Quark Models of Excited Baryons

How are excited N and Δ states constructed in the constituent quark model?

What does this construction imply for the N and Δ spectrum?

How is this modified when we add strange quarks?

Is there a better way to construct the basis if we want to extend it?

Can we deal with momentum and coordinate-space operators in H?

Sample spectrum
Constructing Excited Baryon States

Think first about baryons made of light (u & d) quarks

Quarks have color, flavor, spin and spatial degrees of freedom

If strong interactions of u and d indistinguishable (isospin) then we need total exchange antisymmetry in the wave f’n:

$$\Psi = C_A \sum \psi \chi \phi$$

$$C_A = \frac{1}{\sqrt{6}} (rgb + gbr + brg - rgb - grb - bgr)$$

$$\psi(\vec{r}_1, \vec{r}_2, \vec{r}_3) \quad \chi(s_1, s_2, s_3), \ s_j = \uparrow, \downarrow$$

$$\phi(i_1, i_2, i_3), \ i_j = u, \ d$$

Sum performed to provide total symmetry & good J
Combining representations of $S_3$ exchange group

$S_3 = \{1, (12), (13), (23), (123), (132)\}$

3x2x1 ways of arranging three objects

Representations are totally symmetric $S$, totally antisymmetric $A$, and a mixed symmetry pair $\{M^\rho, M^\lambda\}$ that transform into each other

Action of group elements, e.g.:

\[
\begin{align*}
(12) M^\rho &= -M^\rho, & (12) M^\lambda &= M^\lambda \\
(13) M^\rho &= M^\rho/2 - \sqrt{3} M^\lambda/2 \\
(13) M^\lambda &= -\sqrt{3} M^\rho/2 - M^\lambda/2
\end{align*}
\]
Combining representations of $S_3$ exchange group

Combining two representations:

$S_a \otimes S_b = S_{ab}$

$S_a \otimes A_b = A_{ab}$

$A_a \otimes A_b = S_{ab}$

\[
\frac{1}{\sqrt{2}} (M_\rho^a \otimes M_\rho^b + M_\lambda^a \otimes M_\lambda^b) = S_{ab}
\]

\[
\frac{1}{\sqrt{2}} (M_\rho^a \otimes M_\lambda^b - M_\lambda^a \otimes M_\rho^b) = A_{ab}
\]

\[
\frac{1}{\sqrt{2}} (M_\rho^a \otimes M_\lambda^b + M_\lambda^a \otimes M_\rho^b) = M_{ab}^\rho
\]

\[
\frac{1}{\sqrt{2}} (M_\rho^a \otimes M_\rho^b - M_\lambda^a \otimes M_\lambda^b) = M_{ab}^\lambda
\]
Ground states

Quark-spin wave functions: spin 1/2 have mixed symmetry

\[ \chi^\rho = \frac{1}{\sqrt{2}} (\uparrow \downarrow - \downarrow \uparrow) \quad \chi^\lambda = -\frac{1}{\sqrt{6}} (\uparrow \downarrow \uparrow + \downarrow \uparrow \uparrow - 2 \uparrow \uparrow \downarrow) \]

spin 3/2 are symmetric

\[ \chi^S = \uparrow \uparrow \uparrow \]

Flavor wave functions: N flavor, e.g. proton

\[ \phi^\rho = \frac{1}{\sqrt{2}} (udu - duu) \quad \phi^\lambda = \frac{1}{\sqrt{6}} (udu + duu - 2uud) \]

\[ \Delta \text{ flavor, e.g. } \phi_{\Delta^+}^S = \frac{1}{\sqrt{3}} (uud + udu + duu) \]

For ground state nucleon and \( \Delta \), symmetric sums

\[ \frac{1}{\sqrt{2}} (\chi^\rho \phi^\rho + \chi^\lambda \phi^\lambda) \quad \chi^S \phi^S \]
Ground-state wave functions

Spatial wave functions: separate CM motion by using Jacobi coordinates

\[ \tilde{\rho} = \frac{1}{\sqrt{2}} (\vec{r}_1 - \vec{r}_2) \quad \tilde{\lambda} = \frac{1}{\sqrt{6}} (\vec{r}_1 + \vec{r}_2 - 2\vec{r}_3) \]

