UPDATE OF EXPERIMENT E05-110:
MEASUREMENT OF QUASI-ELASTIC TRANSVERSE AND LONGITUDINAL RESPONSE FUNCTIONS
IN THE RANGE $0.55 < |q| < 1.0 \text{ GeV/c}$
OUTLINE

- Experimental motivation
- Experimental procedure
- Recent developments
- Remaining work
COULOMB SUM RULE

Inclusive electron scattering cross-section:

\[
\frac{d^2\sigma}{d\Omega d\omega} = \sigma_{\text{Mott}} \left[ \frac{q^4}{|q|^4} R_L(\omega, |q|) + \left( \frac{q^2}{2|q|^2} + \tan^2 \frac{\theta}{2} \right) R_T(\omega, |q|) \right]
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- Scattering response due to charge properties
- Scattering response due to magnetic properties
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Coulomb Sum Rule definition:

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S_L(|q|) = \int_{\omega^+} |q| \frac{R_L(\omega, |q|)}{Z\tilde{G}_{Ep}^2(Q^2) + N\tilde{G}_{En}^2(Q^2)}
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If one integrates the charge response divided by the total charge form factor over all available virtual photon energies, naively one might expect the integral to go to unity.
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QUASI-ELASTIC SCATTERING

\[ S_L(|q|) = \int_{\omega^+} |q| \ d\omega \ \frac{R_L(\omega, |q|)}{Z \tilde{G}_{Ep}^2(Q^2) + N \tilde{G}_{En}^2(Q^2)} \]

We want to integrate above the coherent elastic peak:
Quasi-elastic is “elastic” scattering on constituent nucleons inside nucleus.

- Quasi-elastic scattering at intermediate \( Q^2 \) is the region of interest for E05-110:
  - Nuclei investigated:
    - \(^4\text{He}\)
    - \(^{12}\text{C}\)
    - \(^{56}\text{Fe}\)
    - \(^{256}\text{Pb}\)
First group of experiments from Saclay, SLAC, and Bates show a quenching of $S_L$ consistent with medium modified form-factors.

$|q_{\text{eff}}|$ is $|q|$ corrected for a nuclei dependent mean coulomb potential. Methodology agreed on by Andrea Aste, Steve Wallace and John Tjon.
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Very little data above \(|q|\) of 600 MeV/c, where the cleanest signal of medium effects should exist!

Difficult to extract without large angle and energy coverage.

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EXPERIMENTAL DESIGN

- Need $R_L$ → Use Rosenbluth separation!

\[ \text{Slope} = \frac{Q^4}{q^4} R_L \]
\[ \text{Intercept} = \frac{Q^2}{2q^2} R_T \]

- Experiment run at 4 angles per target: 15, 60, 90, 120 degs. Very large lever arm for precise calculation of $R_L$!

- Need data for each angle at a constant $|q|$ over an $\omega$ range starting above the elastic peak up to $|q|$.

- When running a single arm experiment with fixed beam energy and scattering angle, $|q|$ is NOT constant over your momentum acceptance.

- Need to take data at varying beam energies, and “map-out” $|q|$ and $\omega$ space.
If one wants to measure from 100 to 600 MeV \( \omega \) at constant \( |q| = 650 \text{ MeV/c} \)
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- Take data at different beam energies, and interpolate to determine cross-section at constant $|q|$.
EXPERIMENTAL DESIGN

- If one wants to measure from 100 to 600 MeV $\omega$ at constant $|q| = 650$ MeV/c
- Take data at different beam energies, and interpolate to determine cross-section at constant $|q|$.
- $|q|$ can be selected between 550 and 1000 MeV/c

Repeat this “mapping” for 60, 90, and 120 degree spectrometer central angles.
EXPERIMENTAL SPECIFICS

- **E05-110:**
  - Data taken from October 23rd 2007 to January 16th 2008
  - 4 central angle settings: 15, 60, 90, 120 degs.
  - Many beam energy settings: 0.4 to 4.0 GeV
  - Many central momentum settings: 0.1 to 4.0 GeV
  - LHRS and RHRS independent (redundant) measurements for most settings
  - 4 targets: $^4\text{He}$, $^{12}\text{C}$, $^{56}\text{Fe}$, $^{256}\text{Pb}$. 
Both LHRS and RHRS agree well:
- with world data on $^{12}$C elastic form factors
- with each other for $^{56}$Fe quasi-elastic cross-section.
- Each spectrometer arm is an independent measurement.
- Agreement shows a good handle on acceptance and radiative corrections.

Analysis by Hamza Atac, Temple Graduate Student
Preliminary Results

56Fe Longitudinal Response Function

Analysis by Dr. Yoomin Oh
PhD Graduate of Seoul National University

$S_L^{\text{measured}} = 0.592 \pm 0.007 \pm 0.071$

- $R_L$ has been calculated with precision at low $\omega$. 

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PRELIMINARY RESULTS

$^{56}$Fe LONGITUDINAL RESPONSE FUNCTION

$S_L^{measured} = 0.592 \pm 0.007 \pm 0.071$

- $R_L$ has been calculated with precision at low $\omega$.
- at larger $\omega$, where $R_L$ becomes small, there is a high sensitivity to the quality of the linear fit for Rosenbluth separation.
- Recent efforts have been focused at systematically investigating these large $\omega$ bins.

