Dalitz plot analysis of three-body Charmonium Decays at BABAR

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Outline

- Dalitz plot analysis of $J/\psi \to \pi^+ \pi^- \pi^0$ and $J/\psi \to K^+ K^- \pi^0$(*).
- Measurement of the I=1/2 $K\pi$ $S$-wave amplitude from a Dalitz plot analysis of $\eta_c \to K\bar{K}\pi$ in two-photon interactions(**).

All the results presented here are new and preliminary.

(*) Work done in collaboration with M. Pennington and A. Szczepaniak
(**) Work done in collaboration with M. Pennington

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Charmonium decays can be used to obtain new information on light meson spectroscopy.

In $e^+e^-$ interactions, samples of charmonium decays can be obtained using different processes.

In two-photon interactions we select events in which the $e^+$ and $e^-$ beam particles are scattered at small angles and remain undetected.

Only resonances with $J^{PC} = 0^{\pm+}, 2^{\pm+}, 3^{++}, 4^{\pm+}, \ldots$ can be produced.

In the Initial State Radiation (ISR) process, we reconstruct events having a (mostly undetected) fast forward $\gamma_{ISR}$.

Only $J^{PC} = 1^{-+}$ states can be produced.
Dalitz plot analysis of \(J/\psi \rightarrow \pi^+\pi^-\pi^0\) and \(J/\psi \rightarrow K^+K^-\pi^0\)

- Only a preliminary result exists, to date, on a Dalitz-plot analysis of \(J/\psi\) decays to \(\pi^+\pi^-\pi^0\) (SLAC-PUB-5674, (1991)).

- While large samples of \(J/\psi\) decays exist, some branching fractions remain poorly measured. In particular the \(J/\psi \rightarrow K^+K^-\pi^0\) branching fraction has been measured by MarkII using only 25 events.

- The BES III experiment has performed an angular analysis of \(J/\psi \rightarrow K^+K^-\pi^0\). The analysis requires the presence of a broad \(J^{PC} = 1^{--}\) state in the \(K^+K^-\) threshold region, which is interpreted as a multiquark state (Phys. Rev. Lett. 97, 142002 (2006)).
Data selection

□ We study the following reactions:
\[ e^+e^- \to \gamma_{\text{ISR}} \pi^+\pi^-\pi^0, \quad e^+e^- \to \gamma_{\text{ISR}} K^+K^-\pi^0 \]
where \( \gamma_{\text{ISR}} \) indicate the (undetected) ISR photon.
□ Select events having only two tracks and one (mass constrained) \( \pi^0 \).
□ We compute \( M_{\text{rec}}^2 \equiv (p_{e^-} + p_{e^+} - p_{h^+} - p_{h^-} - p_{\pi^0})^2 \), where \( h = \pi/K \).
□ This quantity should peak near zero for ISR events.
□ Plot of \( M_{\text{rec}}^2 \) in the \( J/\psi \) signal region. In red are Monte Carlo simulations.
We select events in the ISR region by requiring $|M_{\text{rec}}^2| < 2 \text{ GeV}^2/c^4$ and obtain the $J/\psi$ signals.

We fit the mass spectra using the Monte Carlo resolution functions described by a Crystal Ball+Gaussian functions and obtain the yields:

<table>
<thead>
<tr>
<th>$J/\psi$ decay mode</th>
<th>$\chi^2/NDF$</th>
<th>$J/\psi$ mass (MeV)</th>
<th>Signal region events</th>
<th>Purity</th>
</tr>
</thead>
<tbody>
<tr>
<td>$J/\psi \rightarrow \pi^+\pi^-\pi^0$</td>
<td>84/115</td>
<td>$3099.8 \pm 0.2$</td>
<td>21974</td>
<td>$(86.1 \pm 1.3)%$</td>
</tr>
<tr>
<td>$J/\psi \rightarrow K^+K^-\pi^0$</td>
<td>111/95</td>
<td>$3101.0 \pm 0.2$</td>
<td>2393</td>
<td>$(87.8 \pm 0.7)%$</td>
</tr>
</tbody>
</table>
Efficiency and Branching fraction

- The efficiency is mapped and fitted on the \((m(h^+h^-), \cos \theta_h)\) plane, where \(\theta_h\) is the \(h^+\) helicity angle in the \(J/\psi\) rest frame.

