The nΛ scattering length and the nnΛ resonance

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Brief Outline

• What do we know about nΛ scattering?
• Experimental data in hand. (pΛ, CSB)
• An nnΛ bound state? A resonance?
• What is possible theoretically? Our model.
• nnΛ resonance properties.
• Summary
• JLab $^3\text{H}(e,e' K^+)nnΛ$ tritium experiment.
• Nothing about nΛ scattering has been measured.
• We have inferred something from pΛ scattering plus the binding energies of few-body Λ hypernuclei.
• Starting from the sparse pΛ data we have inferred a Charge Symmetry Breaking (CSB) difference between pΛ and nΛ interactions from the A=4 mirror hypernuclei binding. However, we do not know whether the observed CSB arises from the fundamental two-body NΛ interaction or from a possible NNA three-body force.
• The HypHI collaboration reported seeing a $^3_\Lambda n$ bound state. Given our knowledge of the nn interaction, a $^3_\Lambda n$ bound state would provide a strong nΛ interaction constraint. Moreover, JLab would be an ideal facility to check the claim using the $^3H(e,e'K^+)^3_\Lambda n$ reaction.

Theoretically, the possibility of a $^3_\Lambda n$ bound state seems remote. However, the coming tritium experiment by Tang and collaborators should provide a definitive answer.
Question: How can we model the nΛ interaction, when we have only limited data regarding pΛ scattering?

Few-body hypernuclei:

- Λ hypernuclei provide weak constraints
  - $^3\Lambda H$ is weakly bound [$B_{\Lambda}(^3\Lambda H) = 0.13 \pm 0.05$ MeV]; small separation energy implies that $^3\Lambda H$ is one of our largest halo nuclei.
  - The A=4 isodoublet seems to exhibit significant Charge Symmetry Breaking, some 2-3 times that in the $^3H$-$^3He$ isodoublet.
  - The uncertainty in the sparse pΛ data implies a potentially wide range of variation in the nΛ interaction.

- Recent experiments have suggested a decrease in the apparent size of the A=4 CSB.
  - Esser et al., PRL 114, 232501 (2015) report a value for $B_{\Lambda}(^4\Lambda H)$ of $2.12 \pm 0.01$ (stat.) $\pm 0.09$ (syst.).
  - Yamamoto et al., PRL 115, 222501 (2015) have measured the gamma decay in $^4\Lambda H$ and obtained a value of $1.406 \pm 0.02 \pm 0.02$ MeV, indicating that CSB in the $1^+$ excited states is quite small.
FIG. 1 (color online). Level schemes of the mirror hypernuclei, $^{3}_{1}H$ and $^{4}_{1}He$. A binding energies ($B_\Lambda$) of $^{4}_{1}H(0^+)$ and $^{4}_{1}He(0^+)$ are taken from past emulsion experiments [2]. $B_\Lambda(^{4}_{1}He(1^+))$ and $B_\Lambda(^{4}_{1}H(1^+))$ are obtained using the present data and past $\gamma$-ray data [6-8], respectively. Recently, $B_\Lambda(^{4}_{1}H(0^+)) = 2.12 \pm 0.01 \, \text{(stat)} \pm 0.09 \, \text{(syst)} \, \text{MeV}$ was obtained with an independent technique [5].
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  - The A=4 isodoublet seems to exhibit significant Charge Symmetry Breaking, some 2-3 times that in the \(^3\text{H}-^3\text{He}\) isodoublet.
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  - Yamamoto et al., PRL 115, 222501 (2015) have measured the gamma decay in \(^4\Lambda\)H and obtained a value of \(1.406 \pm 0.02 \pm 0.02\) MeV, indicating that CSB in the 1\(^+\) excited states is quite small.

Nevertheless, we do not understand CSB in the mirror Λ hypernuclei. Therefore, we cannot accurately model the nΛ interaction.
Question: How can we model the nΛ interaction, when we have only limited data regarding pΛ scattering?

A $^3\Lambda n$ hypernucleus?

- The HypHI collaboration reported a bound nnΛ system ($^3\Lambda n$).
  - They observed both two-body and three-body decay modes.
  - $^3\Lambda n$ would be the lightest neutron-rich hypernucleus observed.
  - Such a bound state would provide a significant constraint on the nΛ interaction, because the nn interaction is well known.
  - Such a bound state could be observed directly in a $^3\text{H}(e,e'K^+)^3\Lambda n$ experiment at JLab, although a weakly bound system would imply a small cross section.
  - Alternative reactions at J-PARC would be $^3\text{H}(K^-,\pi^0)^3\Lambda n$ and $^3\text{He}(K^-,\pi^+)^3\Lambda n$. The latter, being a double-charge-exchange reaction, suggests a very small cross section.
A $^3_\Lambda n$ hypernucleus?

A $^3_\Lambda n$ bound state has been strongly questioned:


Simple physics suggests that one would not expect a bound state.

- The hypertriton is barely bound and its core is a deuteron.
- A $^3_\Lambda n$ bound state would have as a core an unbound di-neutron.
A $^{3}\Lambda n$ hypernucleus?

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Could there instead exist an nn$\Lambda$ three-body resonance?

If so, then one could still utilize the JLab electro-production reaction (or perhaps the J-PARC strangeness-exchange reaction) to constrain the n$\Lambda$ interaction.
Where do we stand?