For ground state nucleon and \( \Delta(1232) \), example symmetric spatial wave f’n (exact for HO potential) with \( L^P=0^+ \)

\[ \psi^S = \frac{a^3}{\pi^3} \exp \left\{ -a^2 (\rho^2 + \lambda^2) \right\} \]

Recall \( \rho \rho + \lambda \lambda \sim S \)

\[ |N^2 S_{S_2}^{1^+} \rangle = C_A \phi_0^S \frac{1}{\sqrt{2}} (\phi_N^S \chi_{\frac{1}{2}}^S + \phi_N^S \chi_{\frac{1}{2}}^S) \]

\[ |\Delta^1 S_{S_2}^{3^+} \rangle = C_A \phi_2^S \psi_S \chi_{\frac{3}{2}}^S \]

N and \( \Delta \), in \([56,0^+]\)

|Flavor L symmetry S J^P\rangle
Orbital excited-state wave functions

Simplest (lowest energy) excited states have one unit of orbital angular momentum in either $\rho$ or $\lambda$ oscillator

$$L_P = 1^- \otimes \{ S = \frac{1}{2} \text{ or } S = \frac{3}{2} \}$$

$$\psi_{1m}^\rho = \{ \rho_-, \sqrt{2}\rho_0, -\rho_+ \} \psi^S$$

$$\psi_{1m}^\lambda = \{ \lambda_-, \sqrt{2}\lambda_0, -\lambda_+ \} \psi^S$$

gives $J^P = 1/2^-, 3/2^-, 5/2^-$

mixed symmetry spatial wave f’ns
mixed symmetry (N) or symmetric (Δ) flavor
mixed symmetry (S=1/2) or symmetric (S=3/2) spin
Orbital excited-state wave functions

**Quark-spin 3/2 Nucleon states**

\[ |N^4P_M(\frac{1}{2}, \frac{3}{2}, \frac{3}{2})\rangle = C_A\chi^{\frac{S}{2}}\frac{1}{\sqrt{2}}(\phi_n^p\psi_{1M}^p + \phi_n^\lambda\psi_{1M}^\lambda) \]

**Quark-spin 1/2 Nucleon states**

\[ |N^2P_M(\frac{1}{2}, \frac{1}{2})\rangle = C_A\frac{1}{2}\left\{\phi_n^p[\psi_{1M}^p + \psi_{1M}^\lambda] + \phi_n^\lambda[\psi_{1M}^p - \psi_{1M}^\lambda]\right\} \]

**Quark-spin 1/2 Δ states**

\[ |\Delta^2P_M(\frac{1}{2}, \frac{3}{2})\rangle = C_A\phi_n^S\frac{1}{\sqrt{2}}(\psi_{1M}^p + \psi_{1M}^\lambda) \]
Lowest orbital excitations

N. Isgur & G. Karl

Sample spectrum: boxes are masses, with uncertainties, from analyses of data (all states seen)

\[ \delta = M_{\Delta} - M_N \sim 300 \text{ MeV} \]

Degeneracy broken by tensor interaction

Also mixes \( S=1/2 \), \( S=3/2 \) wave f’ns, can explain \( N(1535) \) to \( N\eta \)
Radial (doubly-excited) excitations

Radial excitations have a radial node in either the $\rho$ or $\lambda$ oscillator, with $L^p=0^+$

\[ \sqrt{\frac{2}{3}} \left\{ \frac{3}{2} - \alpha^2 \lambda^2 \right\} \psi^S \]

not states of definite symmetry

Additional true 3-body state with $l_\rho=1$ and $l_\lambda=1$ combined to $L^p=0^+$

\[ \psi_{00}^{M^p} = -\frac{2}{\sqrt{3}} \frac{\alpha^5}{\pi^{3/2}} \rho \cdot \lambda e^{-\frac{\alpha^2}{2}(\rho^2 + \lambda^2)} \]

