# From QCD to Physical Resonances

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# Overview of Project

- Extrapolation to the physical point of HadSpec $\pi\pi$ scattering in ρ-channel at $m_\pi = 236$ MeV
- Bolton, Briceno, and Wilson (2015) arXiv:1507.07928 [hep-ph]
- Disclaimer: I am not part of HadSpec!

Raúl Briceño
David Wilson

Daniel Bolton
(with Jacob Emerick, Chris Madrid, and Keith Robertson)

# HadSpec: $\pi\pi$ in ρ Channel

- Single ensemble of anisotropic Clover gauge field configurations

$$\left(\frac{L}{a_s}\right)^3 \times \left(\frac{T}{a_t}\right) = 32^3 \times 256$$

$a_s \sim 0.12$ fm and $\frac{a_s}{a_t} \sim 3.5$

- 2+1 flavors with physical strange quark
- 5 boosts: $(0,0,0), (0,0,1), (0,0,2), (1,1,0), (1,1,1)$
- Multiple irreps depending on boost: $T_1^-$, $A_1$, $E_2$, $B_1$, $B_2$
- At $m_\pi = 236$ MeV, the ρ resonance lies **below** the $K\bar{K}$ and $4\pi$ kinematic thresholds ($K\bar{K}$ mixing was also studied)

Wilson, Briceño, Dudek, Edwards, and Thomas (2015) arxiv:1507.02599 [hep-ph]

# Lüscher's Method

- Finite-sized box: boundary conditions result in discrete spectrum
- Quantization condition relates finite-volume spectrum to infinite-volume phase shift.

$$p \cot \delta(p) = \frac{1}{\pi L} \mathrm{S}\left(\left(\frac{pL}{2\pi}\right)^2\right) \quad \text{where} \quad \mathrm{S}(\eta) = \sum_{\boldsymbol{j}}^{\Lambda_{\boldsymbol{j}}} \frac{1}{|\boldsymbol{j}|^2 - \eta} - 4\pi\Lambda_{\boldsymbol{j}}$$

