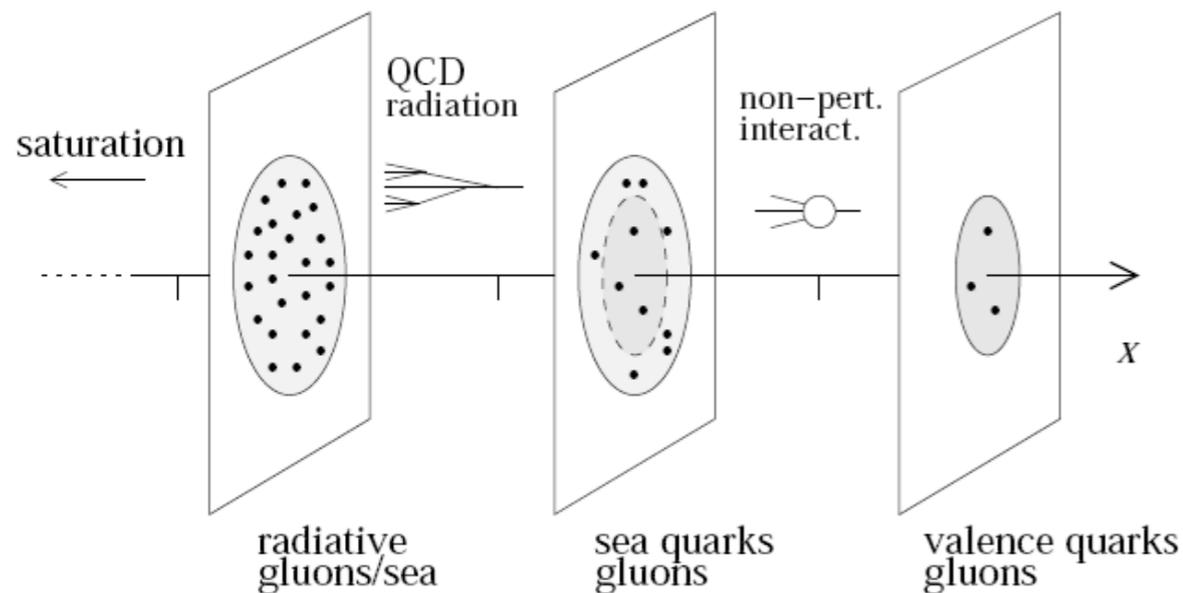
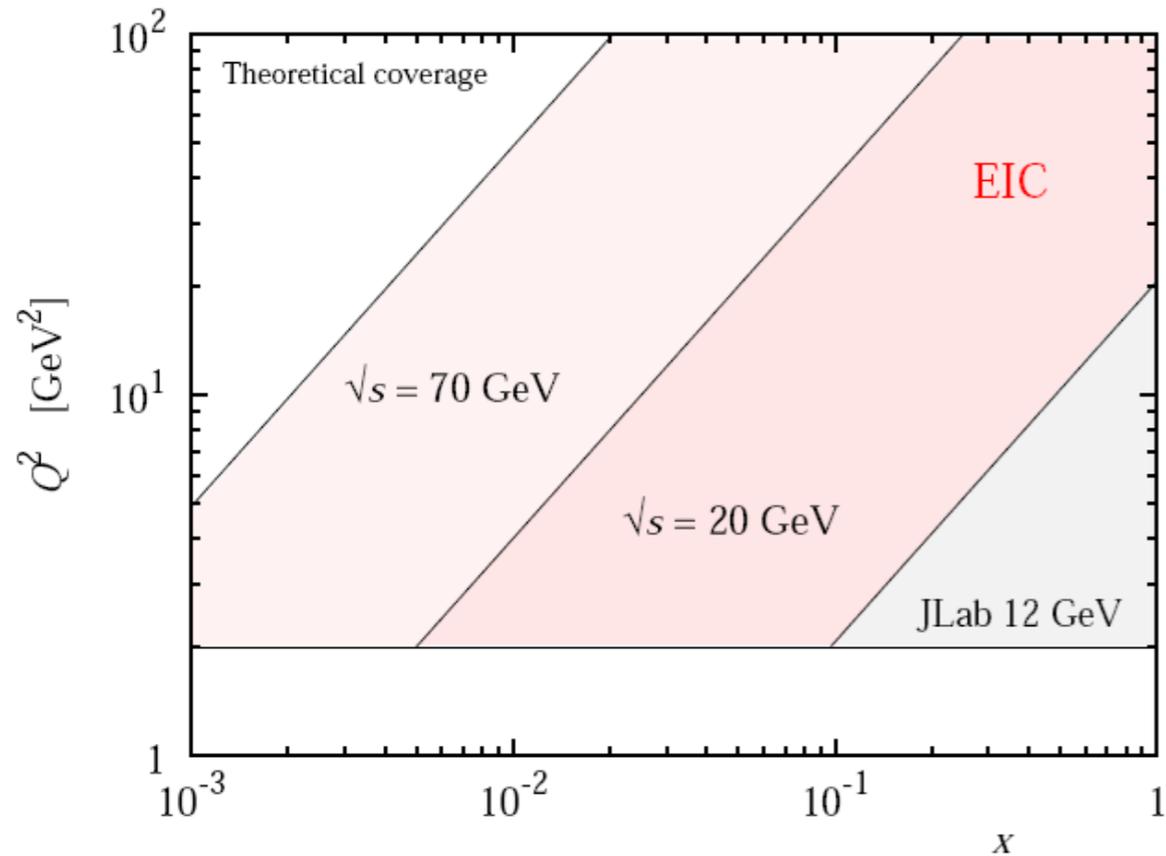


# TMD Factorization in the 12 GeV era

Alexei Prokudin



**PennState**  
Berks



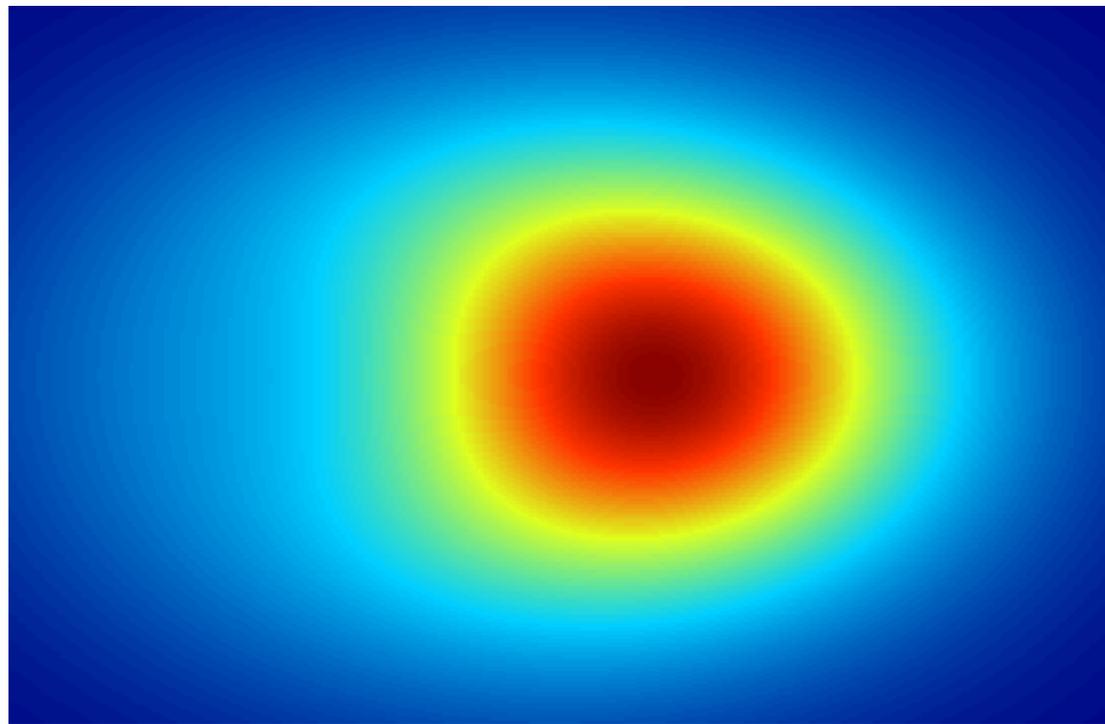
Nucleon is a many body dynamical system of quarks and gluons

Changing  $x$  we probe different aspects of nucleon wave function

How partons move and how they are distributed in space is one of the directions of development of nuclear physics

Technically such information is encoded into Generalised Parton Distributions (GPDs) and Transverse Momentum Dependent distributions (TMDs)

These distributions are also referred to as 3D (three-dimensional) distributions



The polarized proton in momentum space as “seen” by the virtual photon

Factorization theorems help us to relate functions that describe the hadron structure and the experimental observables

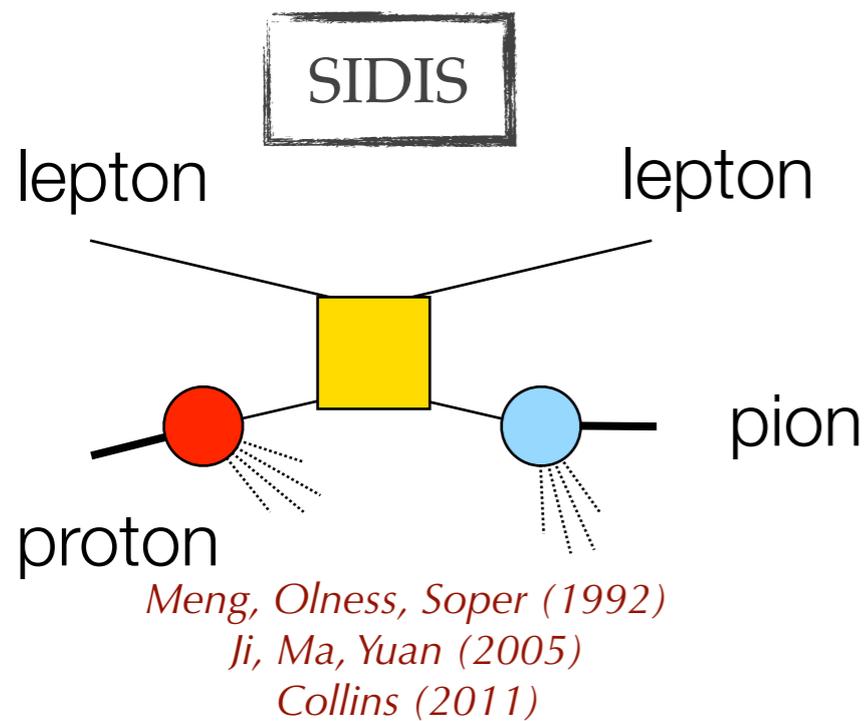
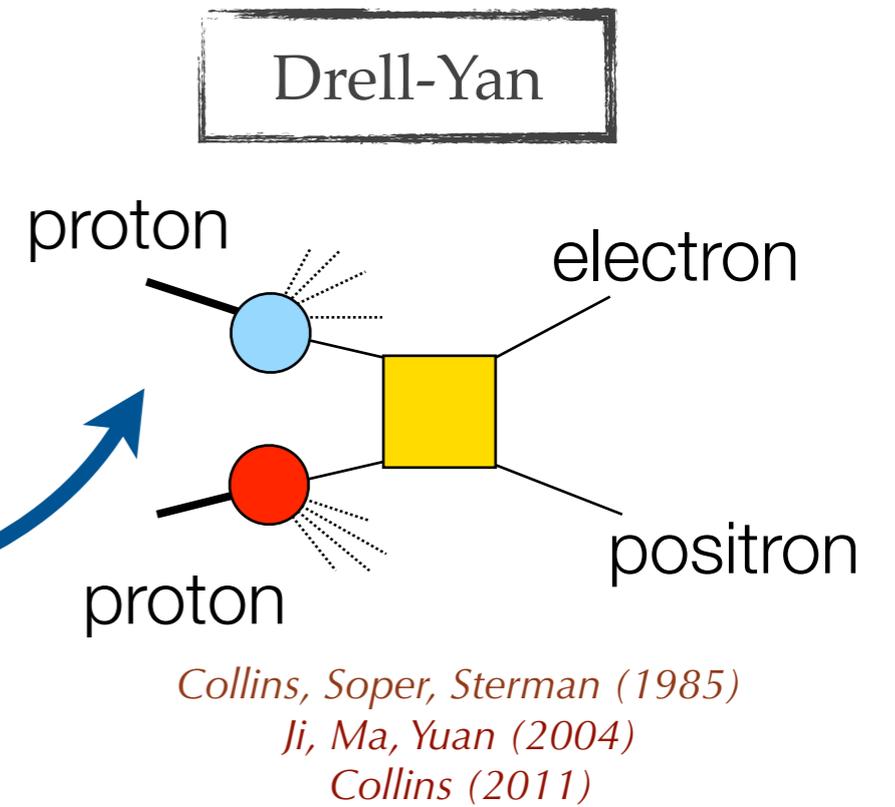
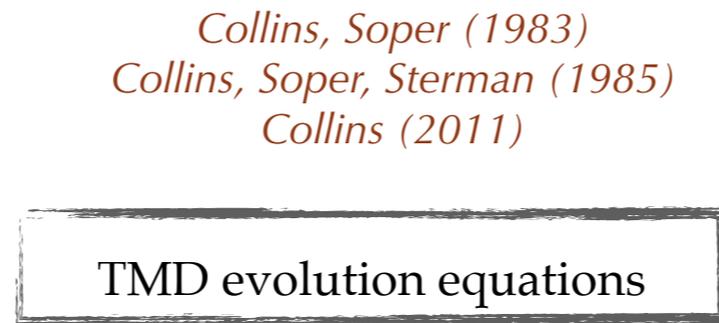
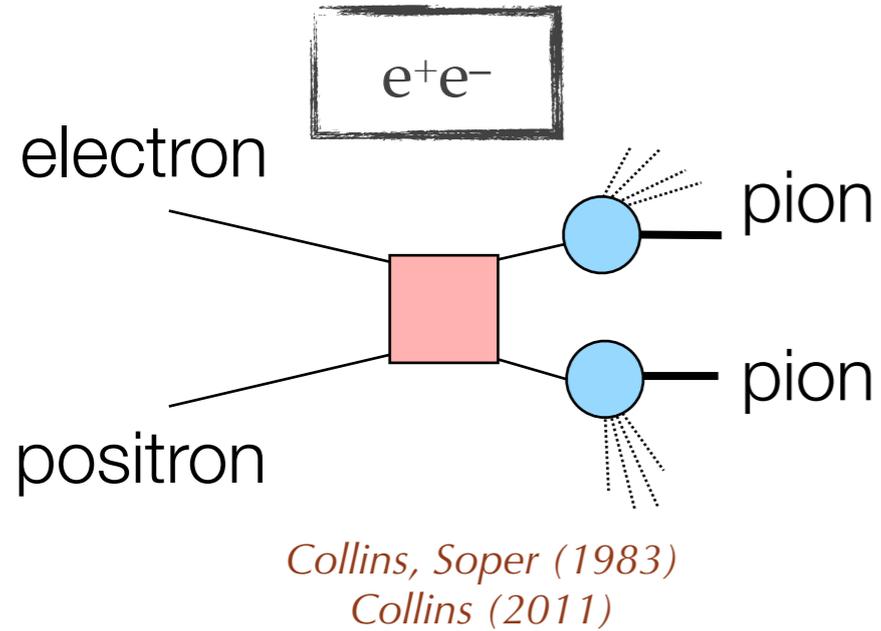
Factorization is a *controllable approximation* and the goal of theorists and phenomenologists is to test and improve the region of applicability of factorization and/or construct new factorization theorems