has $M^p$ symmetry
States of definite symmetry

breathing (lowest frequency) mode

\[ \psi_{00}^{S'} = \sqrt{\frac{2}{3 \pi^{\frac{3}{2}}}} \frac{1}{\sqrt{2}} \left( \rho^2 - \frac{3}{2 \alpha^2} + \lambda^2 - \frac{3}{2 \alpha^2} \right) e^{-\frac{\alpha^2}{2} (\rho^2 + \lambda^2)} \]

antisymmetric (higher frequency) mode

\[ \psi_{00}^{M^\lambda} = \sqrt{\frac{2}{3 \pi^{\frac{3}{2}}}} \frac{1}{\sqrt{2}} (\rho^2 - \lambda^2) e^{-\frac{\alpha^2}{2} (\rho^2 + \lambda^2)} \]

paired with

\[ \psi_{00}^{M^\rho} = -\frac{2}{\sqrt{3 \pi^{\frac{3}{2}}}} \rho \cdot \lambda e^{-\frac{\alpha^2}{2} (\rho^2 + \lambda^2)} \]
L=0 positive-parity excited-state wave functions

Radial recurrences of N (J^p=1/2^+) and Δ (J^p=3/2^+) ground states

\[ |N^2 S_{1/2}^{1^+}\rangle = C_A \psi_{1/2} \frac{1}{\sqrt{2}} (\phi_N^0 \chi_{1/2}^0 + \phi_N^1 \chi_{1/2}^1) \]

\[ |\Delta^4 S_{3/2}^{3^+}\rangle = C_A \phi_{3/2} \psi_{3/2} \chi_{3/2}^0 \]

Other L^p=0^+ states (J^p=1/2^+, 3/2^+)

\[ |N^4 S_{1/2}^{3^+}\rangle = C_A \chi_{1/2} \frac{1}{\sqrt{2}} (\phi_N^0 \psi_{00}^0 + \phi_N^1 \psi_{00}^1) \]

\[ |\Delta^2 S_{3/2}^{3^+}\rangle = C_A \phi_{3/2} \psi_{00}^0 \chi_{3/2}^0 + \psi_{00}^1 \chi_{3/2}^1 \]

\[ |N^2 S_{3/2}^{1^+}\rangle = C_A \frac{1}{2} \left\{ \phi_N^0 \left[ \psi_{00}^0 \chi_{1/2}^0 + \psi_{00}^1 \chi_{1/2}^1 \right] + \phi_N^1 \left[ \psi_{00}^0 \chi_{1/2}^0 - \psi_{00}^1 \chi_{1/2}^1 \right] \right\} \]
Doubly excited-state wave functions with $L=1,2$

Can also form spatial wave f’ns with $L^p=1^+$ and $2^+$, from

$$l_\rho = 0 \otimes l_\lambda = 2, \quad l_\rho = 2 \otimes l_\lambda = 0, \text{ and } l_\rho = 1 \otimes l_\lambda = 1$$

$$\psi^{A}_{11} = -\frac{\alpha^5}{\pi^2} (\rho_+ \lambda_0 - \rho_0 \lambda_+) e^{-\frac{2}{T}(\rho^2 + \lambda^2)}$$

**Antisymmetric $L^p=1^+$**

$$\psi^{S}_{22} = \frac{1}{\sqrt{2}} \frac{\alpha^5}{\pi^2} (\rho_+^2 + \lambda_+^2) e^{-\frac{2}{T}(\rho^2 + \lambda^2)}$$

$$\psi^{M_\lambda}_{22} = \frac{1}{\sqrt{2}} \frac{\alpha^5}{\pi^2} (\rho_+^2 - \lambda_+^2) e^{-\frac{2}{T}(\rho^2 + \lambda^2)}$$

$$\psi^{M_\rho}_{22} = \frac{\alpha^5}{\pi^2} \rho_+ \lambda_+ e^{-\frac{2}{T}(\rho^2 + \lambda^2)},$$

