Hyperon forces from QCD on lattice and their applications

Takashi Inoue @Nihon Univ.

for

the HALQCD Collaboration
Introduction

★ Nuclear physics

• Theories have been developed extensively from 1930's
  • mean field theory, shell model, few-body technique etc.
• Properties of nuclei are explained or predicted.
• But, we need input data from experiment for nuclear force.
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  - mean field theory, shell model, few-body technique etc.
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- But, we need input data from experiment for nuclear force.

🌟 Quantum Chromodynamics
- is the fundamental theory of the strong interaction,
- has no free parameter almost,
- must explain everything, e.g. hadron spectrum, mass of nuclei.
- But, that is difficult to do because of the non-perturbative nature of QCD. One way to handle the nature of QCD is ....
Lattice QCD

\[ L = -\frac{1}{4} G^a_{\mu \nu} G^{\mu \nu}_a + \bar{q} \gamma^\mu \left( i \partial_\mu - g t^a A^a_\mu \right) q - m \bar{q} q \]

quarks \( q \) on the sites
gluons \( U = e^{i a A_\mu} \) on the links

Vacuum expectation value

\[ \langle O(\bar{q}, q, U) \rangle = \int dU d\bar{q} d q \ e^{-S(\bar{q}, q, U)} O(\bar{q}, q, U) \]
\[ = \int dU \det D(U) e^{-S_U(U)} O(D^{-1}(U)) \]
\[ = \lim_{N \to \infty} \frac{1}{N} \sum_{i=1}^{N} O(D^{-1}(U_i)) \]

\( \{ U_i \} : \) ensemble of gauge conf. \( U \) generated w/ probability of \( \det D(U) e^{-S_U(U)} \)

**Well defined (regularized)**  **Fully non-perturbative**
**Manifest gauge invariance**  **Highly predictive**
Lattice QCD

- LQCD simulations with the physical quark were done.
  - BMW, JHEP 1108 (2011) 148

- Mass of (ground state) hadrons are well reproduced!

Summary by Kronfeld, arXive 1203.1204
Lattice QCD

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Mass of (ground state) hadrons are well reproduced!

- What about nuclei and hypernuclei from LQCD?

Summary by Kronfeld, arXive 1203.1204
Various approaches in nuclear phys.

- Lattice simulation
  - Potential
  - Mean field
    - Variational method etc.

QCD

- Lagrangian of N sys.
  - BS eq. w/ a special power counting

- Symmetry

Data

- Lagrangian of N sys.
  - Lattice simulation

- LEC

- Lagrangian of N sys.
  - ChPT

- PWA, Meson-ex. model etc.

Nuclei and N.M.

- Nuclei (A ≤ 4)

- Nuclei and Nuclear Matter
Various approaches in nuclear phys.

Most traditional. Many success.
Various approaches in nuclear phys.

QCD

Lattice simulation → Potential → Mean field Variational method etc.

Nuclei and Nuclear Matter

Nuclei (A ≤ 4)

Lagrangian of N sys. → BS eq. w/ a special power counting

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LEC

Lagrangian of N sys. → Lattice simulation

ChPT → Potential

Mean field Variational method etc.

DATA

PWA, Meson-ex. model etc. → Potential

Very popular today. Let's say chiral approach.
Various approaches in nuclear phys.

Very challenging. Let's call LQCD direct approach.
Various approaches in nuclear phys.

Our approach. I focus on this one in this talk.
HAL QCD Approach

• Good points
  • Based on the fundamental theory QCD, hence provides information independent of experiments and models.
  • Feasible. Direct one must be very difficult for large nuclei.
  • Can utilize established nuclear theories at the 2nd stage.
  • Easy to extend to strange sector, charm sector etc.
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• Challenging points
  1. Demand long time and huge money at the 1\textsuperscript{st} stage.
     • We had to deal with un-physical QCD world before.
       (Un-realistically heavy u,d quark, far from chiral symmetry)
  2. Depend on method/approximation used at the 2\textsuperscript{nd} stage.
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• Today, in this talk, I want to show
  • results of HALQCD approach to strange nuclear physics and want to demonstrate that our approach is promising.
Outline