- We obtain the weighted efficiencies:
  \[
  \epsilon_{h^+h^-\pi^0} = \frac{\sum_{i=1}^{N} f_i}{\sum_{i=1}^{N} f_i / \epsilon(m_i, \cos \theta_i)}
  \]
  where negative weights \(f_i\) are assigned to sidebands events.

- We obtain the following preliminary result:
  \[
  \mathcal{R} = \frac{\mathcal{B}(J/\psi \rightarrow K^+K^-\pi^0)}{\mathcal{B}(J/\psi \rightarrow \pi^+\pi^-\pi^0)} = 0.0929 \pm 0.002 \pm 0.002
  \]

- The PDG reports \(\mathcal{B}(J/\psi \rightarrow K^+K^-\pi^0) = 55.2 \pm 0.12 \times 10^{-4}\), based on 25 events, and \(\mathcal{B}(J/\psi \rightarrow \pi^+\pi^-\pi^0) = 2.11 \pm 0.07 \times 10^{-2}\).

- These values give a ratio \(\mathcal{R} = 0.262 \pm 0.057\), which differs from our result by 3\(\sigma\).
\( J/\psi \rightarrow \pi^+\pi^-\pi^0 \) Dalitz plot and projections

- Dominated by three \( \rho(770)\pi \) contributions.
- Dalitz plot analysis performed using:
  - Isobar model using Zemach tensors;
    C. Zemach, Phys Rev. **133**, B1201 (1964),
  - Veneziano model.
- Dalitz plot projections.

- Shaded is the background interpolated by sidebands.
The Veneziano model deals with trajectories rather than single resonances.

The complexity of the model is related to $n$, the number of Regge trajectories included in the fit.

The fit requires $n=5$.

Combinatorial $\pi$ helicity angle vs. $m(\pi\pi)$.

$m(\pi\pi)$ mass projection for $|\cos\theta_\pi| < 0.2$.

<table>
<thead>
<tr>
<th>Final state</th>
<th>Isobar fraction %</th>
<th>Phase (radians)</th>
<th>Veneziano fraction %</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\rho(770)\pi$</td>
<td>119.0 ± 1.1 ± 3.3</td>
<td>0.</td>
<td>120.0 ± 1.9</td>
</tr>
<tr>
<td>$\rho(1460)\pi$</td>
<td>16.9 ± 2.0 ± 3.1</td>
<td>3.92 ± 0.05 ± 0.11</td>
<td>1.53 ± 0.13</td>
</tr>
<tr>
<td>$\rho(1700)\pi$</td>
<td>0.1 ± 0.1 ± 0.2</td>
<td>1.01 ± 0.35 ± 0.79</td>
<td>0.84 ± 0.08</td>
</tr>
<tr>
<td>$\rho(2150)\pi$</td>
<td>0.04 ± 0.05 ± 0.02</td>
<td>1.89 ± 0.30 ± 0.48</td>
<td>2.03 ± 0.17</td>
</tr>
<tr>
<td>$\rho_3(1690)\pi$</td>
<td>0.09 ± 0.02</td>
<td></td>
<td>0.09 ± 0.02</td>
</tr>
<tr>
<td>Sum</td>
<td>136.0 ± 2.3 ± 4.3</td>
<td></td>
<td>124.5 ± 2.3</td>
</tr>
<tr>
<td>$\chi^2/\nu$</td>
<td>764/552</td>
<td></td>
<td>780/554</td>
</tr>
</tbody>
</table>

The two models give almost similar data representation, but different fractions.
\( J/\psi \rightarrow K^+K^-\pi^0 \) Dalitz plot analysis

- Clear \( K^{*+} \) and \( K^{*-} \) bands.
- Broad structure in the low \( K^+K^- \) mass region.
- We make use of the Isobar model only.