**Symmetric and mixed symmetry $L^p=2^+$**
L=1,2 positive-parity excited-state wave functions

$L^p=2^+$ states with $S=1/2, 3/2$  All $J$ values up to 7/2

\[56, 2^+\] : \[\Delta^4 D_S(\frac{1}{2}^+, \frac{3}{2}^+, \frac{5}{2}^+, \frac{7}{2}^+)\] = \[C_A \phi_S^N \psi_{2M} \lambda_{\frac{3}{2}}^S\]
\[\Delta^2 D_S(\frac{3}{2}^+, \frac{5}{2}^+)\] = \[C_A \phi_S^{\frac{3}{2}} \frac{1}{\sqrt{2}} (\phi_N^\frac{3}{2} \lambda_{\frac{1}{2}}^N + \phi_N^{\frac{3}{2}} \lambda_{\frac{1}{2}}^N)\]

\[70, 2^+\] : \[\Delta^4 D_M(\frac{1}{2}^+, \frac{3}{2}^+, \frac{5}{2}^+, \frac{7}{2}^+)\] = \[C_A \lambda_{\frac{3}{2}} \frac{1}{\sqrt{2}} (\phi_N^\frac{3}{2} \psi_{2M}^\frac{1}{2} + \phi_N^{\frac{3}{2}} \psi_{2M}^{\frac{1}{2}})\]
\[\Delta^2 D_M(\frac{3}{2}^+, \frac{5}{2}^+)\] = \[C_A \phi_S^{\frac{3}{2}} (\phi_N^\frac{3}{2} \lambda_{\frac{1}{2}}^N + \psi_{2M}^{\frac{1}{2}})\]
\[\Delta^2 D_M(\frac{3}{2}^+, \frac{5}{2}^+)\] = \[C_A \lambda_{\frac{3}{2}} \frac{1}{\sqrt{2}} \left(\phi_N^\frac{3}{2} (\psi_{2M}^{\frac{1}{2}} + \psi_{2M}^{\frac{1}{2}} + \phi_N^{\frac{3}{2}} (\psi_{2M}^{\frac{1}{2}} - \psi_{2M}^{\frac{1}{2}})\right)\]

$L^p=1^+$ states with $S=1/2$  $J=1/2, 3/2$

\[20, 1^+\] : \[\Delta^2 P_A(\frac{1}{2}^+, \frac{3}{2}^+)\] = \[C_A \psi_{1M} \lambda_{\frac{1}{2}}^N \frac{1}{\sqrt{2}} (\phi_N^\frac{3}{2} \lambda_{\frac{1}{2}}^N - \phi_N^{\frac{3}{2}} \lambda_{\frac{1}{2}}^N)\]
Pattern of splitting of positive-parity excited states

Isgur & Karl: first order perturbation theory in anharmonicity

\[ U = \sum_{i<j} U_{ij} \]

(E.g. \( U_{ij} = b r_{ij} - 3 K r_{ij}^2/2 \), \( r_{ij} = |r_i - r_j| \), but don’t need to specify)

Starting point

<table>
<thead>
<tr>
<th>State</th>
<th>Energy ((\omega))</th>
</tr>
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<tbody>
<tr>
<td>[56', 0^+]</td>
<td>(5(\omega))</td>
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<tr>
<td>[70, 0^+]</td>
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<tr>
<td>[56, 2^+]</td>
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<tr>
<td>[20, 1^+]</td>
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</tr>
<tr>
<td>[70, 1^-]</td>
<td>(4(\omega))</td>
</tr>
<tr>
<td>[56, 0^+]</td>
<td>(3(\omega))</td>
</tr>
</tbody>
</table>
Positive-parity excited-state spectrum

In $[56,0^+]$ ground states

$$\langle \phi^+_5 \phi^+_0 \rangle \sum_{i<j} U_{ij} \langle \phi^+_5 \phi^+_0 \rangle \frac{\alpha^3}{\pi^2} \int d^3 \rho U(\sqrt{2} \rho) e^{-\alpha^2 \rho^2} = a$$

In $[70,1^-]$ orbitally-excited states

$$\langle \frac{1}{\sqrt{2}} (\phi^+_5 \psi^0_{1M} + \phi^+_1 \psi^1_{1M}) \sum_{i<j} U_{ij} \frac{1}{\sqrt{2}} (\phi^+_5 \psi^0_{1M} + \phi^+_1 \psi^1_{1M}) \rangle$$

$$= \frac{3}{2} \delta_{MM} \left\{ \langle \psi^0_{1M} | U(\sqrt{2} \rho) | \psi^0_{1M} \rangle + \langle \psi^1_{1M} | U(\sqrt{2} \rho) | \psi^1_{1M} \rangle \right\}$$

$$b := \frac{3 \alpha^5}{\pi^2} \int d^3 \rho \rho^2 U(\sqrt{2} \rho) e^{-\alpha^2 \rho^2}$$

$$c := \frac{3 \alpha^7}{\pi^2} \int d^3 \rho \rho^4 U(\sqrt{2} \rho) e^{-\alpha^2 \rho^2}$$
Positive-parity excited-state spectrum