$p$ = relative momentum of pions

$E = 2\sqrt{p^2 + m^2}$

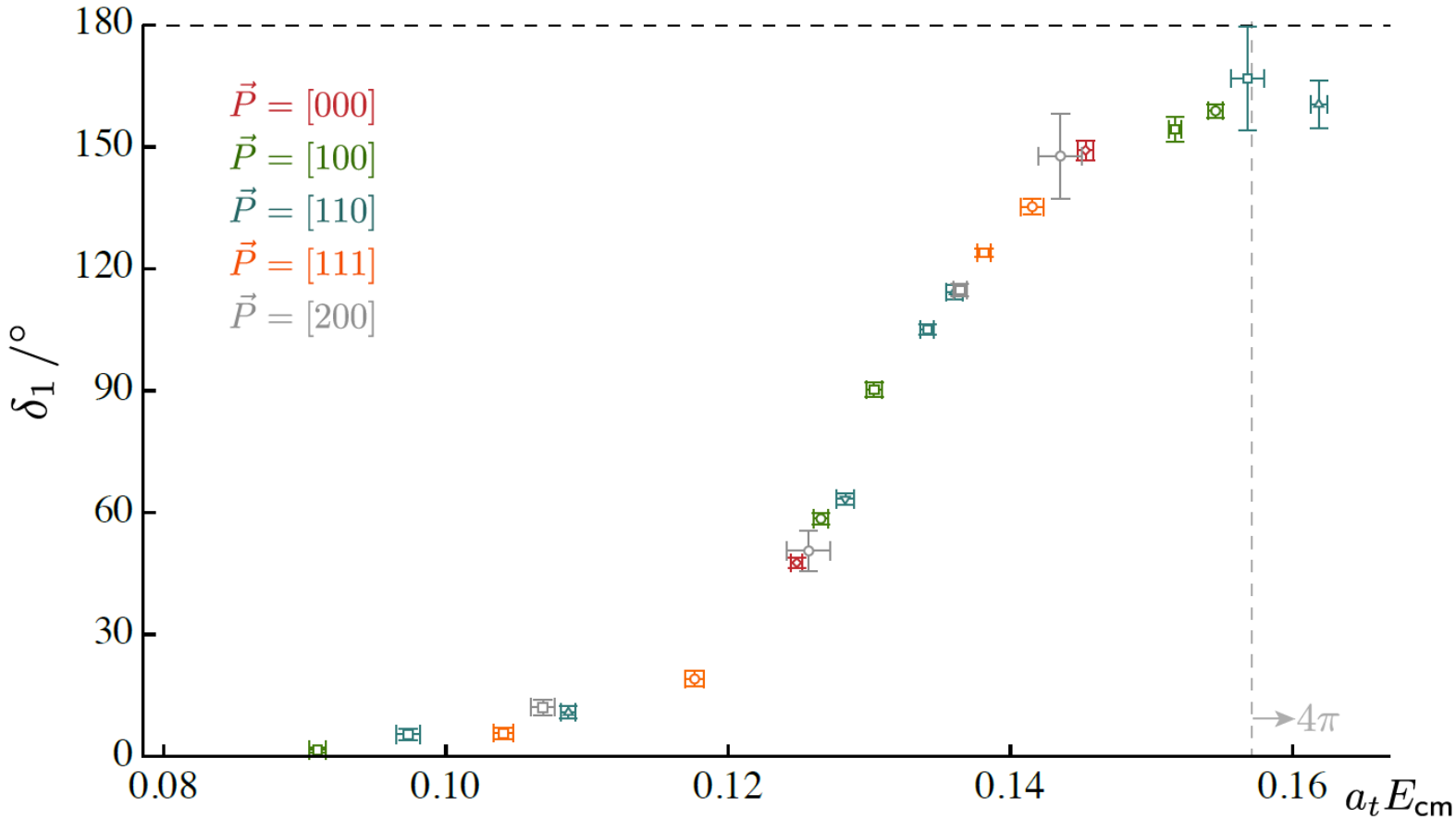
Sum is over integer triplets $\boldsymbol{j}$ such that $|\boldsymbol{j}| < \Lambda_{\boldsymbol{j}}$

- HadSpec: variety of boosts, irreps map out $\delta(E)$ with 22 points
- Our work: $p \cot \delta(p)$ expressed as ChPT amplitude with parameters that are *fit* to LQCD spectrum

# References for Lüscher's Method

- Lüscher (1986)
- Lüscher (1991)
- Relativistic and Boosted
  - Rummukainen and Gottlieb (1995)
  - Kim, Sachrajda, and Sharpe (2005)
  - Christ, Kim, and Yamazaki (2005)

# HadSpec Results (tomorrow's plenary)




Wilson, Briceño, Dudek, Edwards, and Thomas (2015)

# Extraplolation to Physical Point

- Chiral Perturbation Theory (ChPT), Gasser and Leutweyler (1983)

$$A_{LO} = \frac{s - {m_0}^2}{{f_0}^2} \quad A_{NLO} = \text{function of } s, t, u \text{ and } m_0, f_0, \alpha_1, \alpha_2$$

- Low Energy Constants (LECs)

