Heavy Hidden-Flavour Molecules in a Finite Volume

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Outline

Part 1: Formulation of the EFT for hidden-charm meson antimeson molecules

Part 2: The EFT in a finite volume

Part 1

Formulation of the EFT for hidden-charm meson antimeson molecules

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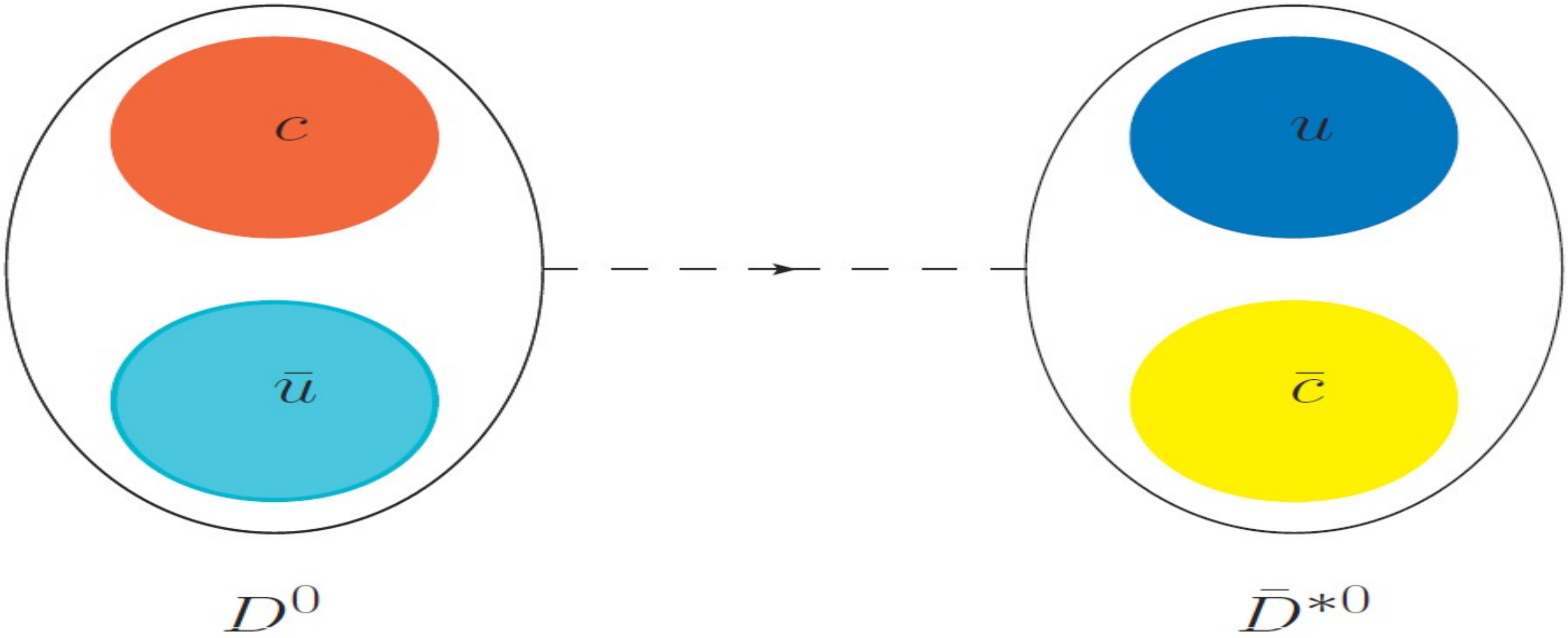
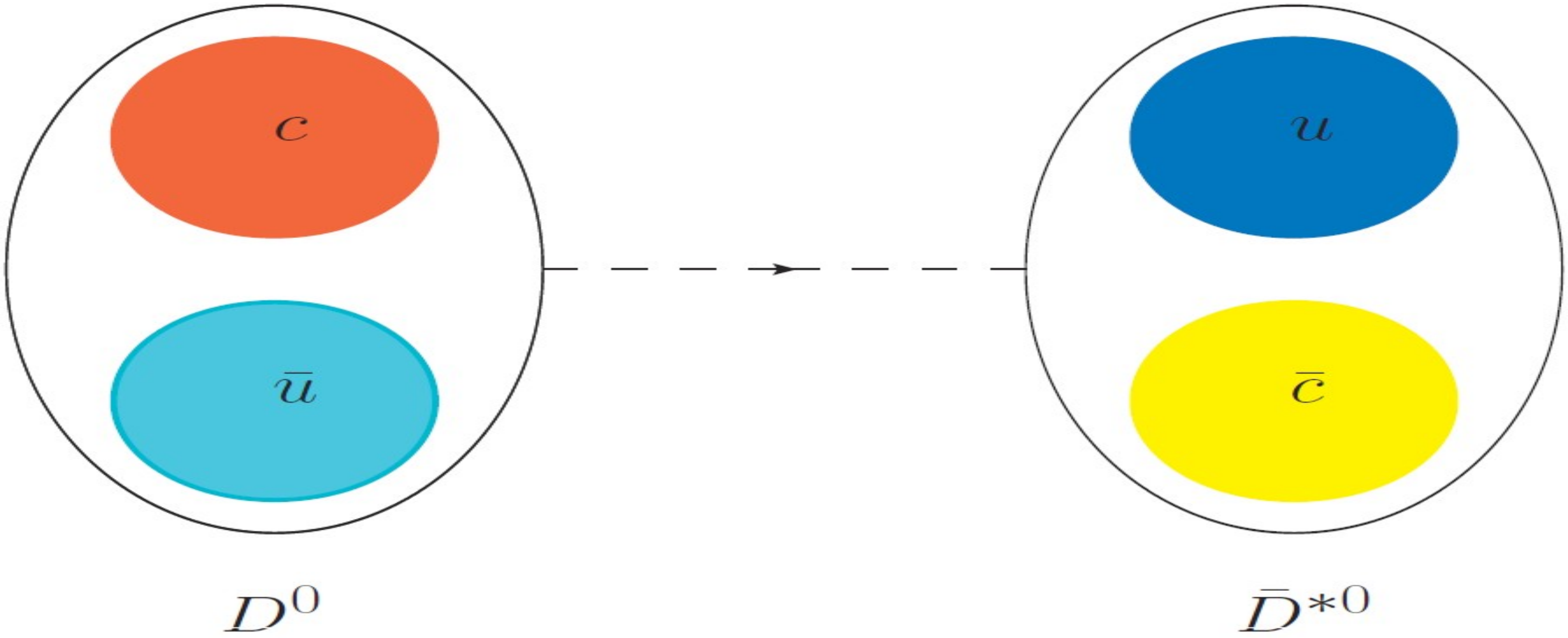
Meson-Antimeson Molecules



