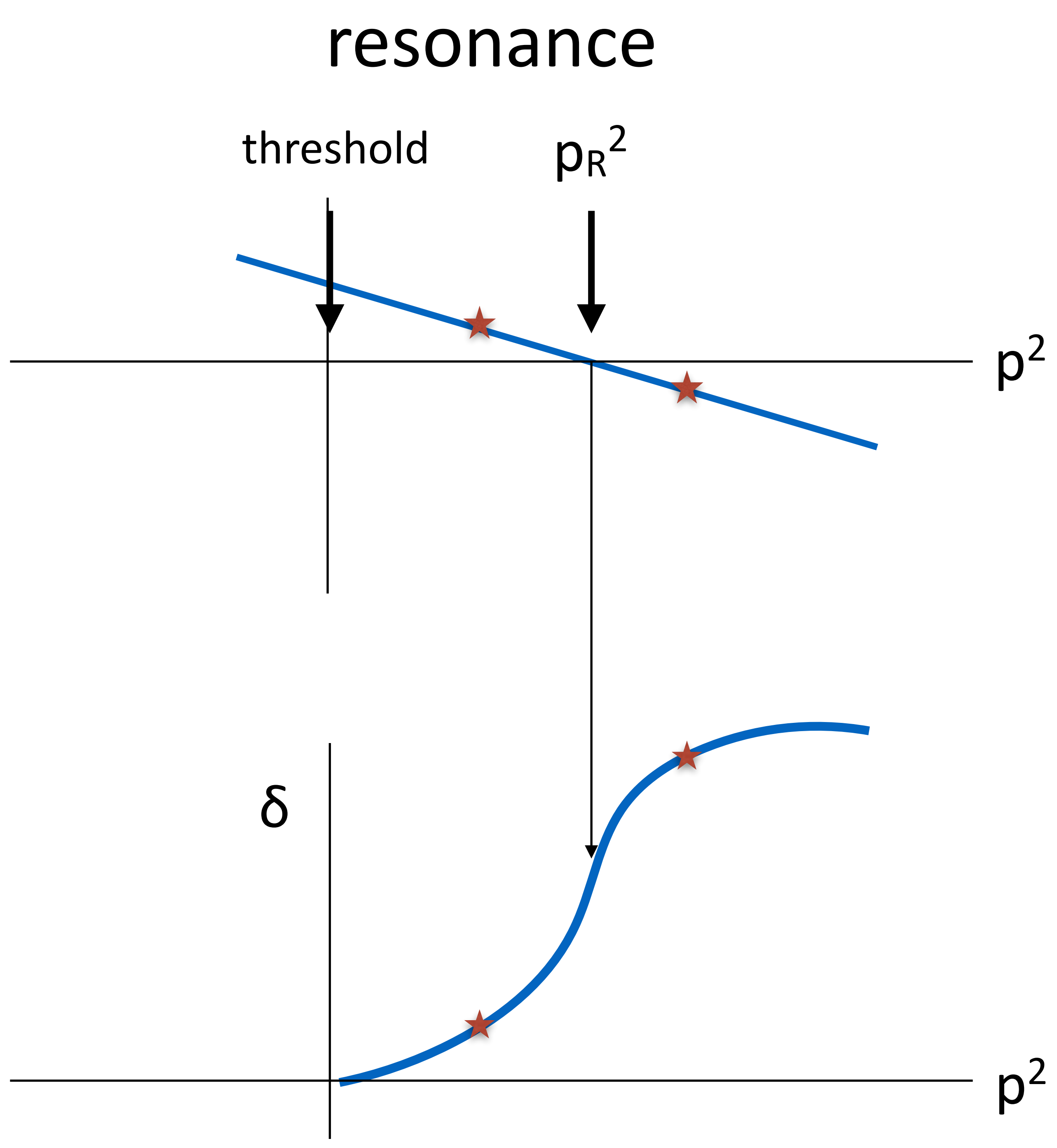
→ irreps of little groups of the cubic group

Above threshold (elastic regime)

$$t^{-1} = \begin{cases} k^{-1} - \mathrm{i}p & \text{for } p^2 > 0 \\ k^{-1} + |p| & \text{for } p^2 < 0 \end{cases}$$

$$\boxed{\mathrm{Re}(t^{-1}) \; - \; c\,\mathcal{Z}_{00}\left(1; \left(\frac{pL}{2\pi}\right)^2\right) = 0}$$