$$\alpha_1 = -2{l_1}^r + {l_2}^r$$

$$\alpha_2 = {l_4}^r$$

- $m_0, f_0$ related to the physical $m_\pi, f_\pi$

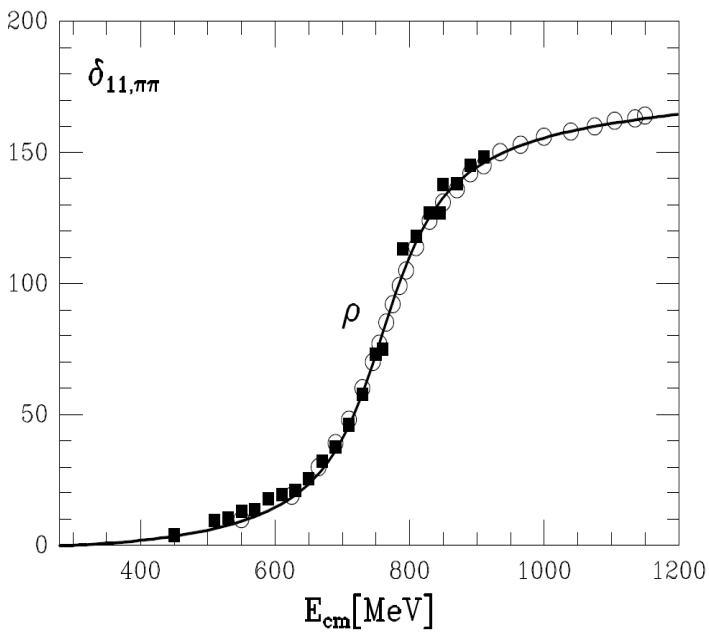
$${m_\pi}^2 = {m_0}^2 \left(1 + \frac{{m_0}^2}{{f_0}^2}\left[\frac{1}{32\pi^2}\ln\frac{{m_0}^2}{\mu^2} + 2{l_3}^r(\mu)\right]\right)$$

$$f_\pi = f_0\left(1 - \frac{{m_0}^2}{{f_0}^2}\left[\frac{1}{16\pi^2}\ln\frac{{m_0}^2}{\mu^2} - {l_4}^r(\mu)\right]\right)$$

renormalization scale

# Unitarized Chiral Perturbation Theory

- Inverse Amplitude Method: $A_{U\chi PT} = A_{LO} \frac{1}{A_{LO} - A_{NLO}} A_{LO}$
- Extended range: $\sim$1.2 GeV
- Resonances: poles in scattering amplitude