Hadron structure is the ultimate goal of measurements and phenomenology

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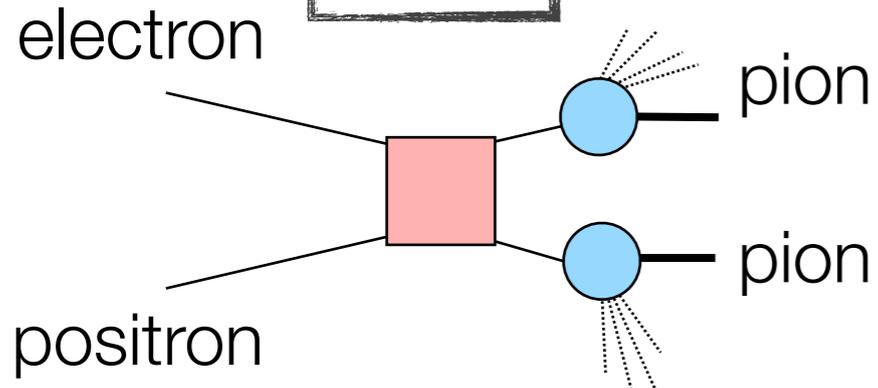
# Why TMDs, factorization, and evolution

# TMD factorization



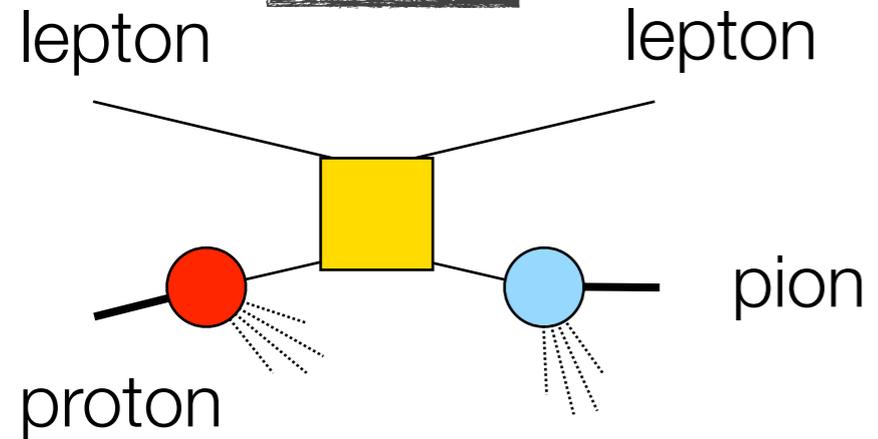
# TMD factorization

$e^+e^-$



*Collins, Soper (1983)*  
*Collins (2011)*

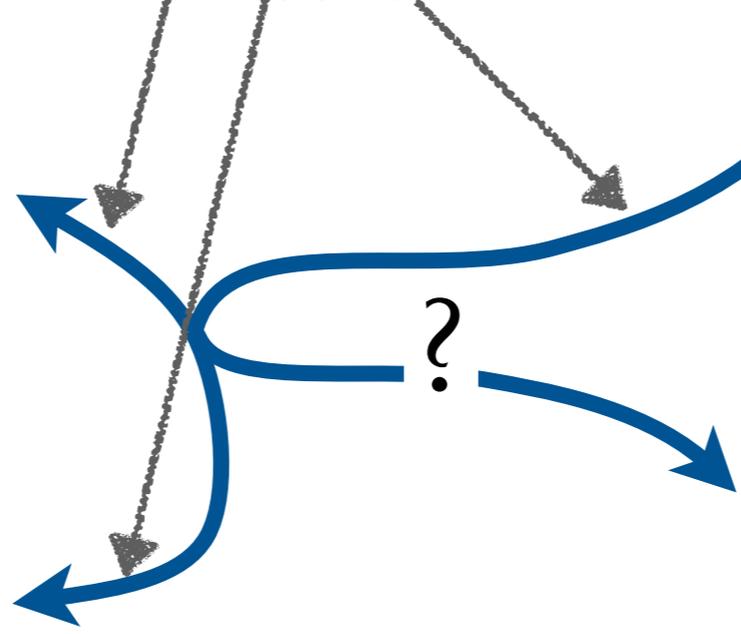
SIDIS



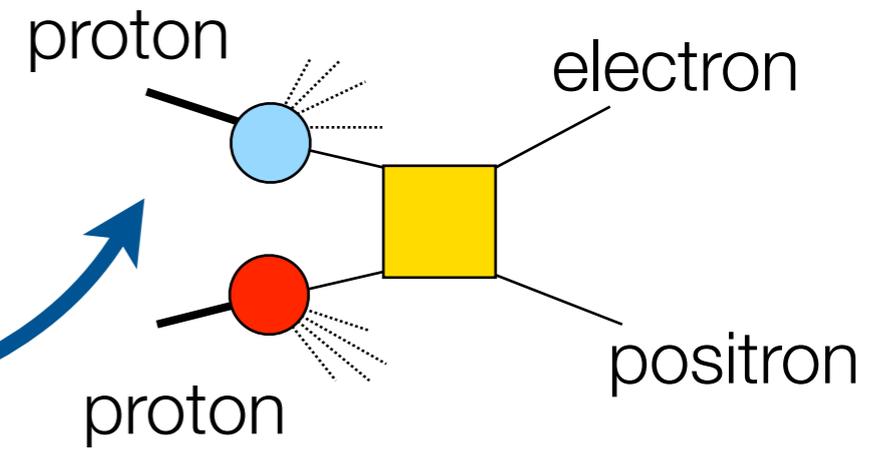
*Meng, Olness, Soper (1992)*  
*Ji, Ma, Yuan (2005)*  
*Collins (2011)*

*Collins, Soper (1983)*  
*Collins, Soper, Serman (1985)*  
*Collins (2011)*

TMD evolution equations

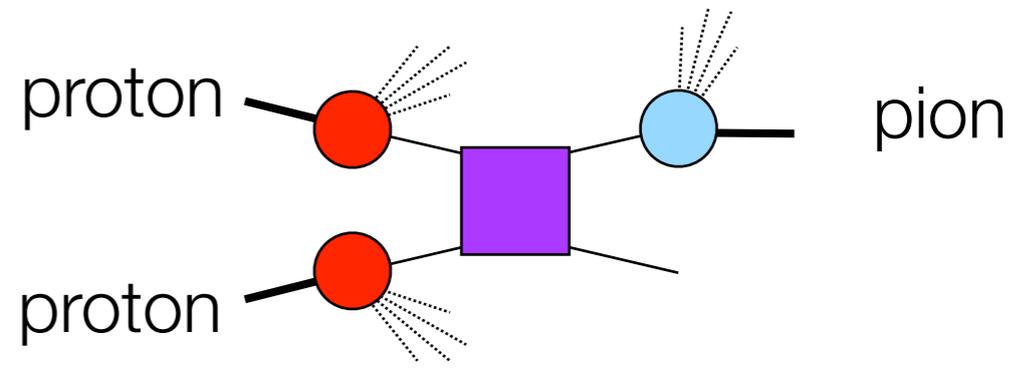


Drell-Yan



*Collins, Soper, Serman (1985)*  
*Ji, Ma, Yuan (2004)*  
*Collins (2011)*

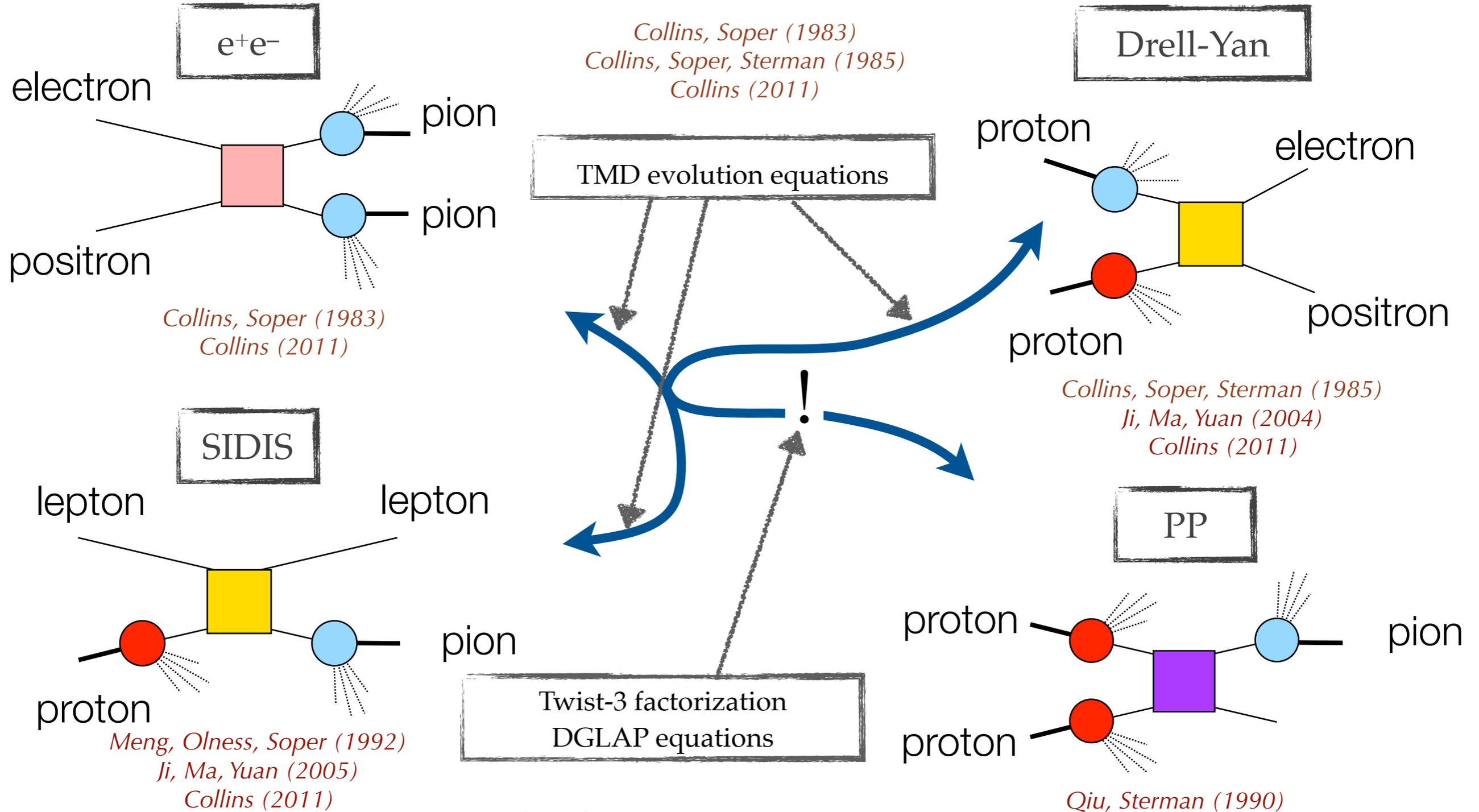
PP



*Qiu, Serman (1990)*

Only one scale is measured in PP  
TMD factorization is not applicable?

# TMD factorization



- Twist-3 functions are related to TMD via OPE
- TMD and twist-3 factorizations are related in high  $QT$  region
- Global analysis of TMDs and twist-3 is possible:  
All four processes can be used.
- Data are from HERMES, COMPASS, JLab, BaBar, Belle, RHIC, LHC, Fermilab

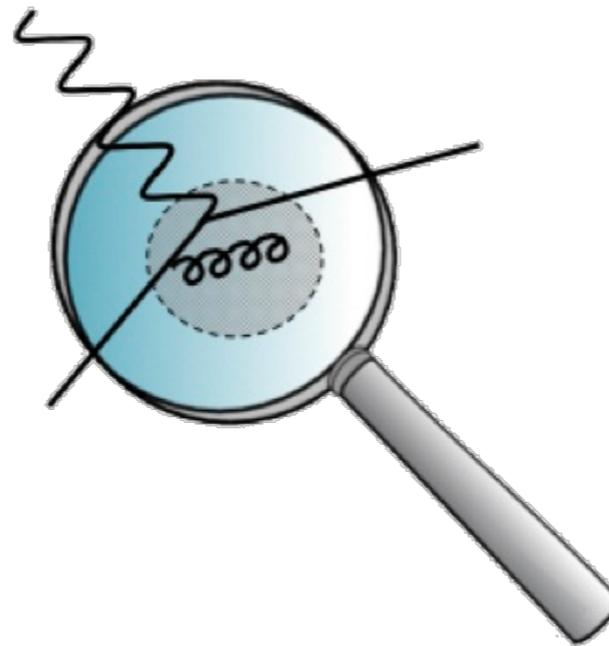
Global fit is needed.  
Work in progress

Just like collinear PDFs, TMDs also depend on the scale of the probe  
= evolution

Collinear PDFs

$$F(x, Q)$$

- ✓ DGLAP evolution
- ✓ Resum  $[\alpha_s \ln(Q^2/\mu^2)]^n$
- ✓ Kernel: purely **perturbative**



TMDs

$$F(x, k_{\perp}; Q)$$

- ✓ Collins-Soper/rapidity evolution equation
- ✓ Resum  $[\alpha_s \ln^2(Q^2/k_{\perp}^2)]^n$
- ✓ Kernel: can be **non-perturbative** when

$$k_{\perp} \sim \Lambda_{\text{QCD}}$$

$$F(x, Q_i)$$

$$\downarrow R^{\text{coll}}(x, Q_i, Q_f)$$

$$F(x, Q_f)$$

$$F(x, k_{\perp}, Q_i)$$

$$\downarrow R^{\text{TMD}}(x, k_{\perp}, Q_i, Q_f)$$

$$F(x, k_{\perp}, Q_f)$$

# TMD evolution and non-perturbative component

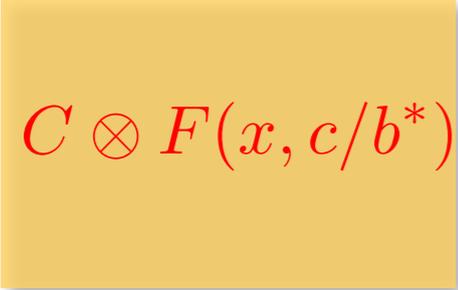
- Fourier transform back to the momentum space, one needs the whole  $b$  region (large  $b$ ): need some non-perturbative extrapolation
  - Many different methods/proposals to model this non-perturbative part

$$F(x, k_{\perp}; Q) = \frac{1}{(2\pi)^2} \int d^2b e^{ik_{\perp} \cdot b} F(x, b; Q) = \frac{1}{2\pi} \int_0^{\infty} db b J_0(k_{\perp} b) F(x, b; Q)$$

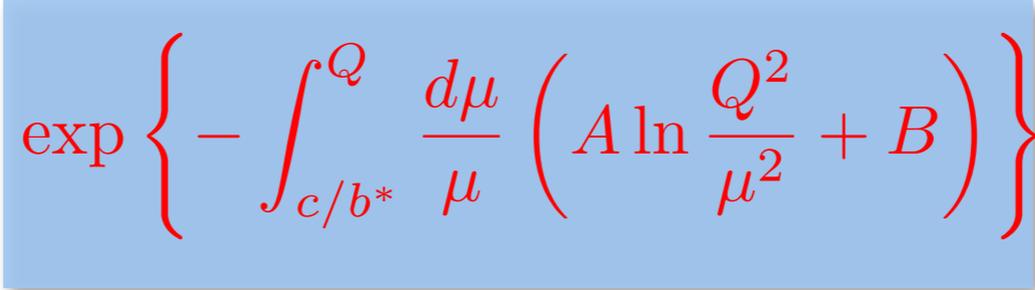
Collins, Soper, Sterman 85, ResBos, Qiu, Zhang 99, Echevarria, Idilbi, Kang, Vitev, 14, Aidala, Field, Gamberg, Rogers, 14, Sun, Yuan 14, D'Alesio, Echevarria, Melis, Scimemi, 14, Rogers, Collins, 15, Vladimirov, Scimemi 17...