$$k^{-1} = p \cot\delta(p) \quad \text{for } p^2 > 0$$



Near threshold

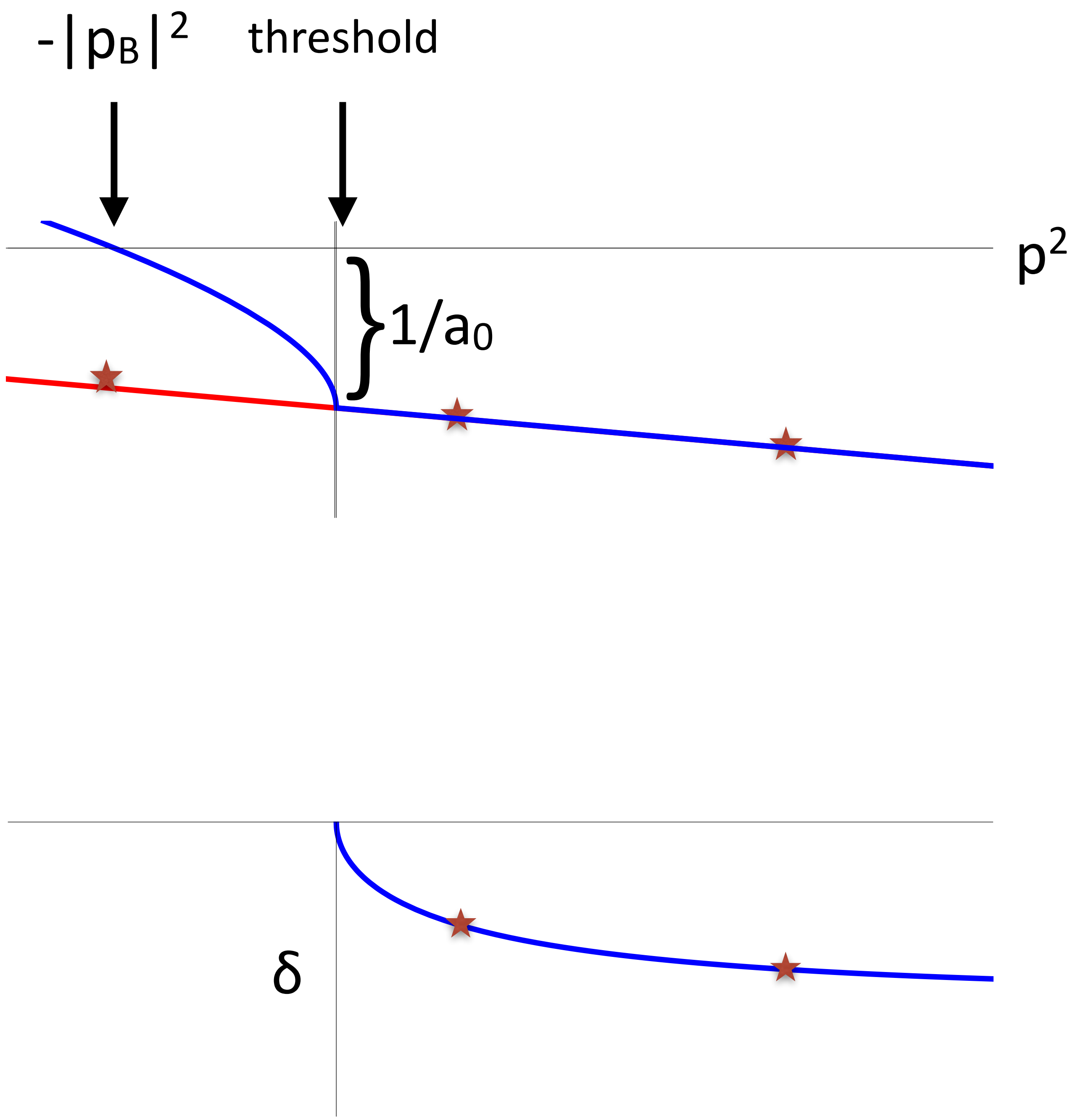
$$t^{-1} = \begin{cases} k^{-1} - ip & \text{for } p^2 > 0 \\ k^{-1} + |p| & \text{for } p^2 < 0 \end{cases}$$

$$\boxed{\mathrm{Re}(t^{-1}) \; - \; c\,\mathcal{Z}_{00}\left(1; \left(\frac{pL}{2\pi}\right)^2\right) = 0}$$

$$k^{-1} = p \cot\delta(p) \quad \text{for } p^2 > 0$$

$$k^{-1} \approx \frac{1}{a_0} + \frac{1}{2} r_0 p^2 \quad \text{for } p^2 \approx 0$$

bound state



coupled channels

scattering parameters

3 particle scattering

resonances

lattice operators

Scattering amplitude

cf Dudek, Wed.

cf Doering, 5A3, Thu.

Extension to several coupled channels

$$\det\left[T^{-1} - Z\right] = 0$$

Bernard et al ., JHEP 1101 (2011) 019 [arXiv:1010.6018]

Briceno et al ., Phys. Rev. D 88, 034502 (2013) two nucleons

Briceno et al , Phys. Rev. D 88, 094507 (2013) moving multichannels

Briceno et al ., Phys. Rev. D 89, 074507 (2014) arbitrary spin

Hansen & Sharpe, Phys.Rev. D86 (2012) 016007[arXiv:1204.0826]

Briceno et al., Phys. Rev. D 91, 034501 (2015) $1 \rightarrow 2$ transitions

Extension to 3-particle channels

Hansen & Sharpe,Phys. Rev. D 90, 116003 (2014) [arXiv:1408.5933] quantization

Meißner et al.,Phys.Rev.Lett. 114, 091602 (2015) [arXiv:1412.4969] shallow bound states

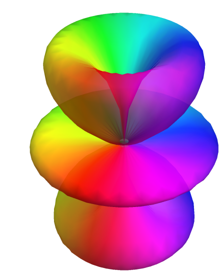
Hansen & Sharpe, [arXiv:1504.04248]

Lattice operators (interpolators) X_i

cf Dudek, Wed.

Irreps of cubic group and its little groups contribute to different angular momenta in continuum

Moore & Fleming , Phys. Rev. D 73, 014504 (2006)
Leskovec, & Prelovsek, PR D85 (2012) 114507
Göckeler et al., PR D 86, 094513 (2012)



J	irreps
0	$A_1(1)$
1	$T_1(3)$
2	$T_2(3) \oplus E(2)$
3	$T_1(3) \oplus T_2(3) \oplus A_2(1)$
4	$A_1(1) \oplus T_1(3) \oplus T_2(3) \oplus E(2)$

Construction of lattice operators by projection from continuum (subduction)

Dudek et al. (HSC), Phys. Rev. D 82, 034508 (2010)

Construction of multi-particles states

Moore et al., Phys. Rev. D 74, 054504 (2006)

Thomas et al. (HSC),Phys. Rev. D 85, 014507 (2012)

Wallace [arXiv:1506.05492]

Example phase shifts

cf Wilson, 1C1, Tue.

