Baryon spectroscopy from lattice QCD

- **Long-term goal**: Solve QCD to determine the hadron mass spectrum.

- **Part I. Recent progress on** $N$, $\Delta$, $\Omega$ **excited state spectra**

- **Part II. Spin identification for baryon states**

- **Phenomenology**

- **Conclusions**
Matrices of correlation functions and smearing of quark fields

$$C_{ij}(t, t') = \sum_{xy} \langle B_i(x, t) B_j^\dagger(y, t') \rangle$$

$$B_i(x, t) = C_i^{\alpha\beta\gamma} \epsilon^{abc} q_i^{\alpha f_1}(x, t) q_j^{\beta f_2}(x, t) q_j^{\gamma f_3}(x, t).$$

Smearing: Project to eigenvectors of Laplacian

$$q_\alpha^a(x, t) \longrightarrow \sum_k v_{\alpha x}^{(k)} \tilde{q}_{\alpha}^{(k)}(t).$$

$$(- \nabla^2)_{xy}^{ab} v_{b, y}^{(k)} = \lambda_k v_{\alpha x}^{(k)}$$

$$C_{ij}(t, t') = \Phi_{i, k\ell m}^{\alpha\beta\gamma}(t) \left\langle \tilde{q}_{\alpha}^{(k)}(t) \tilde{q}_{\beta}^{(\ell)}(t) \tilde{q}_{\gamma}^{(m)}(t) \right\rangle \Phi_{j, k\ell m}^{\bar{\alpha}\bar{\beta}\bar{\gamma}}(t')$$
Determine energies

Calculate eigenvectors at \( t^* = t_0 + 1 \)

\[
\overline{C}(t^*)V(t^*) = \overline{C}(t_0)V(t^*)\Lambda(t^*)
\]

Rotate matrices to basis of eigenvectors, calculate diagonal elements

\[
\tilde{\lambda}_n(t) = \left( V^\dagger(t^*)C(t)V(t^*) \right)_{nn}
\]

Two-exponential fits of diagonal elements

\[
\lambda_{fit}(t) = (1 - A)e^{-E(t-t_0)} + Ae^{-E'(t-t_0)}
\]
Nucleon $G_{1g}$ effective energies: $m_\pi = 392(4)$ MeV
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Patterns of Nucleon Spectra

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Patterns of Delta Spectra

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Patterns of Omega Spectra
Part II. Spin identification

- Rotational symmetry is broken at $O(a^2)$ by lattice action
- Typical lattice spacing is 0.1 fm
- Typical hadron size is 1 fm
- $O(a^2) \approx \left( \frac{0.1 \text{ fm}}{1.0 \text{ fm}} \right)^2 \approx 0.01$
- For hadrons, rotational symmetry should be broken weakly.
Fresh start: Construction of operators with good J in continuum

- **Mesons:** Dudek, *et al.*, Phys.Rev.D80:054506,2009

- **Baryons:** Color singlet structure for 3 quarks, symmetric in space & spin

  - $J = L + S$ with
    - $S = \frac{1}{2}$ or $\frac{3}{2}$ from quark spins
    - $L = 1$ or $2$ from covariant derivatives
    - $J = \frac{1}{2}, \frac{3}{2}, \frac{5}{2}$ and $\frac{7}{2}$

- **Lots of operators** $O^{[J,M]}$ with good spin in continuum limit

- **Feynman, Kislinger and Ravndal formalism** for quark states applied to operator construction.
Subduction to IRs of cubic group

- Why? Because lattice IRs provide orthogonal basis, not the J,M IRs

- In quantum mechanics, subduction is a change of basis $|J, M\rangle \rightarrow |\Lambda, r; J\rangle$.

- $|\Lambda, r; J\rangle = \sum_M |J, M\rangle \langle J, M|\Lambda, r; J\rangle$
  
  $= \sum_M |J, M\rangle S_{\Lambda,r}^{J,M}$.