Define \( \Omega = \omega - a/2 + b/3, \Delta = -5a/4 + 5b/3 - c/3 \)

\[
\Omega = \omega - a/2 + b/3 = E_0 + 2\Omega
\]

\[
\Delta = -5a/4 + 5b/3 - c/3
\]

Roper-like states below negative-parity: \( \Omega \approx \Delta \approx 440 \text{ MeV} \),
but \( \omega = 250 \text{ MeV} \), so first-order perturbation theory not justified
Positive-parity excited-state spectrum

Can then apply your favorite flavor and spin-dependent short-range interaction: Isgur and Karl used one-gluon exchange (OGE), minus spin-orbit interactions from both OGE, and Thomas precession in confining potential

Break SU(6) spin-flavor symmetry, so all of the states mix up

Because of the multiplicity of states and near-denegeracies:

- Lots of strong mixing
- L and S no longer good quantum numbers
- Beware of any model/algebra that identifies states by their L and S
How is this modified when we add strange quarks?

NOT advantageous to use a basis where the wave f’n (minus color) is symmetric under exchange of \{u,d\} and s

\[ m_s - (m_u + m_d)/2 \] substantial compared to quark momenta, SU(3)_f symmetry broken

E.g. ground state \( \Lambda \) wave f’n not same as that of p and n

Use ‘uds’ basis: don’t symmetrize s with u and d

\[
\begin{align*}
\phi_\Lambda^0 &= \frac{1}{\sqrt{2}} (ud - du)s \\
\phi_{\Sigma^+}^{-} &= uus, \frac{1}{\sqrt{2}} (ud + du)s, dds \\
\phi_{\Xi^{-}} &= sss \\
\phi_{\Omega^{-}} &= sss
\end{align*}
\]
How is this modified when we add strange quarks?

Sums now built symmetric only under (12) exchange

$$\Psi = C_A \sum \psi \chi \phi$$

Break symmetry between $\rho$ and $\lambda$ oscillators in wave f'ns

$$\psi_{10} = \frac{\alpha_\rho^2 \alpha_\lambda^2}{\pi^2} e^{-\frac{\alpha_\rho^2}{2} \frac{\alpha_\lambda^2}{2}}$$

Ground states

$$\psi_{1\pm 1}^\rho = \frac{\alpha_\rho^2 \alpha_\lambda^2}{\pi^2} \rho_{\pm} e^{-\frac{\alpha_\rho^2}{2} \rho^2 + \frac{\alpha_\lambda^2}{2} \lambda^2}$$

Orbitally excited states

$$\psi_{1\pm 1}^\lambda = \frac{\alpha_\rho^2 \alpha_\lambda^2}{\pi^2} \lambda_{\pm} e^{-\frac{\alpha_\rho^2}{2} \rho^2 + \frac{\alpha_\lambda^2}{2} \lambda^2}$$

Similarly for positive-parity excited states

$$\psi_{10}^\rho = \frac{\alpha_\rho^2 \alpha_\lambda^2}{\pi^2} \sqrt{2} \rho \lambda e^{-\frac{\alpha_\rho^2}{2} \rho^2 + \frac{\alpha_\lambda^2}{2} \lambda^2}$$
Highly-excited states