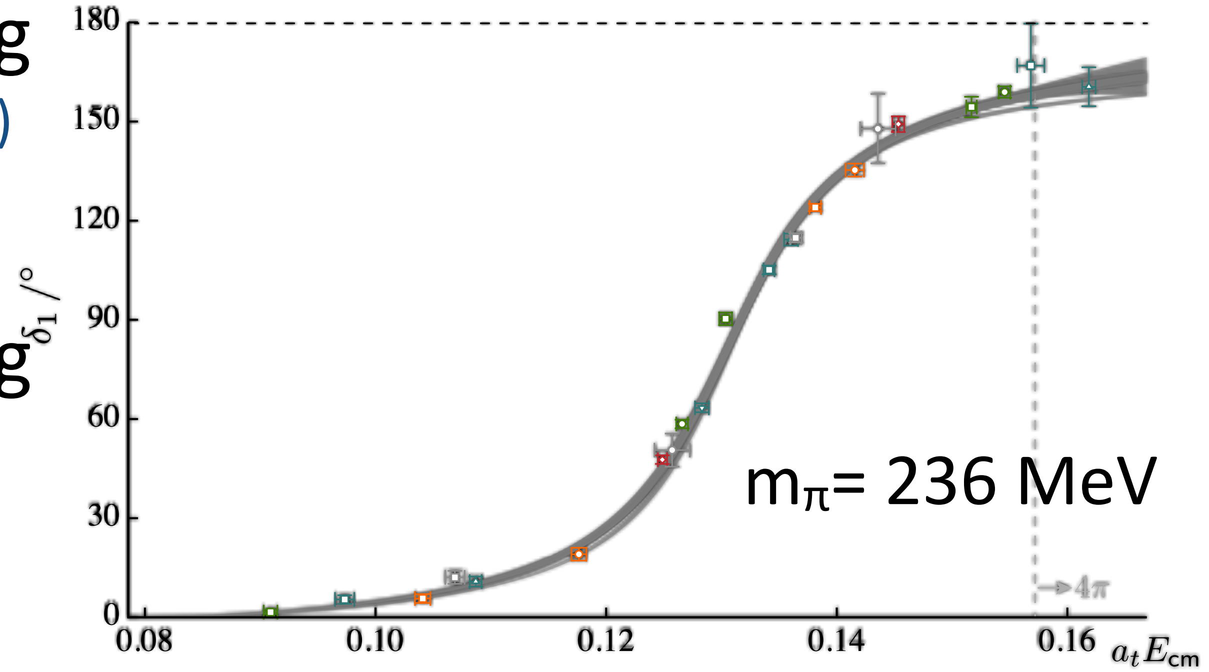
cf Bolton, 1D5, Tue..

Model calculations

Bernard et al., JHEP08(2008)024
Guo et al. Phys. Rev. D 88, 014501 (2013)

Resonances in coupled πK,ηK scattering

Dudek et al.(HSC) Phys. Rev. Lett. 113, 182001 (2014) [arXiv:1406.4158]

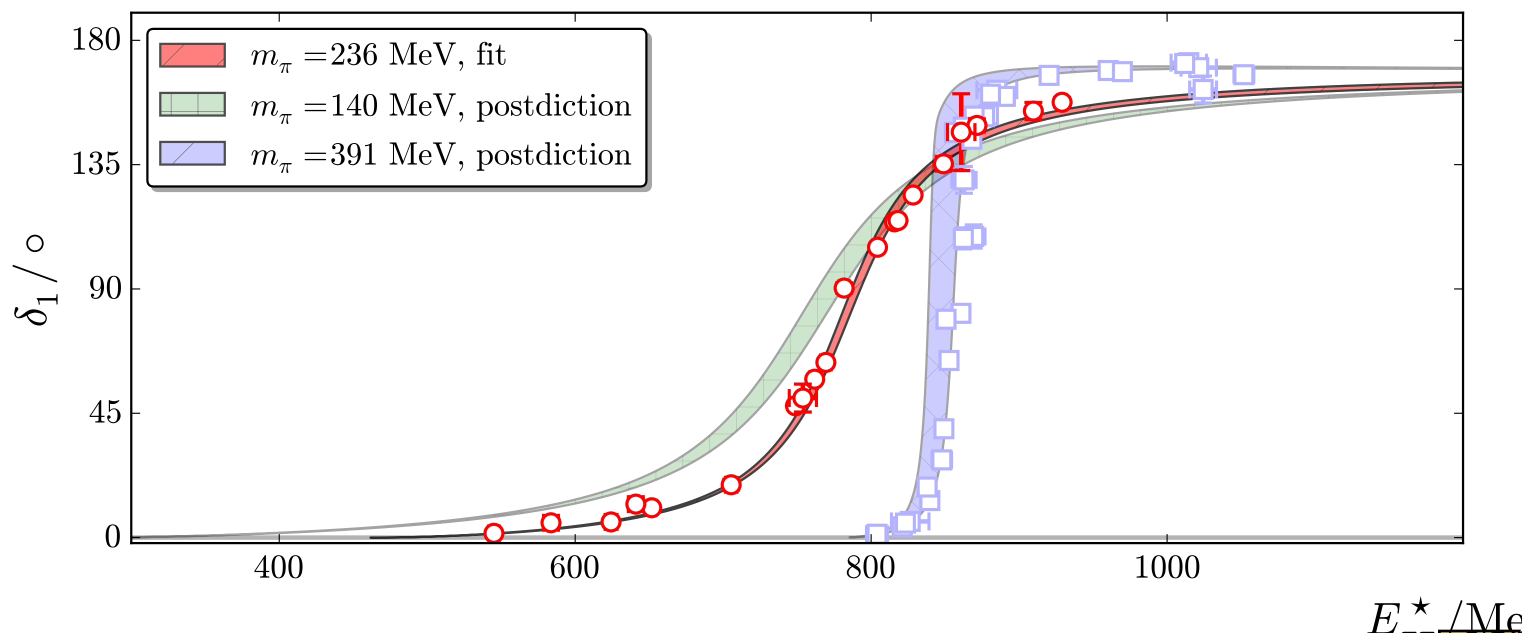


Resonances in coupled ππ,KK scattering

Wilson et al., (HSC) [arXiv:1507.02599]