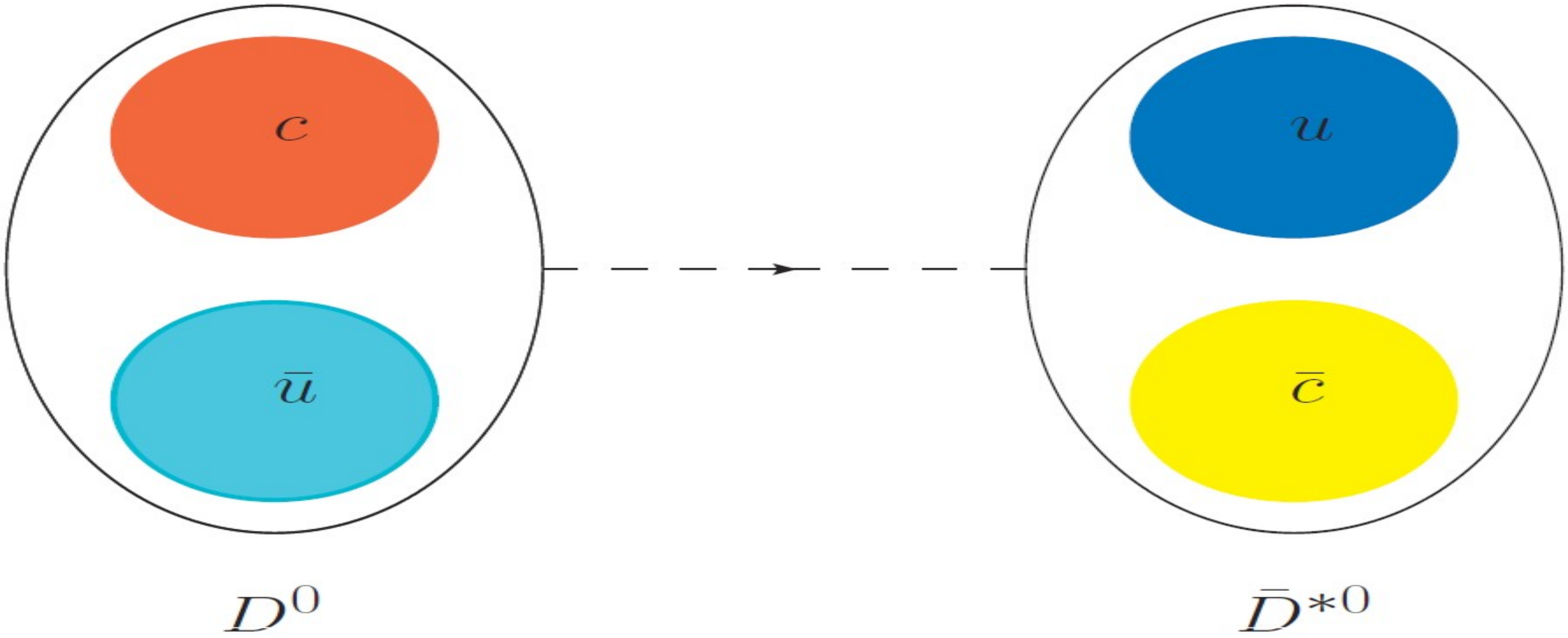
Diagramatic representation of a heavy meson-antimeson molecular system

Meson-Antimeson Molecules



- The mass of the heavy (anti-)quark in the (anti-)meson.
- The size of the mesons.