1. Our approach and method
   - Introduction
   - HAL QCD method
   - BB interactions from QCD

2. Application to strange nuclear physics
   - Hyperon single-particle potentials
   - Hyperon onset in high density matter

3. Summary and outlook
HAL QCD method
Multi-hadron in LQCD

- Direct: utilize temporal correlator and eigen-energy
  - Lüscher's finite volume method for phase-shifts
  - Infinite volume extrapolation for bound states

- HAL: utilize spatial correlation and “potential” $V(r) + \ldots$

$$V(\vec{r}) = \frac{1}{2\mu} \frac{\nabla^2 \psi(\vec{r}, t)}{\psi(\vec{r}, t)} - \frac{\partial}{\partial t} \frac{\psi(\vec{r}, t)}{\psi(\vec{r}, t)} - 2 M_B$$

- Advantages
  - No need to separate E eigenstate.
  - Just need to measure
  - Then, potential can be extracted.
  - Demand a minimal lattice volume.
  - No need to extrapolate to $V=\infty$.
  - Can output many observables.
Multi-hadron in LQCD

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  - Then, potential can be extracted.
  - Demand a minimal lattice volume. No need to extrapolate to \( V=\infty \).
  - Can output more observables.
  - We can attack hyper-nuclei too!!

\[ N = S_0 \]

Need to check validity of the leading term \( V(r) \)

\( a = 3.23 \pm 1.59 \text{ [fm]} \)
\( r = 2.65 \pm 0.24 \text{ [fm]} \)
\( \kappa_{ud,s} = 0.13840 \text{ (m}_s=469 \text{ [MeV]}) \)

\[ E_{lab} \text{ [MeV]} \]

\[ \delta \text{ [Deg]} \]
HAL method


NBS wave function

\[ \phi_{\vec{k}}(\vec{r}) = \sum_{\vec{\tilde{x}}} \langle 0 | B_i(\vec{\tilde{x}} + \vec{r}, t) B_j(\vec{\tilde{x}}, t) | B = 2, \vec{k} \rangle \]

Define a common “potential” \( U \) for all \( E \) eigenstates via “Schrödinger” eq.

\[
\begin{bmatrix} -\frac{\nabla^2}{2\mu} \end{bmatrix} \phi_{\vec{k}}(\vec{r}) + \int d^3 \vec{r}' U(\vec{r}, \vec{r}') \phi_{\vec{k}}(\vec{r}') = E_{\vec{k}} \phi_{\vec{k}}(\vec{r})
\]

Non-local but energy independent below inelastic threshold

Measure 4-point function in LQCD

\[ \psi(\vec{r}, t) = \sum_{\vec{\tilde{x}}} \langle 0 | B_i(\vec{\tilde{x}} + \vec{r}, t) B_j(\vec{\tilde{x}}, t) J(t_0) | 0 \rangle = \sum_{\vec{k}} A_{\vec{k}} \phi_{\vec{k}}(\vec{r}) e^{-W_{\vec{k}}(t - t_0)} + \cdots \]

\[
\begin{bmatrix} 2 M_B - \frac{\nabla^2}{2\mu} \end{bmatrix} \psi(\vec{r}, t) + \int d^3 \vec{r}' U(\vec{r}, \vec{r}') \psi(\vec{r}', t) = -\frac{\partial}{\partial t} \psi(\vec{r}, t)
\]

\[ U(\vec{r}, \vec{r}') = \delta(\vec{r} - \vec{r}') V(\vec{r}, \nabla) = \delta(\vec{r} - \vec{r}')[V(\vec{r}) + \nabla + \nabla^2 + \cdots] \]

Therefor, in the leading

\[
V(\vec{r}) = \frac{1}{2\mu} \frac{\nabla^2 \psi(\vec{r}, t)}{\psi(\vec{r}, t)} - \frac{\partial}{\partial t} \frac{\psi(\vec{r}, t)}{\psi(\vec{r}, t)} - 2 M_B
\]
Source and sink operator