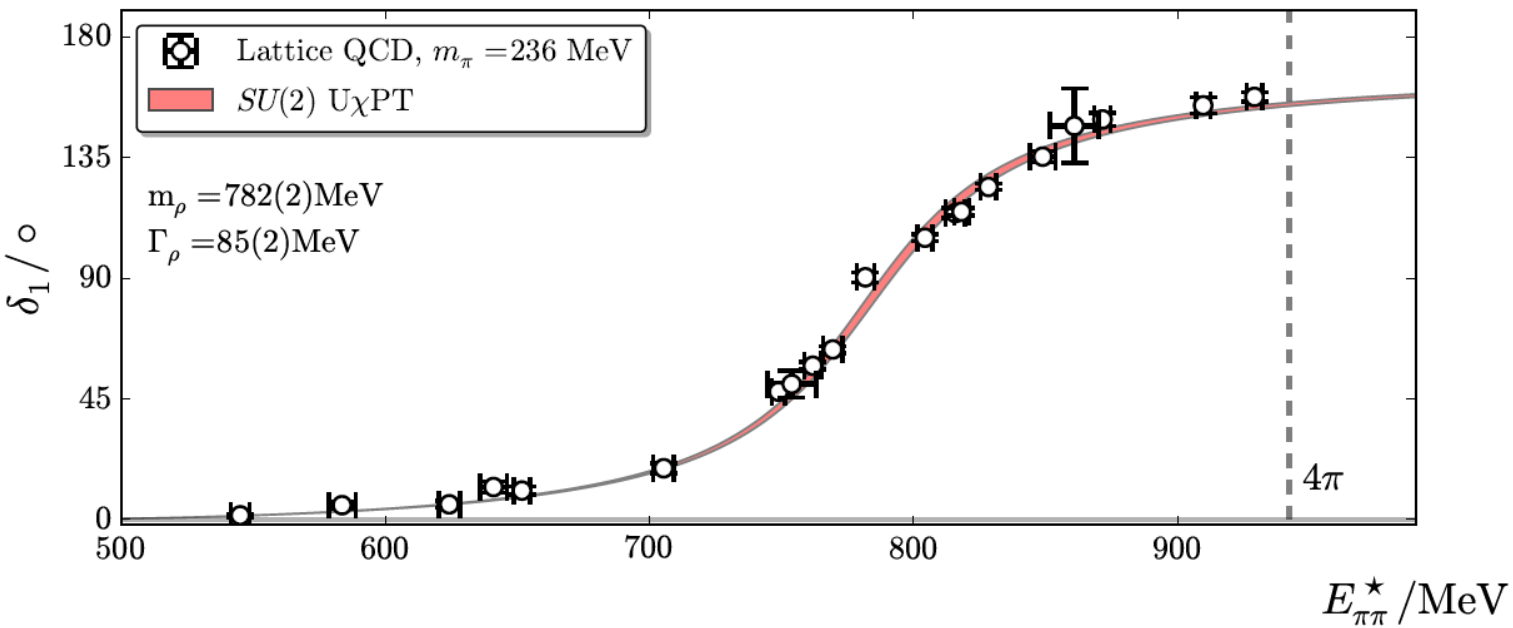

J. A. Oller, E. Oset, and J. R. Pelaez (1998)

# Fit Results

- Scale setting: $a_t