From QCD to Physical Resonances

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Overview of Project

• Extrapolation to the physical point of HadSpec $\pi\pi$ scattering in $\rho$-channel at $m_\pi = 236$ MeV
• Disclaimer: I am not part of HadSpec!

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(with Jacob Emerick, Chris Madrid, and Keith Robertson)
HadSpec: $\pi\pi$ in $\rho$ Channel

- Single ensemble of anisotropic Clover gauge field configurations
  $$\left(\frac{L}{a_s}\right)^3 \times \left(\frac{T}{a_t}\right) = 32^3 \times 256$$
  $$a_s \sim 0.12 \text{ fm} \quad \text{and} \quad \frac{a_s}{a_t} \sim 3.5$$

- 2+1 flavors with physical strange quark

- 5 boosts: (0,0,0), (0,0,1), (0,0,2), (1,1,0), (1,1,1)

- Multiple irreps depending on boost: $T_1^-$, $A_1$, $E_2$, $B_1$, $B_2$

- At $m_\pi = 236$ MeV, the $\rho$ resonance lies below the $K\bar{K}$ and $4\pi$ kinematic thresholds ($K\bar{K}$ mixing was also studied)

Lüscher’s Method

- Finite-sized box: boundary conditions result in discrete spectrum
- Quantization condition relates finite-volume spectrum to infinite-volume phase shift.

\[ p \cot \delta(p) = \frac{1}{\pi L} S \left( \left( \frac{pL}{2\pi} \right)^2 \right) \] where \( S(\eta) = \sum_{j}^{\Lambda_j} \frac{1}{|j|^2 - \eta} - 4\pi \Lambda_j \)

- \( p \) = relative momentum of pions
- \( E = 2\sqrt{p^2 + m^2} \)
- Sum is over integer triplets \( j \) such that \( |j| < \Lambda_j \)

- HadSpec: variety of boosts, irreps map out \( \delta(E) \) with 22 points
- Our work: \( p \cot \delta(p) \) expressed as ChPT amplitude with parameters that are \textit{fit} to LQCD spectrum
References for Lüscher’s Method

• Lüscher (1986)
• Lüscher (1991)
• Relativistic and Boosted
  • Rummukainen and Gottlieb (1995)
  • Kim, Sachrajda, and Sharpe (2005)
  • Christ, Kim, and Yamazaki (2005)
HadSpec Results (tomorrow’s plenary)

Wilson, Briceño, Dudek, Edwards, and Thomas (2015)
Extrapolation to Physical Point

• Chiral Perturbation Theory (ChPT), Gasser and Leutweyler (1983)

\[ A_{LO} = \frac{s-m_0^2}{f_0^2} \quad A_{NLO} = \text{function of } s, t, u \text{ and } m_0, f_0, \alpha_1, \alpha_2 \]

• Low Energy Constants (LECs)

\[ \alpha_1 = -2l_1 r + l_2 r \]
\[ \alpha_2 = l_4 r \]

• \( m_0, f_0 \) related to the physical \( m_\pi, f_\pi \)

\[ m_\pi^2 = m_0^2 \left( 1 + \frac{m_0^2}{f_0^2} \left[ \frac{1}{32\pi^2} \ln \frac{m_0^2}{\mu^2} + 2l_3 r(\mu) \right] \right) \]
\[ f_\pi = f_0 \left( 1 - \frac{m_0^2}{f_0^2} \left[ \frac{1}{16\pi^2} \ln \frac{m_0^2}{\mu^2} - l_4 r(\mu) \right] \right) \]
Unitarized Chiral Perturbation Theory

• Inverse Amplitude Method: \( A_{UXPT} = A_{LO} \frac{1}{A_{LO} - A_{NLO}} A_{LO} \)

• Extended range: \( \sim 1.2 \) GeV

• Resonances: poles in scattering amplitude

**Graph:**

![Graph showing \( \delta_{1\pi\pi} \) and \( \rho \) vs. \( E_{cm} \) in MeV with a fit line and data points.]

Fit Results

• Scale setting: $a_t m_\Omega^{\text{latt}} = 0.2789(16)$

• UChPT parameters ($\alpha_1, \alpha_2$) fit to LQCD spectrum:

\[
\begin{align*}
\alpha_1(770 \text{ MeV}) &= 14.7(4)(2)(1) \times 10^{-3} \\
\alpha_2(770 \text{ MeV}) &= -28(6)(3)(1111) \times 10^{-3}
\end{align*}
\]

\[
\begin{bmatrix}
1 & -0.98 \\
1 - \chi^2 \\
d.o.f
\end{bmatrix} = 1.26
\]

• HadSpec (Breit-Wigner): $m_\rho = 783(2) \text{ MeV}$ and $\Gamma_\rho = 85(2) \text{ MeV}$
Chiral Extrapolations: SU(2) vs SU(3)

- This plot: stat bands only
- SU(3) expected to have much larger systematics!
Chiral Extrapolations: SU(2) stat+sys

Wilson, Briceño, Dudek, Edwards, and Thomas (2015)

Dudek, Edwards, Thomas (2013)
Comparison with Experiment


PDG: \( m_\rho = 775.26 \pm 0.25 \) MeV; \( \Gamma_\rho = 147.8 \pm 0.9 \) MeV

- No estimate of systematic due \( m_q \)-dependence of LECs (from NLO truncation)
Summary

- HadSpec spectrum used to constrain ChPT
- Post-diction shows agreement with experimental $\delta_{\pi\pi}(E)$
- Analytic continuation of $\delta_{\pi\pi}(E)$ yields $\rho$ pole
- More complicated systems?
Fit Parameters
Scale Setting

\[ a_t = 0.1630(14) \text{GeV}^{-1} \]