- Eventually evolved TMDs in  $b$ -space

$$F(x, b; Q) \approx C \otimes F(x, c/b^*) \times \exp \left\{ - \int_{c/b^*}^Q \frac{d\mu}{\mu} \left( A \ln \frac{Q^2}{\mu^2} + B \right) \right\} \times \exp \left( -S_{\text{non-pert}}(b, Q) \right)$$



longitudinal/collinear part



transverse part



✓ Non-perturbative: fitted from data  
 ✓ The key ingredient –  $\ln(Q)$  piece is spin-independent

Since the polarized scattering data is still limited kinematics, we can use unpolarized data to constrain/extract key ingredients for the non-perturbative part

## Quark TMDs

$N \backslash q$	U	L	T
U			
L			
T			

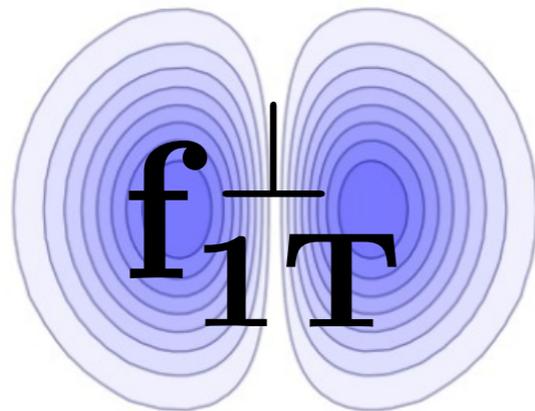
8 functions in total (at leading twist)

Each represents different aspects of partonic structure

Each depends on Bjorken- $x$ , transverse momentum, the scale

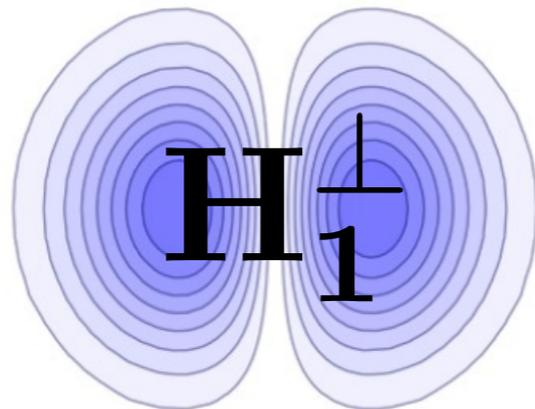
Each function is to be studied

Kotzinian (1995), Mulders, Tangerman (1995), Boer, Mulders (1998)



## Sivers function

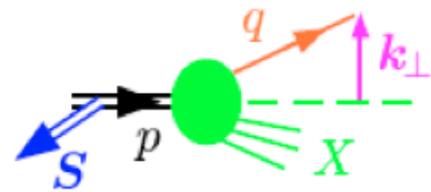
Non universal



## Collins function

Universal

Sivers function: unpolarized quark distribution inside a transversely polarized nucleon

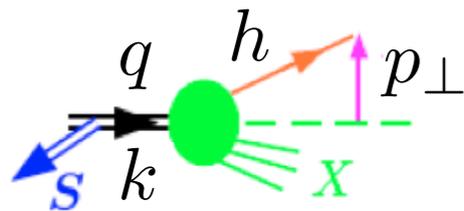


Sivers 1989

$$f_{q/h}^{\uparrow}(x, \vec{k}_{\perp}, \vec{S}) = f_{q/h}(x, k_{\perp}^2) - \frac{1}{M} f_{1T}^{\perp q}(x, k_{\perp}^2) \vec{S} \cdot (\hat{P} \times \vec{k}_{\perp})$$

Spin independent Spin dependent

Collins function: unpolarized hadron from a transversely polarized quark



Collins 1992

$$D_{q/h}(z, \vec{p}_{\perp}, \vec{S}_q) = D_{q/h}(z, p_{\perp}^2) + \frac{1}{zM_h} H_1^{\perp q}(z, p_{\perp}^2) \vec{S}_q \cdot (\hat{k} \times \vec{p}_{\perp})$$

Sivers function:  $f_{1T}^{\perp q}$  describes strength of correlation

$$\vec{S} \cdot (\hat{P} \times \vec{k}_{\perp})$$

Sivers 1989

Collins function:  $H_1^{\perp q}$  describes strength of correlation

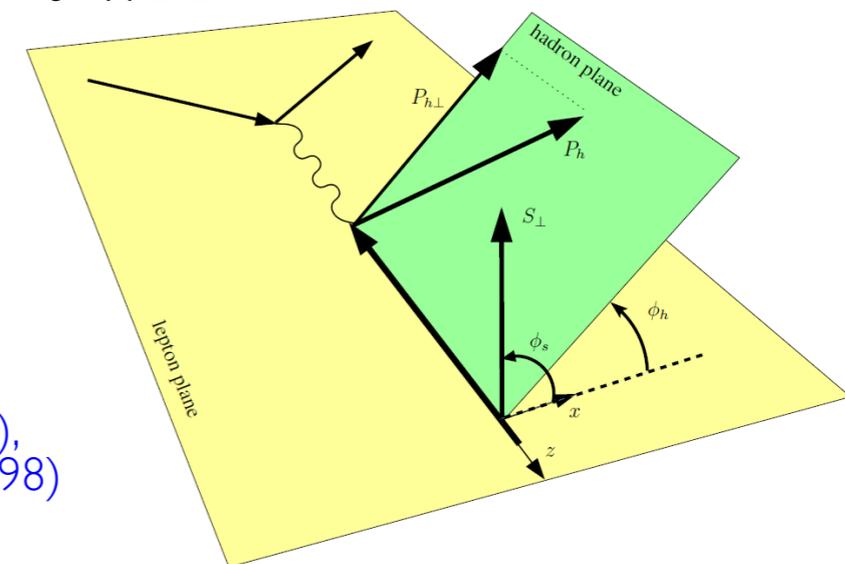
$$\vec{s}_q \cdot (\hat{k} \times \vec{p}_{\perp})$$

Collins 1992

Both functions extensively studied experimentally, phenomenologically, theoretically

Sivers function and Collins function can give rise to Single Spin Asymmetries in scattering processes. For instance in Semi Inclusive Deep Inelastic process

$$\ell P \rightarrow \ell' \pi X$$



Kotzinian (1995),  
Mulders,  
Tangerman (1995),  
Boer, Mulders (1998)

$$d\sigma(S) \sim \sin(\phi_h + \phi_S) h_1 \otimes H_1^{\perp} + \sin(\phi_h - \phi_S) f_{1T}^{\perp} \otimes D_1 + \dots$$

# Sivers function

---

Large  $-N_c$  result  $f_{1T}^{\perp u} = -f_{1T}^{\perp d}$

Pobylitsa 2003

→ Confirmed by phenomenological extractions

→ Confirmed by experimental measurements

Relation to GPDs (E) and anomalous magnetic moment

Burkardt 2002

$$f_{1T}^{\perp q} \sim \kappa^q$$

→ Predicted correct sign of Sivers asymmetry in SIDIS

→ Shown to be model-dependent

Meissner, Metz, Goeke 2007

→ Used in phenomenological extractions

Bacchetta, Radici 2011

Sum rule

Burkardt 2004

→ Conservation of transverse momentum

→ Average transverse momentum shift of a quark inside a transversely polarized nucleon

$$\langle k_T^{i,q} \rangle = \varepsilon_T^{ij} S_T^j f_{1T}^{\perp(1)q}(x)$$

$$f_{1T}^{\perp(1)q}(x) = \int d^2 k_{\perp} \frac{k_{\perp}^2}{2M^2} f_{1T}^{\perp q}(x, k_{\perp}^2)$$

→ Sum rule

$$\sum_{a=q,g} \int_0^1 dx \langle k_T^{i,a} \rangle = 0 \quad \sum_{a=q,g} \int_0^1 dx f_{1T}^{\perp(1)a}(x) = 0$$

# Sivers function

## Extractions

→ Many extractions without taking into account TMD evolution

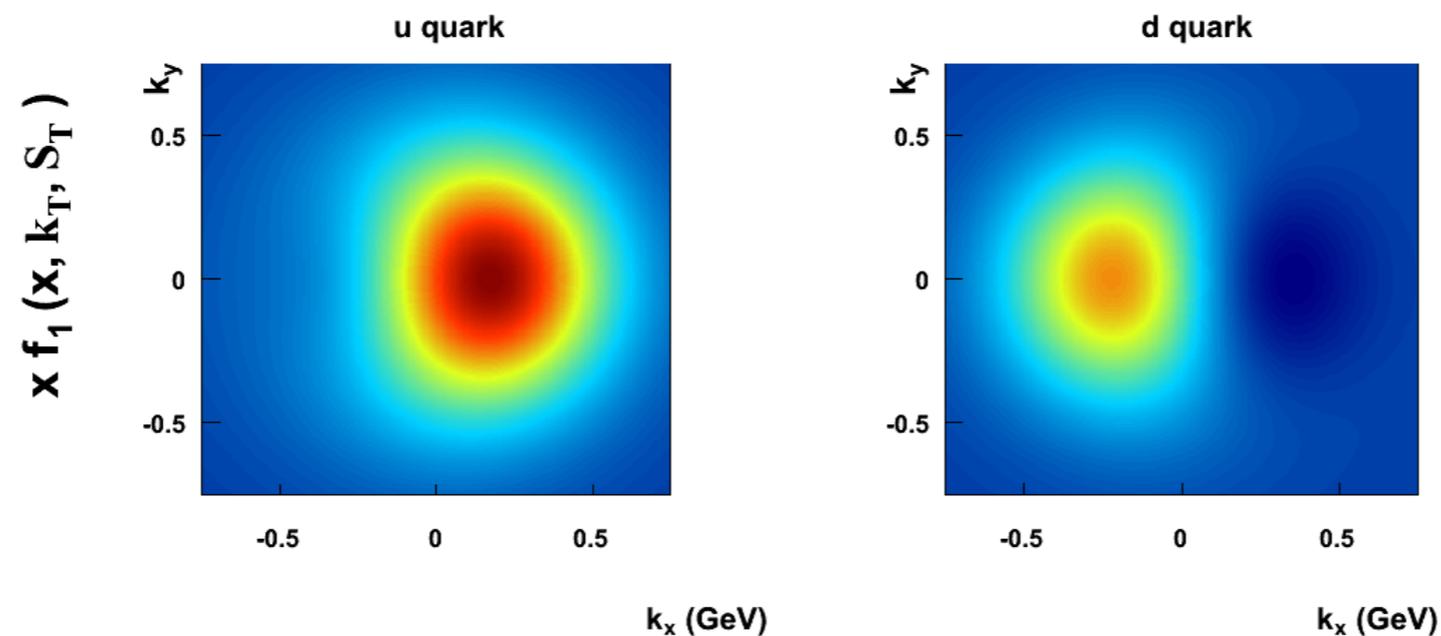
Efremov et al 2005, Vogelsang, Yuan 2005, Anselmino et al 2005,  
Collins et al 2006, Anselmino et al 2009, 2011, 2016, Bacchetta Radici 2011

→ Extractions with TMD evolution

Echevarria et al 2014, Sun Yuan 2013

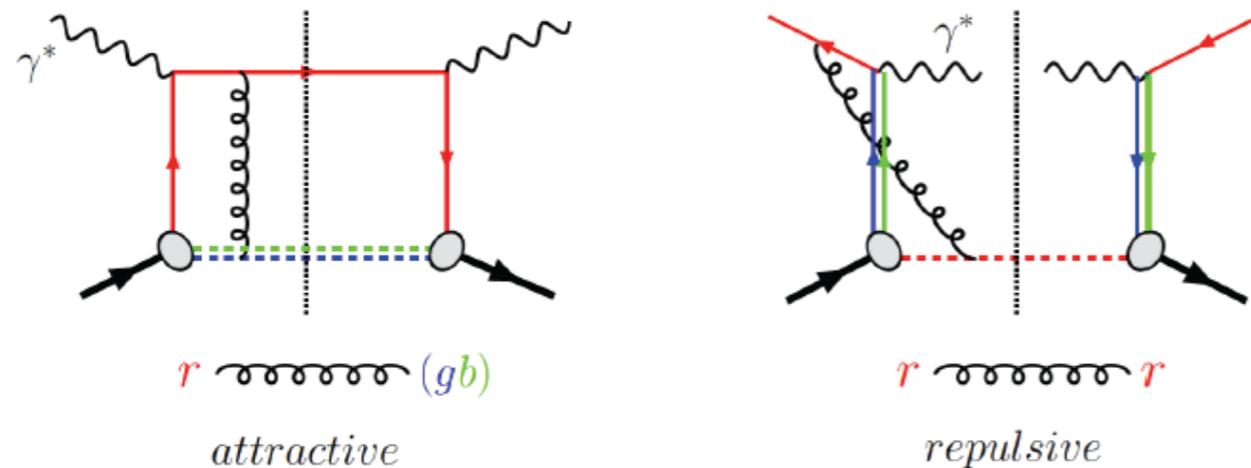
→ Relation to the tomography of the nucleon

Anselmino et al 2011



→ Agreement with the sum rule and large  $N_c$  prediction

Colored objects are surrounded by gluons, profound consequence of gauge invariance:  
Sivers function has opposite sign when gluon couple after quark scatters (SIDIS) or before quark annihilates (Drell-Yan)



Brodsky, Hwang, Schmidt;  
Belitsky, Ji, Yuan;  
Collins;  
Boer, Mulders, Pijlman;  
Kang, Qiu;  
Kovchegov, Sievert;  
etc

$$f_{1T}^{\perp \text{SIDIS}} = -f_{1T}^{\perp \text{DY}}$$

Crucial test of TMD factorization and collinear twist-3 factorization  
Several labs worldwide aim at measurement of Sivers effect in Drell-Yan

BNL, CERN, GSI, IHEP, JINR, FERMILAB etc

Barone et al., Anselmino et al., Yuan, Vogelsang, Schlegel et al., Kang, Qiu, Metz, Zhou etc

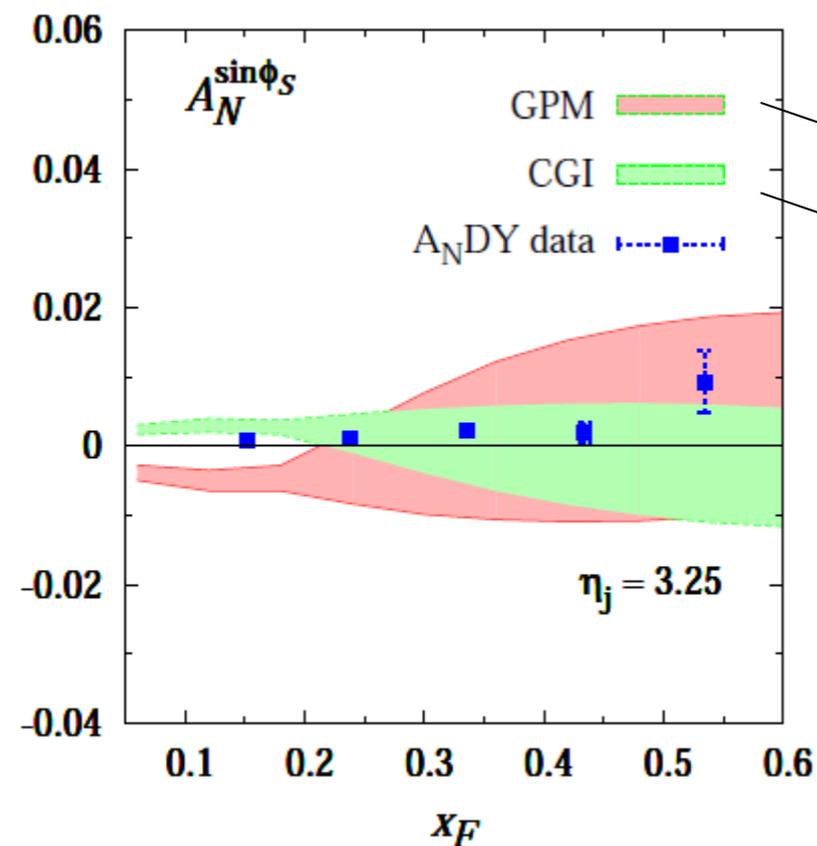
The verification of the sign change is an NSAC (DOE and NSF) milestone

