Partners of the $X(3872)$ and HQSS breaking

*Newport News, September 2015*

**D.R. Entem, P.G. Ortega, F. Fernández**

University of Salamanca
Outline

- The $X(3872)$ and its partners
- The Chiral Quark Model
- Coupling two meson and one meson states: the $^3P_0$ model
- Heavy Quark Spin Symmetry relations
- Predictions of the Chiral Quark model in the charmonium and bottomonium sectors
- Summary
The $X(3872)$ was discovered by Belle in 2003 and very soon confirmed by CDFII, D0 and BaBar

The Quantum numbers have been measured by LHCb in 2014 to be $J^{PC} = 1^{++}$

The mass is very close to the $D\bar{D}^*$ threshold so it is a well accepted candidate for a $D\bar{D}^*$ molecule

One would expect partners with other quantum numbers in the charmonium and bottomonium sectors.

Experimental search of the $X_b$: bottom partner in the $1^{++}$ sector

- Phys. Lett. B 727, 57 (2013). CMS didn’t find it in the $\Upsilon(1S)\pi^+\pi^-$ decay channel in $pp$ collisions
- Phys. Lett. B 740, 199 (2015). ATLAS also didn’t find it in the same decay channel
- Phys. Rev. Lett. 113, 142001 (2014). Belle didn’t find it in $\omega \Upsilon(1S)$ in $e^+e^-$ collisions between 10.55 and 10.65 GeV
**X(3872) partners**

- Heavy Quark Spin Symmetry (HQSS), independence of the heavy quark spin
- Heavy Flavour Symmetry (HFS), independence on the \( c \) or \( b \) flavours

**HQSS expectation: Charm sector**

J. Nieves and M. Pavón-Valderrama, Phys. Rev. D 86, 056004

- HQSS implies the existence of a \( 2^{++} \) \( D^*D^* \) partner \( X(4012) \)
- Assuming the \( X(3915) \) to be the \( 0^{++} \) partner a total of six-molecular states.
  - Spin independent \((C_{0a})\) and spin dependent \((C_{0b})\) terms.
  - Small coupled channel effects.
  - No coupling with \( c\bar{c} \) states.

**HQSS and HFS expectations: Charm and Bottom sector relations**

Feng-Kun Guo et al., Phys. Rev. D 88, 054007

- A \( 1^{++} \) isoscalar \( BB^* \) state at 10.58 GeV \( V_{DD^*}^{LO}(1^{++}) = V_{BB^*}^{LO}(1^{++}) \)
- A \( 2^{++} \) isoscalar \( B^*B^* \) state at 10.6 GeV \( V_{DD^*}^{LO}(2^{++}) = V_{BB^*}^{LO}(2^{++}) \)
- Isovector partners of \( Z_b \) in the chamonium sectors are predicted
The Chiral Quark Model


- Spontaneous Chiral Symmetry Breaking →
  - Golstone bosons

\[ \mathcal{M} = \bar{\Psi} (i \gamma^\mu \partial_\mu - M U^\gamma_5) \Psi \]

\[ U^\gamma_5 = e^{i \pi a \lambda^a \gamma_5 / f_\pi} \sim 1 + \frac{1}{f_\pi} \gamma_5 \lambda^a \pi^a - \frac{1}{2 f_\pi^2} \pi^a \pi^a \]

- Goldstone bosons exchange
- Scalar boson exchanges

- Gluon coupling

\[ \mathcal{L}_{gqq} = i \sqrt{4 \pi \alpha_s} \bar{\Psi} \gamma_\mu G^{\mu}_c \lambda^c \Psi \]

- One gluon exchange

- Confinement

- Interactions:

\[ V_{q_i q_j} = \begin{cases} 
  q_i q_j = nn & \Rightarrow V_{CON} + V_{OGE} + V_{GBE} + V_{SBE} \\
  q_i q_j = nQ & \Rightarrow V_{CON} + V_{OGE} \\
  q_i q_j = QQ & \Rightarrow V_{CON} + V_{OGE}
\end{cases} \]
The $M_1M_2$ system

