Thoughts on the nature of hadrons

• Great to be here on the eve of a new era in hadron physics
• The light quark sector of QCD: a perfect problem
• Ordinary and extraordinary hadrons guided by large $N_c$
• Full-disclosure: not new, but perhaps not well-known
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And more recently:  
  • Weinberg, PRL 110 261601 (2013)
  • Knecht & Peris, PRD 88 036016 (2013)
  • Cohen, Llanes-Estrada, Pelaez, & Ruiz de Elvira, PRD 90 036003 (2014)
Light Quark QCD

\[ \mathcal{L} = -\frac{1}{4} \text{Tr} F_{\mu\nu} F^{\mu\nu} + \bar{q} (i\gamma_\mu D^\mu - m) q \]

- QCD with massless quarks: zero parameter, strongly coupled, non-Abelian gauge theory.
- Important flavor (esp. chiral) symmetries.
- All phenomena (hadrons, nuclei, etc.) are emergent.
- Prototype for a fundamental theory.
- Dynamics that confines quarks and builds hadrons is still obscure.
- Lattice QCD allows us to compute and perform “theoretical experiments” by deforming the theory.
The nature of hadronic resonances

- Ancient question: How should enhancements and resonances be described in hadronic scattering processes?
- How should the results of theory/model calculations with no open decay channels be compared with experiment?
The nature of hadronic resonances

• Ancient question: How should enhancements and resonances be described in hadronic scattering processes?

• How should the results of theory/model calculations with no open decay channels be compared with experiment?

• Are “hadrons” necessarily associated with poles in the S-matrix?

• Is there a useful “unitarization scheme” which begins with zero-width parameterization and then couples to open channels?

• Are hadronic resonances generated by “forces” between the scattering hadrons?

• Does one size fit all?
Summary – take aways...

• “Ordinary” hadrons are (Fano)-Feshbach resonances

  Bound states in confined channels that appear as resonances in the continuum of scattering channels.

  (Meson) widths vanish as $N_c \to \infty$

  A zero width approximation (eg, even as simple as a K-matrix parameterization) is a good starting point, at least conceptually
But these are not the only kind of hadrons

“Extraordinary” hadrons --- Resemble the unitarized potential resonances of the (very) old bootstrap.

Generally disappear(!) as $N_c \to \infty$

Zero-width unitarization scheme does not seem appropriate.

$\{f_0(500), K_0^*(800), a_0(980), f_0(980)\}$ and XYZ mesons are candidates.

No fundamental distinction between “tetraquarks” and “meson-meson bound states”.
Expectations at large $N_c$

Rules

![Diagram](image)

- $\frac{1}{\sqrt{N_c}}$
- $\frac{1}{\sqrt{N_c}}$
- $N_c$
Expectations at large $N_c$

Rules

\[ \frac{1}{\sqrt{N_c}} \]

\[ \frac{1}{\sqrt{N_c}} \]

\[ N_c \]

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\[ \frac{1}{\sqrt{N_c}} \]

\[ N_c \]
Expectations at large $N_c$

Rules

\[
\begin{align*}
\frac{1}{\sqrt{N_c}} & \\
\frac{1}{\sqrt{N_c}} & \\
N_c & \\
\left[\frac{1}{\sqrt{N_c}}\right]^4 & \rightarrow 1
\end{align*}
\]
Expectations at large $N_c$

Rules

\[
\begin{align*}
\frac{1}{\sqrt{N_c}} & \quad \frac{1}{\sqrt{N_c}} \\
N_c & \\
N_c \left[ \frac{1}{\sqrt{N_c}} \right]^4 & \rightarrow 1 \\
N_c \left[ \frac{1}{\sqrt{N_c}} \right]^6 & \rightarrow \frac{1}{N_c}
\end{align*}
\]
Ordinary Mesons

“Classic” example:
\[ \pi \pi \rightarrow \rho \rightarrow \pi \pi \]

“In” and “out” states are ordinary hadrons, forming a continuum, and only couple to the “confined” channel by quark-antiquark annihilation, which is suppressed at large \( N_c \)

Resonance formation takes place by transition from channel with continuum spectrum (meson-meson, multiquark) to a channel with a discrete spectrum (confined channel) that has no asymptotic states

Confined channel, not in the Hilbert space of asymptotic states
Classic results:

- Meson widths vanish $\Gamma \sim \mathcal{O}(1/N_c)$ as $N_c \to \infty$
- Quark content becomes pure $Q\bar{Q}$
- Meson-meson scattering vanishes as $\mathcal{O}(1/N_c)$; ordinary mesons appear as narrow $s$-channel resonances dual to $t$-channel exchanges.
Ordinary hadrons as Feshbach resonances

Scattering particles couple to bound (confined) state in the continuum

\[ \Psi = \begin{pmatrix} \psi_1 \\ \psi_2 \end{pmatrix} \rightarrow \pi \pi \rightarrow \rho \]

Mass

\[ \pi \pi \text{-channel} \rightarrow Q\bar{Q} \ (\rho) \text{ channel} \]
Non-relativistic example is easy to interpret

\[ \Psi = \begin{pmatrix} \psi_1 \\ \psi_2 \end{pmatrix} \leftrightarrow \pi \pi \]

\[ H_0 \psi_1 + \frac{1}{\sqrt{N_c}} V \psi_2 = E \psi_1 \]

\[ H_0 \psi_2 + \frac{1}{\sqrt{N_c}} V \psi_1 + [V_{\text{conf}}] \psi_2 = E \psi_2 \]