- **NBS wave function and 4-point function**
  \[
  \phi_{\vec{k}}(\vec{r}) = \sum_{\vec{x}} \langle 0 | B_i(\vec{x}+\vec{r}, t) B_j(\vec{x}, t) | B=2, \vec{k} \rangle \quad \text{QCD eigenstate}
  \]
  \[
  \psi(\vec{r}, t) = \sum_{\vec{x}} \langle 0 | B_i(\vec{x}+\vec{r}, t) B_j(\vec{x}, t) J(t_0) | 0 \rangle = \sum_{\vec{k}} A_{\vec{k}} \phi_{\vec{k}}(\vec{r}) e^{-W_{\vec{k}}(t-t_0)} + \cdots
  \]

- **Point type octet baryon field operator at sink**
  \[
  p_\alpha(x) = \epsilon_{c_1c_2c_3} (C \gamma_5)_{\beta_1\beta_2} \delta_{\beta_3\alpha} u(\xi_1) d(\xi_2) u(\xi_3) \quad \text{with} \quad \xi_i = \{c_i, \beta_i, x\}
  \]
  \[
  \Lambda_\alpha(x) = -\epsilon_{c_1c_2c_3} (C \gamma_5)_{\beta_1\beta_2} \delta_{\beta_3\alpha} \left[\frac{1}{6} \left[d(\xi_1) s(\xi_2) u(\xi_3) + s(\xi_1) u(\xi_2) d(\xi_3) - 2 u(\xi_1) d(\xi_2) s(\xi_3)\right]\right]
  \]

- **Wall type quaruk source of two-baryon state**
  \[
  \text{e.g.} \quad BB^{(1)} = -\sqrt{\frac{1}{8}} \overline{\Lambda} \Lambda + \sqrt{\frac{3}{8}} \overline{\Sigma} \Sigma + \sqrt{\frac{4}{8}} \overline{N} \Xi \quad \text{for flavor-singlet}
  \]
Hyperon interactions from QCD
LQCD simulation setup

• Nf = 2+1 full QCD
  • Clover fermion + Iwasaki gauge w/ stout smearing
  • Volume $96^4 \approx (8 \text{ fm})^4$ large enough to accommodate $BB$ interaction
  • $1/a = 2333 \text{ MeV, } a = 0.0845 \text{ fm}$

• $M_\pi \approx 146, M_K \approx 525 \text{ MeV}$
• $M_N \approx 956, M_\Lambda \approx 1121, M_\Sigma \approx 1201, M_\Xi \approx 1328 \text{ MeV}$

• Collaboration in HPCI Strategic Program Field 5 Project 1

• Measurement
  • 4pt correlators: 52 channels in 2-octet-baryon (+ others)
  • Wall source w/ Coulomb gauge fixing
  • Dirichlet temporal BC to avoid the wrap around artifact
  • $\#\text{data} = 414 \text{ confs} \times 4 \text{ rot} \times (96,96) \text{ src.}$

We've accomplished at last!
$S = -2, \ I = 0, \ BB \ potentials$

$V(r) [\text{MeV}]$

$\Lambda\Lambda - \Lambda\Lambda$

$V(r) [\text{MeV}]$

$N\Xi - \Lambda\Lambda$

$V(r) [\text{MeV}]$

$\Sigma\Lambda - \Lambda\Lambda$

$V(r) [\text{MeV}]$

$N\Xi - N\Xi$

$V(r) [\text{MeV}]$

$\Sigma\Xi - \Lambda\Lambda$

$V(r) [\text{MeV}]$

$\Sigma\Sigma - \Sigma\Xi$

$(96,96) \ src \ t-t_0 = 12$

$^3 SD_1$

$^1 S_0$
$S=-2, I=1, BB$ potentials

$^1S_0 \quad ^3SD_1$

$(96,96) \text{ src} \quad t-t_0 = 12$
\[ S = -2, \ I = 2, \ BB \text{ potential} \]

- HAL QCD results of \( S = -1 \) BB potentials (\( \Lambda N, \Sigma N \) forces) will be presented by Dr. Nemura on Thursday.

- It is difficult to grasp feature of interactions from those figures.