# Process dependence of Sivers function

→ Indication on process dependence of Sivers functions from analysis of  $A_N$  in  $\ell N^\uparrow \rightarrow \ell X$

Metz et al 2012

→ Indication on process dependence from AnDY data on  $A_N$  in  $p^\uparrow p \rightarrow jet X$



Gamberg, Kang, AP 2013  
D'Alesio et al 2013

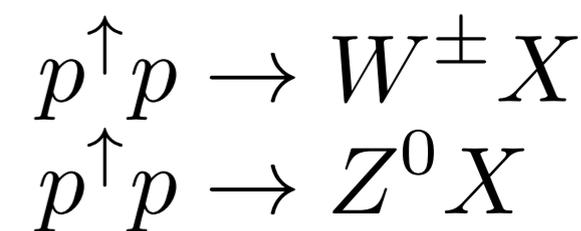
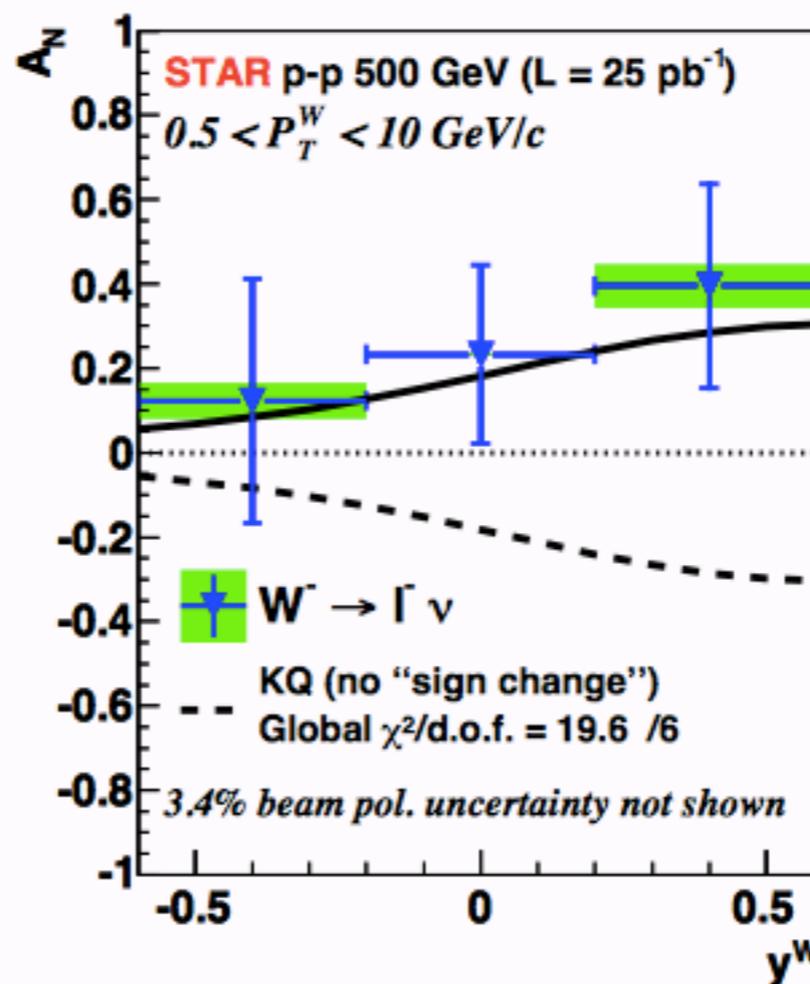
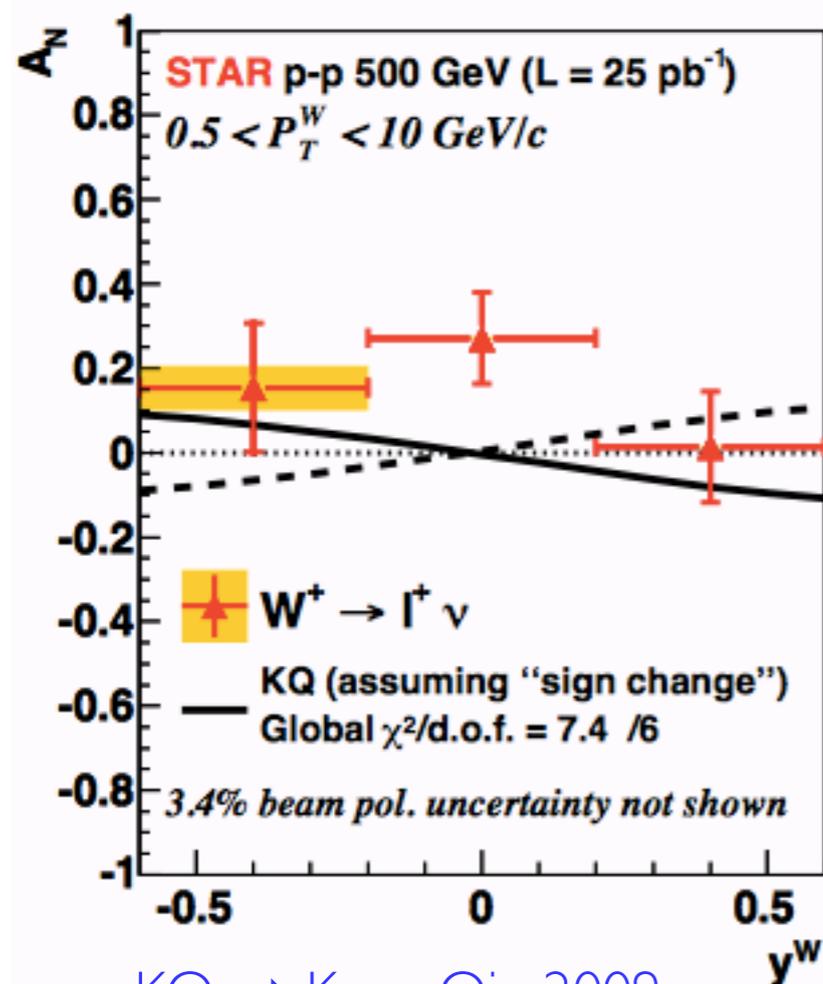
No sign change  
Sign change

# Process dependence of Sivers function

STAR 2016

→ First experimental hint on the sign change:  $A_N$  in W and Z production

STAR Collab. Phys. Rev. Lett. 116, 132301 (2016)



KQ → Kang, Qiu 2009

→ No sign change

$$\chi^2/d.o.f \sim 1.2$$

$$\chi^2/d.o.f \sim 3.2$$

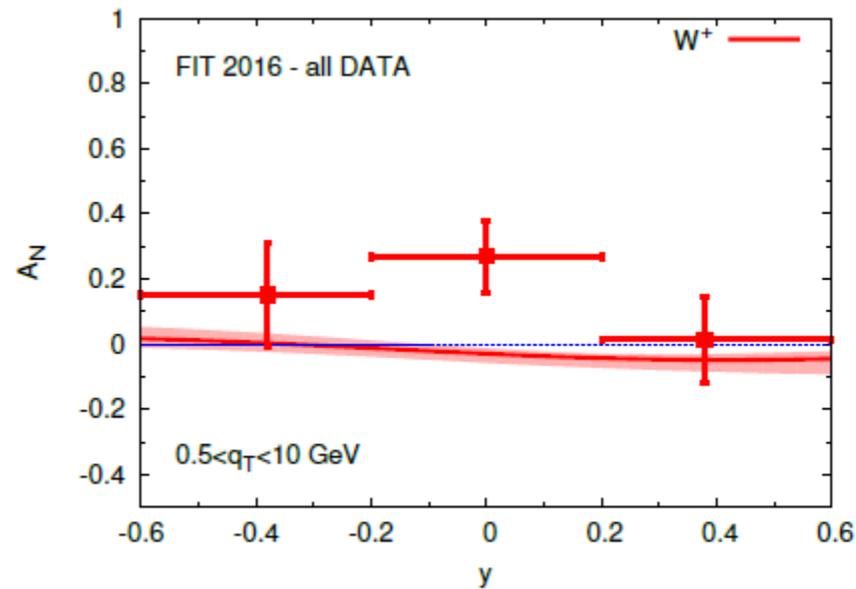
→ Large uncertainties of predictions

→ No antiquark Sivers functions

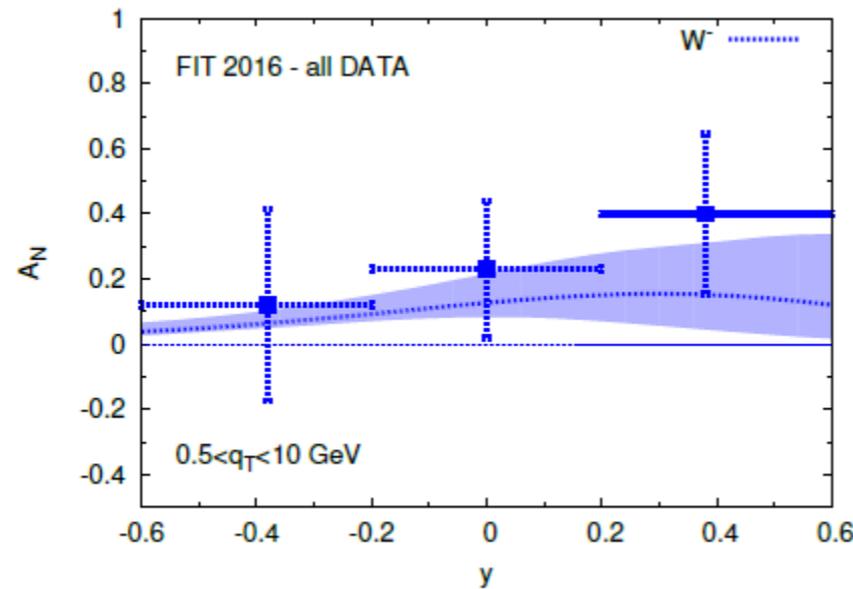
# Process dependence of Sivers function

Anselmino et al 2016

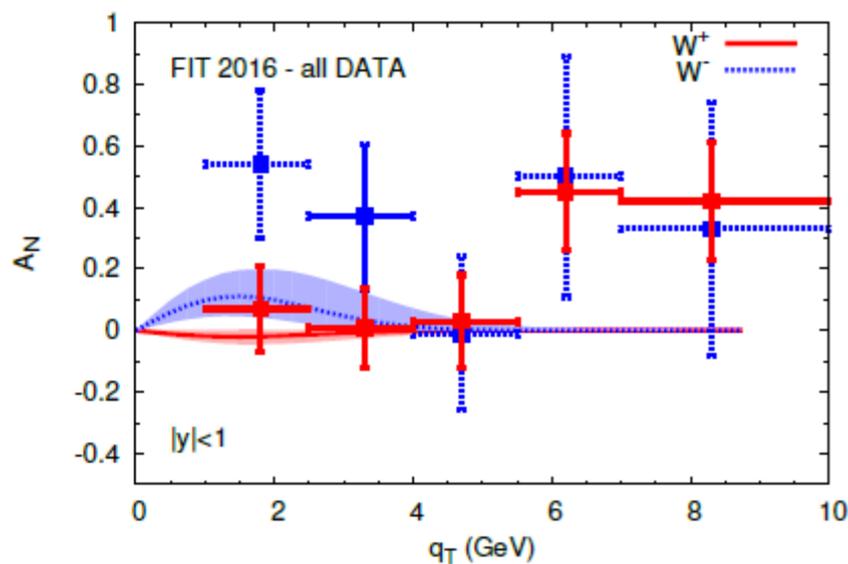
→ First experimental hint on the sign change:  $A_N$  in  $W$  and  $Z$  production



(a)



$$p^\uparrow p \rightarrow W^\pm X$$

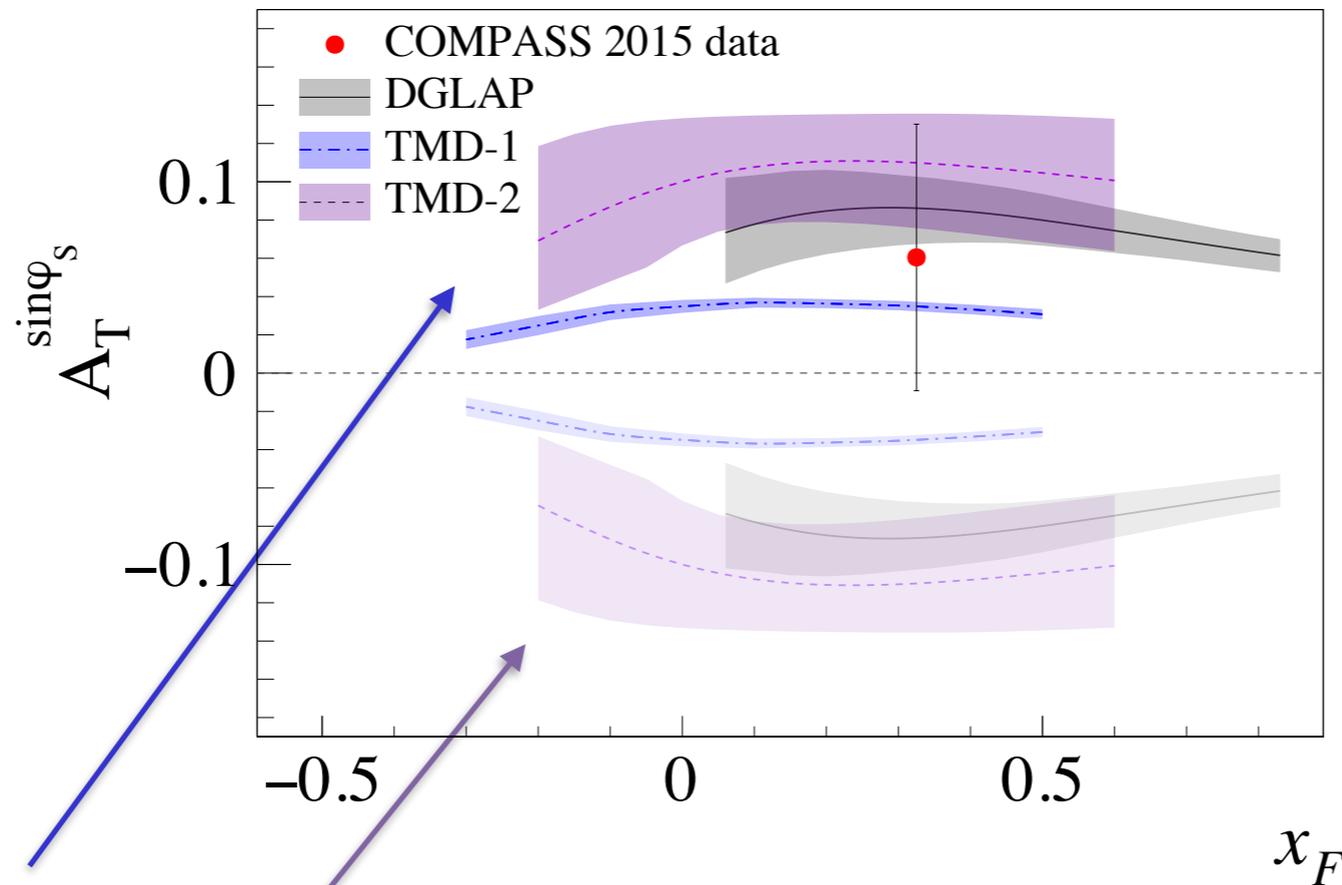


- Results with sign change
- No TMD evolution
- Antiquark Sivers functions included
- STAR results hint on sign change