Exactly the structure that leads to Feshbach resonance, a bound state in the continuum, whose lifetime goes to infinity (width to zero) as

Only a discrete spectrum

\[ \mathcal{H} = \begin{pmatrix} h_0 & V \\ V & h \end{pmatrix} \]

\[ h |\phi\rangle = E_0 |\phi\rangle \]

\[ g(E) = \frac{1}{E - h} \approx \frac{|\phi \rangle \langle \phi|}{E - E_0} \]

\[ h_\ell |u_\ell\rangle + V \frac{|\phi \rangle \langle \phi|}{E - E_0} V |u_\ell\rangle = E |u_\ell\rangle. \]

Confined channel “bound state” generates infinitely strong effective interaction in continuum

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But ordinary $Q\bar{Q}$ meson resonances are not the only effect that remains at $\mathcal{O}(1/N_c)$...

**Extraordinary Mesons**

Closer look at meson-meson scattering at large $N_c$.

Generic normalized two meson source

$$D(x) \equiv \frac{1}{N_c} \bar{q}q\bar{q}q(x)$$

$$\langle 0|D(x)D(0)|0\rangle \sim 1 \quad \text{as } N_c \to \infty$$
All meson-meson interactions are $\mathcal{O}(1/N_c)$. t-channel exchange of narrow ordinary mesons build up narrow s-channel ordinary-meson resonances.

Additionally – quark exchange diagrams are also $\mathcal{O}(1/N_c)$ but do not couple to ordinary (narrow) meson resonances. Off the narrow ordinary-meson resonances, quark exchange is the dominant meson-meson interaction as $N_c \to \infty$. 

$$\left[ \frac{1}{N_c} \right]^2 \left[ \frac{1}{\sqrt{N_c}} \right]^4 \rightarrow \frac{1}{N_c}$$
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\[ \left( \frac{1}{N_c} \right)^2 \left( \frac{1}{\sqrt{N_c}} \right)^4 \to \frac{1}{N_c} \]
Quark exchange is the dominant force between mesons off resonance at large $N_c$

Implications

- In any fixed basis, e.g., $M_{12}M_{34}$ quark exchange mixes color octet components into the wavefunction, so the force is fundamentally chromodynamic.
- The range of the force is determined by the distance at which hadrons overlap, of order 1 fermi.
- Attractive, repulsive, capable of generating bound/virtual states.
- No coupling to confined channels, so the interactions are “potential-like.” Non-relativistic analog would be simply the Schrödinger equation with an open channel potential.
- Extraordinary hadrons —— if they exist at all —— disappear as $N_c \rightarrow \infty$; they merely subside into the hadron-hadron continuum.
• No one knows how to calculate scattering amplitudes in QCD, so how can these ideas be tested?

• Meson-meson scattering can be studied as a function of $N_c$ using unitized chiral dynamics$^\dagger$  

\[ |t_{00}| \]

\[ |t_{11}| \]

\[ \rho(770) \]

\begin{align*}
N_c &= 3 \\
N_c &= 5 \\
N_c &= 10
\end{align*}

$^\dagger$I understand that there is model dependence here!

$^\dagger$J. Pelaez & collaborators

$N_c$ dependence of $\rho$ consistent with “ordinary meson”

$N_c$ dependence of $f_0(500)$ is not consistent with “ordinary meson”
Miscellaneous Thoughts on the Description of Extraordinary Hadrons

I. The low energy meson-meson s-wave is very special

$0^{++}$ quantum numbers

$\bar{q}q$ $p$-wave ($s$-wave has negative parity)

$\pi\pi$ or $\bar{q}qqq$ $s$-wave

Diquark correlations can be important in the $\bar{q}qqq$ $s$-wave

$f_0(600)$ $\kappa(800)$ $f_0(980)$ $a_0(980)$

Contrast the $1^{--}$ channel:

$\bar{q}q$ $s$-wave

$\pi\pi$ or $\bar{q}qqq$ $p$-wave

Generically the meson-meson, meson-baryon, and baryon-baryon $s$-wave is where unusual enhancements are expected and often seen
Trajectories of poles in the complex k-plane as a function of coupling

S-wave

Ordinary and extraordinary resonances could hardly be more different!
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II. There is no clear distinction between a meson-meson molecule and a $\bar{q}qqq$ state

$f_0(980)$ or $X(2873)$ a meson-meson molecule or $\bar{q}qqq$ state?

could (will) be debated forever, but the distinction is not meaningful when a state is near or above its prominent decay threshold.

Consider a (non-relativistic) system with binding energy $B$

$$B \to 0 \quad \text{kinematics requires} \quad \sqrt{\langle r^2 \rangle} \to \sqrt{\frac{2MB}{\hbar^2}}$$

Even if all the (attractive) interaction comes from a chromodynamic force that has no simple parameterization in terms of meson exchange potentials --- nevertheless the system has nearly unit probability to be found as separate mesons.

Simple and elegant model, where hadrons interact only by quark exchange forces, but meson molecules $\text{meson molecules} \iff \bar{q}qqq$ states

F. Lenz, J.T. Londergan, E.J. Moniz, R. Rosenfelder, M. Stingl, K. Yazaki

Quark Confinement And Hadronic Interactions, Annals Phys.170:65,1986

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