Meson-Antimeson Molecules



- The mass of the heavy (anti-)quark in the (anti-)meson.
- The size of the mesons.

- The meson-antimeson distance (order Λ_{QCD})
- The total momentum of the molecular system.

Symmetries

- Our approach for the study of heavy mesonic molecular systems will be based on,
  - **Heavy Quark Spin Symmetry (HQSS).** the dynamics is invariant under separate spin rotations of the heavy quark and antiquark.
  - **Heavy Flavour Symmetry (HFS).** Spectrum in the charm sector must be similar to the spectrum in the bottom sector.
  - **Heavy Antiquark-Diquark Symmetry (HADS).** Heavy diquark behaves as a heavy antiquark.

Symmetries

- Our approach for the study of heavy mesonic molecular systems will be based on,
  - **Chiral symmetry** contains pion exchange interactions.
  - **SU(3)-light flavour symmetry**: Heavy molecules also come in SU(3)-light flavour multiplets.

- HQS has a spin-flavour SU($2N_h$) symmetry.

- HQET eigenstates are "would-be" hadrons composed by a heavy quark with light antiquarks and gluons, which, assuming SU(3) light-flavour symmetry, will be described into triplets, e.g. $D = (D^0, D^+, D_s)$

EFT Lagrangian

- The heavy and light degrees of freedom of the whole heavy meson-heavy antimeson system can take the following values:

$$S_L = 0,1$$
$$S_H = 0,1$$

EFT Lagrangian

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$$\mathbf{S_L = 0,1}$$

$$S_H = 0,1$$

- Since the dynamics does not depend on the heavy quark spin, the Lagrangian has to contain two different Low Energy Constants, C_A and C_B.

- Taking into account the degrees of freedom related to isospin, the Lagrangian has to include four LECs.

EFT Lagrangian

- At Leading Order, the most general potential that respects HQSS takes the form,

$$
\begin{aligned}
V_4 \;=\; & +\frac{C_A}{4}\, Tr\left[\bar{H}^{(Q)} H^{(Q)} \gamma_\mu\right] Tr\left[H^{(\bar{Q})} \bar{H}^{(\bar{Q})} \gamma^\mu\right] + \\
& +\frac{C_A^\lambda}{4}\, Tr\left[\bar{H}_a^{(Q)} \lambda_{ab}^i H_b^{(Q)} \gamma_\mu\right] Tr\left[H_c^{(\bar{Q})} \lambda_{cd}^i \bar{H}_d^{(\bar{Q})} \gamma^\mu\right] + \\
& +\frac{C_B}{4}\, Tr\left[\bar{H}^{(Q)} H^{(Q)} \gamma_\mu \gamma_5\right] Tr\left[H^{(\bar{Q})} \bar{H}^{(\bar{Q})} \gamma^\mu \gamma_5\right] + \\
& +\frac{C_B^\lambda}{4}\, Tr\left[\bar{H}_a^{(Q)} \lambda_{ab}^j H_b^{(Q)} \gamma_\mu \gamma_5\right] Tr\left[H_c^{(\bar{Q})} \lambda_{cd}^j \bar{H}_d^{(\bar{Q})} \gamma^\mu \gamma_5\right]
\end{aligned}
$$

- From now on, we refer to the LECs as C_{0a}, C_{0b}, C_{1a} and C_{1b}.

Lippmann-Schwinger Equation

- Once we have determined V, we find bound states by solving the LSE equation for each spin, isospin and charge-conjugation sector:

$$T = V + V\ G\ T$$

$$\langle \vec{p}\ |T|\vec{p}\ '\rangle = \langle \vec{p}\ |V|\vec{p}\ '\rangle + \int d^3\vec{k}\frac{\left\langle \vec{p}\ |V|\vec{k}\right\rangle \left\langle \vec{k}|T|\vec{p}\ '\right\rangle}{E - m_1 - m_2 - \frac{k^2}{2\mu}}$$

- Bound states of this model will appear as poles in the T-matrix.
- Ultraviolet divergences are regularized/renormalized introducing a Gaussian regulator Λ:

$$\langle \vec{p}|V|\vec{p}\ '\rangle = V(\vec{p},\vec{p}\ ') = v\ e^{-\vec{p}^2/\Lambda^2}\ e^{-\vec{p}'^2/\Lambda^2} \quad \Rightarrow \quad G = \int \frac{d^3\vec{k}}{(2\pi)^3}\frac{e^{-2\vec{k}^2/\Lambda^2}}{E-m_1-m_2-\frac{k^2}{2\mu}}$$

Determination of the LECs

- To determine the LECs, we have made use of the following assumptions.

  - X(3917) is a $D^*\overline{D}^*$ bound state with $J^{PC} = 0^{++}$.
  - Y(4140) is a $D^*_s\overline{D}_s{}^*$ bound state with $J^{PC} = 0^{++}$.
  - X(3872) is $D\overline{D}^*$ bound state with $J^{PC} = 1^{++}$.
  - The fourth condition will be obtained from the "isospin violation" observed in the X(3872) decays.