# Process dependence of Sivers function

COMPASS 2017

→ First experimental hint on the sign change in Drell-Yan



→ Sign change

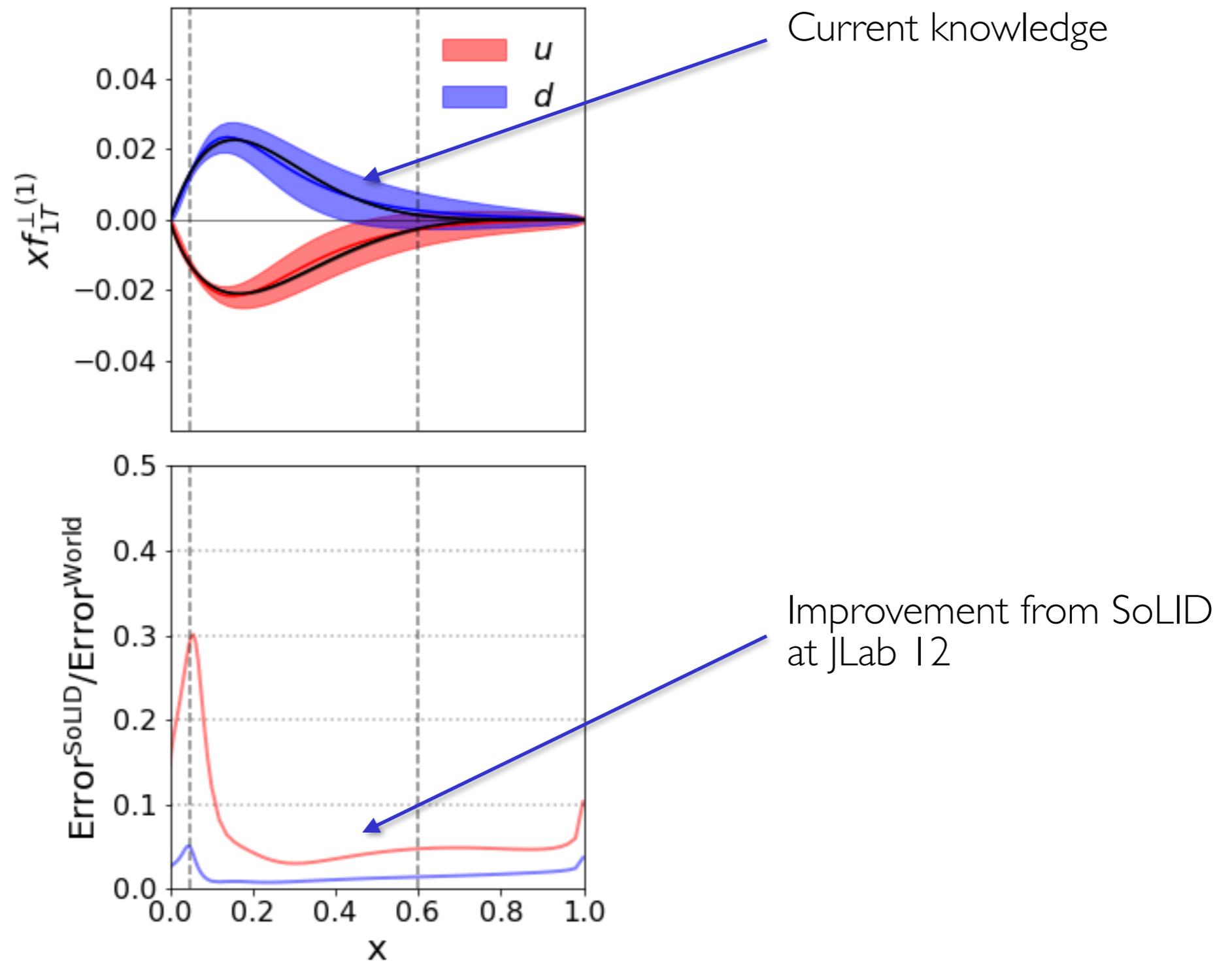
→ No sign change

→ COMPASS results hint on sign change

# Sivers function

Expectation of JLab 12

Sato et al 2018



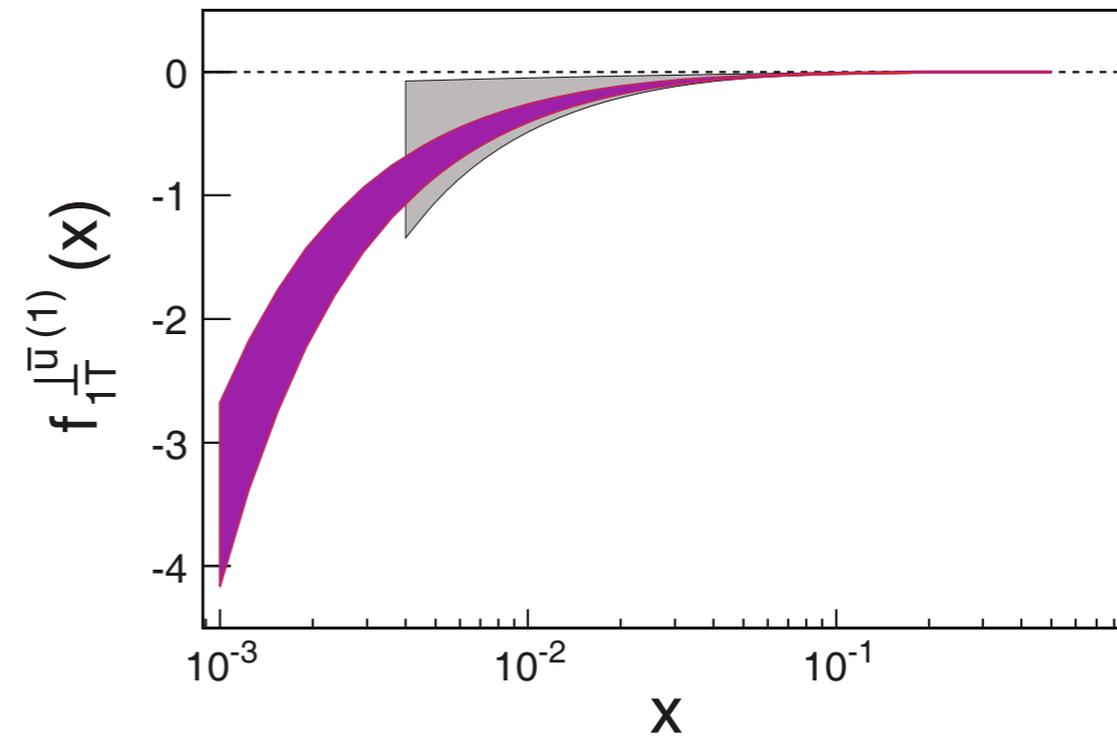
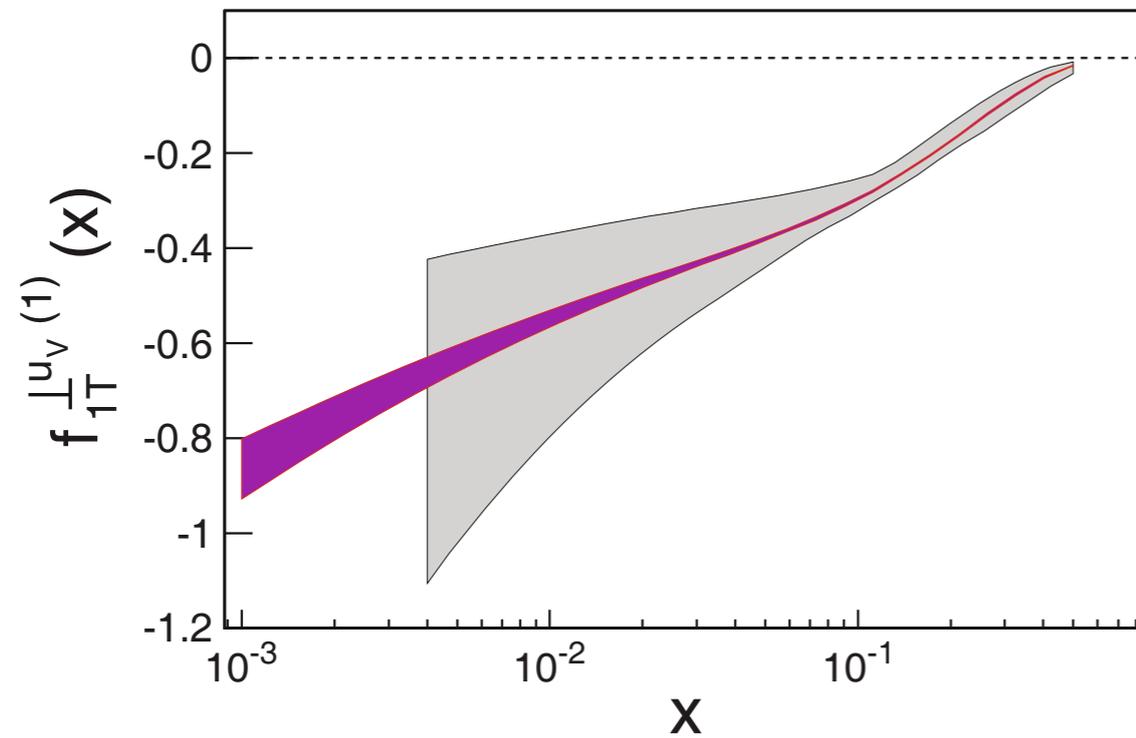
Current knowledge

Improvement from SoLID at JLab 12

# Sivers function

Expectation of EIC

AP 2010



Update of this estimate is needed and work is in progress

# Collins function

Schafer-Teryaev sum rule

Schafer Teryaev 1999  
Meissner, Metz, Pitonyak 2010

→ Conservation of transverse momentum

$$\langle P_T^i(z) \rangle \sim H_1^{\perp(1)}(z) \quad H_1^{\perp(1)}(z) = \int d^2 p_{\perp} \frac{p_{\perp}^2}{2z^2 M_h^2} H_1^{\perp}(z, p_{\perp}^2)$$

→ Sum rule

$$\sum_h \int_0^1 dz \langle P_T^i(z) \rangle = 0$$

→ If only pions are considered  $H_1^{\perp fav}(z) \sim -H_1^{\perp unf}(z)$

Universality of TMD fragmentation functions

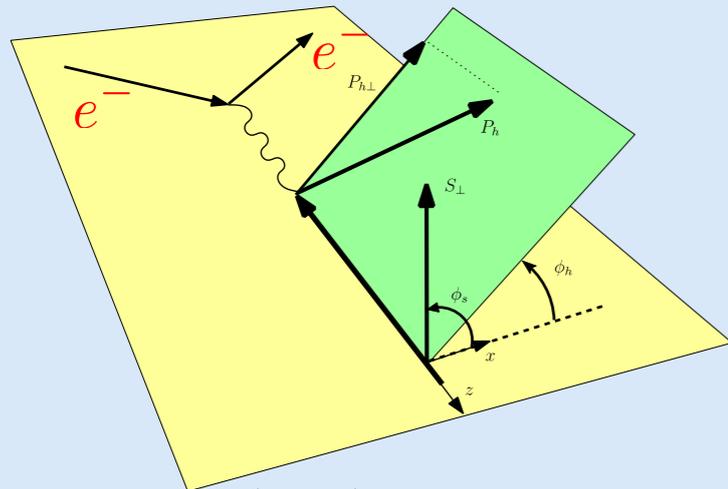
Metz 2002, Metz, Collins 2004, Yuan 2008  
Gamberg, Mukherjee, Mulders 2011  
Boer, Kang, Vogelsang, Yuan 2010

$$H_1^{\perp}(z)|_{SIDIS} = H_1^{\perp}(z)|_{e^+e^-} = H_1^{\perp}(z)|_{pp}$$

→ Very non trivial results

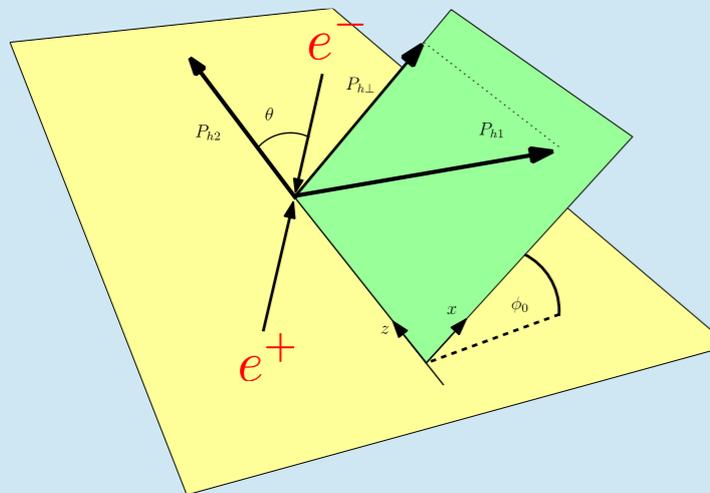
→ Agrees with phenomenology, allows global fits

- SIDIS and e+e-: combined global analysis



$$F_{UT}^{\sin(\phi_h + \phi_s)} \sim \underbrace{h_1(x_B, k_\perp)}_{\text{transversity}} H_1^\perp(z_h, p_\perp)_{\text{Collins function}}$$

