Ab Initio Hypernuclear Structure

- precise data on ground states & spectroscopy of hypernuclei
- ab initio few-body and phenomen. shell-model, mean-field or cluster-model calculations done so far
- chiral YN & YY interactions at (N)LO are available
Ab Initio Hypernuclear Structure

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- chiral YN & YY interactions at (N)LO are available

constrain YN interactions with hypernuclear spectroscopy

time to transfer ab initio toolbox to hypernuclei
Ab Initio Hypernuclear Structure

- Lattice QCD can be a game changer in hypernuclear physics
- extract YN & YY phase shifts or potentials from Lattice QCD, possibly YNN
- compute light hypernuclei directly on the lattice

Extensive study of s-shell nuclei and hypernuclei, and baryon-baryon interactions at SU(3) symmetric point


(Layer)Nuclei from QCD

Ab initio Hypernuclear Structure

[Diagram showing various hypernuclear states and mass shifts]

- \( m_\pi \sim 800 \text{ MeV} \)

[Legend for hypernuclear states and mass shifts]
Ab Initio Hypernuclear Structure

- Lattice QCD can be a game changer in hypernuclear physics
- Extract YN & YY phase shifts or potentials from Lattice QCD, possibly YNN
- Compute light hypernuclei directly on the lattice

Lattice data to determine YN, YY, YNN interactions

Structure theory for consistency check and access to heavier hypernuclei
Ab Initio Toolbox

Nuclear Structure & Reaction Observables

Many-Body Solution
No-Core Shell Model,…

Pre-Processing
Similarity Renormalization Group

Hamiltonian
Chiral Effective Field Theory

Low-Energy QCD
Ab Initio Toolbox

Nuclear Structure & Reaction Observables

Many-Body Solution
No-Core Shell Model,…

Pre-Processing
Similarity Renormalization Group

Hamiltonian
Chiral Effective Field Theory

Low-Energy QCD

- systematic and improvable input for all ab initio calculations
- NN & NNN interactions at N3LO give robust description of p-shell nuclei
- YN interaction only at LO, NLO with large uncertainties
Ab Initio Toolbox

Nuclear Structure & Reaction Observables

Many-Body Solution
- No-Core Shell Model,

Pre-Processing
- Similarity Renormalization Group

Hamiltonian
- Chiral Effective Field Theory

Low-Energy QCD

- accelerate convergence of many-body calculation, tame correlations
- induced many-nucleon interactions are sizeable, but under control
- induced YNN interactions hold some surprises
Ab Initio Toolbox

Nuclear Structure & Reaction Observables

- different many-body methods for different mass regions and different observables
- light nuclei: No-Core Shell Model
- inclusion of explicit $\Lambda$, $\Sigma^+$, $\Sigma^0$, $\Sigma^-$ degrees-of-freedom increases complexity

Many-Body Solution
No-Core Shell Model,…

Pre-Processing
Similarity Renormalization Group

Hamiltonian
Chiral Effective Field Theory

Low-Energy QCD
Chiral EFT for Nuclear Interactions

- low-energy **effective field theory** for relevant degrees of freedom ($\pi, N$) based on symmetries of QCD
- explicit long-range **pion dynamics**
- unresolved short-range physics absorbed in **contact terms**, low-energy constants fit to experiment
- **well constrained** in NN and 3N sector up to N3LO and beyond

<table>
<thead>
<tr>
<th></th>
<th>NN</th>
<th>3N</th>
<th>4N</th>
</tr>
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<tbody>
<tr>
<td>LO</td>
<td>X</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>NLO</td>
<td>X</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>N2LO</td>
<td>X</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>N3LO</td>
<td>X</td>
<td>+</td>
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</tr>
</tbody>
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- low-energy **effective field theory** for relevant degrees of freedom ($\pi, N$) based on symmetries of QCD
- explicit long-range **pion dynamics**
- unresolved short-range physics absorbed in **contact terms**, low-energy constants fit to experiment
- **well constrained** in NN and 3N sector up to N3LO and beyond
- extension to **hyperon-nucleon sector** by Jülich group (Haidenbauer, Polinder et al.)
- **problem**: number of LECs in relation to available data; already difficult to constrain NLO interaction