- So, let us rotate \( S = -2 \) BB potential data into the flavor irreducible-representation basis.
BB S-wave potentials

\[ 8 \times 8 = 27 + 8s + 1 + 10^* + 10 + 8a \]

\[ ^1S_0 \quad ^3S_1, ^3D_1 \]
BB S-wave potentials

- Functions fitted to data
  
  $V_C(r) = a_1 e^{-a_2 r^2} + a_3 e^{-a_4 r^2} + a_5 \left[ (1 - e^{-a_6 r^2}) \frac{e^{-a_7 r}}{r} \right]^2$

  $V_T(r) = a_1 \left(1 - e^{-a_2 r^2} \right)^2 \left(1 + \frac{3}{a_3 r} + \frac{3}{(a_3 r)^2} \right) \frac{e^{-a_3 r}}{r} + a_4 \left(1 - e^{-a_5 r^2} \right)^2 \left(1 + \frac{3}{a_6 r} + \frac{3}{(a_6 r)^2} \right) \frac{e^{-a_6 r}}{r}$

- Since $SU(3)\not{\equiv}$ is broken at the physical point (K-conf.), there exist flavor base off-diagonal potentials.
- But, to begin with, let’s ignore them, and apply $V_{YN}$, $V_{YY}$ constructed with these diagonal potentials only.
BB S-wave potentials

- Qualitatively reasonable NN forces are obtained from QCD.
- Features can be understood by the quark Pauli + OGE.
BB S-wave potentials

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Application to strange nuclear physics
Introduction

Hyperon is a serious subject in physics of NS.
  • Does hyperon appear inside neutron star core?
  • How EoS of NS mater can be so stiff with hyperon?
    cf. PSR J1614-2230 1.97±0.04 \( M_\odot \)

Tough problem due to ambiguity of hyperon forces
  • comes form difficulty of hyperon scattering experiment.
Introduction

• However, nowadays, we can study or predict hadron-hadron interactions from QCD.
  • measure h-h NBS w.f. in lattice QCD simulation.  
  • define & extract interaction “potential” from the w.f.
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• Today, we study hyperons in nuclear matter by basing on \(YN,YY\) interactions extracted from QCD.
  • We calculate hyperon single-particle potential \(U_Y(k;\rho)\)
  • defined by \(e_Y(k;\rho) = \frac{k^2}{2M_Y} + U_Y(k;\rho)\)
  • \(U_Y\) is crucial for hyperon chemical potential.
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    • defined by $e_Y(k;\rho) = \frac{k^2}{2M_Y} + U_Y(k;\rho)$

• $U_Y$ is crucial for hyperon chemical potential.

• Hypernuclear experiment suggest that
  \[
  U_{\Lambda}^{\text{Exp}}(0) \approx -30 \text{, } \quad U_\Xi(0)^{\text{Exp}} \approx -10 \text{ ?}, \quad U_\Sigma^{\text{Exp}}(0) \geq +20 \text{ ? [MeV]} \]

  @ $\rho=0.17 \text{ [fm}^{-3}]$, $x=0.5$

  attraction, attraction small, repulsion
Nuclear matter

• Uniform matter consisting an infinite number of nucleon interacting each other via nuclear force

• Theories
  • Brueckner Hartree Fock
  • Relativistic Mean Field
  • Fermi Hyper-Netted Chain
  • Cupled Cluster
  • Self-consistent Green’s function
  • Quantum Monte Carlo
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Brueckner-Hartree-Fock


- Ground state energy in BHF framework

\[ E_0 = \gamma \sum_k k_F^2 \frac{k^2}{2M} + \frac{1}{2} \sum_i \sum_{k,k'} \text{Re} \langle G_i (e(k) + e(k')) \rangle \]

- Bethe Goldston eq.

\[ \langle k_1 k_2 | G(\omega) | k_3 k_4 \rangle = \langle k_1 k_2 | V | k_3 k_4 \rangle + \]

\[ \sum_{k_5, k_6} \frac{\langle k_1 k_2 | V | k_5 k_6 \rangle Q(k_5, k_6) \langle k_5 k_6 | G(\omega) | k_3 k_4 \rangle}{\omega - e(k_5) - e(k_6)} \]