- Subduction coefficients: $S_{\Lambda,r}^{J,M}$

- Operators: $O^{[\Lambda,r;J]} = \sum_M O^{[J,M]} S_{\Lambda,r}^{J,M}$

- If rotational symmetry is broken weakly,
  
  $\langle 0| O^{[\Lambda,r;J]}(t) O^{[\Lambda,r;J']\dagger}(0)|0 \rangle \approx \delta_{J,J'}$

  is block diagonal in $J$. 
Figure 4: Magnitude of matrix elements in a matrix of correlation functions at timeslice 5.
Test 2 for $28G_{1g}$ energies
Test 2 for $^{48}H_g$ energies

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Test 2 for $20 G_{2g}$ energies
How were the spins identified?

\[ C_{ik}(t) = \sum_n \langle 0 | O_i(0) | n \rangle e^{-E_n t} \langle n | O_k^\dagger(0) | 0 \rangle = \sum_n Z^*_n e^{-E_n t} Z_{nk} \]

Spin weights

\[ W_{nJ} = \sum_{k \subseteq J} \frac{|Z_{nk}|^2}{\sum_k |Z_{nk}|^2} \]

\( W_{nJ} \) is the relative weight for operators subduced from spin \( J \) in the creation of state \( |n\rangle \).
How well do weights identify the spins?

Table 1: Spin weights in % for ten $G_{1g}$ energy levels.

<table>
<thead>
<tr>
<th>$E_n$</th>
<th>$\frac{1}{2}$</th>
<th>$\frac{3}{2}$</th>
<th>$\frac{5}{2}$</th>
<th>$\frac{7}{2}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$G_{1g}^0$</td>
<td>0.2081(16)</td>
<td>99.9</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>$G_{1g}^{-1}$</td>
<td>0.3752(52)</td>
<td>99.9</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>$G_{1g}^{-2}$</td>
<td>0.3830(66)</td>
<td>99.6</td>
<td>0</td>
<td>0.3</td>
</tr>
<tr>
<td>$G_{1g}^{-3}$</td>
<td>0.3922(78)</td>
<td>99.9</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>$G_{1g}^{-4}$</td>
<td>0.3944(71)</td>
<td>99.7</td>
<td>0</td>
<td>0.2</td>
</tr>
<tr>
<td>$G_{1g}^{-5}$</td>
<td>0.4263(103)</td>
<td>99.6</td>
<td>0</td>
<td>0.3</td>
</tr>
<tr>
<td>$G_{1g}^{-6}$</td>
<td>0.4398(41)</td>
<td>0.5</td>
<td>0</td>
<td>99.4</td>
</tr>
<tr>
<td>$G_{1g}^{-7}$</td>
<td>0.5003(166)</td>
<td>97.9</td>
<td>0</td>
<td>2</td>
</tr>
<tr>
<td>$G_{1g}^{-8}$</td>
<td>0.5020(114)</td>
<td>80.2</td>
<td>0</td>
<td>19.7</td>
</tr>
<tr>
<td>$G_{1g}^{-9}$</td>
<td>0.5060(167)</td>
<td>99.7</td>
<td>0</td>
<td>0.2</td>
</tr>
</tbody>
</table>
Positive parity nucleon spectrum

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Negative parity nucleon spectrum

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Parity + and - nucleon spectrum

$E/m$ vs $m_\pi = 392$

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Nucleon states similar to 'quark-model' pattern

Phenomenology: Nucleon spectrum

Discern structure: wave-function overlaps

[20,1^+]
P-wave

[70,2^+]
D-wave

[56,2^+]
D-wave

Looks like quark model?

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$m_{\pi} \sim 520\text{MeV}$
\( \Delta \) states also similar to 'quark-model' pattern

Spin identified \( \phi \) spectrum

Spectrum slightly higher than nucleon

**[56,2^+]**
D-wave

**[70,1^-]**
P-wave
Conclusions

- The patterns of lattice baryonic states are similar to the patterns of physical resonance states.

- Spin identification based on subduction of continuum \( j \) works well.

- Lots of baryonic states, but no sign of chiral restoration.

The path forward

- Multiparticle operators are needed to include scattering states (e.g., \( \pi N \)).

- Multiple volumes are needed for determination of phase shifts using Luscher’s formalism.