m_\Omega{}^{latt} = 0.2789(16)$

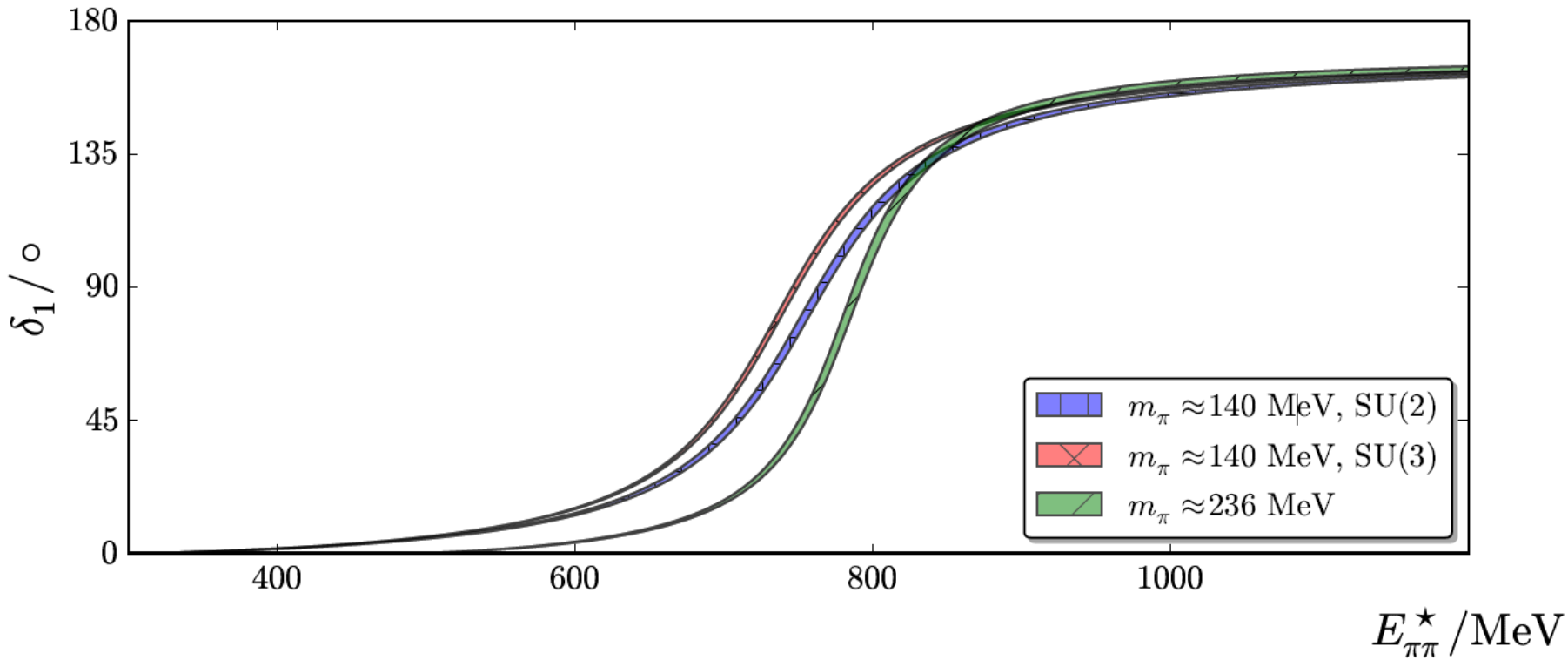
- UChPT parameters $(\alpha_1, \alpha_2)$ fit to LQCD spectrum:

$$\begin{aligned} \alpha_1(770\ \text{MeV}) &= 14.7(4)(2)(1) \times\ 10^{-3} \\ \alpha_2(770\ \text{MeV}) &= -28(6)(3)\binom{01}{11} \times\ 10^{-3} \end{aligned} \quad \begin{bmatrix} 1 & -0.98 \\ & 1 \end{bmatrix}, \quad \frac{\chi^2}{d.o.f} = 1.26$$



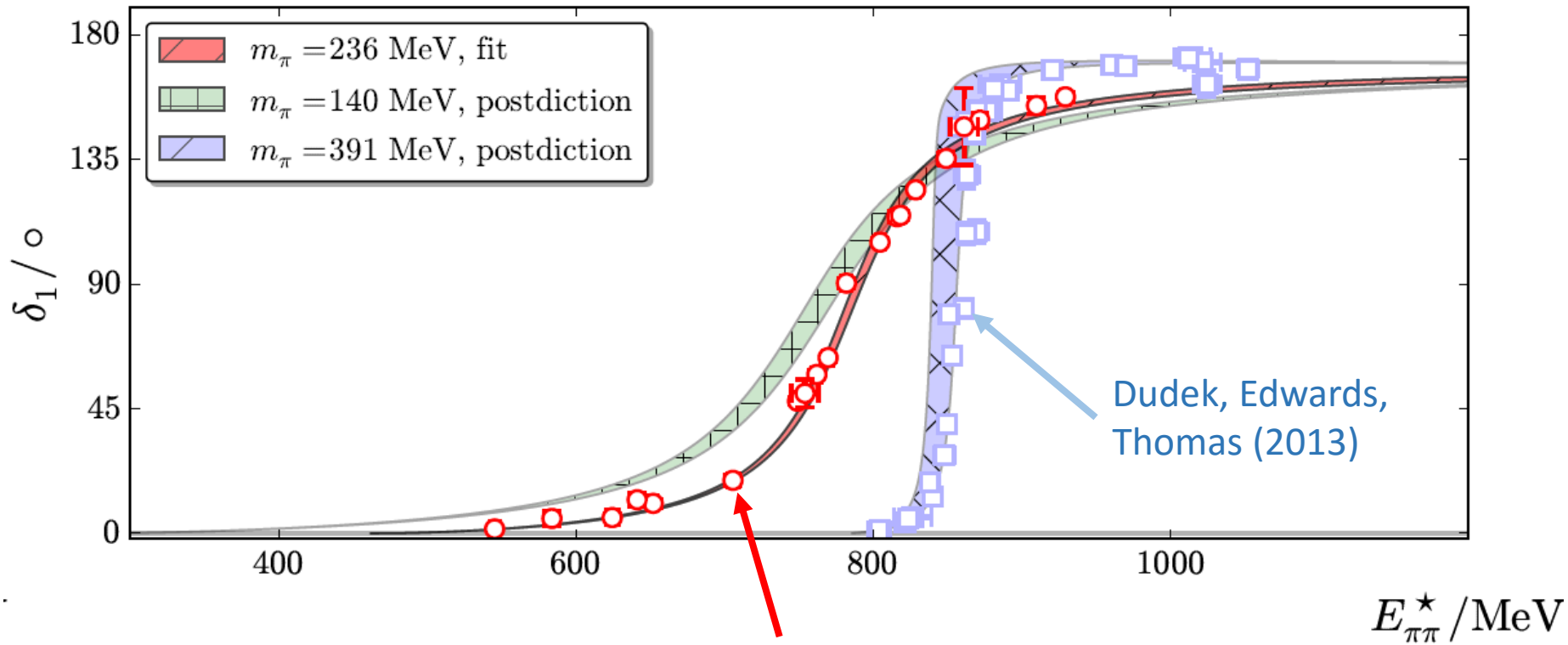

- HadSpec (Breit-Wigner): $m_\rho = 783(2)$ MeV and $\Gamma_\rho = 85(2)$ MeV

# Chiral Extrapolations: SU(2) vs SU(3)



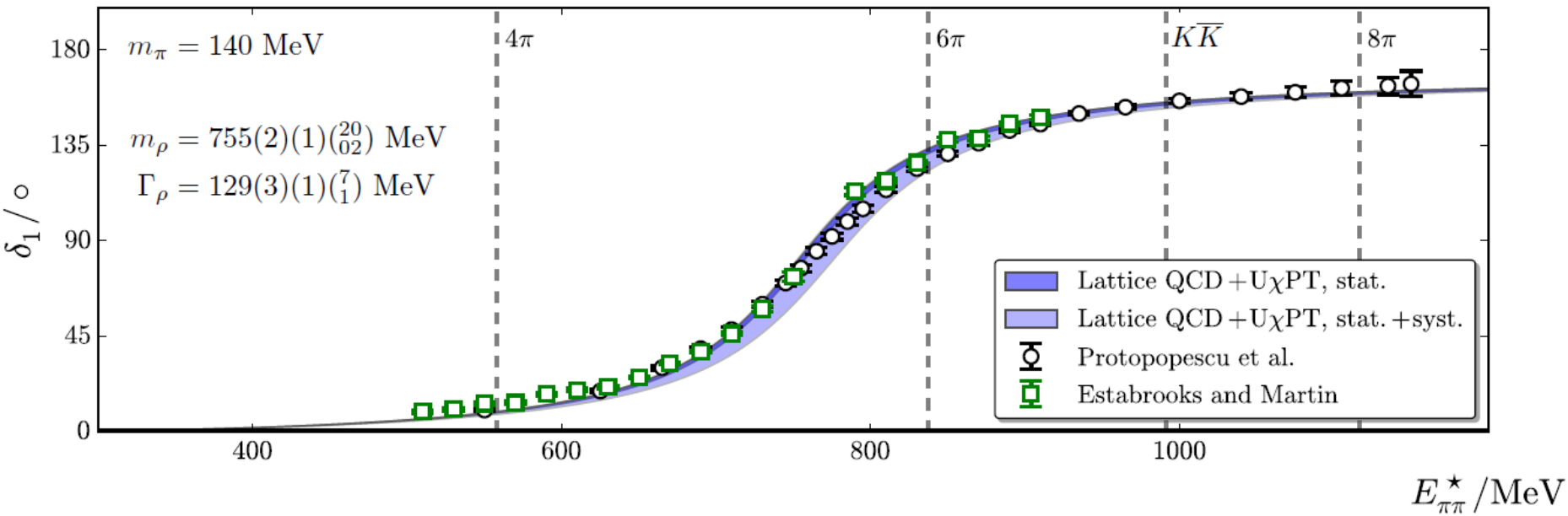

- This plot: stat bands only
- SU(3) expected to have much larger systematics!