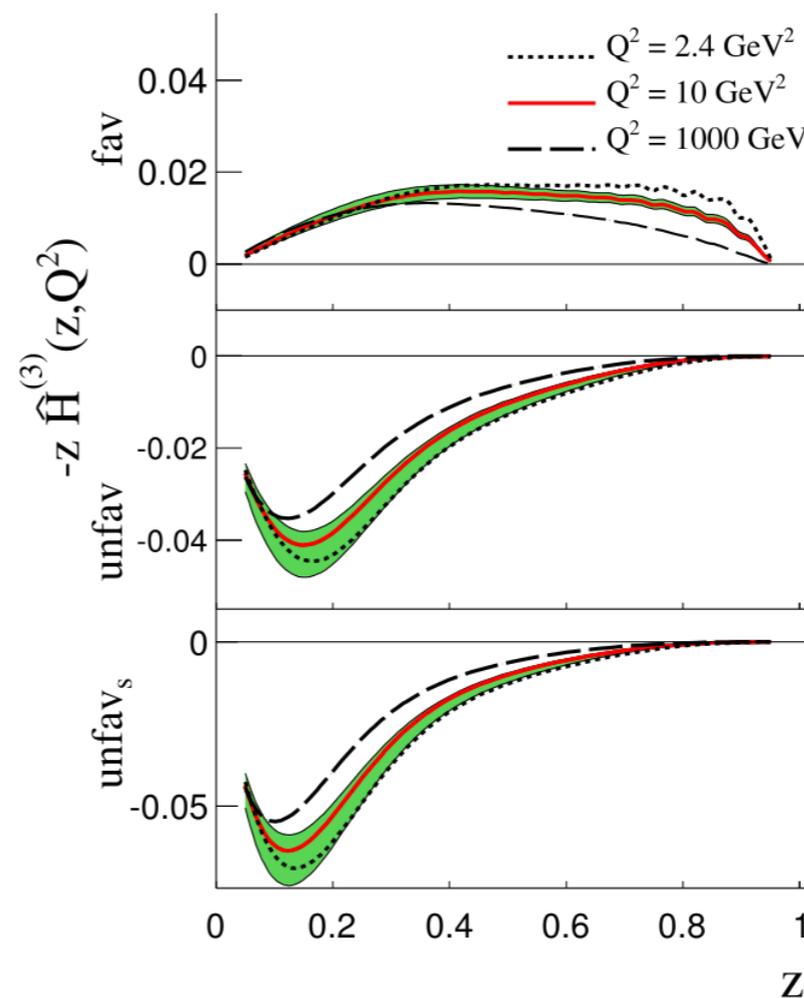
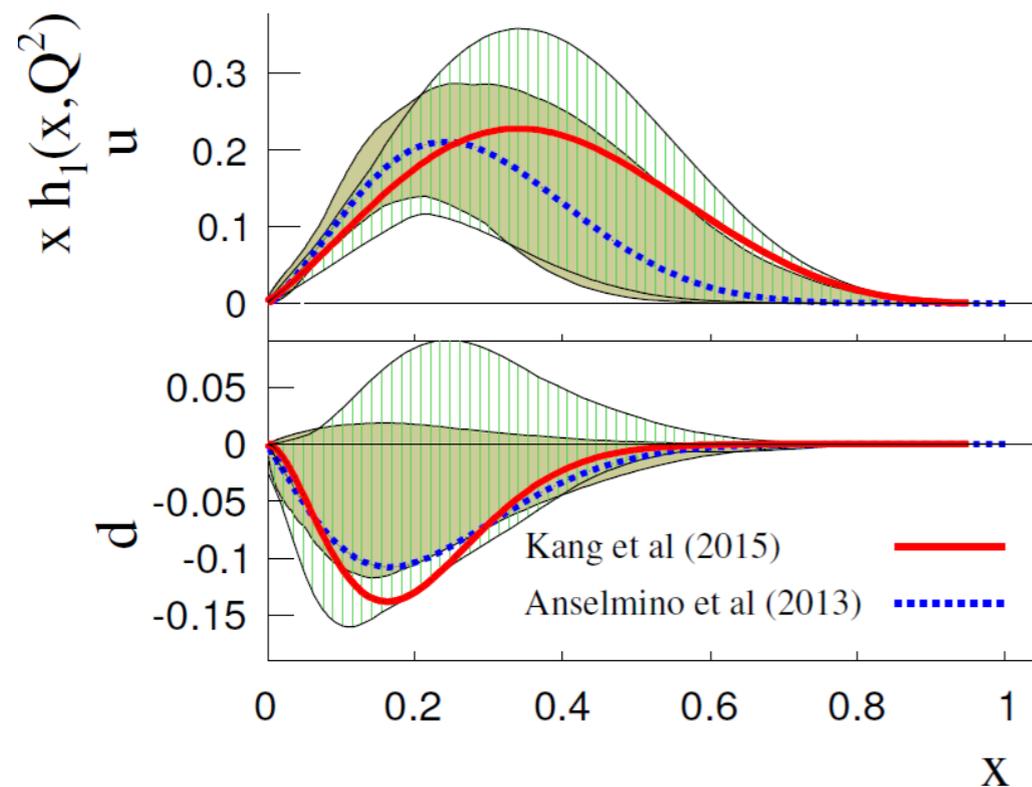
$$\frac{d\sigma(S_\perp)}{dx_B dy dz_h d^2 P_{h\perp}} = \sigma_0(x_B, y, Q^2) \left[ F_{UU} + \sin(\phi_h + \phi_s) \frac{2(1-y)}{1+(1-y)^2} F_{UT}^{\sin(\phi_h + \phi_s)} + \dots \right]$$



$$Z_{\text{collins}}^{h_1 h_2} \sim \underbrace{H_1^\perp(z_1, p_{1\perp})}_{\text{Collins function}} \underbrace{H_1^\perp(z_2, p_{2\perp})}_{\text{Collins function}}$$

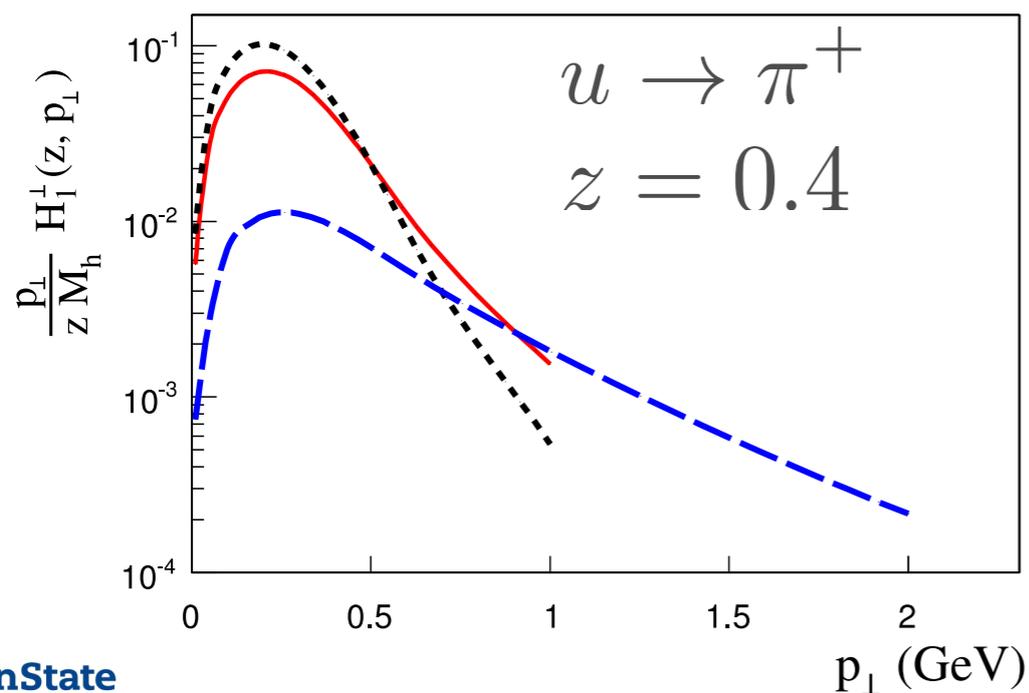
$$\frac{d\sigma^{e^+ e^- \rightarrow h_1 h_2 + X}}{dz_{h1} dz_{h2} d^2 P_{h\perp} d \cos \theta} = \frac{N_c \pi \alpha_{\text{em}}^2}{2Q^2} \left[ (1 + \cos^2 \theta) Z_{uu}^{h_1 h_2} + \sin^2 \theta \cos(2\phi_0) Z_{\text{collins}}^{h_1 h_2} \right]$$

- Fitted quark transversity and Collins function:  $x(z)$ -dependence



fav :  $u \rightarrow \pi^+$   
unfav :  $d \rightarrow \pi^+$   
unfav<sub>s</sub> :  $s \rightarrow \pi$

- Collins function:  $p_T$ -dependence



Compatible with LO extraction  
Anselmino et al 2009, 2013, 2015

Precision of extraction depends on precision of calculations

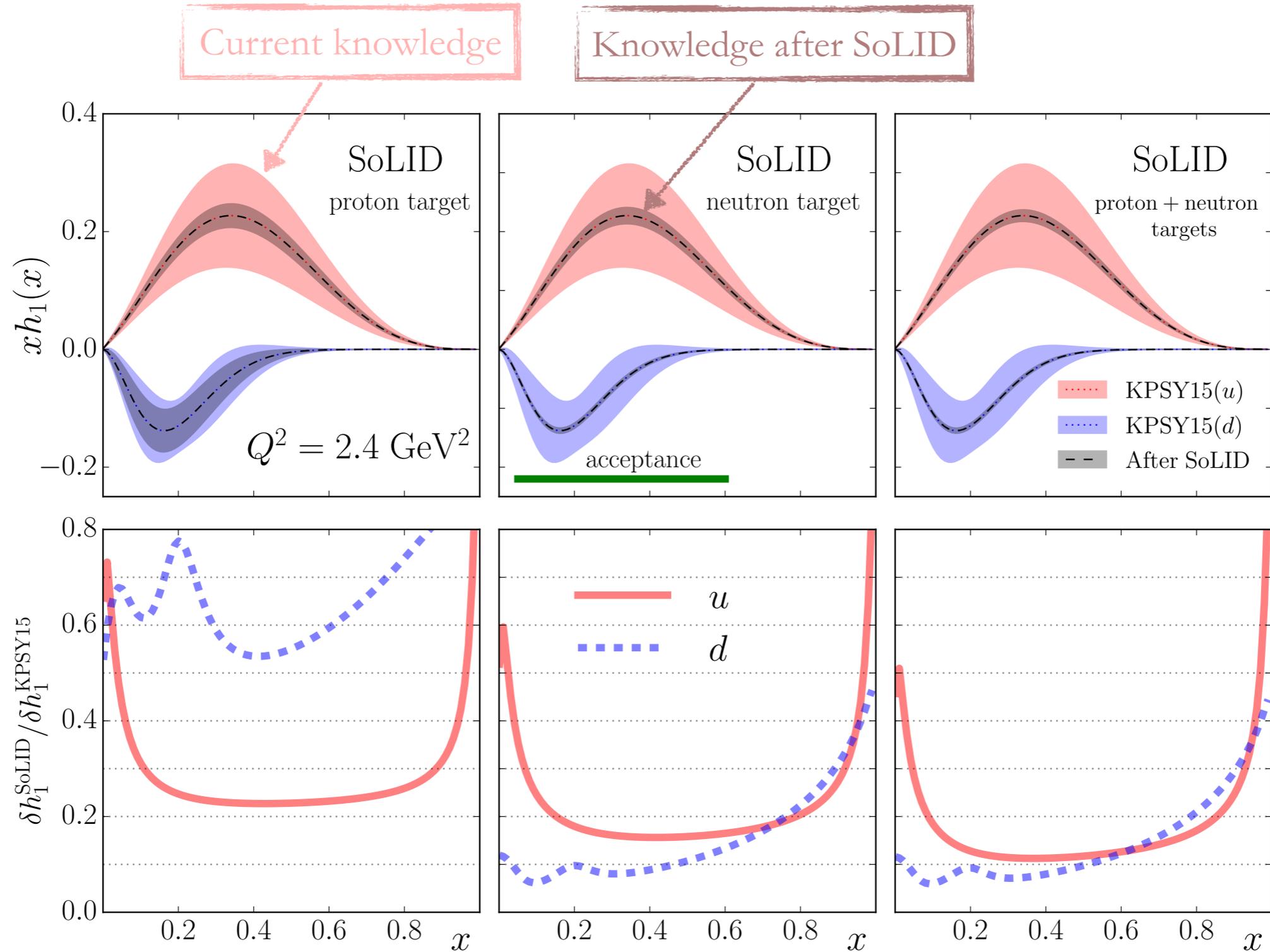
Leading Log (LL):	$A^{(1)}$		
Next-to Leading Log (NLL):	$A^{(1,2)}$	$B^{(1)}$	$C^{(1)}$
Next-to-Next-to Leading Log (NNLL):	$A^{(1,2,3)}$	$B^{(1,2)}$	$C^{(1)}$

Kang, AP, Sun, Yuan 2015  
 Echevarria, Scimemi, Vladimirov 2016

Precision is important!

$C^{(1)}$  means that one should use NLO collinear distributions

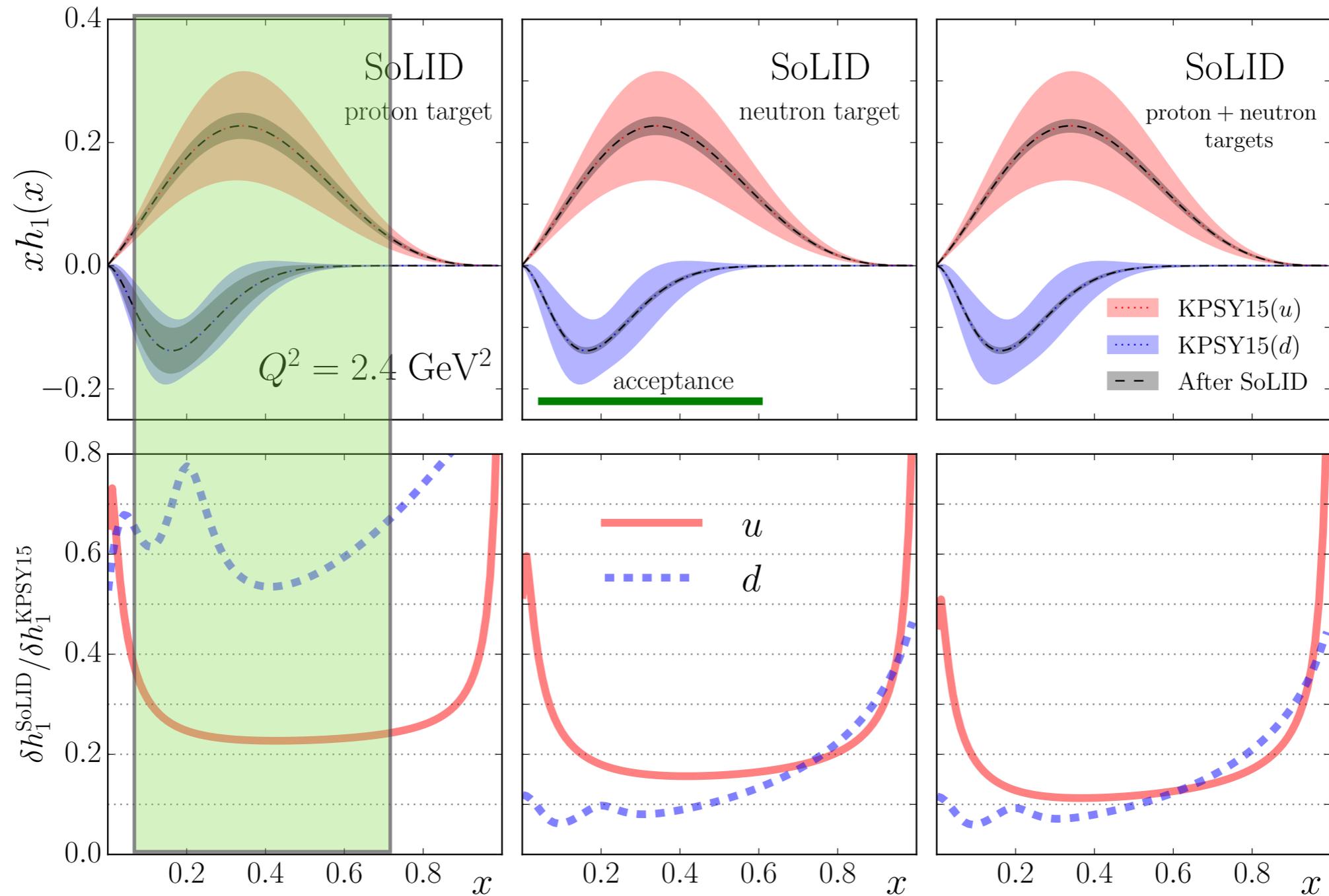
# What do we expect from JLab 12?