Features of the EFT

- Light flavour symmetry and HQSS in heavy meson-antimeson systems, along with the determination of four LECs, provides a systematic study of a whole family of hidden charm molecules.
- Pion exchanges and coupled channels should be considered. However, according to previous studies, these effects are small and smaller than those expected from HQSS breaking terms.
- Important consequences:
  - Charm and bottom dynamics are similar.
  - $J^{PC} = 1^{++}$ $D\bar{D}^*$ and $J^{PC} = 2^{++}$ $D^*\bar{D}^*$ have the same dynamics.
  - $J^{PC} = 1^{+-}$ $D\bar{D}^*$ and $J^{PC} = 1^{+-}$ $D^*\bar{D}^*$ are degenerate too.

Part 2

The EFT in a finite volume

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LQCD

- Since QCD is non-perturbative at low energies, perturbative methods cannot be directly used. LQCD computes path integrals in a finite volume. This formalism allows the analysis of QCD at low energies.

- There exists a connection between LQCD with the infinite volume real world. The *Lüscher method* [C.Mat.Phys., 105,153('86); NP,B354,531('91)] translates energy levels calculated in LQCD to hadron-hadron phase shifts of binding energy.

- This method was generalized and simplified in [Döring et al., EPJA47, 139 (2011)].

(Generalized) Lüscher approach

- In a finite box (with periodic bound conditions), momenta are quantized.

$$\vec{q} = \frac{2\pi}{L}\vec{n},\ \vec{n} \in \mathbb{Z}^3$$

- It is possible to rewrite the amplitude in the box by replacing the integrals with sums ([Döring, Meiβner, Oset, Rusetsky, EPJ,A47, 139 (2011)]). In our EFT model,

$$T^{-1}(E) = V^{-1}(E) - G(E)$$

$$G = \int_{|\vec{q}|<\Lambda} \frac{\mathrm{d}^3\vec{q}}{(2\pi)^3} \frac{e^{-2(q^2-k^2)/\Lambda^2}}{E - m_1 - m_2 - \frac{\vec{q}^2}{2\mu} + i0^+}$$

(Generalized) Lüscher approach

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- It is possible to rewrite the amplitude in the box by replacing the integrals with sums ([Döring, Meißner, Oset, Rusetsky, EPJ,A47, 139 (2011)]). In our EFT model,

$$\widetilde{T}^{-1}(E) = V^{-1}(E) - \widetilde{G}(E)$$

$$\widetilde{G}(E) = \frac{1}{L^3}\sum_{\vec{q}} \frac{e^{-2(\vec{q}^{\,2}-k^2)/\Lambda^2}}{E - m_1 - m_2 - \vec{q}^{\,2}/2\mu}$$

(Generalized) Lüscher approach

- Therefore, the energy levels in a finite volume are given by, $\widetilde{T}^{-1}(E_n) = 0$

- The relation of the finite volume amplitude with its infinite volume counter-part reads then (notice the explicit dependence on the cutoff),

$$T^{-1}(E_n) = V^{-1} - G = \widetilde{G} - G \propto e^{-2i\delta(E_n)}$$

- The Lüscher formula is recovered when $\Lambda \to \infty$:

$$\sqrt{4\pi}\mathcal{Z}_{00}(1,\hat{k}^2) = -\frac{L}{2\pi}\frac{(2\pi)^3}{2\mu}\delta G_L(E), \quad \hat{k}^2 = \frac{k^2 L^2}{(2\pi)^2}$$

$$\delta G_L = \lim_{\Lambda\to\infty} \widetilde{G} - G$$

(Generalized) Lüscher approach

- A very useful way to compute the Lüscher function is then obtained.