- Lower \( m_\pi \) is needed in order to approach the physical limit.
Lattice parameters

- $N_f = 2+1$ QCD
  - Gauge action: Symanzik-improved
  - Fermion action: Clover-improved Wilson
- Anisotropic: $a_s = 0.122$ fm, $a_t = 0.035$ fm

<table>
<thead>
<tr>
<th>ensemble</th>
<th>1</th>
<th>2</th>
<th>3</th>
</tr>
</thead>
<tbody>
<tr>
<td>$m_\ell$</td>
<td>−.0840</td>
<td>−.0830</td>
<td>−.0808</td>
</tr>
<tr>
<td>$m_s$</td>
<td>−.0743</td>
<td>−.0743</td>
<td>−.0743</td>
</tr>
<tr>
<td>Volume</td>
<td>$16^3 \times 128$</td>
<td>$16^3 \times 128$</td>
<td>$16^3 \times 128$</td>
</tr>
<tr>
<td>$N_{cfgs}$</td>
<td>344</td>
<td>570</td>
<td>481</td>
</tr>
<tr>
<td>$t_{sources}$</td>
<td>4</td>
<td>4</td>
<td>4</td>
</tr>
<tr>
<td>$m_\pi$</td>
<td>0.0691(6)</td>
<td>0.0797(6)</td>
<td>0.0996(6)</td>
</tr>
<tr>
<td>$m_K$</td>
<td>0.0970(5)</td>
<td>0.1032(5)</td>
<td>0.1149(6)</td>
</tr>
<tr>
<td>$m_\Omega$</td>
<td>0.2951(22)</td>
<td>0.3040(8)</td>
<td>0.3200(7)</td>
</tr>
<tr>
<td>$m_\pi$ (MeV)</td>
<td>392(4)</td>
<td>438(3)</td>
<td>521(3)</td>
</tr>
</tbody>
</table>
Part I. $N$, $\Delta$ and $\Omega$ spectra

- Many interpolating field operators in each IR of octahedral group: Prune to $\approx 10$


- Matrices of correlation functions: Diagonalize them at $t^* \approx 8$, Fix eigenvectors at $t^*$.

- Diagonal correlation functions: Fit them & extract six energies

- Lattice spectra: Compare patterns with experimental resonance spectra.
Limitations

- **Three-quark operators:**
  - No multiparticle operators
  - Scant evidence for scattering states

- **One (small) volume:** No extrapolations or δ’s

- $m_\pi = 392, 438, 521$ MeV : Energies generally are high.

- **Spins:** A single $J^P = \frac{5}{2}^-$ pattern is seen. Patterns for higher spins are ambiguous.
Computational Resources

- USQCD allocations
- Jefferson Laboratory clusters
- Fermi National Accelerator Lab clusters
- and the Chroma software system (Edwards et al.)
Subduction of $J$ to $O_D$

<table>
<thead>
<tr>
<th>IR</th>
<th>Parity</th>
<th>Dimension</th>
<th>$\frac{1}{2}$</th>
<th>$\frac{3}{2}$</th>
<th>$\frac{5}{2}$</th>
<th>$\frac{7}{2}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$G_{1g}$</td>
<td>+1</td>
<td>2</td>
<td>1</td>
<td></td>
<td></td>
<td>1</td>
</tr>
<tr>
<td>$H_g$</td>
<td>+1</td>
<td>4</td>
<td></td>
<td>1</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>$G_{2g}$</td>
<td>+1</td>
<td>2</td>
<td></td>
<td>1</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>$G_{1u}$</td>
<td>-1</td>
<td>2</td>
<td>1</td>
<td></td>
<td></td>
<td>1</td>
</tr>
<tr>
<td>$H_u$</td>
<td>-1</td>
<td>4</td>
<td></td>
<td>1</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>$G_{2u}$</td>
<td>-1</td>
<td>2</td>
<td></td>
<td>1</td>
<td>1</td>
<td>1</td>
</tr>
</tbody>
</table>

- **Isolated** $G_1$ state $\rightarrow$ Spin $\frac{1}{2}$
- **isolated** $H$ state $\rightarrow$ Spin $\frac{3}{2}$
- **Degenerate** $G_2$ and $H$ states $\rightarrow$ Spin $\frac{5}{2}$
- **Degenerate** $G_1$, $H$ and $G_2$ states $\rightarrow$ Spin $\frac{7}{2}$
Nucleon $G_{1u}$ effective energies: $m_\pi = 392(4)$ MeV
Nucleon $H_g$ effective energies: $m_\pi = 392(4) \text{ MeV}$

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Nucleon $H_u$ effective energies: $m_\pi = 392(4)$ MeV
Nucleon $G_{2g}$ effective energies: $m_\pi = 392(4)$ MeV
Nucleon $G_{2u}$ effective energies: $m_\pi = 392(4)$ MeV

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Summary of Part I.