Bolton et al. [arXiv:1507.07928]

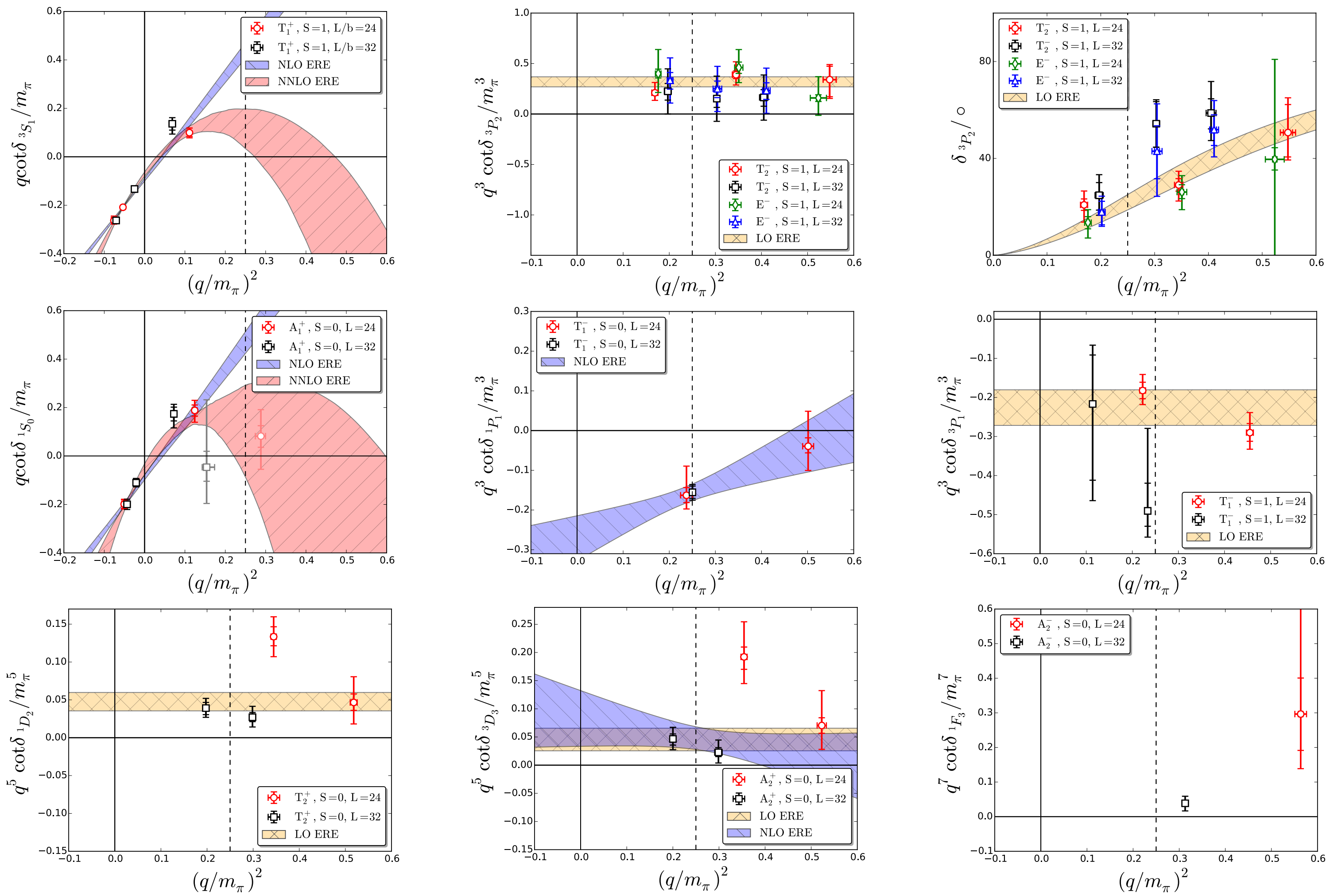
extrapolation to the physical point



Last month: Two nucleon scattering

Berkowitz et al. (CalLat),[arXiv:1508.00886]

m_π=800 MeV (u,d,s flavor symmetric limit)
spatial extent up to 4.6 fm
partial-waves: S, P, D, F



Charmonium: "Level hunting"

Search for $1^+(1^{+-})$ ($Z_c^+(3900)$ $c\underline{c}u\underline{d}$)

Prelovsek et al. , Phys. Rev. D 91 (2015) 014504; arXiv:1405.7623v2

18 interpolators of meson-meson type
covering all imaginable states up to 4.1 GeV
(small volume bonus)
4 tetraquark operators

$\boxed{\rightarrow \text{No signal for } Z_c^+(3900)}$

Lee et al. (FNAL/MILC), [arXiv:1411.1389]

Chen et al.,(CLQCD) Phys. Rev. D 89, 094506 (2014)

DD* weakly repulsive!