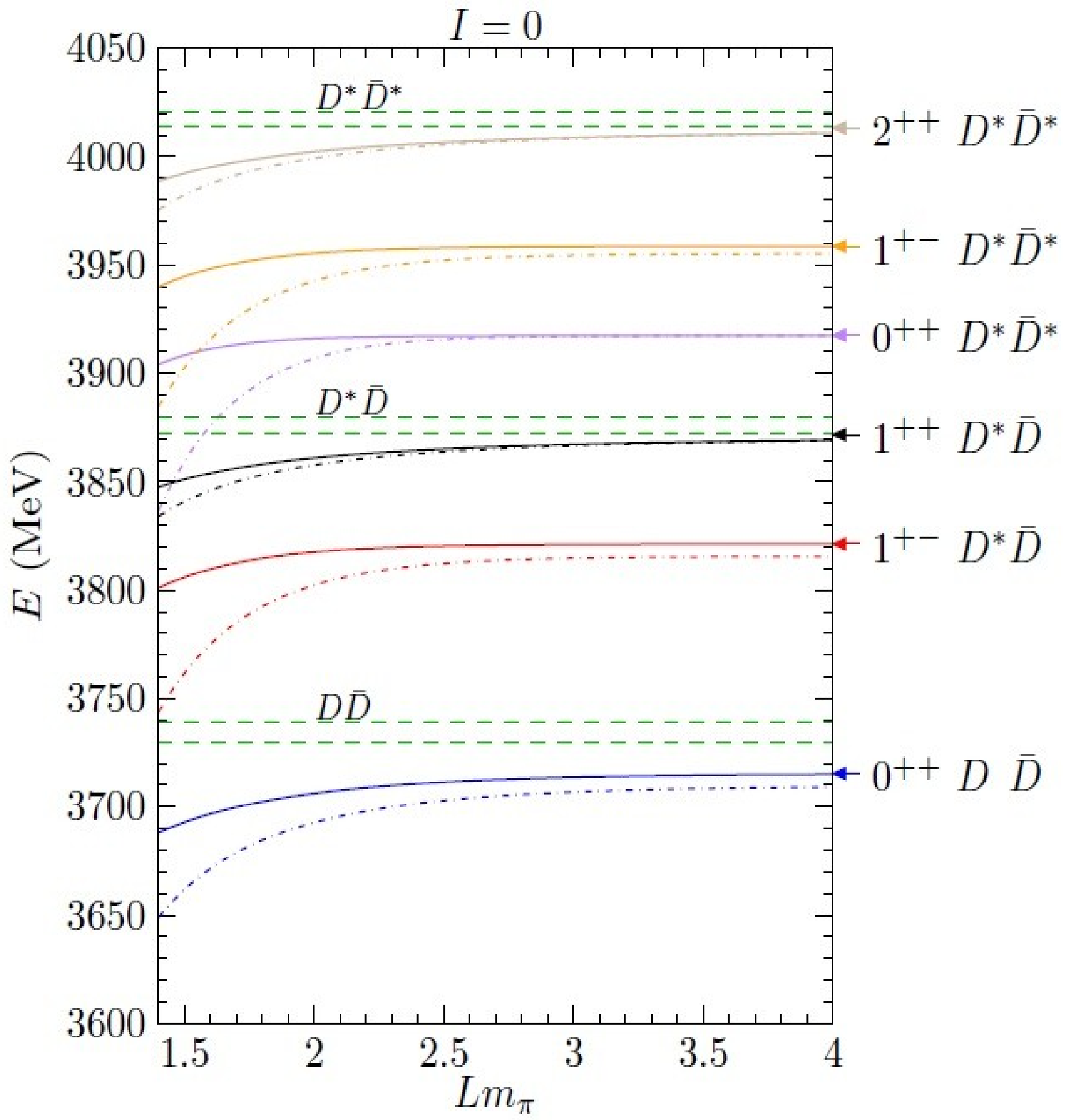
$$\delta G(E;\Lambda) = \tilde{G}(E;\Lambda) - G(E;\Lambda) = \left(\frac{1}{L^3}\sum_{\vec{q}} - \int \frac{\mathrm{d}^3\vec{q}}{(2\pi)^3}\right) \frac{e^{-2(\vec{q}^{\,2}-k^2)/\Lambda^2}}{\frac{\vec{k}^{\,2}}{2\mu} - \frac{\vec{q}^{\,2}}{2\mu} + i0^+}$$

$$= \underbrace{\left(\frac{1}{L^3}\sum_{\vec{q}} - \int \frac{\mathrm{d}^3\vec{q}}{(2\pi)^3}\right) \frac{e^{-2(\vec{q}^{\,2}-k^2)/\Lambda^2} - 1}{\frac{\vec{k}^{\,2}}{2\mu} - \frac{\vec{q}^{\,2}}{2\mu} + i0^+}}_{\delta G_\Lambda(E;\Lambda)} + \underbrace{\left(\frac{1}{L^3}\sum_{\vec{q}} - \int \frac{\mathrm{d}^3\vec{q}}{(2\pi)^3}\right) \frac{1}{\frac{\vec{k}^{\,2}}{2\mu} - \frac{\vec{q}^{\,2}}{2\mu} + i0^+}}_{\delta G_L(E)}$$

- For a finite Λ,

$$\delta G(E;\Lambda) = \delta G_L(E) + \frac{24\mu}{(2\pi)^{3/2}} \frac{e^{-\frac{\Lambda^2 L^2}{8}}}{\Lambda L^2} \left[1 + \frac{2(k^2L^2-2)}{L^2\Lambda^2} + \mathcal{O}\left(\Lambda^{-4}\right)\right] + \cdots$$

The EFT in a finite box



- Attractive potentials generate energy levels. Are they bound states?