- **Quark interactions** → **Cluster interaction** (Resonanting Group Method)

- For the $DD^*$ system only **direct RGM Potential**:

\[
 RGM V_D(\vec{P'}, \vec{P}_i) = \sum_{i \in A, j \in B} \int d\vec{\xi}_A d\vec{\xi}'_A d\vec{\xi}_B d\vec{\xi}'_B \\
 \phi^*_A(\vec{p}_{\xi'_A}) \phi^*_B(\vec{p}_{\xi'_B}) V_{ij}(\vec{P'}, \vec{P}_i) \phi_A(\vec{p}_{\xi_A}) \phi_B(\vec{p}_{\xi_B})
\]

- $\phi_C(\vec{p}_C)$ is the wave function for cluster $C$ solution of Schrödinger’s equation using Gaussian Expansion Method.
The $M_1 M_2$ system

- **Quark interactions** → **Cluster interaction** (Resonanting Group Method)

- **For the $DD^*$ system only direct RGM Potential:**

$$ RGM_{VD}(\vec{P}', \vec{P}_i) = \sum_{i \in A, j \in B} \int d\vec{p}_{\xi_A} d\vec{p}_{\xi'_B} d\vec{p}_{\xi_A} d\vec{p}_{\xi_B} \phi^*_A(\vec{p}_{\xi'_A}) \phi^*_B(\vec{p}_{\xi'_B}) V_{ij}(\vec{P}', \vec{P}_i) \phi_A(\vec{p}_{\xi_A}) \phi_B(\vec{p}_{\xi_B}) $$

- **$\phi_C(\vec{p}_C)$** is the wave function for cluster $C$ solution of Schrödinger’s equation using Gaussian Expansion Method.

**Rearrangement processes (like $DD^* \rightarrow J/\psi\omega$)**
\(3P_0 \) model

Running coupling

\[
\gamma(\mu) = \frac{\gamma_0}{\log\left(\frac{\mu}{\mu_0}\right)}
\]

\(\gamma_0 = 0.81 \pm 0.02\)

\(\mu_0 = (49.84 \pm 2.58) \text{ MeV}\)

- Fitted to 6 states in the \(c\bar{n}, c\bar{s}, c\bar{c}\) and \(b\bar{b}\) sectors.
- Overall good description of the widths of other states in these sectors.
- Good description of the \(b\bar{n}\) sector not included in the fit.
Coupling $q\bar{q}$ and $q\bar{q}q\bar{q}$ sectors

- **Hadronic state:**
  \[ |\Psi\rangle = \sum_{\alpha} c_\alpha |\psi\rangle + \sum_{\beta} \chi_\beta(P) |\phi_{M1}\phi_{M2}\beta\rangle \]

- **Solving the coupling with $c\bar{c}$ states → Schrödinger type equation:**
  \[
  \sum_\beta \int \left( H_{\beta'\beta}^{M1M2}(P', P) + V_{\beta'\beta}^{eff}(P', P) \right) \chi_\beta(P) P^2 dP = E \chi_{\beta'}(P')
  \]

  with

  \[
  V_{\beta'\beta}^{eff}(P', P) = \sum_\alpha \frac{h_{\beta'\alpha}(P') h_{\alpha\beta}(P)}{E - M_\alpha}
  \]

- **The $c\bar{c}$ amplitudes are given by,**
  \[
  c_\alpha = \frac{1}{E - M_\alpha} \sum_\beta \int h_{\alpha\beta}(P) \chi_\beta(P) P^2 dP
  \]
Resonance states