<table>
<thead>
<tr>
<th>Final state</th>
<th>fraction %</th>
<th>phase</th>
</tr>
</thead>
<tbody>
<tr>
<td>( K^*(892)K )</td>
<td>87.8 ± 2.0 ± 1.7</td>
<td>0.</td>
</tr>
<tr>
<td>( \rho(1450)^0\pi^0 )</td>
<td>11.5 ± 2.1 ± 2.1</td>
<td>-2.81 ± 0.25 ± 0.36</td>
</tr>
<tr>
<td>( K^*(1410)K )</td>
<td>1.7 ± 0.7 ± 1.1</td>
<td>2.89 ± 0.35 ± 0.08</td>
</tr>
<tr>
<td>( K_2^*(1430)K )</td>
<td>3.8 ± 1.4 ± 0.5</td>
<td>-2.42 ± 0.22 ± 0.07</td>
</tr>
<tr>
<td>( \rho(1700)^0\pi^0 )</td>
<td>0.9 ± 1.0 ± 0.6</td>
<td>1.06 ± 0.20 ± 0.7</td>
</tr>
<tr>
<td>Total</td>
<td>105.6 ± 3.4 ± 3.0</td>
<td>( \chi^2/\nu = 94/92 )</td>
</tr>
</tbody>
</table>

- Leaving free the \( \rho(1450) \) parameters:
  \[ m(\rho(1450)) = 1361 \pm 43 \text{ MeV}/c^2 \]
  \[ \Gamma(\rho(1450)) = 479 \pm 63 \text{ MeV} \]
  in the range of other \( \rho(1450) \) measurements.

- Dalitz plot projections:

- Shaded is the background.
\[\rho(1450) \text{ branching fraction}\]

\[\Box \text{ We find the parameters of the low mass } K^+K^- \text{ structure consistent for being associated to } \rho(1450).\]

\[\Box \text{ We have measured the ratio}\]

\[\mathcal{R} = \frac{\mathcal{B}(J/\psi \rightarrow K^+K^-\pi^0)}{\mathcal{B}(J/\psi \rightarrow \pi^+\pi^-\pi^0)} = 0.0929 \pm 0.002 \pm 0.002\]

\[\Box \text{ From the Dalitz-plot analysis of } J/\psi \rightarrow \pi^+\pi^-\pi^0 \text{ we obtain:}\]

\[\mathcal{B}_1 = \mathcal{B}(J/\psi \rightarrow \rho(1450)\pi^0) = \frac{[(16.9 \pm 2.0 \pm 3.1)/3.]}{\%} = (5.63 \pm 0.67 \pm 1.03)\%\]

\[\Box \text{ From the Dalitz-plot analysis of } J/\psi \rightarrow K^+K^-\pi^0 \text{ we obtain:}\]

\[\mathcal{B}_2 = \mathcal{B}(J/\psi \rightarrow \rho(1450)\pi^0) = (11.5 \pm 2.1 \pm 2.1)\%\]

\[\Box \text{ We therefore obtain:}\]

\[\frac{\mathcal{B}(\rho(1450)^0 \rightarrow K^+K^-)}{\mathcal{B}(\rho(1450)^0 \rightarrow \pi^+\pi^-)} = \frac{\mathcal{B}_2}{\mathcal{B}_1} \cdot \mathcal{R} = 0.190 \pm 0.042 \pm 0.049\]
**Previous work**

- The BaBar Dalitz plot analysis of the $\eta_c \to K^+ K^- \eta$ and $\eta_c \to K^+ K^- \pi^0$ has provided the unexpected observation of $K^*_0(1430) \to K\eta$ (Phys. Rev. D89 (2014) 11, 112004).