4.14 GeV $J/\psi(11)\pi(-11)$

$\psi_3(0)\pi(0)$

$D(1)D^*(-1)$

$\eta_c(1)\rho(-1)$

$\psi_{3700}(0)\pi(0)$

$D^*(0)D^*(0)$

$\psi_{2S}(0)\pi(0)$

$D(0)D^*(0)$

$J/\psi(1)\pi(-1)$

$\eta_c(0)\rho(0)$

3.36 GeV $J/\psi(0)\pi(0)$

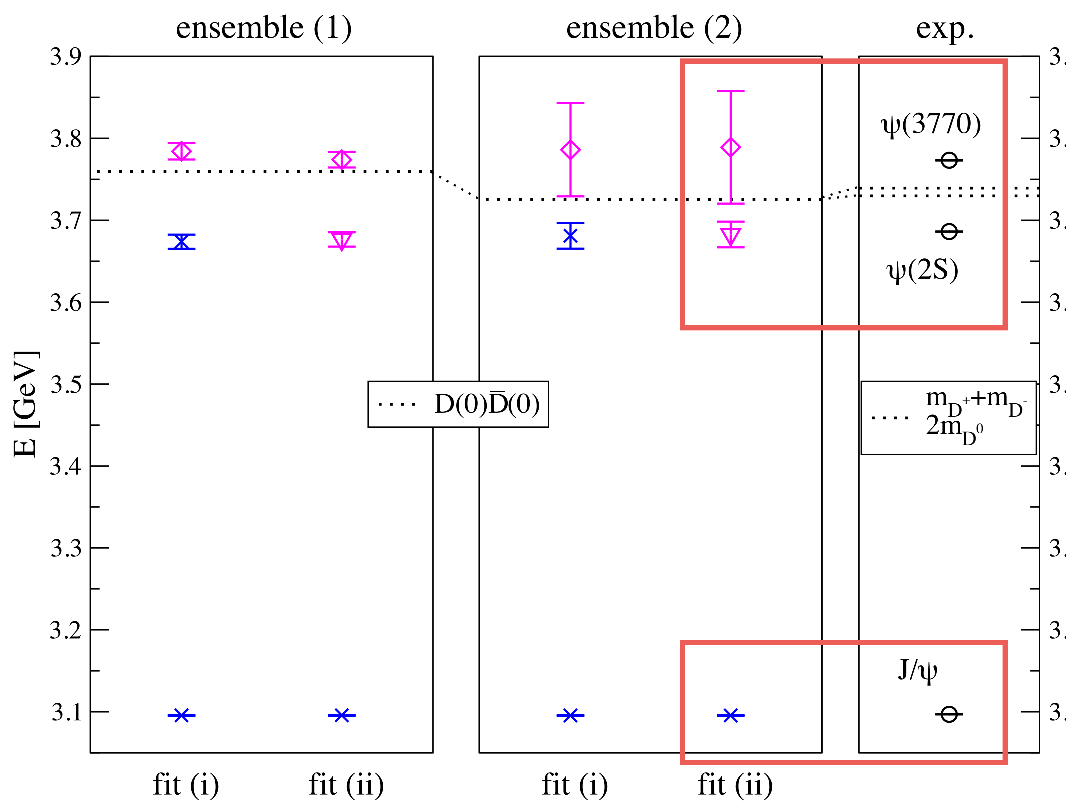
Charmonium

ψ(3770): resonance close to D$\underline{\text{D}}$ threshold

Lattice study: D$\underline{\text{D}}$ scattering on two volumes and m_π=266 and 157 MeV

15 interpolators of c$\underline{\text{c}}$ type
2 operators of type D$\underline{\text{D}}$

CBL et al., JHEP (2015) [arXiv:1503.05363]



[same paper:
η_{c0}(2P) or X(3915): 0^{++}
controversial signal]

	ψ(3770), m_R	g(no unit)	ψ(2S), m_R
m_π=266 MeV	3774(6)(10)	9.7(1.4)	3676(6)(9)
m_π=157 MeV	3789(68)(10)	28(21)	3682(13)(9)
Exp.	3773.15(33)	18.7(1.4)	3686.11(1)

X(3872)

X(3872) $0^+(1^{++})$

cf Santoro, Swanson, Wed.

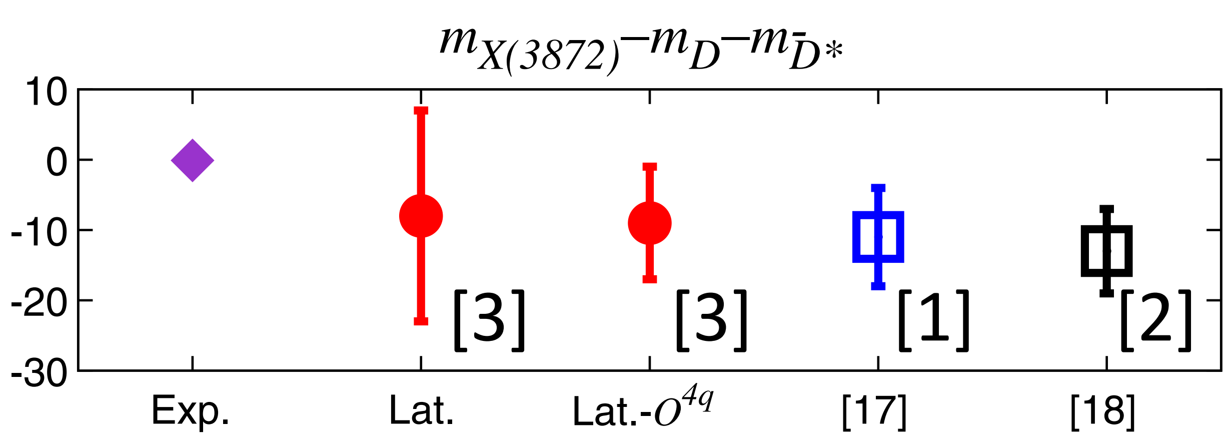
[1] Prelovsek/Leskovec, Phys. Rev. Lett. 111, 192001 (2013)