Analysis by Dr. Yoomin Oh
PhD Graduate of Seoul National University
RECENT DEVELOPMENTS

- Interpolation of $|q|$
- Rosenbluth separation can be sensitive to the results of the interpolation*. 

Note: The radiative corrections also use interpolation techniques, but are not very sensitive to the method used.

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RECENT DEVELOPMENTS

- Interpolation of $|q|$

  - Rosenbluth separation can be sensitive to the results of the interpolation*.

- Currently investigating 120 degree data where Rosenbluth looks most non-linear.

Note: The radiative corrections also use interpolation techniques, but are not very sensitive to the method used.
RECENT DEVELOPMENTS

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  - Could go along a constant $\omega$ line. Not very useful.
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  - Could go along a constant $\omega$ line. Not very useful.
  - Better: use a constant $y$ line, which will follow the trend of quasi-elastic peak.
  - Alternative: use a constant $W$ line, which should follow the $\Delta$ peak.
  - or even a combination of $y$ and $W$. 
RECENT DEVELOPMENTS

- Alternative methods for interpolation of $|q|$
  - Neural Network can predict cross-section based on training sample and relevant kinematic variables like $|q|$, $\omega$, $y$ and $W$.
  - “Kriging” statistical estimator, useful for mapping of 2D of trends.

Sample neural network map from data
RECENT DEVELOPMENTS

- Backward angle data:

Analysis by Hamza Atac, Temple Graduate Student
RECENT DEVELOPMENTS

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  - At low $\omega$, cross-section seems to hit a floor where it should continue to drop to zero.
  - Early investigations seem to suggest a combination of backgrounds.
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  - At low $\omega$, cross-section seems to hit a floor where it should continue to drop to zero.
  - Early investigations seem to suggest a combination of backgrounds.

Still, largest uncertainties in $R_L$ calculation come from high $\omega$ region.
RECENT DEVELOPMENTS – OPTICS

- A large range of central-momentum settings for spectrometer arms was used.
  - **High momentum optics** (1100 MeV/c) were well optimized using standard procedures.
  - **Low momentum optics** (400 MeV/c) are proving difficult to calibrate:
    - No pointing survey information (surveys were not performed for every new spectrometer setting because of the large number of setting changes)
    - NMR probe doesn’t work well below 450 MeV/c, and gaussmeter has larger uncertainties.
  - **Intermediate momentum optics** runs:
    - No pointing survey information.
    - Raster was on!

---

**High momentum 1100 MeV/c**

\[ \Delta \pm \sigma = (-0.0 \pm 2.2) \times 10^{-4} \]

**Factor of 2 difference in the Delta scan peak width**

**Low momentum 400 MeV/c**

\[ \Delta \pm \sigma = (1.4 \pm 4.2) \times 10^{-4} \]

Analysis by Kai Jin, University of Virginia Graduate Student
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**High ω analysis sensitive to low momentum optics!!**

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![Graph](image-url)
CONCLUSIONS AND WORK LEFT:

- Most regions of $S_L$ calculation are well understood.
  - Working systematically through all contributions to $R_L$
    - carefully investigating the large $\omega$ region where Rosenbluth separation is most sensitive.
- Completing studies and calibration of low and intermediate momentum optics.
- Final results are just around the corner (fingers crossed).
VARIOUS DEFINITIONS AND CORRECTIONS

Basic kinematic definitions:

\[ \epsilon(|q|, \omega, \theta) = \left[ 1 + \frac{2q^2}{q^2 - \omega^2} \tan^2 \frac{\theta}{2} \right]^{-1} \]

\[ Q^2 = q^2 - \omega^2 \]

\[ \frac{d\sigma}{d\Omega}_{\text{Mott}} = \frac{\alpha^2 \cos^2(\theta/2)}{4E^2 \sin^4(\theta/2)} \]

\[ W^2 = M_N^2 + 2M_N \omega - Q^2 \]

Relativistic correction to nucleon form-factor:

\[ \tilde{G}_E^2 = G_E^2 \frac{1 + Q^2/4M^2}{1 + Q^2/2M^2} \]
MORE OPTICS COMPARISONS

All Delta scans - momentum - 400 MeV/c
MORE OPTICS COMPARISONS

All Delta scans - momentum - 1100 MeV/c
MORE OPTICS COMPARISONS

All Delta scans - Sieve / angle - 400 MeV/c
MORE OPTICS COMPARISONS

All Delta scans - Sieve / angle - 1100 MeV/c
VERIFICATION OF RADIATIVE CORRECTIONS

$^{56}\text{Fe, } \theta = 15^\circ \text{ E} = 2448 \text{ MeV}$

Graph showing the differential cross section $\frac{d\sigma}{d\Omega dE}$ for $^{56}\text{Fe}$ at an angle of $15^\circ$ and an energy of 2448 MeV. The graph compares data before and after radiative corrections, with different markers indicating different correction methods.

Graph below showing the percentage difference $(\sigma - \sigma_{W})/\sigma$ and $(\sigma_{W}/E - \sigma_{W})/\sigma_{W}/E$ as a function of $\omega$ (in MeV).
TARGET FRAME ISSUES AT 60 DEGS

$^{56}\text{Fe, } \theta = 60, E = 645 \text{ MeV}$

LHRS
RHRS

60 degs
RHRS
PEOPLE (A LITTLE OUT OF DATE)


and

**Hall-A Collaboration**

**Students**  **Post-docs**  **Run Coordinators**  **Collaborators**  **Spokespersons**