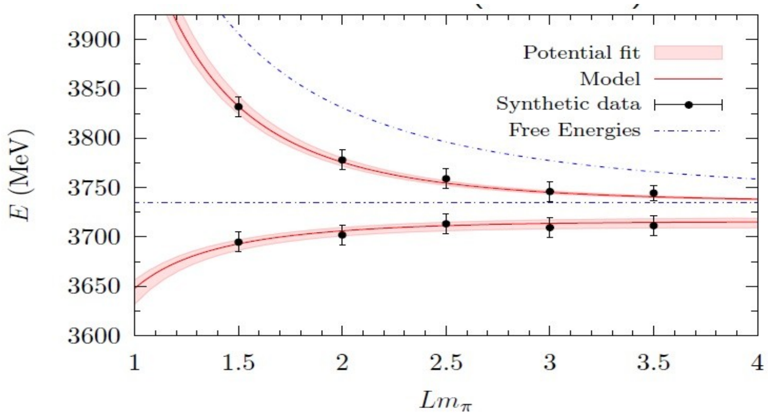
- There are some cases where the answer is clear but others are more uncertain.

- Algorithms to analyze the energy levels are then required.

INVERSE PROBLEM

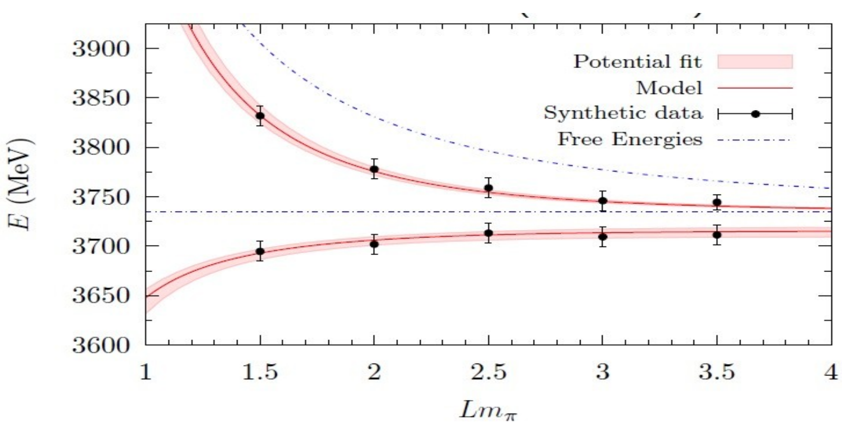
Inverse Analysis

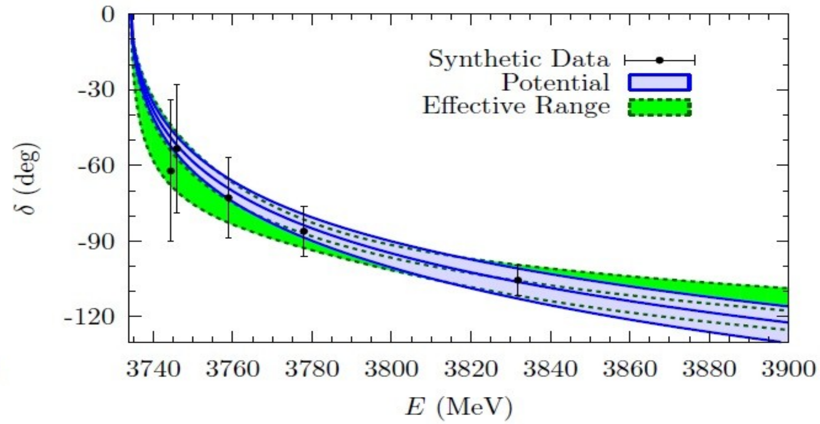
- We generate "synthetic" levels of energy.



- Three algorithms are tested in two cases, the $D\bar{D}$ with $J^{PC} = 0^{++}$ and the $D^*\bar{D}^*$ with $J^{PC} = 2^{++}$:
    - The phase shift analysis (level above threshold).
    - A potential fit (above and below threshold).
    - An effective range analysis (above and below threshold).

I.A.: Phase Shifts ($D\bar{D}$, 0^{++})



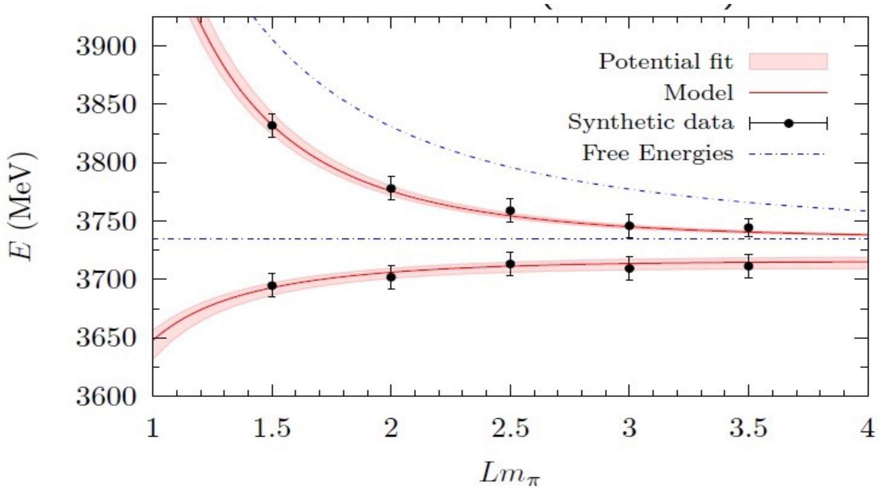


- Lüscher method transforms energy levels (E_n) into phase shifts $\delta(E_n)$

$$k \cot \delta = -\frac{1}{a} + \frac{1}{2} r k^2 = -\frac{2\pi}{\mu} \lim_{\Lambda \to \infty} \mathrm{Re}\left(\widetilde{G}(E) - G(E)\right) = \frac{4}{\sqrt{4\pi} L} \mathcal{Z}_{00}(1, \hat{k}^2)$$