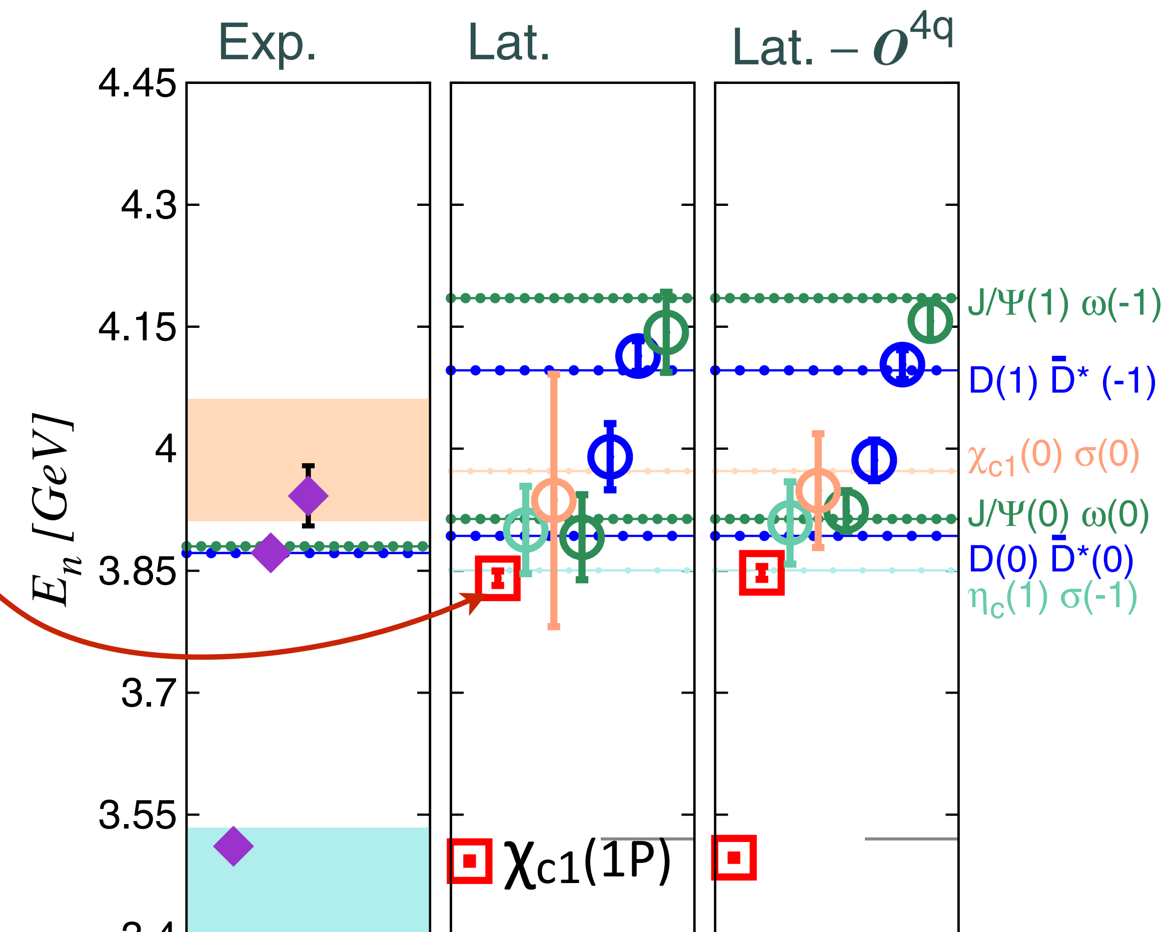
[2] Lee et al. (FNAL/MILC), [arXiv:1411.1389]

[3] Padmanath et al, Phys. Rev. D 92 (2015) 034501 [arXiv:1503.03257]

at m_π=266 MeV
22 $c\underline{c}$ and $c\underline{c}u\underline{u}$, $c\underline{c}u\underline{d}$,..interpolators
for I=0 and 1
($D\underline{D}^*$, $J/\psi\ \rho$, $J/\psi\ \omega$, $\eta_c\ \sigma$, $\chi_{c0}\ \pi$, $\chi_{c2}\ \pi$, 4q)

all observe X(3872) closely below $D\underline{D}^*$ (with strong $c\underline{c}$ component)





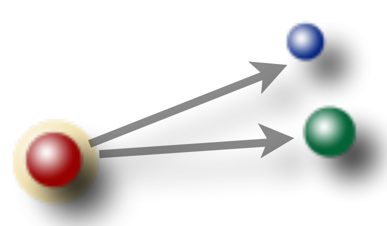
(a) $I=0$: $\bar{c}c$ ($\bar{u}u + \bar{d}d$) & $\bar{c}c$

(large scatt.length 1.1 fm)

Heavy quark sector: D_s (0^+, 1^+, 2^+)