- **First** excited baryon spectrum based on $N_f = 2+1$ QCD using anisotropic lattices

- 6 lowest energy $N$, $\Delta$ and $\Omega$ states in each IR for $m_\pi = 392(4), 438(3)$ and 521(3) MeV.

- Patterns of lowest energies are similar to the patterns of lowest physical resonance states.

- Spin identification is very difficult. Degeneracies allow several subduction patterns to be compatible with results.

- Degenerate states in $G_1$, $H$ and $G_2$: Could be a $J = \frac{7}{2}$ state or degenerate $J = \frac{1}{2}$ and $J = \frac{5}{2}$ states?
Test 2: Spectra with and without couplings between operators subduced from different J’s

- Small couplings can mix different J’s when states are degenerate

- Compare energies based on $C_{ij}(t)$ using all operators in an IR (include $J \neq J'$ couplings)

- and energies based on $C_{ij}(t)$ using only operators subduced from a single J (omit $J \neq J'$ couplings)

- For example, we have 28 $G_{1g}$ operators in all. They include 24 subduced from $J = \frac{1}{2}$ and 4 subduced from $\frac{7}{2}$.

- Are the $J = \frac{1}{2}$ energies using all 28 $G_{1g}$ operators similar to those using only the 24 operators subduced from $J = \frac{1}{2}$?
Test 2 for 28 $G_{1u}$ energies
Test 2 for 48 $H_u$ energies

<table>
<thead>
<tr>
<th>$E/m\Omega$</th>
<th>$H_u^{3/2}$</th>
<th>$H_u^{5/2}$</th>
<th>$H_u^{7/2}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.4</td>
<td>ops</td>
<td>ops</td>
<td>ops</td>
</tr>
<tr>
<td>0.6</td>
<td>$3^{-}$</td>
<td>$5^{-}$</td>
<td>$7^{-}$</td>
</tr>
<tr>
<td>0.8</td>
<td>$3^{-}$</td>
<td>$5^{-}$</td>
<td>$7^{-}$</td>
</tr>
<tr>
<td>1.0</td>
<td>$3^{-}$</td>
<td>$5^{-}$</td>
<td>$7^{-}$</td>
</tr>
<tr>
<td>1.2</td>
<td>$3^{-}$</td>
<td>$5^{-}$</td>
<td>$7^{-}$</td>
</tr>
<tr>
<td>1.4</td>
<td>$3^{-}$</td>
<td>$5^{-}$</td>
<td>$7^{-}$</td>
</tr>
<tr>
<td>1.6</td>
<td>$3^{-}$</td>
<td>$5^{-}$</td>
<td>$7^{-}$</td>
</tr>
<tr>
<td>1.8</td>
<td>$3^{-}$</td>
<td>$5^{-}$</td>
<td>$7^{-}$</td>
</tr>
<tr>
<td>2.0</td>
<td>$3^{-}$</td>
<td>$5^{-}$</td>
<td>$7^{-}$</td>
</tr>
</tbody>
</table>
Test 2 for 20 $G_{2u}$ energies
How well do weights identify the spins?

Table 2: Spin weights in % for ten $H_g$ energy levels.