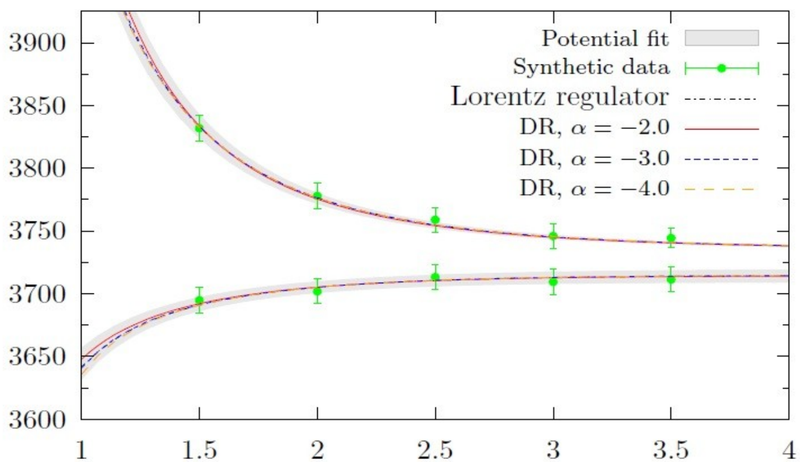
Observable	Analysis	Theory
a (fm)	$1.6^{+1.0}_{-0.5}$	1.38
r (fm)	0.53 ± 0.18	0.52
M (MeV)	3721^{+10}_{-25}	3715

I.A.: Potential fit ($D\bar{D}$, 0^{++})



Observable	Analysis	Theory
C_{0a} (fm^2)	$-1.08^{+0.19}_{-0.29}$	-1.024
Λ (GeV)	0.97 ± 0.13	1.00
M (MeV)	3715^{+3}_{-6}	3715

More accurate predictions!



Similar results with different regulators!

- Lorentzian Regulator:

$$e^{-2(q^2-k^2)/\Lambda^2} \Rightarrow \left(\frac{k^2+\Lambda^2}{q^2+\Lambda^2}\right)^2$$

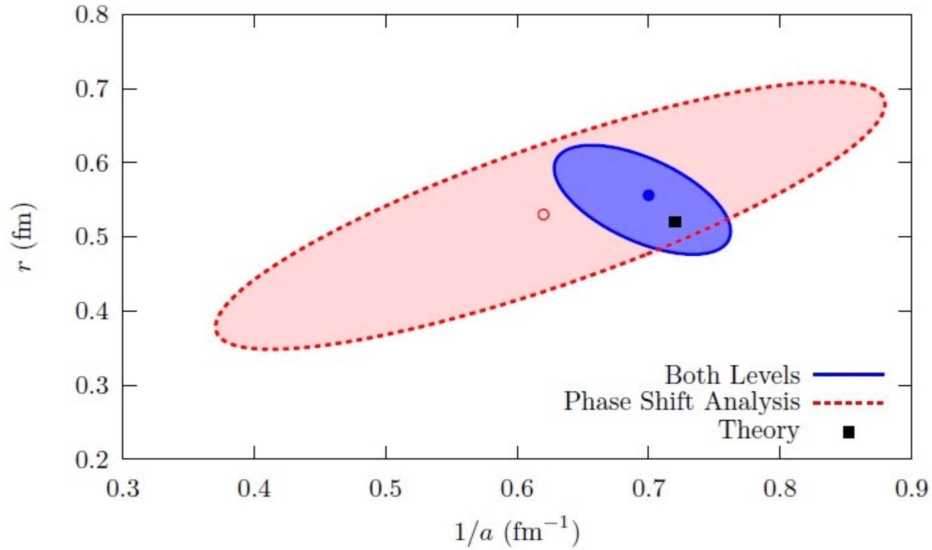
- Relativistic amplitude, once subtracted dispersion relation

$$V = a + bk^2,$$
$$16\pi^2 G = \alpha + \log\frac{m^2}{\mu^2} - \sigma(s)\log\frac{\sigma(s)-1}{\sigma(s)+1}$$

I.A.: Effective Range ($D\bar{D}$, 0^{++})