Lippman-Schwinger equation

\[ T^{\beta' \beta}(E; P', P) = V_T^{\beta' \beta}(P', P) + \sum_{\beta''} \int dP'' P''^2 V_T^{\beta' \beta''}(P', P'') \frac{1}{E - E_{\beta''}(P'')} T^{\beta'' \beta}(E; P'', P) \]

with \( V_T^{\beta' \beta}(P', P) = V^{\beta' \beta}(P', P) + V_{\text{eff}}^{\beta' \beta}(P', P) \), \( V_{\text{eff}}^{\beta' \beta}(P', P) = \sum_{\alpha} \frac{h^{\beta' \alpha}(P') h_{\alpha \beta}(P)}{E - M_{\alpha}} \)
Resonance states

Lippman-Schwinger equation

\[ T_{\beta'}^{\beta}(E; P', P) = V_{T}^{\beta'}(P', P) + \sum_{\beta''} \int dP'' P''^2 V_{T}^{\beta' \beta''}(P', P'') \frac{1}{E - E_{\beta''}(P'')} T_{\beta''}^{\beta'}(E; P'', P) \]

with \( V_{T}^{\beta'}(P', P) = V^{\beta'}(P', P) + V_{eff}^{\beta'}(P', P), V_{eff}^{\beta'}(P', P) = \sum_{\alpha} \frac{h^{\beta'}_{\alpha}(P') h_{\alpha}(P)}{E - M_{\alpha}} \)


\[ T_{\beta'}^{\beta}(E; P', P) = T_{V}^{\beta'}(E; P', P) + \sum_{\alpha, \alpha'} \phi^{\beta'}_{\alpha}(E; P') \Delta_{\alpha'}^{\alpha}(E) \phi^{\alpha}_{\beta}(E; P) \]

- Non resonant contribution
- Resonant contribution

with

\[ T_{V}^{\beta'}(E; P', P) = V^{\beta'}(P', P) + \sum_{\beta''} \int dP'' P''^2 V_{V}^{\beta' \beta''}(P', P'') \frac{1}{z - E_{\beta''}(P'')} T_{V}^{\beta''}(E; P'', P) \]
Resonance states


\[ T^{\beta' \beta}(E; P', P) = T^V_{\beta' \beta}(E; P', P) + \sum_{\alpha, \alpha'} \phi^{\beta' \alpha'}(E; P') \Delta_{\alpha' \alpha}^{-1}(E) \tilde{\phi}^{\alpha \beta}(E; P) \]

- Non resonant contribution
- Resonant contribution

with

\[ \phi^{\alpha \beta'}(E; P) = h_{\alpha \beta'}(P) - \sum_{\beta} \int T^V_{\beta' \beta}(E; P, q) h_{\alpha \beta}(q) \frac{q^2 / 2 \mu - E}{2} dq, \]

\[ \tilde{\phi}^{\alpha \beta}(E; P) = h_{\alpha \beta}(P) - \sum_{\beta'} \int h_{\alpha \beta'}(q) T^V_{\beta' \beta}(E; q, P) \frac{q^2 / 2 \mu - E}{2} dq \]
Resonance states


\[ T^{\beta' \beta}(E; P', P) = T_V^{\beta' \beta}(E; P', P) + \sum_{\alpha, \alpha'} \phi^{\beta' \alpha'}(E; P') \Delta_{\alpha' \alpha}^{-1}(E) \bar{\phi}^{\alpha \beta}(E; P) \]

- Non resonant contribution
- Resonant contribution

with

\[ \Delta_{\alpha' \alpha}(E) = \left\{ (E - M_\alpha) \delta_{\alpha' \alpha} + \mathcal{G}_{\alpha' \alpha}(E) \right\} \]

\[ \mathcal{G}_{\alpha' \alpha}(E) = \sum_\beta \int dq q^2 \frac{\phi^{\alpha \beta}(q, E) h_{\beta \alpha'}(q)}{q^2/2\mu - E} \]
Resonance states

■ Resonance mass (pole position)

\[
\left| \Delta^\alpha\alpha'(\bar{E}) \right| = \left| (\bar{E} - M_\alpha)\delta^{\alpha\alpha'} + g^{\alpha\alpha'}(\bar{E}) \right| = 0
\]