It is not a good idea to try to symmetrize the basis if you plan to extend it; e.g. Karl and Obryk did $S_3$ group theory ("fairly tediously" appears several times in their paper)

\[ N = 3 \]
\[ \psi_{3,3;S} = -y_+^\lambda y_+^\lambda y_+^\lambda + 3 y_+^\rho y_+^\rho y_+^\rho \]
\[ \psi_{3,3;M} = [y_+^\rho y_+^\rho y_+^\lambda + y_+^\lambda y_+^\lambda y_+^\lambda, y_+^\rho y_+^\rho y_+^\rho + y_+^\lambda y_+^\lambda y_+^\rho] \]
\[ \psi_{3,3;A} = y_+^\rho y_+^\rho y_+^\rho - 3 y_+^\lambda y_+^\lambda y_+^\rho \]
\[ \psi_{3,2;M} = [(y_+^\lambda y_+^\rho - y_+^\rho y_+^\lambda) y_+^\rho, -(y_+^\lambda y_+^\rho - y_+^\rho y_+^\lambda) y_+^\lambda] \]
\[ \psi_{3,1;S} = (q^2 - \lambda^2) y_+^\lambda + 2 \lambda \cdot q y_+^\rho \]
\[ \psi_{3,1;A} = (q^2 - \lambda^2) y_+^\rho - 2 \lambda \cdot q y_+^\lambda \]
\[ \psi_{3,1;M} = [(q^2 + \lambda^2) y_+^\lambda, (q^2 + \lambda^2) y_+^\rho] \]
\[ \psi_{3,1;M} = [2 \lambda \cdot q y_+^\rho - (q^2 - \lambda^2) y_+^\lambda, (q^2 - \lambda^2) y_+^\rho + 2 \lambda \cdot q y_+^\lambda] \]
Highly-excited states

\[ N = 4 \]

\[ \psi_{4,4;S} = (\eta \eta^0 \eta^0 + \eta^0 \eta^0 \eta^0)(\eta \eta^0 \eta^0 + \eta^0 \eta^0 \eta^0) \]

\[ \psi_{4,4;M} = \left((\eta^{\lambda}\eta^{\lambda})^2 Y_0 + 3(\eta^{\lambda}\eta^{\lambda}) Y_0^2 \right) Y_0^\lambda, \left((\eta^{\lambda}\eta^{\lambda}) Y_0^2 \right) Y_0^\lambda \right] \]

\[ \psi_{4,4;A} = \left((\eta \eta^0 + \eta^0 \eta^0)(\eta^{\lambda}\eta^{\lambda}) Y_0^2 + \eta^{\lambda}\eta^{\lambda} Y_0^2 \right) Y_0^\lambda \]

\[ \psi_{4,3;A} = \left((\eta \eta^0 + \eta^{\lambda}\eta^{\lambda}) Y_0^2 \right) Y_0^\lambda, \left((\eta \eta^0 + \eta^{\lambda}\eta^{\lambda}) Y_0^2 \right) Y_0^\lambda \right] \]

\[ \psi_{4,2;S} = \left((\eta \eta^0 + \eta^{\lambda}\eta^{\lambda}) Y_0^2 \right) Y_0^\lambda, \left((\eta \eta^0 + \eta^{\lambda}\eta^{\lambda}) Y_0^2 \right) Y_0^\lambda \right] \]

\[ \psi_{4,2;A} = \left((\eta \eta^0 + \eta^{\lambda}\eta^{\lambda}) Y_0^2 \right) Y_0^\lambda, \left((\eta \eta^0 + \eta^{\lambda}\eta^{\lambda}) Y_0^2 \right) Y_0^\lambda \right] \]

\[ \psi_{4,1;A} = \left((\eta \eta^0 + \eta^{\lambda}\eta^{\lambda}) Y_0^2 \right) Y_0^\lambda, \left((\eta \eta^0 + \eta^{\lambda}\eta^{\lambda}) Y_0^2 \right) Y_0^\lambda \right] \]

\[ \psi_{4,0;S} = \left((\eta \eta^0 + \eta^{\lambda}\eta^{\lambda}) Y_0^2 \right) Y_0^\lambda, \left((\eta \eta^0 + \eta^{\lambda}\eta^{\lambda}) Y_0^2 \right) Y_0^\lambda \right] \]

\[ \psi_{4,0;A} = \left((\eta \eta^0 + \eta^{\lambda}\eta^{\lambda}) Y_0^2 \right) Y_0^\lambda, \left((\eta \eta^0 + \eta^{\lambda}\eta^{\lambda}) Y_0^2 \right) Y_0^\lambda \right] \]
Extending the basis to highly-excited states