- Single particle spectrum & potential

\[ e(k) = \frac{k^2}{2M_N} + U(k) \]

\[ U(k) = \sum_i \sum_{k' \leq k_F} \text{Re} \langle k k' | G_i (e(k) + e(k')) | k k' \rangle \]

- Partial wave decomposition

\[ ^{2S+1}L_J = ^1S_0 , ^3S_1 , ^3D_1 , ^1P_1 , ^3P_J \cdots \]

- Continuous choice w/ effective mass approx. Angle averaged Q-operator
Brueckner-Hartree-Fock

- Hyperon single-particle potential

\[
U_Y(k) = \sum_{N=n,p} \sum_{SLJ} \sum_{k' \leq k_F} \langle kk' | G_{YN(YN)}^{SLJ} | e_Y(k) + e_N(k') \rangle |kk'\rangle
\]

\[
2S+1 \quad L_J = \begin{cases} 
1 & S_0, \ 3 & S_1, \ 3 & D_1, \\
\text{in our study} & 1 & P_1, \ 3 & P_J \end{cases} \ldots
\]

- YN G-matrix using \( M_{N,Y}^{\text{Phys}} \), \( U_{n,p}^{\text{AV18+UIX}} \), \( V_{LQCD}^{S=-1} \) and, \( U_{LQCD}^{Y} \)

\[
\begin{cases}
Q=0 \quad G_{(\Lambda n)(\Lambda n)}^{SLJ} & G_{(\Lambda n)(\Sigma^0 n)} & G_{(\Lambda n)(\Sigma p)} \\
G_{(\Sigma^0 n)(\Lambda n)} & G_{(\Sigma^0 n)(\Sigma^0 n)} & G_{(\Sigma^0 n)(\Sigma^p)} \\
G_{(\Sigma^p)(\Lambda n)} & G_{(\Sigma^p)(\Sigma^0 n)} & G_{(\Sigma^p)(\Sigma^p)}
\end{cases}
\]

\[
\begin{cases}
Q=+1 \quad G_{(\Lambda p)(\Lambda p)}^{SLJ} & G_{(\Lambda p)(\Sigma^0 p)} & G_{(\Lambda p)(\Sigma^+ n)} \\
G_{(\Sigma^0 p)(\Lambda p)} & G_{(\Sigma^0 p)(\Sigma^0 p)} & G_{(\Sigma^0 p)(\Sigma^+ n)} \\
G_{(\Sigma^+ n)(\Lambda p)} & G_{(\Sigma^+ n)(\Sigma^0 p)} & G_{(\Sigma^+ n)(\Sigma^+ n)}
\end{cases}
\]

\[
\begin{cases}
Q=-1 \quad G_{(\Sigma n)(\Sigma n)}^{SLJ} \\
Q = +2 \quad G_{(\Sigma^+ p)(\Sigma^+ p)}^{SLJ}
\end{cases}
\]
Brueckner-Hartree-Fock

- Hyperon single-particle potential

\[
U_\Xi(k) = \sum_{N=n,p} \sum_{\text{SLJ}} \sum_{k' \leq k_F} \langle kk'| G^{SLL}_{(\Xi N)\Xi N} | e_\Xi(k) + e_N(k') | kk' \rangle
\]

- \(\Xi N\) G-matrix using \(M^\text{Phys}_{N,Y}\), \(U^\text{AV18+UIX}_{n,p}\), \(U^\text{LQCD}_{\Lambda,\Sigma}\), \(V^\text{LQCD}_{S=-2}\), \(U^\text{LQCD}_\Xi\)

Flavor symmetric \(^1S_0\) sectors

<table>
<thead>
<tr>
<th>(Q=0)</th>
<th>(G^{SLL}_{(\Xi^0 n)\Xi n})</th>
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<td>(G_{(\Sigma^-\Lambda)\Sigma^-})</td>
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Brueckner-Hartree-Fock