# Chiral Extrapolations: SU(2) stat+sys




Wilson, Briceño, Dudek, Edwards, and Thomas (2015)

# Comparison with Experiment




Experiments: Protopopescu *et al* (1973) and Estabrooks and Martin (1974)

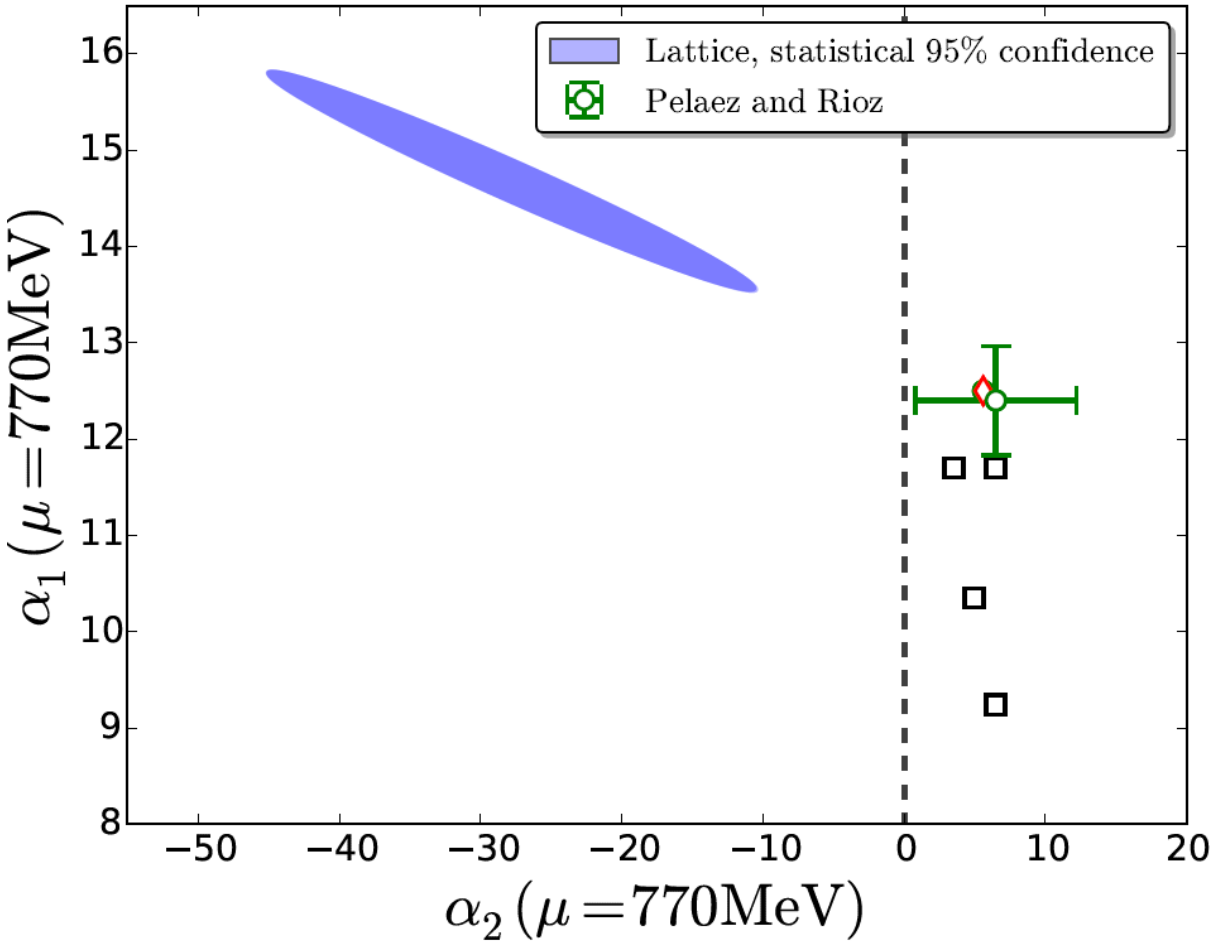
PDG: $m_\rho = 775.26 \pm 0.25$ MeV; $\Gamma_\rho = 147.8 \pm 0.9$ MeV

- No estimate of systematic due $m_q$-dependence of LECs (from NLO truncation)

# Summary

- HadSpec spectrum used to constrain ChPT
- Post-diction shows agreement with experimental $\delta_{\pi\pi}(E)$
- Analytic continuation of $\delta_{\pi\pi}(E)$ yields $\rho$ pole
- More complicated systems?

# Fit Parameters



# Scale Setting