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Polinder et al.; NPA 779, 244 (2006)
Continuous unitary transformation driving Hamiltonian towards diagonal form

- Unitary transformation via flow equation
  \[ H_\alpha = U_\alpha^\dagger H_0 U_\alpha \quad \rightarrow \quad \frac{d}{d\alpha} H_\alpha = [\eta_\alpha, H_\alpha] \]

- Dynamic generator determines physics of transformation
  \[ \eta_\alpha = (2\mu)^2 [T_{\text{int}}, H_\alpha] \]

- Solve flow equation using matrix representation in two- and three-body space

- Flow parameter \( \alpha \) determines how far to go
Similarity Renormalization Group

**pro:** improves convergence of many-body calculations

**con:** induces many-body interactions

- need to truncate evolved Hamiltonian
  \[ H_\alpha = H_\alpha^{[1]} + H_\alpha^{[2]} + H_\alpha^{[3]} + H_\alpha^{[4]} + \cdots \]

- variation of flow parameter provides diagnostic for omitted many-body terms

- truncations used in the following:
  - **NN+3N**\textsubscript{full}
    use initial NN+3N, keep evolved NN+3N
  - **YN**\textsubscript{only} vs. **YN+YNN**\textsubscript{ind}
    explore role of induced YNN interactions
No-Core Shell Model

No-Core Shell Model is the most powerful and universal ab initio method for light nuclei

- **idea**: solve eigenvalue problem of Hamiltonian represented in model space of HO Slater determinants truncated w.r.t. HO excitation energy $N_{\text{max}}\hbar\Omega$
No-Core Shell Model

No-Core Shell Model is the most powerful and universal ab initio method for light nuclei

- **idea**: solve eigenvalue problem of Hamiltonian represented in model space of HO Slater determinants truncated w.r.t. HO excitation energy $N_{\text{max}}\hbar\Omega$

$$
\begin{bmatrix}
\vdots \\
C_{i'}^{(n)} \\
\vdots \\
\end{bmatrix}
= E_n
\begin{bmatrix}
\vdots \\
C_i^{(n)} \\
\vdots \\
\end{bmatrix}
$$
No-Core Shell Model

- **idea**: solve eigenvalue problem of Hamiltonian represented in model space of HO Slater determinants truncated w.r.t. HO excitation energy $N_{\text{max}}\hbar\Omega$
  - convergence of observables w.r.t. $N_{\text{max}}$ is the only limitation and source of uncertainty

- **Importance-Truncated NCSM**: reduce NCSM model space to physically relevant basis states and extrapolate to full space a posteriori
  - increases the range of applicability of NCSM significantly

- **Hypernuclear NCSM**: add explicit ($\Lambda$, $\Sigma^+$, $\Sigma^0$, $\Sigma^-$) degrees of freedom into model space and solve full coupled-channel problem
  - model-space dimension increases by $\sim$2 orders of magnitude compared to parent nucleus

No-Core Shell Model is the most powerful and universal ab initio method for light nuclei
Ab Initio Toolbox for Hypernuclei

Wirth et al.; PRL 113, 192502 (2014); PRL 117, 182501 (2016); PRC 97, 064315 (2018)

- **Hamiltonian from chiral EFT**
  - NN+NNN: chiral N3LO+N2LO interactions (Entem-Machleidt, Navratil)
  - YN: chiral LO interaction (Polinder, Heidenbauer et al.), NLO in progress

- **Similarity Renormalization Group**
  - consistent SRG evolution of NN, NNN, YN interactions
  - including induced NNN, YNN terms
  - using particle basis and fully including $\Lambda-\Sigma$ conversion (large matrices)
  - $\Lambda-\Sigma$ mass difference and $p-\Sigma^\pm$ Coulomb interactions included consistently