<table>
<thead>
<tr>
<th>$E_n$</th>
<th>$\frac{1}{2}$</th>
<th>$\frac{3}{2}$</th>
<th>$\frac{5}{2}$</th>
<th>$\frac{7}{2}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$H_g$-0</td>
<td>0.3705(90)</td>
<td>0</td>
<td>98.2</td>
<td>1.6</td>
</tr>
<tr>
<td>$H_g$-1</td>
<td>0.3816(38)</td>
<td>0</td>
<td>5.1</td>
<td>94.7</td>
</tr>
<tr>
<td>$H_g$-2</td>
<td>0.4005(48)</td>
<td>0</td>
<td>0.8</td>
<td>98.9</td>
</tr>
<tr>
<td>$H_g$-3</td>
<td>0.4013(61)</td>
<td>0</td>
<td>96.4</td>
<td>3.4</td>
</tr>
<tr>
<td>$H_g$-4</td>
<td>0.4030(43)</td>
<td>0</td>
<td>99.1</td>
<td>0.8</td>
</tr>
<tr>
<td>$H_g$-5</td>
<td>0.4113(42)</td>
<td>0</td>
<td>99.3</td>
<td>0.4</td>
</tr>
<tr>
<td>$H_g$-6</td>
<td>0.4237(60)</td>
<td>0</td>
<td>96.1</td>
<td>3.6</td>
</tr>
<tr>
<td>$H_g$-7</td>
<td>0.4267(35)</td>
<td>0</td>
<td>3.2</td>
<td>96.4</td>
</tr>
<tr>
<td>$H_g$-8</td>
<td>0.4414(38)</td>
<td>0</td>
<td>0.6</td>
<td>0.3</td>
</tr>
<tr>
<td>$H_g$-9</td>
<td>0.5050(224)</td>
<td>0</td>
<td>91.3</td>
<td>5.9</td>
</tr>
</tbody>
</table>
How well does the spin identification work?

Table 3: Spin weights in % for ten $G_{2g}$ energy levels.

<table>
<thead>
<tr>
<th>$E_n$</th>
<th>$1\over 2$</th>
<th>$3\over 2$</th>
<th>$5\over 2$</th>
<th>$7\over 2$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$G_{2g}^{-0}$</td>
<td>0.3717(54)</td>
<td>0.0000</td>
<td>0.0000</td>
<td>0.0000</td>
</tr>
<tr>
<td>$G_{2g}^{-1}$</td>
<td>0.4088(50)</td>
<td>0.0000</td>
<td>0.0000</td>
<td>0.0000</td>
</tr>
<tr>
<td>$G_{2g}^{-2}$</td>
<td>0.4151(49)</td>
<td>0.0000</td>
<td>0.0000</td>
<td>0.0000</td>
</tr>
<tr>
<td>$G_{2g}^{-3}$</td>
<td>0.4307(58)</td>
<td>0.0000</td>
<td>0.0000</td>
<td>0.0000</td>
</tr>
<tr>
<td>$G_{2g}^{-4}$</td>
<td>0.4854(393)</td>
<td>0.0000</td>
<td>0.0000</td>
<td>0.0000</td>
</tr>
<tr>
<td>$G_{2g}^{-5}$</td>
<td>0.5095(158)</td>
<td>0.0000</td>
<td>0.0000</td>
<td>0.0000</td>
</tr>
<tr>
<td>$G_{2g}^{-6}$</td>
<td>0.5178(112)</td>
<td>0.0000</td>
<td>0.0000</td>
<td>0.0000</td>
</tr>
<tr>
<td>$G_{2g}^{-7}$</td>
<td>0.5184(87)</td>
<td>0.0000</td>
<td>0.0000</td>
<td>0.0000</td>
</tr>
<tr>
<td>$G_{2g}^{-8}$</td>
<td>0.5368(108)</td>
<td>0.0000</td>
<td>0.0000</td>
<td>0.0000</td>
</tr>
<tr>
<td>$G_{2g}^{-9}$</td>
<td>0.5480(187)</td>
<td>0.0000</td>
<td>0.0000</td>
<td>0.0000</td>
</tr>
</tbody>
</table>
Results of spin identification analysis for baryon excited states

- The spin of a baryon excited state is equal to $J$ when the state is created predominantly by operators subduced from continuum spin $J$.

- Some baryon excited states that are nearly degenerate can have significant mixings of their $J$ parentage.
Spin $\frac{5}{2}$ and $\frac{7}{2}$ states based on average over $M$

$$C^{[J]}(t) = \frac{1}{2J+1} \sum_{\Lambda,r} C^{[\Lambda,r;J]}(t)$$

$$= \frac{1}{2J+1} \sum_M C^{[J,M]}(t)$$
Nucleon & Delta Spectrum

Suggests spectrum at least as dense as quark model

Change at lighter quark mass? Decays!

\[ m_\pi \sim 400 \text{ MeV} \]
\[ V \sim 2.0^3 \text{ fm}^3 \]