■ Bare $c\bar{c}$ probabilities

\[
\left\{ M_\alpha \delta^{\alpha\alpha'} - g^{\alpha\alpha'}(\bar{E}) \right\} c_{\alpha'}(\bar{E}) = \bar{E} c_\alpha(\bar{E})
\]

■ Molecular wave function

\[
\chi_\beta'(P') = -2\mu_{\beta'} \sum_\alpha \frac{\phi_{\beta'\alpha}(E; P')c_\alpha}{P'r^2 - k_{\beta'}^2}
\]

■ Normalization

\[
\sum_\alpha |c_\alpha|^2 + \sum_\beta <\chi_\beta|\chi_\beta> = 1
\]
HQSS implies the relations (S waves)

\[
\frac{2}{\sqrt{3}} \langle D^* D^* (0^{++}) | H | DD(0^{++}) \rangle = \langle DD(0^{++}) | H | DD(0^{++}) \rangle \\
- \langle D^* D^* (0^{++}) | H | D^* D^* (0^{++}) \rangle \\
2 \langle DD^* (1^{+-}) | H | DD^* (1^{+-}) \rangle = \langle DD(0^{++}) | H | DD(0^{++}) \rangle \\
+ \langle D^* D^* (0^{++}) | H | D^* D^* (0^{++}) \rangle \\
\langle DD^* (1^{++}) | H | DD^* (1^{++}) \rangle = \langle D^* D^* (2^{++}) | H | D^* D^* (2^{++}) \rangle \\
= \frac{3}{2} \left[ \langle DD(0^{++}) | H | DD(0^{++}) \rangle - \frac{1}{3} \langle D^* D^* (0^{++}) | H | D^* D^* (0^{++}) \rangle \right]
\]
We find the relations

\[ \frac{2}{\sqrt{3}} \langle D^* D^* (0^{++}) | H | D D (0^{++}) \rangle = \langle D D (0^{++}) | H | D D (0^{++}) \rangle - \langle D^* D^* (0^{++}) | H | D^* D^* (0^{++}) \rangle \]
We find the relations

\[
\langle DD^*(1^{++})|H|DD^*(1^{++})\rangle = \langle D^* D^*(2^{++})|H|D^* D^*(2^{++})\rangle \\
= \frac{3}{2} \left[ \langle DD(0^{++})|H|DD(0^{++})\rangle - \frac{1}{3} \langle D^* D^*(0^{++})|H|D^* D^*(0^{++})\rangle \right]
\]
HQSS breaking

HQSS slightly broken

Mass

<table>
<thead>
<tr>
<th></th>
<th>Theo.(MeV)</th>
<th>Exp.(MeV)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$D$</td>
<td>1896</td>
<td>1867</td>
</tr>
<tr>
<td>$D^*$</td>
<td>2017</td>
<td>2009</td>
</tr>
<tr>
<td>$B$</td>
<td>5275</td>
<td>5279</td>
</tr>
<tr>
<td>$B^*$</td>
<td>5315</td>
<td>5325</td>
</tr>
</tbody>
</table>

Wave function

$D^{(*)}$

$B^{(*)}$

\[ \Psi(p) \text{ (GeV}^{-3/2}) \]
The $X(3872)$

- $^3 S_1$ and $^3 D_1$ $DD^*$ partial waves included.

- Coupling to $1^{++}$ ground and first excited $c\bar{c}$ states with bare masses within the model:
  
  \[
  c\bar{c}(1^3 P_1) \rightarrow M = 3503.9 \text{ MeV} \quad c\bar{c}(2^3 P_1) \rightarrow M = 3947.4 \text{ MeV} \quad \text{and}
  \]

- Isospin breaking $M_{D\pm} + M_{D^*\mp} \neq M_{D^0} + M_{D^{*0}}$

  Parameter free calculation.