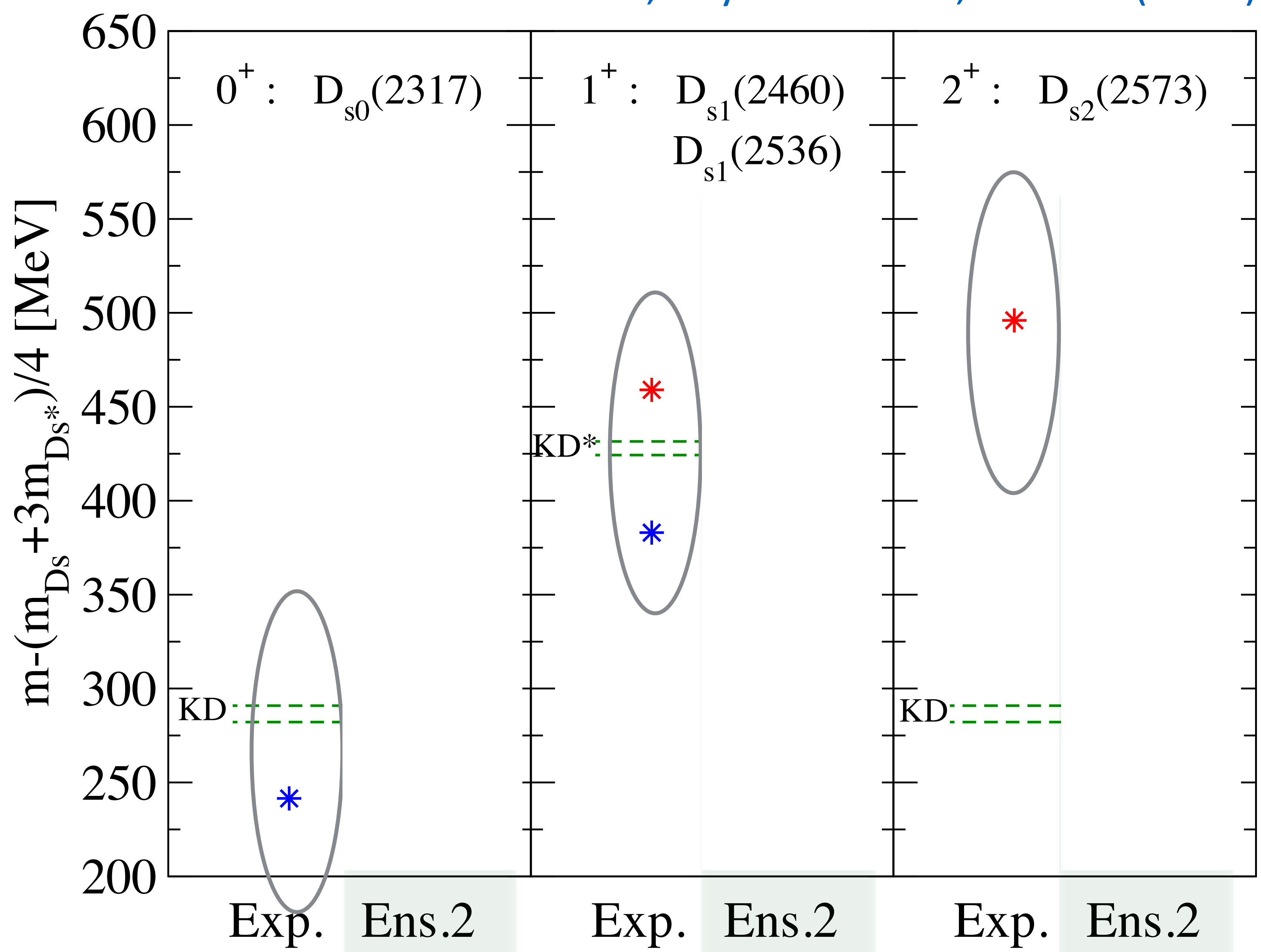
Quark model and LQCD in single hadron approximation: unclear picture, threshold important?

Mohler et al.,PRL. 111, 222001; (2013) [arXiv:1308.3175];
CBL et al., Phys. Rev. D 90, 034510 (2014)



Include meson meson interpolators!

(PACS-CS lattices. m_π=157 MeV)



See also Martinez Torres et al., JHEP 1505 (2015) 153

Heavy quark sector: D_s (0^+, 1^+, 2^+)

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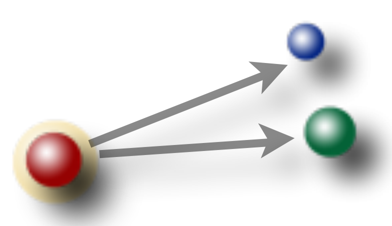
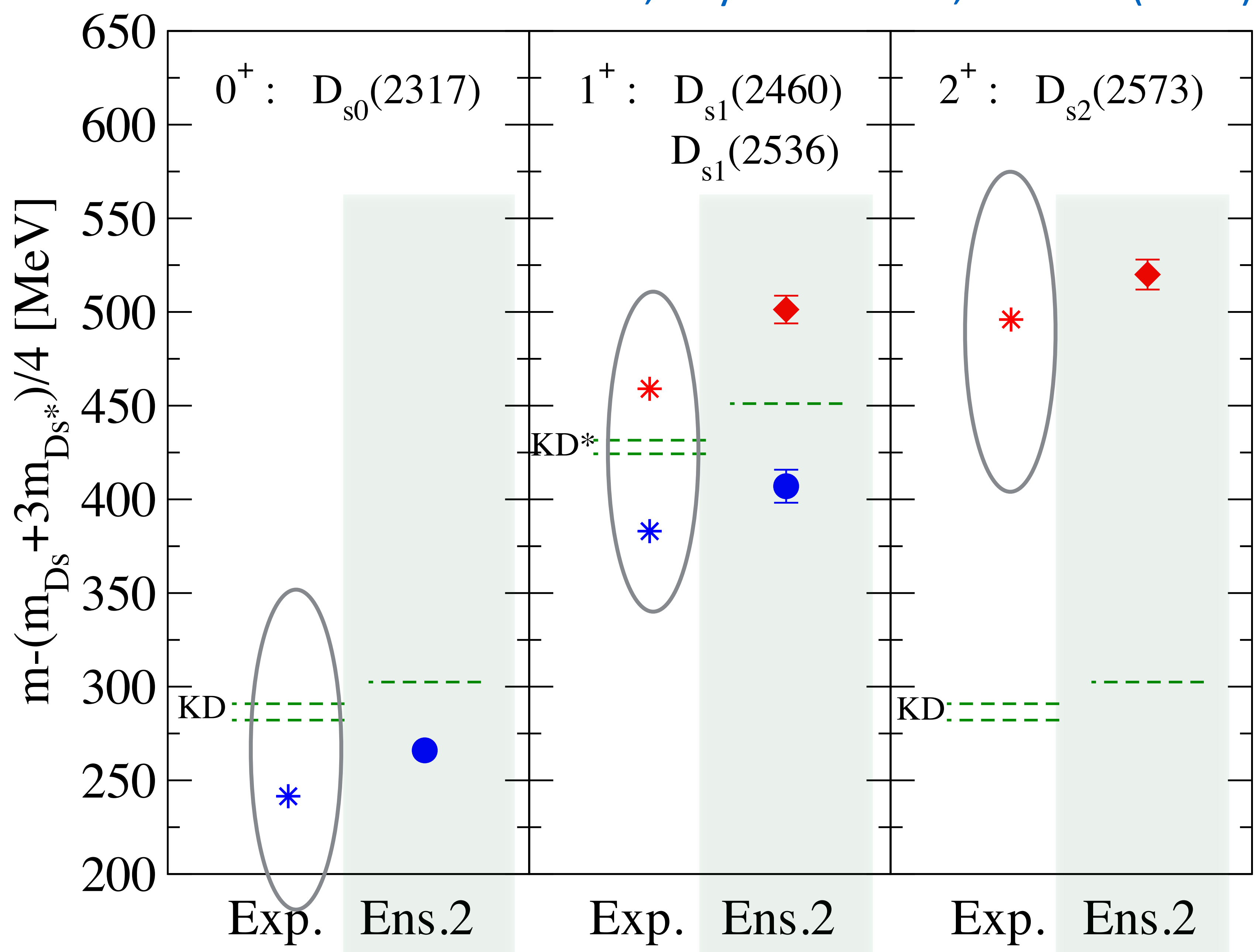
CBL et al., Phys. Rev. D 90, 034510 (2014)





Include meson meson interpolators!