- We measure the $K^*_0(1430)$ branching ratio

\[
\frac{B(K^*_0(1430) \to \eta K)}{B(K^*_0(1430) \to \pi K)} = 0.092 \pm 0.025^{+0.010}_{-0.025}
\]

- We also find that the $\eta_c$ three-body hadronic decays proceed almost entirely through:

\[\eta_c \to \text{pseudoscalar} + \text{scalar}\]

- Therefore three body decays of the $\eta_c$ are a unique window to study the properties of the scalar mesons.
Selection of $\gamma \gamma \rightarrow K\bar{K}\pi$

- We study the reactions:
  - $\gamma \gamma \rightarrow K^0_S K^+ \pi^- \ (*)$
  - $\gamma \gamma \rightarrow K^+ K^- \pi^0 \ (**)$

- Select events having only four tracks.
- $p_T$: transverse momentum of the $K^0_S K^+ \pi^-$ system with respect to the beam axis.

- The signal at low $p_T$ evidences the presence of two-photon events. We require $p_T < 0.08 \text{ GeV}/c$.

- We define $M^2_{\text{rec}}$ as:
  \[ M^2_{\text{rec}} \equiv (p_{e^+e^-} - p_{\text{rec}})^2 \]

- $p_{e^+e^-}$ is the four-momentum of the initial state and $p_{\text{rec}}$ is the four-momentum of the $K^0_S K^+ \pi^-$ system.
- We remove ISR events requiring $M^2_{\text{rec}} > 10 \text{ GeV}^2/c^4$.

(*): Charge conjugation is implied through all this work.
(**): Details will be given only for the $K^0_S K^+ \pi^-$ final state.
The $K\bar{K}\pi$ mass spectra in the $\eta_c$ region

- $\eta_c \to K_S^0 K^+ \pi^-, 12849$ evts with $(64.3 \pm 0.4)\%$ purity.
- $\eta_c \to K^+ K^- \pi^0$, 6494 evts with $(55.2\pm0.6)\%$ purity.

- Residual $J/\psi$ signals from ISR.
- Dalitz plots:

- Dominated by the presence of $K_0^*(1430)$.

Purity: $P = N_{sig} / (N_{sig} + N_{back})$
Dalitz plot analysis of $\eta_c \to K\bar{K}\pi$

- Unbinned maximum likelihood fits.
- Fits performed using:
  - Isobar model: resonances described by Breit-Wigner functions.
  - Model Independent Partial Wave Analysis (MIPWA) (Phys. Rev. D 73, 032004 (2006)).
- The $K\pi$ S-wave ($A_1$) is taken as the reference amplitude.

$$A = A_1 + c_2 A_2 e^{i\phi_2} + c_3 A_3 e^{i\phi_3} + ...$$

- The $K\pi$ mass spectrum is divided into 30 equally spaced mass intervals 60 MeV wide and for each bin we add to the fit two new free parameters, the amplitude and the phase of the $K\pi$ S-wave (constant inside the bin).
- We also fix the $A_1$ amplitude to 1.0 and its phase to $\pi/2$ in an arbitrary interval of the mass spectrum (bin 11 which corresponds to a mass of 1.42 GeV/$c^2$).
- The number of additional free parameters is therefore 58.
Interference between the two $K\pi$ modes is determined by Isospin conservation.

For $\eta_c \rightarrow K_S^0 K^+\pi^-$:

$$A_{S-wave} = \frac{1}{\sqrt{2}} \left( a_j^{K^+\pi^-} e^{i\phi_j^{K^+\pi^-}} + a_j^{K^0\pi^-} e^{i\phi_j^{K^0\pi^-}} \right)$$

where $a^{K^+\pi^-}(m) = a^{K^0\pi^-}(m)$ and $\phi^{K^+\pi^-}(m) = \phi^{K^0\pi^-}(m)$

For $\eta_c \rightarrow K^+ K^-\pi^0$:

$$A_{S-wave} = \frac{1}{\sqrt{2}} \left( a_j^{K^+\pi^0} e^{i\phi_j^{K^+\pi^0}} + a_j^{K^-\pi^0} e^{i\phi_j^{K^-\pi^0}} \right)$$

where $a^{K^+\pi^0}(m) = a^{K^-\pi^0}(m)$ and $\phi^{K^+\pi^0}(m) = \phi^{K^-\pi^0}(m)$

The $K_2^*(1420)$, $a_0(980)$, $a_0(1400)$, $a_2(1310)$, ... contributions are modeled as relativistic Breit-Wigner functions multiplied by the corresponding angular functions.