- $\Xi N$ G-matrix using $M_{N,Y}^{\text{Phys}}, U_{n,p}^{\text{AV18+UIX}}, U_{\Lambda,\Sigma}^{\text{LQCD}}, V_{S=-2}^{\text{LQCD}}, U_{\Xi}^{\text{LQCD}}$

Flavor anti-symmetric $^3S_1, ^3D_1$ sectors

\[
Q=0:
\begin{pmatrix}
G^{SLJ}_{(\Xi^0 n)(\Xi^0 n)} & G_{(\Xi^0 n)(\Xi^- p)} & G_{(\Xi^0 n)(\Sigma^+ \Sigma^-)} & G_{(\Xi^0 n)(\Sigma^0 \Lambda)} \\
G_{(\Xi^- p)(\Xi^0 n)} & G_{(\Xi^- p)(\Xi^- p)} & G_{(\Xi^- p)(\Sigma^+ \Sigma^-)} & G_{(\Xi^- p)(\Sigma^0 \Lambda)} \\
G_{(\Sigma^+ \Sigma^-)(\Xi^0 n)} & G_{(\Sigma^+ \Sigma^-)(\Xi^- p)} & G_{(\Sigma^+ \Sigma^-)(\Sigma^+ \Sigma^-)} & G_{(\Sigma^+ \Sigma^-)(\Sigma^0 \Lambda)} \\
G_{(\Sigma^0 \Lambda)(\Xi^0 n)} & G_{(\Sigma^0 \Lambda)(\Xi^- p)} & G_{(\Sigma^0 \Lambda)(\Sigma^+ \Sigma^-)} & G_{(\Sigma^0 \Lambda)(\Sigma^0 \Lambda)} \\
\end{pmatrix}
\]

\[
Q=+1:
\begin{pmatrix}
G^{SLJ}_{(\Xi^0 p)(\Xi^0 p)} & G_{(\Xi^0 p)(\Sigma^+ \Sigma^-)} & G_{(\Xi^0 p)(\Sigma^+ \Lambda)} \\
G_{(\Sigma^+ \Sigma^-)(\Xi^0 p)} & G_{(\Sigma^+ \Sigma^-)(\Sigma^+ \Sigma^-)} & G_{(\Sigma^+ \Sigma^-)(\Sigma^+ \Lambda)} \\
G_{(\Sigma^+ \Lambda)(\Xi^0 p)} & G_{(\Sigma^+ \Lambda)(\Sigma^+ \Sigma^-)} & G_{(\Sigma^+ \Lambda)(\Sigma^+ \Lambda)} \\
\end{pmatrix}
\]

\[
Q=-1:
\begin{pmatrix}
G^{SLJ}_{(\Xi n)(\Xi^- n)} & G_{(\Xi n)(\Sigma \Sigma^0)} & G_{(\Xi n)(\Sigma \Lambda)} \\
G_{(\Sigma \Sigma^0)(\Xi^- n)} & G_{(\Sigma \Sigma^0)(\Sigma \Sigma^0)} & G_{(\Sigma \Sigma^0)(\Sigma \Lambda)} \\
G_{(\Sigma \Lambda)(\Xi^- n)} & G_{(\Sigma \Lambda)(\Sigma \Sigma^0)} & G_{(\Sigma \Lambda)(\Sigma \Lambda)} \\
\end{pmatrix}
\]
Results
Hyperon single-particle potentials

- obtained by using \(YN,YY\) S-wave forces form QCD.

\[ \rho = 0.17 \text{ [fm}^{-3}\text{]} \]

Vertical vars show statistical error only

PoS INPC2016 277

Preliminary
Hyperon single-particle potentials

- obtained by using $YN, YY$ S-wave forces form QCD.
- Results are compatible with experimental suggestion.

$$U_{\Lambda}^{\text{Exp}}(0) \approx -30\text{, } U_{\Xi}(0)^{\text{Exp}} \approx -10\text{, } U_{\Sigma}^{\text{Exp}}(0) \geq +20\text{ [MeV]}$$

attraction\hspace{2cm} attraction small\hspace{2cm} repulsion

@ $\rho = 0.17\text{ [fm}^{-3}\text{]}$

PoS INPC2016 277
Hyperon single-particle potentials

- obtained by using $YN, YY$ S-wave forces form QCD.
- Results are compatible with experimental suggestion.