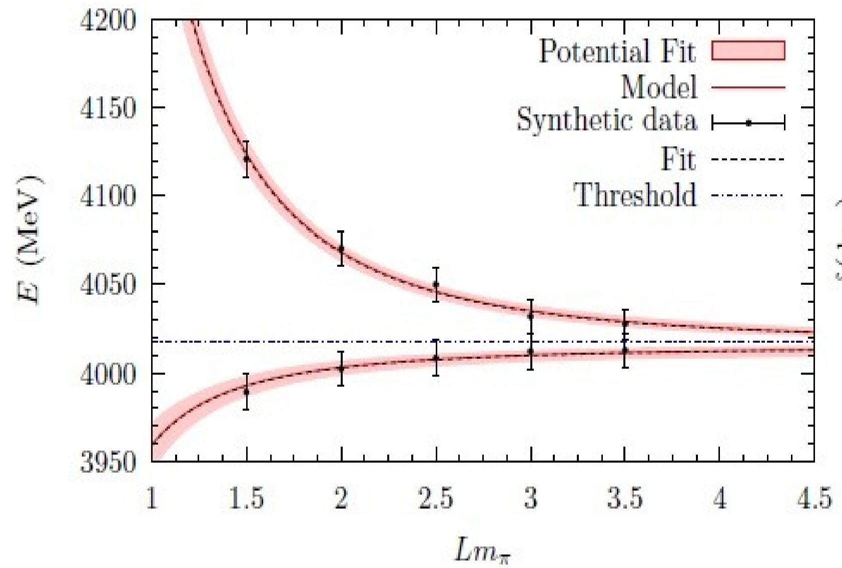
- We parameterize the amplitude as,

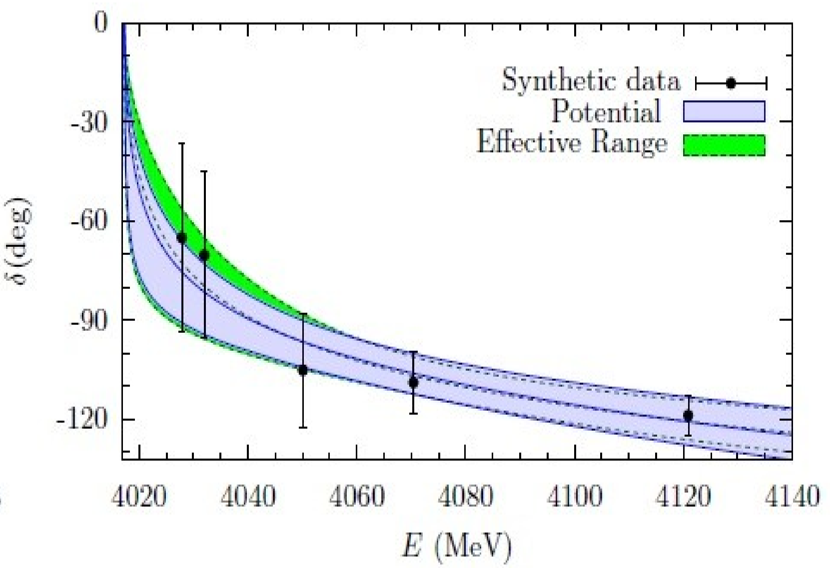
$$T^{-1} = -\frac{1}{a} + \frac{1}{2} r k^2$$



Phase shift and Eff. Range				Potential		
Par.	Phases	Eff. Range	Theory	Par.	Analysis	Theory
a (fm)	$1.6^{+1.0}_{-0.5}$	$1.43^{+0.16}_{-0.13}$	1.38	C_{0a} (fm^2)	$-1.08^{+0.19}_{-0.29}$	-1.024
r (fm)	0.53 ± 0.18	0.56 ± 0.07	0.52	Λ (GeV)	0.97 ± 0.13	1.00
M (MeV)	3721^{+10}_{-25}	3716^{+4}_{-5}	3715	M (MeV)	3715^{+3}_{-6}	3715

Inverse Analysis: ($D^*\overline{D}^*$, 2^{++})





Phase shift and Eff. Range				Potential		
Par.	Phases	Eff. Range	Theory	Par.	Analysis	Theory
a (fm)	$2.4^{+2.4}_{-1.2}$	$2.9^{+2.0}_{-0.9}$	3.0	C_0 (fm^2)	$-0.71^{+0.19}_{-0.39}$	-0.73
r (fm)	0.67 ± 0.19	0.64 ± 0.15	0.58	Λ (GeV)	1.20 ± 0.24	1.00
M (MeV)	4013^{+4}_{-18}	$4014.2^{+2.3}_{-4.8}$	4014.6	M (MeV)	$4014.3^{+2.3}_{-5.4}$	4014.6

Conclusions

- The interaction in a finite volume produces energy levels (above and below threshold). These predictions can be tested in LQCD.
- We have studied the inverse problem: analyze the generated energy levels with different methods. Standard phase-shifts analysis, potential analysis, effective range analysis. Particular emphasis is done in the error analysis.
- ER and potential analyses work best (though ER may be limited to near threshold energies).
- We focus on two $I = 0$ different channels: $D\bar{D}$ with $J^{PC} = 0^{++}$ and $D^*\bar{D}^*$ with $J^{PC} = 2^{++}$.
- An efficient method to compute the Lüscher function is also presented.