Backgrounds are fitted separately and interpolated into the $\eta_c$ signal regions.
An additional $a_0(1950)$ resonance

- The fits improve when an additional high mass $a_0(1950) \to K\bar{K} I=1$ resonance is included with free parameters in both $\eta_c$ decay modes.
- The fits return the following parameters:

<table>
<thead>
<tr>
<th>Final state $\eta_c \to K_S^0 K^+\pi^-$</th>
<th>Mass (MeV/$c^2$)</th>
<th>Width (MeV)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\eta_c \to K^+ K^-\pi^0$</td>
<td>1927 $\pm$ 15 $\pm$ 23</td>
<td>274 $\pm$ 28 $\pm$ 30</td>
</tr>
<tr>
<td>Weighted mean</td>
<td>1931 $\pm$ 14 $\pm$ 22</td>
<td>271 $\pm$ 22 $\pm$ 29</td>
</tr>
</tbody>
</table>

*red line:* no $a_0(1950)$

- Statistical significances for the $a_0(1950)$ effect (including systematics) are 2.5$\sigma$ for $\eta_c \to K_S^0 K^+\pi^-$ and 4.0$\sigma$ for $\eta_c \to K^+ K^-\pi^0$. 
Dalitz plots mass projections

- Dalitz plot projections with fit results for \( \eta_c \rightarrow K^0_S K^+ \pi^- \) (top) and \( \eta_c \rightarrow K^+ K^- \pi^0 \) (bottom)

- Shaded is contribution from the interpolated background.
- \( K^*(890) \) contributions entirely from background.
Legendre polynomial moments: $\eta_c \rightarrow K_S^0 K^+ \pi^-$

- Mass projections weighted by $Y_L^0$ moments and compared with fit results.

$m(K^+ \pi^-) + m(K_S^0 \pi^-)$ projections:

$m(K_S^0 K^+)$ projections:

- Good agreement in all the projections.
- Similarly for $\eta_c \rightarrow K^+ K^- \pi^0$. 
Fit fractions from the MIPWA. Comparison with the Isobar Model

<table>
<thead>
<tr>
<th>Amplitude</th>
<th>$\eta_c \rightarrow K^0_S K^+ \pi^-$</th>
<th>$\eta_c \rightarrow K^+ K^- \pi^0$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$(K\pi \text{ S-wave}) K$</td>
<td>$107.3 \pm 2.6 \pm 17.9$</td>
<td>$125.5 \pm 2.4 \pm 4.2$</td>
</tr>
<tr>
<td>$a_0(1950)\pi$</td>
<td>$3.1 \pm 0.4 \pm 1.2$</td>
<td>$4.4 \pm 0.8 \pm 0.7$</td>
</tr>
<tr>
<td>$K_2^*(1430)^0 K$</td>
<td>$4.7 \pm 0.9 \pm 1.4$</td>
<td>$3.0 \pm 0.8 \pm 4.4$</td>
</tr>
</tbody>
</table>

$\chi^2/N_{cells}$

|                     | $301/254=1.17$ | $283.2/233=1.22$ |

**Isobar Model**

| $(K_0^*(1430)K)^+$ | $73.6 \pm 3.7$ | $63.6 \pm 5.6$ |
| $(K_0^*(1950)K)^+$ |                           |                |
| *Nonresonant*       |                           |                |