*Don’t* antisymmetrize in u and d for \( \Delta \) and N flavor wave f’ns

\[
\Delta^{++} = uuu, \; \{\Delta^+,p\} = uud, \; \{\Delta^0,n\} = ddu, \; \Delta^- = ddd
\]

\[
\Psi = C_A \phi \sum \chi \psi
\]

Require only (12) symmetry (N,\( \Delta, \Sigma, \Xi \)) or antisymmetry (\( \Lambda \)) in sums, and good angular momentum

Build a basis large enough to ensure convergence of expansion of wave functions; harmonic oscillator is convenient, i.e. easily Fourier transformed
Extending the basis to highly-excited states

Solve $H\Psi = E\Psi$ (doesn’t have to be non-relativistic) by diagonalizing $H$ matrix formed by expanding $\Psi$ in large basis $\Psi_a$ (hundreds of sub-states)

Variational calculation (separate for each eigenstate; Hylleraas-Undheim theorem) in oscillator parameter(s)

$u,d$ symmetry of $H$ reflected in eigenfunctions

Costs?

Have to look at wave functions to decide if a state is $\Delta$ ($\sum \chi \psi$ is $S$) or $N$ ($\sum \chi \psi$ is $M^\lambda$)

No longer have $\sum_{i<j} \langle \psi | H_{ij} | \psi \rangle = 3 \langle \psi | H_{12} | \psi \rangle$ (Moshinsky)
Momentum and position-dependence in $H$

If have basis easy to Fourier transform, can deal with momentum and position-dependent terms in $H$

E.g. kinetic energy

$$\langle \psi_a | \sum_i \sqrt{p_i^2 + m_i^2} | \psi_b \rangle$$

simply evaluate in momentum space

Effects of spinor normalization in, e.g. OGE contact interaction

$$\left( \frac{m_i m_j}{E_i E_j} \right)^{1+\epsilon_{\text{norm}}} \frac{8\pi}{3} \alpha_s(r_{ij}) \frac{2}{3} \frac{S_i \cdot S_j}{m_i m_j} \left[ \frac{\sigma_{ij}^2}{\pi^2} e^{-\sigma_{ij}^2 r_{ij}^2} \right] \left( \frac{m_i m_j}{E_i E_j} \right)^{1+\epsilon_{\text{norm}}}$$

insert (nominally) complete sets of states, multiply matrices

$$\sum_{cd} \langle \psi_a | f(p_i) | \psi_c \rangle \langle \psi_c | V(r_{ij}) | \psi_d \rangle \langle \psi_d | f(p_i) | \psi_b \rangle$$
Sample spectrum (SC & N. Isgur)

Hamiltonian: relativistic kinetic energy
  confining potential $b \sum_i l_i$
  + associated spin-orbit
  
  one-gluon exchange (color-Coulomb, contact, tensor, spin-orbit interactions)

  smeared quarks, suppression of potentials at high momentum

Large oscillator basis; 8\textsuperscript{th}-order polynomials (positive parity)
or 7\textsuperscript{th}-order (negative parity)

Calculate all baryon masses & wave f'ns with one consistent set of parameters
Sample spectrum (SC & N. Isgur)

N experimental and model states below 2200 MeV

- Masses: SC and N. Isgur, PRD 34 (1986) 2809

Florida State University	Simon Capstick	Hadronic Workshop @ JLab 2/25/2011
Sample spectrum (SC & N. Isgur)

Lowest few non-strange baryons of either parity up to \( J=\frac{11}{2} \) (bars) vs. PDG mass range (boxes)
More recent work

**Electromagnetic couplings**
- photocouplings in corrected NR approach
- Compton scattering, with B. Keister
- Electroproduction amplitudes in light-cone dynamics, with B. Keister

**Strong-decay amplitudes**, with W. Roberts

**Flux-tube model of hybrid baryons**, with P. Page

**Effects of baryon-meson loops**, with D. Morel

**Baryons as resonances in effective Lagrangian model**
\((N_\gamma, N\pi, N\eta, N\pi\pi, N\pi\eta,...)\), with A. Kiswandhi