- **Importance-Truncated No-Core Shell Model**
  - include explicit $p, n, \Lambda, \Sigma^+, \Sigma^0, \Sigma^-$ in model space with physical masses
  - larger model spaces easily tractable with importance truncation
  - all single-strange $s$ and $p$-shell hypernuclei accessible
Application: $^7\Lambda$Li

**NCSM**

**chiral NN+3N**
- standard
- N3LO+N2LO
- $\Lambda_{3N}=500$ MeV
- $\alpha=0.08$ fm$^4$

**chiral YN**
- LO
- $\Lambda_{YN}=700$ MeV
- $\alpha=0.08$ fm$^4$

$\hbar\Omega=20$ MeV

---

$Wirth \ et\ et.; \ PRL \ 113, \ 192502 \ (2014); \ PRL \ 117, \ 182501 \ (2016)$
Application: $^7\Lambda\text{Li}$

$\Lambda$N + 3N\text{full}

$^6\text{Li}$

$^7\Lambda\text{Li}$

$\Lambda$N + 3N\text{full}

YN\text{only}

NCSM

chiral NN+3N

standard

N3LO+N2LO

$\Lambda_{3N}=500$ MeV

$\alpha=0.08$ fm$^4$

chiral YN

LO

$\Lambda_{YN}=700$ MeV

$\alpha=0.08$ fm$^4$

$h\Omega=20$ MeV

Wirth et al.; PRL 113, 192502 (2014); PRL 117, 182501 (2016)
Application: $^7_ΛLi$

- **NN+3N**
  - Full
  - Standard N3LO+N2LO
  - $\Lambda_{3N}=500$ MeV
  - $\alpha=0.08$ fm$^4$

- **YN only**
  - $\Lambda_{YN}=700$ MeV
  - $\alpha=0.08$ fm$^4$

- **YN+YNN**
  - Ind
  - $\hbar\Omega=20$ MeV

Wirth et al.; PRL 113, 192502 (2014); PRL 117, 182501 (2016)
Application: $^7\Lambda$Li

NCSM

chiral NN+3N
standard
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$\hbar\Omega=20$ MeV

Wirth et al.; PRL 113, 192502 (2014); PRL 117, 182501 (2016)
Application: $^9\Lambda$Be

Wirth et al.; PRL 113, 192502 (2014); PRL 117, 182501 (2016)

NCSM

chiral NN+3N
standard
N3LO+N2LO
$\Lambda_{3N}=500$ MeV
$\alpha=0.08$ fm$^4$

chiral YN
LO
$\Lambda_{YN}=700$ MeV
$\alpha=0.08$ fm$^4$

$\hbar\Omega=20$ MeV
Application: $^{13}\Lambda$C

WNCSM

chiral NN+3N
standard
N3LO+N2LO
$\Lambda_{3N}=500$ MeV
$\alpha=0.08$ fm$^4$

chiral YN
LO
$\Lambda_{YN}=700$ MeV
$\alpha=0.08$ fm$^4$

$\hbar\Omega=20$ MeV

Wirth et al.; PRL 113, 192502 (2014); PRL 117, 182501 (2016)
Light Neutron-Rich Hypernuclei

\[ \Lambda \text{YN} = 600 \text{ MeV} \]
\[ \Lambda \text{YN} = 700 \text{ MeV} \]

- 4He
- 5He
- 6He
- 7He
- 8He
- 9He

NCSM

**NN+3N\text{full}**
- Standard
- N3LO+N2LO

\[ \Lambda_{3N} = 500 \text{ MeV} \]
\[ \alpha = 0.08 \text{ fm}^4 \]

**YN+YNN\text{ind}**
- LO
\[ \alpha = 0.08 \text{ fm}^4 \]

\[ \hbar \Omega = 20 \text{ MeV} \]

- J=0,1/2
- J=1,3/2
- J=2,5/2

\[ E \text{ [MeV]} \]