\[ U_\Lambda^{\text{Exp}}(0) \approx -30, \quad U_{\Xi}(0)^{\text{Exp}} \approx -10, \quad U_{\Sigma}^{\text{Exp}}(0) \geq +20 \text{ [MeV]} \]
Hyperon single-particle potentials

- **Skybule** curves show $U_\Xi(k)$ w/ original $S=-2$ $BB$ potentials including explicit $SU(3)_F$ breaking.
- We see that the flavor symmetric approximation used in the blue ones is reasonable for $\Xi N$, $YY$ forces.
Hyperon single-particle potentials

- Breakdown of $U_Y(\theta ; \rho_0)$ in SNM including spin, iso-spin multiplicity

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Hyperon single-particle potentials

- Breakdown of $U_Y(0; \rho_0)$ in SNM including spin, iso-spin multiplicity

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<td>(^3D_1)</td>
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<td>7.43</td>
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<table>
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<td>(^3D_1)</td>
<td>(^1S_0)</td>
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<td>7.12 &amp; -2.41 &amp; (-0.08)</td>
<td>(-4.11)</td>
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$U_\Xi$ w/ original $BB$ potentials including explicit flavor $SU(3)$ breaking
Chemical potentials in NSM

- Neutron Star Matter: \( \text{ANM} + e^- , \mu^- \@ Q=0, \beta\text{-eq.} \)

- Parabola approx. for ANM

\[
\begin{align*}
\mu_p(\rho; \beta) &= \mu_{SNM}^N(\rho) + \beta^2 \frac{dE^\text{sym}(\rho)}{d\rho} - \beta(\beta+2)E^\text{sym}(\rho) \\
\mu_n(\rho; \beta) &= \mu_{SNM}^N(\rho) + \beta^2 \frac{dE^\text{sym}(\rho)}{d\rho} - \beta(\beta-2)E^\text{sym}(\rho) \\
4E^\text{sym}(\rho) &= \mu_{P\text{NM}}^n(\rho) - \mu_{P\text{NM}}^p(\rho), \quad \beta = 1 - 2x_p
\end{align*}
\]

- Hyperon chemical in NSM \( \mu_Y(\rho) \approx M_Y - M_N + U^\text{ANM}_Y(0; \rho) \)

- Hyperons appear as \( n \rightarrow Y^0 \) when \( \mu_n > \mu_{Y^0} \)

\( nn \rightarrow pY^- \) when \( 2\mu_n > \mu_p + \mu_Y \)
Hyperon onset in NSM  (just for fun)

- Result indicate $\Lambda$, $\Sigma^-$, $\Xi^-$ appear around $\rho = 3.0 - 4.0 \rho_0$
- However,
  - $YN_{L=1,2}$... and $YNN$ force could be important at high density.
  - We may need to compare with more sophisticated $\mu_n, \mu_p$ than BHF.
Summary and Outlook
Summary and Outlook

★ We've explained our goal and approach
  • Want to do (strange) nuclear physics starting from QCD.
  • Extract $BB$ interaction potentials in lattice QCD simulation.
  • Then, apply potentials to many-body theories and so on.

★ We've introduced HAL QCD method
  • Utilize spatial correlation containing information of interaction.
  • This method avoid difficulty in the temporal plateau approach to multi-hadron system in lattice QCD.

★ We've shown HAL QCD $BB$ potentials
  • We obtain QCD prediction of hyperon interactions.
  • We obtain (qualitatively) reasonable two-nucleon force.
  • We reveal nature of general $BB$ S-wave interactions.
Summary and Outlook

★ Results of application

• We studied hyperon s.p. potentials w/ the $YN, YY$ forces.
  • First, I used rotated diagonal data in the flavor basis.
• We obtained $UY$ compatible with experiment!
  • In SNM, $\Lambda$ and $\Xi$ feel attraction, while $\Sigma$ feels repulsion.
  • This is a remarkable success and very encouraging.
  • Recall that we've never used any experimental data about hyperon interactions, but we used only QCD.
• Dr. Hiyama will talk on $\Xi$-hypernuclcus with HALQCD $V_{\Xi N}$.