Bayesian statistics is used to estimate the improvement from new data

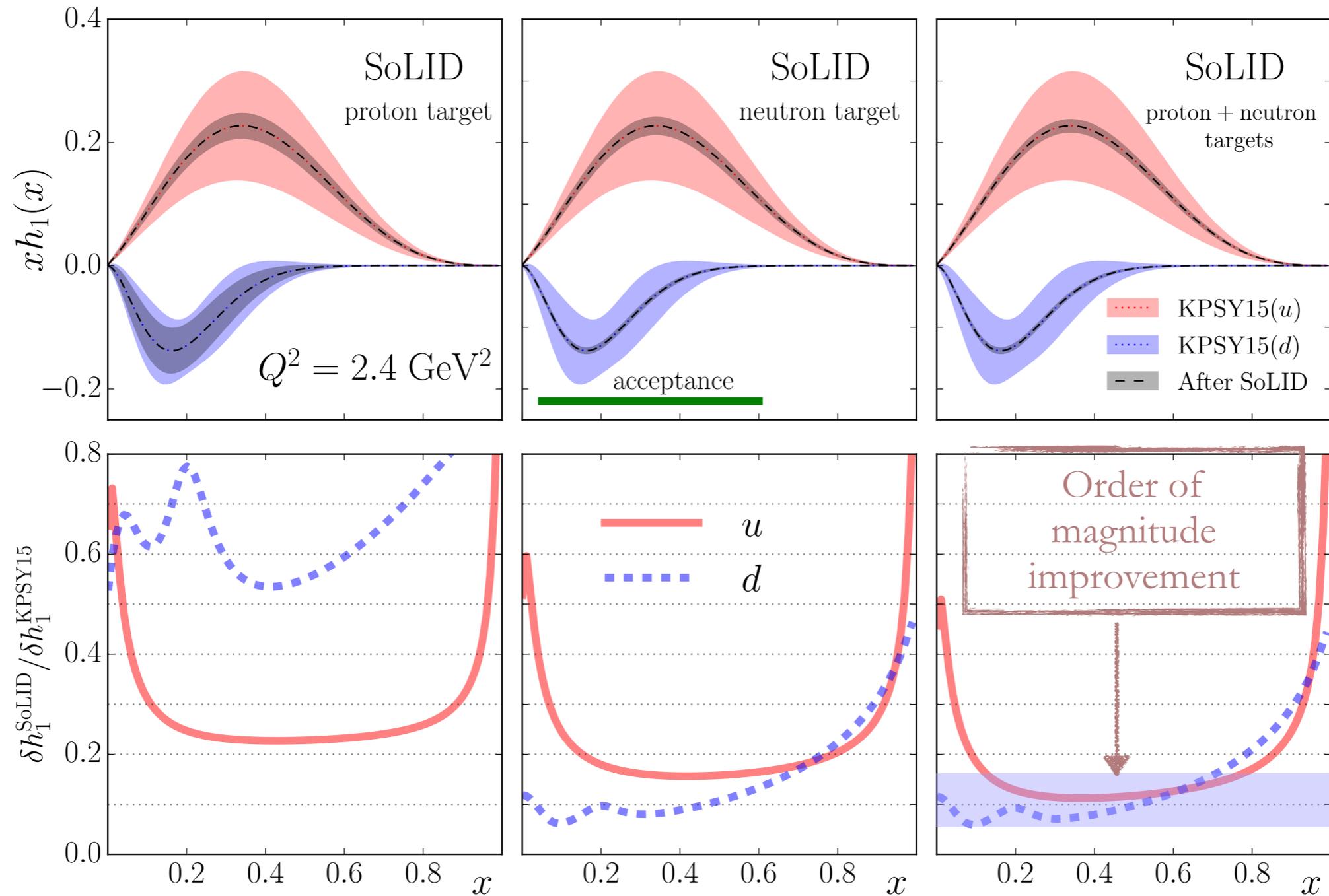
Current knowledge corresponds to a fit with TMD evolution *Kang et al., P.R. D93 (16) 014009*

## Kinematical coverage of SoLID



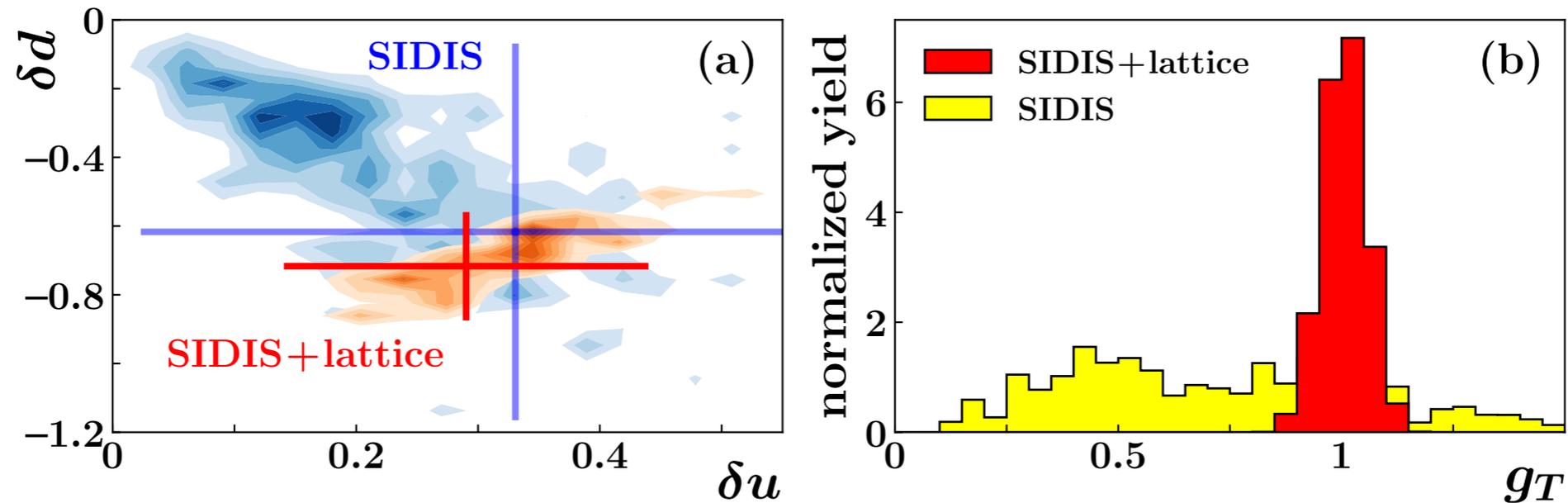
The errors grow outside of the future data region as expected

# What do we expect from JLab 12?



Only combination of proton and neutron target measurements will ensure similar improvement for both u and d quark transversity

$$g_T = \delta u - \delta d \quad \text{isovector tensor charge}$$

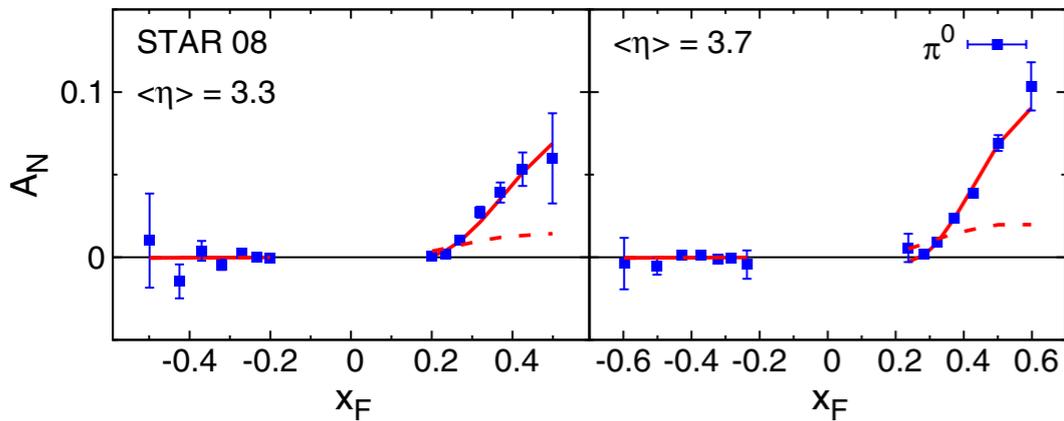


First combined new methodology fir of SIDIS data using lattice constraints:  
Lattice and SIDIS data are compatible and including lattice data improves extraction of  $g_T$

*Lin, Melnitchouk, AP, Sato 2017*

# RHIC: the process $p + p^\uparrow \rightarrow \pi + X$

## Twist-3 factorization, fragmentation contributions



$$\frac{E_h d\sigma^{Frag}(S_P)}{d^3\vec{P}_h} = -\frac{4\alpha_s^2 M_h}{S} \epsilon^{P'PP_h S_P} \sum_i \sum_{a,b,c} \int_0^1 \frac{dz}{z^3} \int_0^1 dx' \int_0^1 dx \delta(\hat{s} + \hat{t} + \hat{u})$$

$$\times \frac{1}{\hat{s}(-x'\hat{t} - x\hat{u})} h_1^a(x) f_1^b(x') \left\{ \left[ H_1^{\perp(1),\pi/c}(z) - z \frac{dH_1^{\perp(1),\pi/c}(z)}{dz} \right] S_{H_1^\perp}^i + \frac{1}{z} H^{\pi/c}(z) S_H^i \right.$$

$$\left. + \frac{2}{z} \int_z^\infty \frac{dz_1}{z_1^2} \frac{1}{\left(\frac{1}{z} - \frac{1}{z_1}\right)^2} \hat{H}_{FU}^{\pi/c,\mathcal{J}}(z, z_1) S_{\hat{H}_{FU}}^i \right\},$$

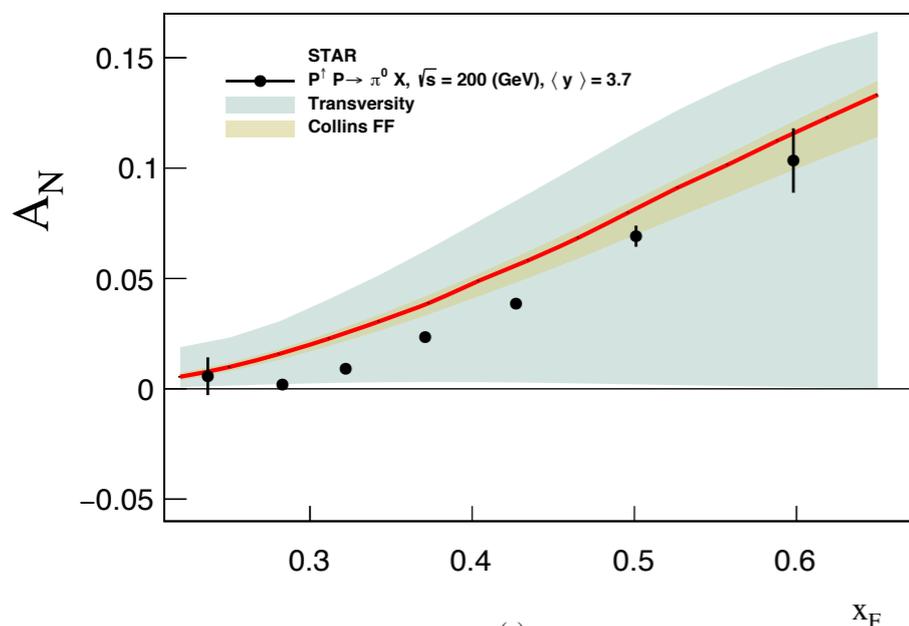
Integration over  $\mathbf{x}$  for transversity, conservation of momenta in  $ab \rightarrow cd$ :  $x_{min} = -(U/z)/(T/z + S)$ .

$$\int_{x_{min}}^1 \frac{dx}{x}$$

RHIC data is sensitive to high- $x$  behavior of transversity quark-gluon channel is dominant contribution for large  $x_F$

More complicated structure of cross-section, additional functions to study

*Kanazawa, Koike, Metz, Pitonyak (2014)*



*Gamberg, Kang, Pitonyak, AP PLB 770 (2017)*

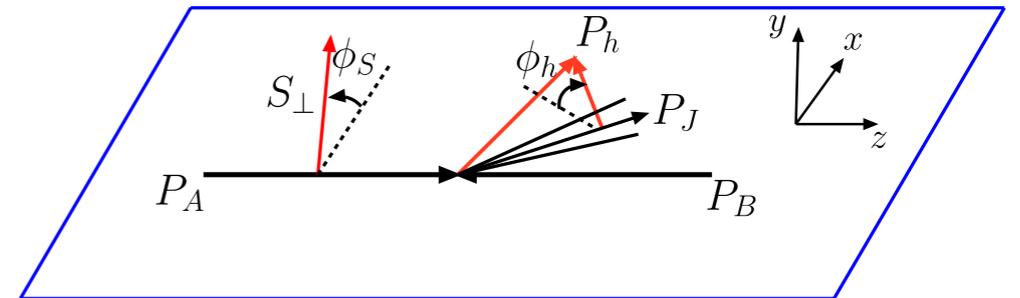
Improving errors in large- $x$  region?  
Analysis in progress.

# Hadron within a jet

- Consider a process  $PP$  scattering where jet is produced and a hadron is measured inside the jet

$$p^\uparrow [\vec{S}_\perp(\phi_S)] + p \rightarrow [\text{jet } h(\phi_H)] + X$$

*Feng Yuan (2008), D'Alesio, Murgia, Pisano (2014)*



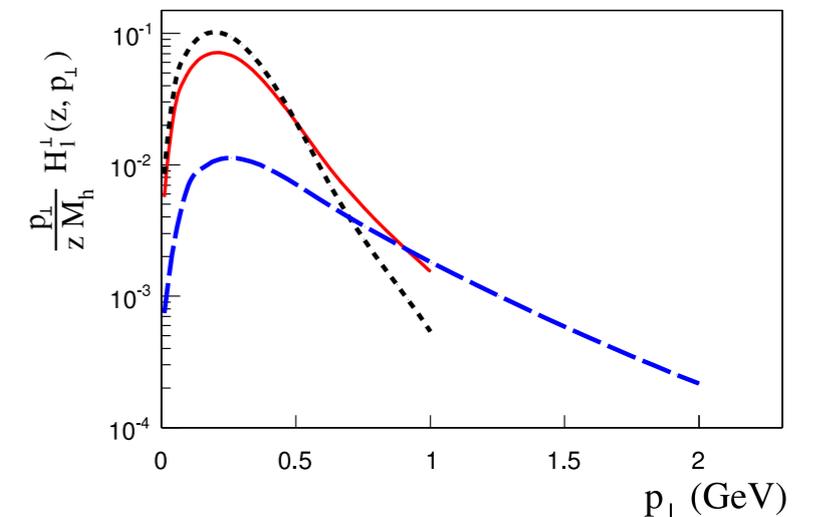
- Azimuthal modulation is related to convolution of Collins FF and transversity

$$\frac{d\sigma}{dy d^2 p_\perp^{\text{jet}} dz d^2 j_T} = F_{UU} + \sin(\phi_S - \phi_H) F_{UT}^{\sin(\phi_S - \phi_H)}$$

*Kang, Prokudin, Sun, Yuan (2015)*

$$F_{UT}^{\sin(\phi_S - \phi_H)} \propto h_1^a(x_1) \otimes f_{b/B}(x_2) \otimes \frac{j_T}{zM_h} H_1^{\perp c}(z, j_T^2) \otimes H_{ab \rightarrow c}^{\text{Collins}}(\hat{s}, \hat{t}, \hat{u})$$