Experiment
Light Neutron-Rich Hypernuclei

\[ E(J) \]
Light Neutron-Rich Hypernuclei

\[ ^4\text{He} \quad ^5\text{He} \quad ^5\text{He} \quad ^6\text{He} \quad ^6\text{He} \quad ^7\text{He} \quad ^7\text{He} \quad ^8\text{He} \quad ^8\text{He} \quad ^9\text{He} \quad ^9\text{He} \quad ^{10}\text{He} \]

\[ E \quad [\text{MeV}] \]

\[ \hbar \Omega = 20 \text{ MeV} \]

\[ J = 0, 1/2 \]

\[ J = 1, 3/2 \]

\[ J = 2, 5/2 \]

\[ \Lambda_{YN} = 600 \text{ MeV} \quad \Lambda_{YN} = 700 \text{ MeV} \]

\[ \Lambda_{3N} = 500 \text{ MeV} \]

\[ \alpha = 0.08 \text{ fm}^4 \]

\[ \text{NCSM} \]

\[ \text{NN+3N} \]

\[ \text{NN}+3N_{\text{full}} \]

\[ \text{standard} \]

\[ \text{N3LO+N2LO} \]

\[ \text{N3LO+N2LO} \]

\[ \text{α}=0.08 \text{ fm}^4 \]

\[ \text{YN+YNN}_{\text{ind}} \]

\[ \text{LO} \]

\[ \text{LO} \]

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\[ \text{NCSM} \]

\[ \text{Experiment} \]

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Light Neutron-Rich Hypernuclei

Wirth et al.; PLB 779, 336 (2018)

NCSM

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LO
\[ \alpha = 0.08 \text{ fm}^4 \]
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\[ J=0,1/2 \]
\[ J=1,3/2 \]
\[ J=2,5/2 \]
Light Neutron-Rich Hypernuclei

Wirth et al.; PLB 779, 336 (2018)

$^6\text{Li}$ $^7\text{Li}$ $^7\Lambda\text{Li}$ $^8\text{Li}$ $^8\Lambda\text{Li}$ $^9\text{Li}$ $^9\Lambda\text{Li}$ $^{10}\text{Li}$ $^{10}\Lambda\text{Li}$ $^{11}\text{Li}$ $^{11}\Lambda\text{Li}$ $^{12}\text{Li}$

$E$ [MeV]

$^6\text{Li}$ $^7\text{Li}$ $^7\Lambda\text{Li}$ $^8\text{Li}$ $^8\Lambda\text{Li}$ $^9\text{Li}$ $^9\Lambda\text{Li}$ $^{10}\text{Li}$ $^{10}\Lambda\text{Li}$ $^{11}\text{Li}$ $^{11}\Lambda\text{Li}$ $^{12}\text{Li}$

NCSM

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$\Lambda_{3N}=500$ MeV
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YN+YN_{ind}
LO
$\alpha=0.08$ fm$^4$

$\hbar\Omega=20$ MeV

$J=0,1/2$
$J=1,3/2$
$J=2,5/2$

experiment

$\cdots\cdots\Lambda_{YN}=600$ MeV

$\cdots\cdots\Lambda_{YN}=700$ MeV
Application: $^7\Lambda Li$

Wirth et al.; PRL 113, 192502 (2014); PRL 117, 182501 (2016)

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  - LO
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- $\hbar\Omega=20$ MeV

$\Lambda$ binding energy and induced $\Lambda$NN interaction are of similar size

WHY?

...something to do with $\Lambda-\Sigma$ conversion?
Suppression of $\Lambda$-$\Sigma$ Conversion

- design SRG-generator that suppresses the $\Lambda$-$\Sigma$ conversion exclusively
- $\Sigma$ admixture in the wave functions eliminated or “integrated out”
- same large induced YNN interactions as in standard SRG
Suppression of $\Lambda$-$\Sigma$ Conversion

- design SRG-generator that **suppresses the $\Lambda$-$\Sigma$ conversion** exclusively
- $\Sigma$ admixture in the wave functions eliminated or “integrated out”
- same large induced YNN interactions as in standard SRG
Suppression of \( \Lambda-\Sigma \) Conversion

\[
E \, [\text{MeV}] \\
\begin{align*}
\Lambda^7\text{Li} \\
\text{YN}_{\text{ind}} \\
\text{YN}_{\text{only}} \\
N_{\text{max}} = 12
\end{align*}
\]
Suppression of $\Lambda$-$\Sigma$ Conversion

Origin of the Induced Terms

Two-body evolution suppresses $\Lambda$-$\Sigma$ conversion. Mechanism for inducing $\text{YNN}_{\text{ind}}$?