<table>
<thead>
<tr>
<th>$M$ (MeV)</th>
<th>$c\bar{c}(1^3 P_1)$</th>
<th>$c\bar{c}(2^3 P_1)$</th>
<th>$D^0 D^{*0}$</th>
<th>$D^\pm D^{*\mp}$</th>
<th>Assignment</th>
</tr>
</thead>
<tbody>
<tr>
<td>3937</td>
<td>0 %</td>
<td>79 %</td>
<td>7 %</td>
<td>14 %</td>
<td></td>
</tr>
<tr>
<td>3863</td>
<td>1 %</td>
<td>30 %</td>
<td>46 %</td>
<td>23 %</td>
<td>$\rightarrow X(3872)$</td>
</tr>
<tr>
<td>3467</td>
<td>95 %</td>
<td>0 %</td>
<td>2.5 %</td>
<td>2.5 %</td>
<td></td>
</tr>
</tbody>
</table>

- Isospin probabilities: $P_{I=0} = 66 \%$, $P_{I=1} = 3 \%$, $P_{c\bar{c}} = 30 \%$.

- Fine tune $^3 P_0 \gamma$ strength parameter to $E_{bind}$. $P_{I=0} \sim 70 \%$, $P_{I=1} \sim 23 \%$, $P_{c\bar{c}} \sim 7 \%$


- M. Takizawa, S. Takeuchi, PTEP 9 (2013) at hadron level
Dependence on $\gamma$

No $X(3872)$ without coupling to $c\bar{c}$ states

No $DD^*$ interaction included.
$DD^*$ interaction included.

$D^0\bar{D}^{*0}$ component
$D^+\bar{D}^{*-}$ component
$c\bar{c}(2P)$ component
$c\bar{c}(1P)$ component
HQSS and HFS breaking

- We investigate the deviation on HQSS and HFS expectations induced by the coupling to $Q\bar{Q}$ states
- Our prescription is to include the states above and below the relevant threshold.
- We only include thresholds where an $S$ wave is present
- In the $0^{++}$ sector we include $DD (BB)$ and $D^* D^* (B^* B^*)$ $^1S_0$ waves
- In the $1^{++}$ sector we include $DD^* (BB^*)$ $^3S_1$ and $^3D_1$ waves
- In the $1^{+-}$ sector we include $DD^* (BB^*)$ and $D^* D^* (B^* B^*)$ $^3S_1$ and $^3D_1$ waves
- In the $2^{++}$ sector we include $D^* D^* (B^* B^*)$ $^5S_2$, $^1D_2$ and $^5D_2$ waves

Additional states to naive quark model results

<table>
<thead>
<tr>
<th></th>
<th>$1^{+-}$</th>
<th>$0^{++}$</th>
<th>$1^{++}$</th>
<th>$2^{++}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Charmonium</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>HQSS and HFS</td>
<td>3815/3955</td>
<td>3710/Input</td>
<td>Input</td>
<td>4012</td>
</tr>
<tr>
<td>coupling $Q\bar{Q}$</td>
<td>No</td>
<td>No</td>
<td>X(3872)</td>
<td>No</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th></th>
<th>$1^{+-}$</th>
<th>$0^{++}$</th>
<th>$1^{++}$</th>
<th>$2^{++}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Bottomonium</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>HQSS and HFS</td>
<td></td>
<td>10580</td>
<td>10600</td>
<td></td>
</tr>
<tr>
<td>coupling $Q\bar{Q}$</td>
<td>No</td>
<td>10621</td>
<td>?</td>
<td>10648</td>
</tr>
</tbody>
</table>
Spectrum

Charmonium

<table>
<thead>
<tr>
<th>1+−</th>
<th>0++</th>
<th>1++</th>
<th>2++</th>
</tr>
</thead>
<tbody>
<tr>
<td>3515</td>
<td>3452</td>
<td>3504</td>
<td>3531</td>
</tr>
<tr>
<td>3956</td>
<td>3909</td>
<td>3947</td>
<td>3969</td>
</tr>
<tr>
<td>4278</td>
<td>4241</td>
<td>4271</td>
<td>4289</td>
</tr>
</tbody>
</table>