$\chi^2/N_{cells}$

|                     | $457/254=1.82$ | $383/233=1.63$ |

- For MIPWA, good agreement between the two $\eta_c$ decay modes.
- $I=1/2$ $(K\pi \text{ S-wave}) K$ amplitude dominant with small contributions from $K_2^*(1430)^0 K$ and $a_0(1950)\pi$ amplitudes.
- Spin-1 resonances consistent to come entirely from background.
- Good description of the data with MIPWA.
- Poorer description of the data with the Isobar Model.
New measurement of the I=1/2 $K\pi$ S-wave

- Fitted amplitude and phase. Average systematic uncertainty is 16%.
- Red: $\eta_c \rightarrow K^+ K^- \pi^0$. Black: $\eta_c \rightarrow K_S^0 K^+ \pi^-$. 
- Clear $K_0^*(1430)$ resonance and corresponding phase motion.
- At high mass broad $K_0^*(1950)$ contribution.

![Graph (a)](image1)
![Graph (b)](image2)

- Dashed lines are $K\eta$ and $K\eta'$ thresholds.
- Good agreement between the two $\eta_c$ decay modes.
Comparison with the LASS and E791 experiments

- Black is $\eta_c \rightarrow K_S^0 K^+ \pi^-$.  

LASS($K^-p$)  

- Normalization is arbitrary.  

- LASS analysis has two solutions above 1.9 GeV.  

- Phases before the $K\eta'$ threshold are similar, as expected from Watson theorem.  

- Amplitudes are very different.

E791($D^+ \rightarrow K^- \pi^+ \pi^+$)  

• We show preliminary results on Dalitz plot analyses of $J/\psi \to \pi^+\pi^-\pi^0$ and $J/\psi \to K^+K^-\pi^0$ produced in Initial State Radiation events using the isobar and Veneziano models.

• We show preliminary results on the Dalitz plot analyses of $\eta_c \to K^0_SK^+\pi^-$ and $\eta_c \to K^+K^-\pi^0$ produced in two-photon interactions.

• We extract for the first time the I=1/2 $K\pi$ $S$-wave amplitude and phase using the MIPWA method. We find a very different amplitude with respect to that measured by previous experiments in different processes.
Backup slides
Efficiency and Background \((\eta_c \to K^0_S K^+\pi^-)\)

- Efficiency evaluated on the \((m(K^+\pi^-), \cos\theta)\) plane, where \(\theta\) is the \(K^+\) helicity angle in the \(K^0_S K^+\pi^-\) rest frame.
- Fitted using Legendre polynomials moments:
  \[
  \epsilon(\cos\theta) = \sum_{L=0}^{12} a_L(m_{K^+\pi^-}) Y_L^0(\cos\theta)
  \]
  in slices of \(m_{K^+\pi^-}\).
- \(a_L(m_{K^+\pi^-})\) fitted with seventh-order polynomials.
- Background estimated from \(\eta_c\) sidebands.

- Asymmetric \(K^*\)'s.
- Interference between I=1 and I=0 contributions.
Test for multiple solutions and Systematic uncertainties

- We have generated and fitted MC simulations with different mixtures of amplitudes.
- We started the fits from random values for the $K\pi$ $S$-wave amplitude and phase.
- We have evaluated the following systematic uncertainties.
  
  - Fit bias. We generate MC simulations according to the fit results and re-fit. The distribution of the absolute value of the fractional residuals is fit with a Gaussian having zero mean and take the $\sigma$ as systematic uncertainty.
  
  - The amplitude and phase are constant within the mass bins in the reference fit. We replace the representation using a cubic spline.
  
  - We remove low significances amplitudes such as $a_0(980)$ and $a_2(1310)$ resonances.
  
  - We vary up and down the purity of the signal.
  
  - The effect of the efficiency variation as a function of the $K\bar{K}\pi$ mass is evaluated by computing separate efficiencies in the regions below and above the $\eta_c$ mass.
  
- Total average systematic uncertainty is of the order of 16%.