(PACS-CS lattices. m_π=157 MeV)

See also Martinez Torres et al., JHEP 1505 (2015) 153

Heavy quark sector: B_s (0^+, 1^+, 2^+)

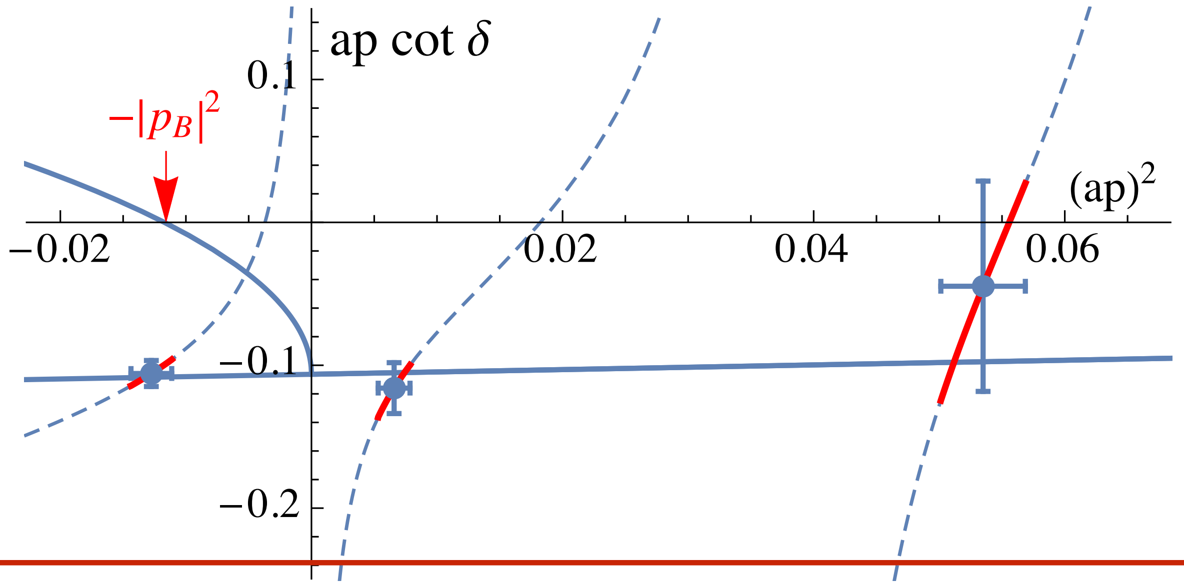
CBL et al., Phys. Lett. B 750 (2015) 17 [arXiv:1501.01646]

BK, B*K scattering (PACS-CS lattices, m_π=157 MeV)

0^+: Bound state B_{s0} with

$m(B_{s0})$ = 5.711(13)(19) GeV

`(prediction)`



1^+: Bound state B_{s1} with $m(B_{s1})$= 5.750(17)(19) GeV `(prediction)`

Close to threshold weakly coupled state B_{s1}' at m=5.831(9)(6) GeV (Exp: B_{s1}(5830) at 5.8287(4) GeV)

Summary (biased)

- Structure
  - g_A improving, deviations still to be understood
  - Form factors need more work and ideas
  - Proton spin: Gluonic contributions?

- Spectroscopy
  - States below decay threshold well determined
  - **Exciting progress: resonances and coupled channel scattering theory** (some modelling necessary)
  - **Heavy quarks: nice results** (need continuum limit)

Recent reviews:

M. Constantinou, PoS LATTICE2014 (2014) 001; [arXiv:1411.0078]

J. Zanotti, LATTICE2015

N. Mathur & M. Padmanath, CHARM2015 [arXiv:1508.07168]

D. Mohler, CHARM2015 [arXiv:1508.02753]

S. Prelovsek, CHARM2015 [arXiv:1508.07322]

Thanks to: Constantia Alexandrou, Raul Briceño, Martha Constatinou, Sara Collins, Christine Davies, Michael Engelhardt, Daniel Mohler, Sasa Prelovsek, M. Padmanath, Gian-Carlo Rossi, Andre Walker-Loud

Disclaimer: time limit of 25 minutes!!!

Acronyms

BMW Budapest-Marseille-Wuppertal
CalLat Clalifornia Lattice Collaboration
ChiQCD Chiral QCD
CLQCD Chinese Lattice QCD
CLS Coordinated Lattice Simulations
CSSM Centre for the Subatomic Structure of Matter
ETMC European Twisted Mass Collaboration
FLAG Flavor Lattice Averaging Group
HALQCD Hadrons to Atomic nuclei from LQCD
HPQCD High precision QCD
HSC Hadron Spectrum Collaboration
ILGTI Indian Lattice Gauge Theory Initiative
JLQCD Japan Lattice QCD
LHPC Lattice Hadron Physics Collaboration
MILC MIMD Lattice Computation
NPLQCD Nuclear Physics with Lattice QCD
PACS-CS Parallel Array Computer System for Computational Sciences
PNDME Precision Neutron Decay Matrix Elements
QCDSF QCD spectral function
RBC RIKEN Brookhaven Columbia
RQCD Regensburg QCD
UKQCD UK QCD
USQCD US QCD