$j_T$  : hadron transverse momentum with respect to the jet direction



*Kang, Prokudin, Ringer, Yuan PLB 774 (2017)*

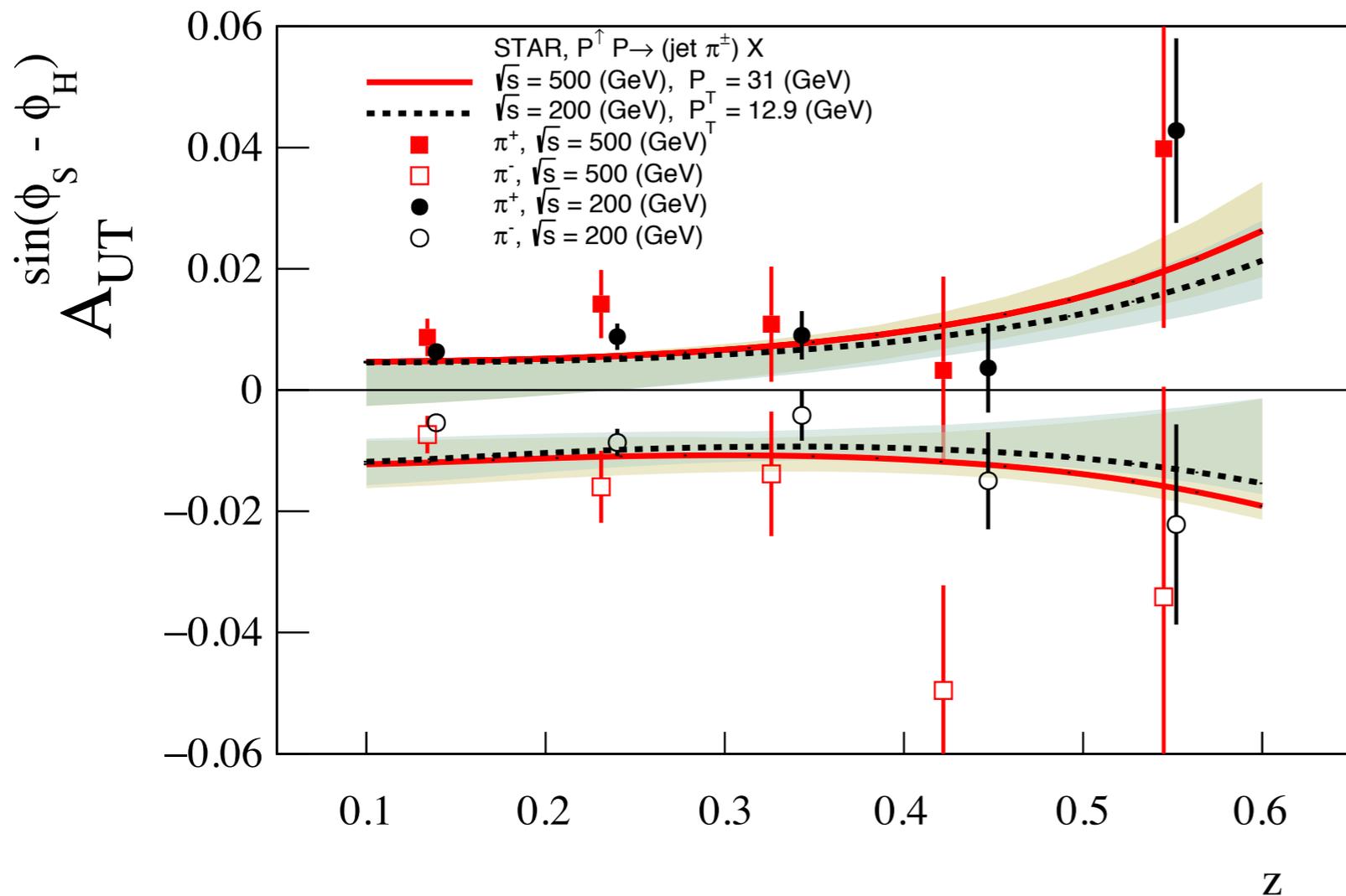
Asymmetry should depend on  $Q^2$

# Test of QCD evolution

*Kang, Prokudin, Ringer, Yuan (2017)*

## ■ Compute the asymmetry without TMD evolution

*functions: Anselmino et al (2015)*

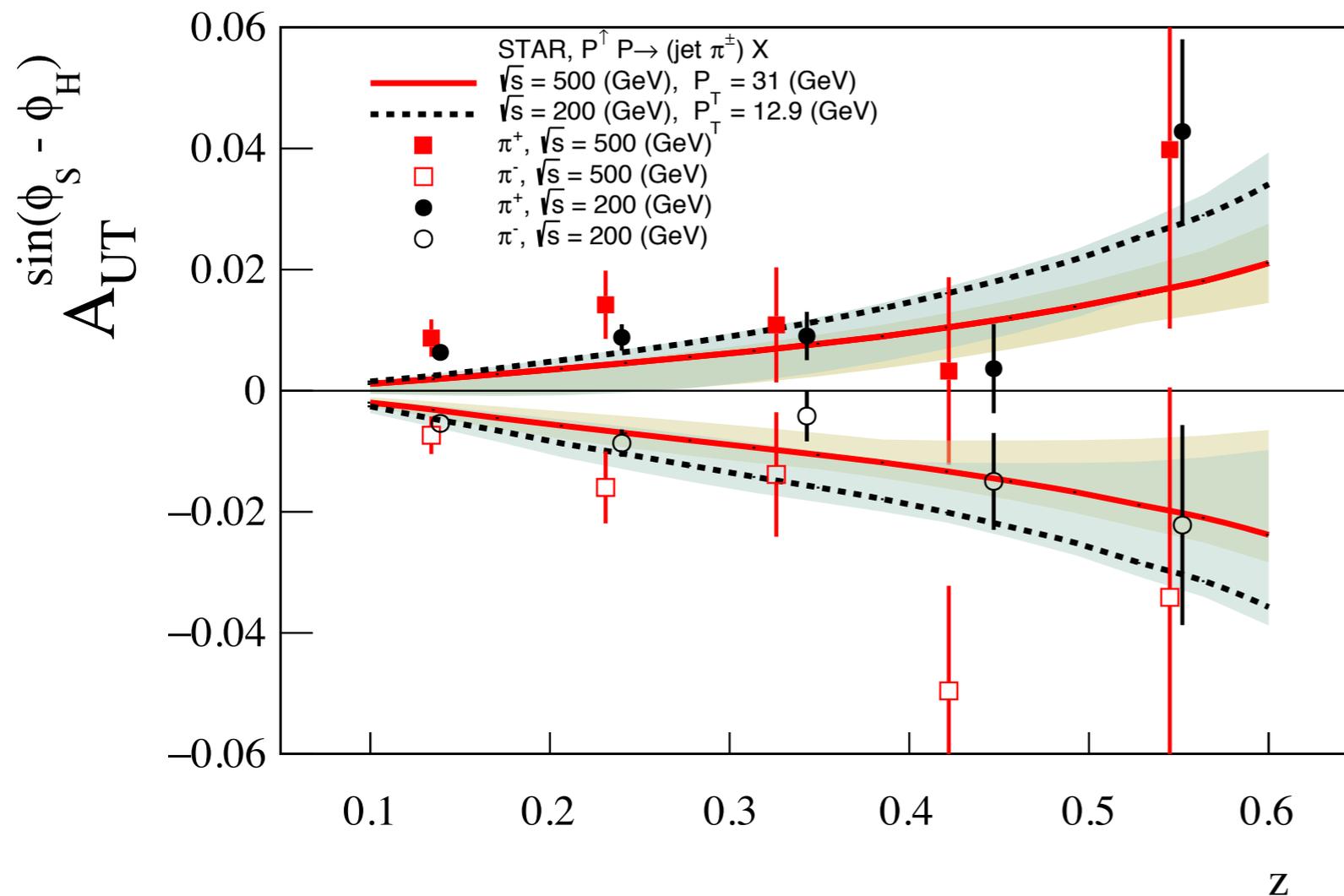


- Similar results for 200 GeV and 500 GeV
- Results are compatible with data within uncertainties

*Kang, Prokudin, Ringer, Yuan (2017)*

## ■ Compute the asymmetry **with** TMD evolution

*functions: Kang, Prokudin, Sun, Yuan (2015)*



- Slight reduction for 500 compared to 200 GeV due to TMD evolution
- Results are compatible with data within uncertainties

# Complementarity of SIDIS, e<sup>+</sup>e<sup>-</sup> and Drell-Yan, and hadron-hadron

Various processes allow study and test of evolution, universality and extractions of distribution and fragmentation functions. We need information from all of them

$$f(x) \otimes D(z)$$

Semi Inclusive DIS –  
convolution of distribution functions and  
fragmentation functions

$$\ell + P \rightarrow \ell' + h + X$$

$$f(x_1) \otimes f(x_2)$$

Drell-Yan – convolution of distribution  
functions

$$P_1 + P_2 \rightarrow \bar{\ell}\ell + X$$

$$D(z_1) \otimes D(z_2)$$

e<sup>+</sup> e<sup>-</sup> annihilation – convolution of  
fragmentation functions

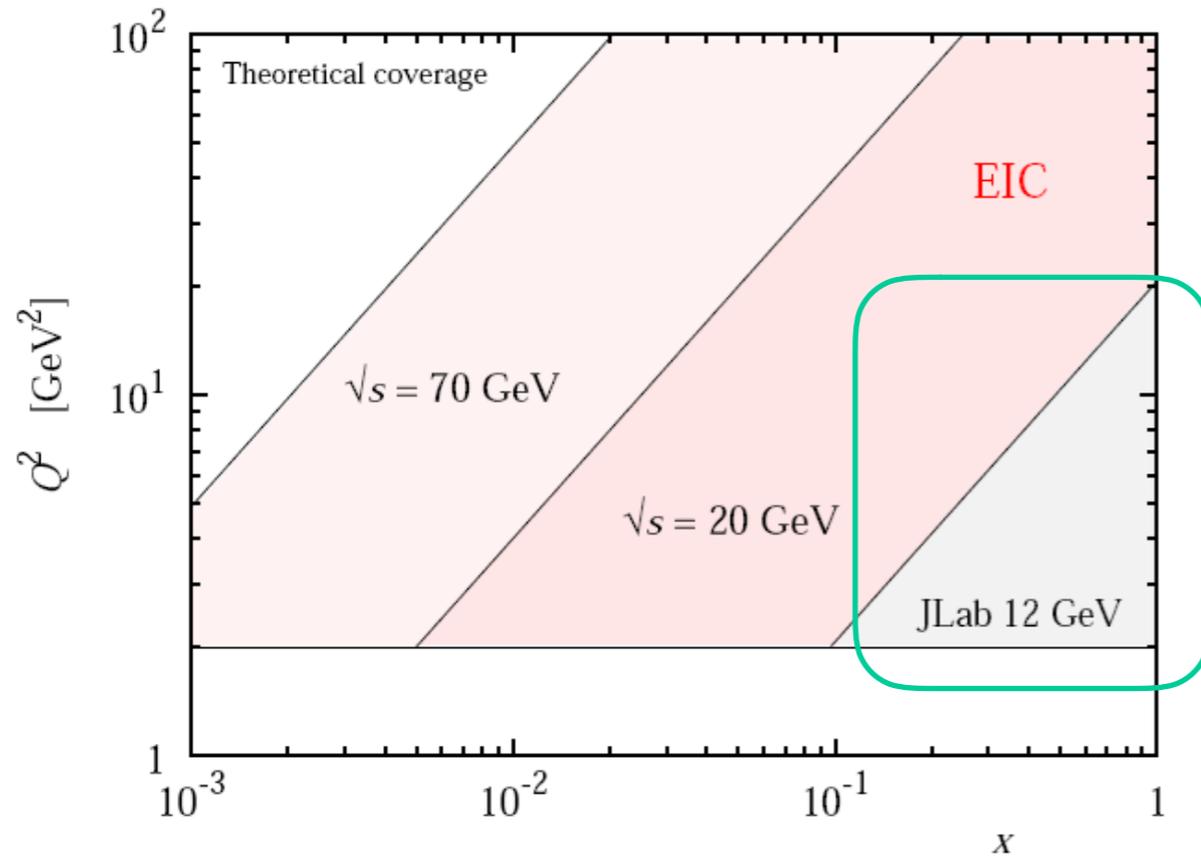
$$\bar{\ell} + \ell \rightarrow h_1 + h_2 + X$$

$$f(x_1) \otimes f(x_2) \otimes D(z)$$

Hadron-hadron – convolutions of PDF and  
fragmentation functions

$$h_1 + h_2 \rightarrow h_3(\gamma, jet, W, \dots) + X$$

Combining measurements from all above is important

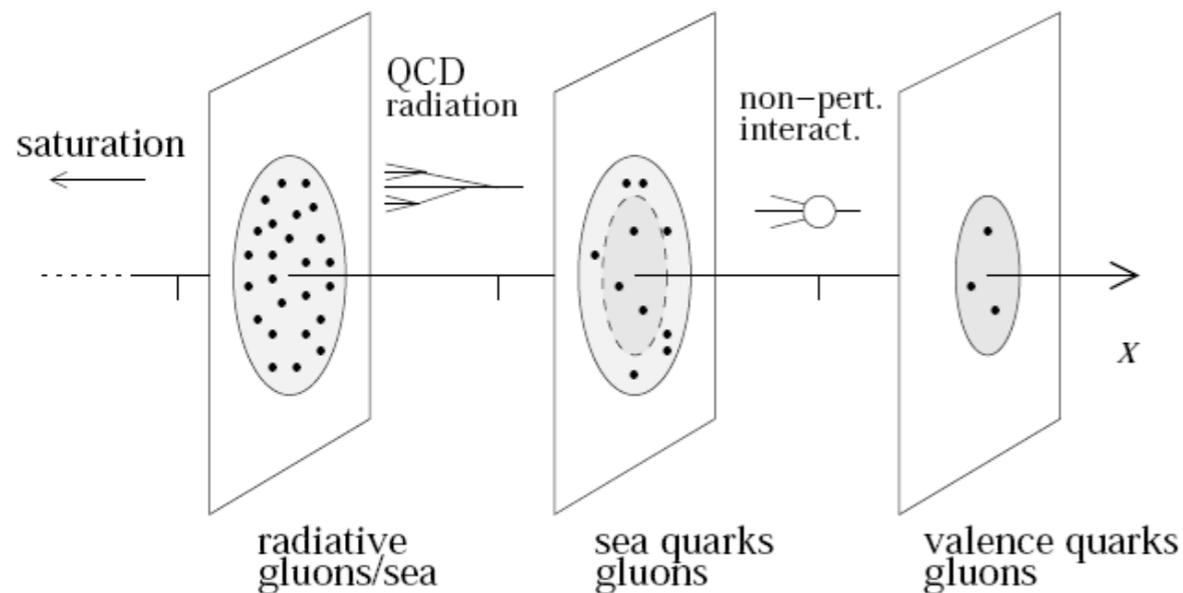


Fixed target experiments will always have admixture of target fragmentation while factorization theorems are formulated for beam fragmentation region

High- $x$  region and low- $Q$  region will have big target mass corrections and big higher twist contributions to be accounted for in phenomenology

High precision data will require both better theoretical understanding and phenomenological methods, mere amount of data will require new computational methods in phenomenology

*All these points are both challenges and opportunities.*



- 
- TMD related studies have been extremely active in the past few years, lots of progress have been made
  - We look forward to the future experimental results from COMPASS, RHIC, Jefferson Lab, LHC, Fermilab, future Electron Ion Collider
  - Many TMD related groups are created throughout the world:

Italy, Netherlands, Belgium, Germany, Japan, China, Russia, and the USA



5 years of funding of **\$2,160,000**

18 institutions

Theory, phenomenology, lattice QCD

Several postdoc positions.

2 tenure track positions: Temple, NMSU

Support of undergraduates.

## The TMD Collaboration

Spokespersons: William Detmold (MIT) and Jianwei Qiu (BNL)

Co-Investigators - (in alphabetical order of institutions):

Jianwei Qiu and Raju Venugopalan (Brookhaven National Laboratory)

Thomas Mehen (Duke University)

Ted Rogers (Jefferson Laboratory and Old Dominion University)

Alexei Prokudin (Jefferson Laboratory and Penn State University at Berks)

Feng Yuan (Lawrence Berkeley National Laboratory)

Christopher Lee and Ivan Vitev (Los Alamos National Laboratory)

William Detmold, John Negele and Iain Stewart (MIT)

Matthias Burkardt and Michael Engelhardt (New Mexico State University)

Leonard Gamberg (Penn State University at Berks)

Andreas Metz (Temple University)

Sean Fleming (University of Arizona)

Keh-Fei Liu (University of Kentucky)

Xiangdong Ji (University of Maryland)

Simonetta Liuti (University of Virginia)

- ◇ 5 years of funding
- ◇ 18 institutions
- ◇ Theory, phenomenology, lattice QCD
- ◇ Several postdoc and tenure track positions are created
- ◇ “To address the challenges of extracting novel quantitative information about the nucleon’s internal landscape”
- ◇ “To provide compelling research, training, and career opportunities for young nuclear theorists”