★ Outlook

• We’ll try to extract hyperon forces in higher partial waves, higher order terms of $\nabla$-expansion, and $BBB$ forces so that we can attack high density matter like NS.
• I hope we can explain hypernuclei from QCD, and we can solve the hyperon puzzle of NS, in near future.
Thank you !!
Backup
Hyperon single-particle potentials

- \( \text{Im}[U_Y] \) are obtained by summing up \( \text{Im}[G_{YN,YN}] \).
- But, \( \text{Im}[U_B] \) are not taken into the Bethe-Goldsone eq.
## Hyperon single-particle potentials

- Breakdown of $U_Y(0; \rho_0)$ in SNM including sign multiplicity

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Nijmegen

Partial wave contributions to $U_\Lambda(\rho_0)^{(a)}$

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Partial wave contributions to $U_\Sigma(\rho_0)$

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## Partial wave contributions to $U_\Xi(\rho_0)$

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# Quark model

Taken from M. Kohno et al.

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Effective two-body potential

\[ \bar{V}(\rho; r) = \tau_1 \cdot \tau_2 [\sigma_1 \cdot \sigma_2 V_C(\rho; r) + S_{12}(\hat{r}) V_T(\rho; r)] + V_R(\rho; r) \]

- obtained by integrating out positon of 3rd nucleon.
- Here, \( \bar{V}(\rho, r) \) is \( \rho \)-propotional due to a fixed defect.
Urbana NNN force is adjusted so that AV18 + Urbana reproduce the “empirical” saturation property of SNM.
In $I=1/2$, $^1S_0$ channel, $\Lambda N$ has an attraction, while $\Sigma N$ is repulsive.
In $I=1/2$, $^3S_1$ channel, both $\Lambda N$ and $\Sigma N$ have an attraction.
In $I=1/2$, strong tensor coupling in flavor off-diagonal.
Many experimentally unknown coupled-channel potentials.

One can see predictive power of the HALQCD method.
FAQ

1. Does your potential depend on the choice of source?

2. Does your potential depend on choice of operator at sink?

3. Does your potential $U(r,r')$ or $V(r)$ depends on energy?
FAQ

1. Does your potential depend on the choice of source?
   ➔ No. Some sources may enhance excited states in 4-point func. However, it is no longer a problem in our new method.

2. Does your potential depend on choice of operator at sink?
   ➔ Yes. It can be regarded as the “scheme” to define a potential. Note that a potential itself is not physical observable. We will obtain unique result for physical observables irrespective to the choice, as long as the potential $U(r,r')$ is deduced exactly.
3. Does your potential $U(r,r')$ or $V(r)$ depends on energy?

→ By definition, $U(r,r')$ is non-local but energy independent. While, determination and validity of its leading term $V(r)$ depend on energy because of the truncation.

However, we know that the dependence in $NN$ case is very small (thanks to our choice of sink operator = point) and negligible at least at $E_{lab.} = 0 – 90$ MeV. We rely on this in our study.

If we find some dependence, we will determine the next leading term of the expansion from the dependence.
FAQ

in SU(3)\(_F\) limit, ie. heavy u,d quark world

4. Is the H a compact **six-quark** object or a tight **BB bound** state?
FAQ

found in $SU(3)_F$ limit, ie. heavy u,d quark world

4. Is the H a compact six-quark object or a tight BB bound state?

> Both.

There is no distinct difference between two in QCD. Note that baryon is made of three quarks in QCD. Imagine a compact 6-quark object in $(0S)^6$ configuration. This configuration can be re-written in a form of $(0S)^3 \times (0S)^3 \times \text{Exp}(-a r^2)$ with relative coordinate $r$. This demonstrate that a compact six-quark object, at the same time, has a BB type configuration. In LQCD simulation at $SU(3)_F$ limits, we've established existence of a $B=2, S=-2, I=0$ stable QCD eigenstate.