Small breaking in $c \bar{c}$ and $b \bar{b}$

Stronger breaking in meson-meson thresholds

Bottomonium

<table>
<thead>
<tr>
<th>1+−</th>
<th>0++</th>
<th>1++</th>
<th>2++</th>
</tr>
</thead>
<tbody>
<tr>
<td>9879</td>
<td>9855</td>
<td>9874</td>
<td>9886</td>
</tr>
<tr>
<td>10240</td>
<td>10221</td>
<td>10236</td>
<td>10246</td>
</tr>
<tr>
<td>10516</td>
<td>10500</td>
<td>10513</td>
<td>10521</td>
</tr>
<tr>
<td>10740</td>
<td>10726</td>
<td>10737</td>
<td>10744</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>1+−</th>
<th>0++</th>
<th>1++</th>
<th>2++</th>
</tr>
</thead>
<tbody>
<tr>
<td>10781</td>
<td>10781</td>
<td>10781</td>
<td>10781</td>
</tr>
</tbody>
</table>
The $X_b \ 1^{++}$ state

- **Channels:** $b\bar{b}(3\,^3P_1), \ b\bar{b}(4\,^3P_1), \ B\bar{B}^* (\,^3S_1), \ B\bar{B}^* (\,^3D_1)$.

- The $b\bar{b}(3\,^3P_1)$ has been measured by LHCb, **JHEP 10, 088 (2014)**

\[
M(3\,^3P_1) = 10515.7^{+2.2+1.5}_{-3.9-2.1} \text{ MeV}
\]

\[
M(3\,^3P_2) - M(3\,^3P_1) = 10.5 \text{ MeV}
\]

- Very shallow bound state with $E_{\text{bind}} = -0.016 \text{ MeV}$ and $\Gamma = 1.7 \text{ MeV}$

- No definite conclusions can be obtained about its existence
The $X_b \, 2^{++}$ state

- **Channels:** $b\bar{b}(3\,^3P_2)$, $b\bar{b}(4\,^3P_2)$, $b\bar{b}(2\,^3F_2)$, $B^*\bar{B}^*(5\,^1S_2)$, $B^*\bar{B}^*(5\,^3D_2)$, $B^*\bar{B}^*(1\,^3D_2)$.

<table>
<thead>
<tr>
<th>$M$ (MeV)</th>
<th>$b\bar{b}(3,^3P_2)$</th>
<th>$b\bar{b}(4,^3P_2)$</th>
<th>$B^<em>\bar{B}^</em>(5,^1S_2)$</th>
<th>$B^<em>\bar{B}^</em>(5,^3D_2)$</th>
<th>$B^<em>\bar{B}^</em>(1,^3D_2)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>10469.2</td>
<td>0.71</td>
<td>1.03</td>
<td>90.7</td>
<td>7.4</td>
<td>0.01</td>
</tr>
</tbody>
</table>
Summary

- We have studied the partners of the $X(3872)$ using a Chiral Quark model
- HQSS relations are almost fulfilled by the interaction Hamiltonian
- HQSS breaking effects are mostly due to threshold effects
- The coupling to $c\bar{c}$ and $b\bar{b}$ states produces discrepancies from HQSS expectations
- The $1^{++}$ state in the charmonium sector is bound due to the coupling with the $2P_c\bar{c}$ state
- In the $2^{++}$ charmonium sector we don’t have a new state, however we have a dressed $c\bar{c}$ state close to the $D^*D^*$ threshold.
- In the bottom sector the $1^{++}$ state have a strong repulsion due to the coupling with the $3P$ state and no definite conclusions about its existence can be obtained
- In the bottom sector the $2^{++}$ state is bounded with and without coupling to $b\bar{b}$ states
- In the $0^{++}$ and $1^{+-}$ channels only a new state appears in the $0^{++}$ botomonium spectrum.
- Coupling with $Q\bar{Q}$ states can induced discrepancies from HQSS and HFS expectations