$N_{\text{max}} = 12$

Wirth et al.; PRL 117, 182501 (2016)

full theory with explicit $\Sigma$ degrees of freedom

no initial $\text{YNN}$ interaction included

\[
\begin{align*}
\text{YNN}_{\text{ind}}
\end{align*}
\]

\[
\begin{align*}
\text{YN}_{\text{only}}
\end{align*}
\]
Origin of the Induced Terms

Two-body evolution suppresses $\Lambda$-$\Sigma$ conversion

Mechanism for inducing YNN?

$\Lambda N$ interaction only

$\Sigma N$ interaction induced

Full theory with explicit $\Sigma$ degrees of freedom

No initial YNN interaction included

Effective $\Lambda$-only theory, $\Sigma$ fully decoupled

Strong repulsive $\Lambda NN$ interaction is induced

Suppression of $\Lambda$-$\Sigma$ Conversion

$Wirth et al.; PRL 117, 182501 (2016)$
Origin of the Induced Terms — Wegner SRG

Two-body evolution suppresses $\Sigma$-$\Lambda$ conversion

Mechanism for inducing YNN?

Induced YNN terms driven by suppression of $\Sigma$-$\Lambda$ conversion?

R. Wirth – 2/2016 – 13

full theory with explicit $\Sigma$ degrees of freedom

no initial YNN interaction included

effective $\Lambda$-only theory, $\Sigma$ fully decoupled

strong repulsive $\Lambda NN$ interaction is induced

SRG evolves full coupled-channel theory to effective $\Lambda$-only theory

Wirth et al.; PRL 117, 182501 (2016)
Implications for the Hyperon Puzzle

- Neutron stars reach densities, where hyperon production should be energetically favorable.
- Including explicit $\Lambda$s with $\Lambda N$ interaction softens EOS - does not support $2M\odot$ neutron star.
- Phenomenological fix: introduce strongly repulsive $\Lambda NN$ interaction.

Lonardoni et al.; PRL 114, 092301 (2014)
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Lonardoni et al.; PRL 114, 092301 (2014)
Conclusions & Outlook

- **established ab initio NCSM for p-shell hypernuclei**
  chiral EFT interactions + SRG + NCSM with explicit $\Lambda$ and $\Sigma$ hyperons

- **focus on light neutron-rich hypernuclei**
  ground-state energies, excitation spectra, electromag. observables,…

- **hyperon puzzle**
  strong repulsive $\Lambda NN$ interaction is inevitable if $\Sigma$ and $\Lambda-\Sigma$ conversion
  is not explicitly included in EOS model

- **update to most recent set of chiral EFT interactions**
  NN, NNN, $YN$, $YNN$ with order and cutoff variation

- **systematic quantification of theory uncertainties**
  both, many-body and chiral EFT uncertainties

- **close feedback loop**
  improve $YN$, $YNN$ interactions based on structure data
Epilogue

- thanks to my group and my collaborators
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  - H. Hergert  
    NSCL / Michigan State University
  - J. Vary, P. Maris  
    Iowa State University
  - E. Epelbaum, H. Krebs & the LENPIC Collaboration  
    Universität Bochum, ...
How do the binding energies of hypernuclei look like with AFDMC?
How do the binding energies of hypernuclei look like with AFDMC?
How do the binding energies of hypernuclei look like with AFDMC?

In ab initio theory with Λ and Σ degrees of freedom there is no need for strong initial YNN interactions...

... and there is no hyperon puzzle!