The Science

• *No published* nΛ data exist! Our numerous NA potential models have never been tested against nΛ data.

• Were a bound $^3_A$Λ hypernucleus to exist, our knowledge of the nn interaction would permit us to significantly constrain the low-energy properties of the nΛ system. *Existence of such a bound state has been thoroughly questioned theoretically!*

• However, either a strong sub-threshold resonance or an actual physical resonance in the nnΛ system should exist. The resonance position and the shape of the spectrum in a $K^+$ electro-production measurement from a tritium target at JLab leading to an nnΛ final state would provide a significant constraint upon the low-energy properties of the heretofore unmeasured nΛ interaction.
Where do we stand?

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*No published nΛ data exist!* Our multiple NA potential models have never been tested against nΛ data. Were a bound $^3_Λn$ hypernucleus to exist, our knowledge of the nn interaction would permit us to significantly constrain the low-energy properties of the nΛ system. *Existence of such a bound state has been thoroughly questioned theoretically!* However, either a strong sub-threshold resonance or an actual physical resonance in the nnΛ system should exist. The resonance position and the shape of the spectrum in a $K^+$ electroproduction measurement from a tritium target leading to an nnΛ final state would provide a significant constraint upon the low-energy properties of the unmeasured nΛ interaction.

The Impact

- Understanding $Λ$ hypernuclei (ground states and single-particle spectra) requires knowledge of the nΛ interaction. Scattering data exist only for pΛ.
- Data on an nnΛ (sub-threshold or physical) resonance would constrain the critical low-energy properties of the nΛ interaction.
- nΛ knowledge would elucidate Charge Symmetry Breaking in the NA interaction, provide a realistic basis for understanding the $Λ$-hypernuclei data, enhance our calculations of neutron-rich hypernuclei, and constrain our modeling of neutron stars.
Our nnΛ Three-Body Model

- pairwise s-wave interactions of rank one separable form
  \[ V(k, k') = g(k)C g(k') ; \quad g(k) = 1/(k^2 + \beta^2) \]
- \( nn \) potential strength and range \( \sim \) effective range parameters:
  \[ a_{nn} = -18.9 \pm 0.4 \text{ fm} \text{ and } r_{nn} = 2.75 \pm 0.11 \text{ fm} \]
- \( n\Lambda \) strength and range fitted to the Nijmegen model D values:
  \[ a_s = -2.03 \pm 0.32 \text{ fm} \text{ and } r_s = 3.66 \pm 0.32 \text{ fm} \]
  \[ a_t = -1.84 \pm 0.10 \text{ fm} \text{ and } r_t = 3.32 \pm 0.11 \text{ fm} \]
- Separable potentials allow us to (simply) analytically continue onto the second sheet of the energy plane.
- We search for resonance poles by examining the eigenvalue spectrum of the kernel of the Faddeev equations for the \( nn\Lambda \) system.
- We previously used such a technique to explore \( \Lambda - d \) scattering:
Searching for Resonances in the nnΛ System

We must analytically continue the Faddeev equations onto the second energy sheet.

• For two identical Fermions interacting via Yamaguchi pairwise potentials, the homogeneous integral equation is of the form

\[ \lambda_n(E) \phi_{n,k \alpha}(q, E) = \sum_{k \beta} \int_{0}^{\infty} Z_{k \alpha,k \beta}^{JT}(q, q'; E) \, \tau_{k \beta}[E - \epsilon_{\beta}(q')] \times \phi_{n,k \beta}(q', E) \, q'^2 \, dq'. \]

• We analytically continue onto the second energy sheet by utilizing the transformation

\[ q \rightarrow q e^{-i \theta} \quad q' \rightarrow q' e^{-i \theta} \quad \text{with} \quad \theta > 0. \quad (1) \]

• The rotation angle \( \theta \) is limited by kernel singularities. The Born amplitude \( Z_{k \alpha,k \beta}^{JT} \) requires that \( \theta < \frac{\pi}{2} \), which gives us the region \( \Im(E) < 0 \) on the second Riemann sheet. The other source of singularities is the quasi-particle propagator \( \tau_{k \beta}[E - \epsilon_{\beta}(q')] \); because there are no two-body bound states, the rotation is not limited.
Results of the Eigenvalue Search

Consider the specific example utilizing the nn and the $^{1}S_{0}$ and $^{3}S_{1}$ nA potentials defined previously.

- **nn potential strength and range ~ effective range parameters:**
  \[ a_{nn} = -18.9 \pm 0.4 \text{ fm} \quad \text{and} \quad r_{nn} = 2.75 \pm 0.11 \text{ fm} \]

- **nA strength and range fitted to the Nijmegen model D values:**
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1. **We searched the complex energy plane for the largest eigenvalue of the kernel = 1 and found a pole at:**

   \[ E = -0.154 - 0.753 i \text{ MeV} \quad \text{with eigenvalue} \quad \lambda(E) = 1.0000 - 0.0001 i . \]

2. **Because \( \Re(E) < 0 \), this pole corresponds to a sub-threshold resonance, one that lies below the breakup threshold in a region inaccessible by experiment.**
Results of the Eigenvalue Search

We considered the example utilizing the $nn$ and the $^1S_0$ and $^3S_1$ nΛ potentials previously defined.

- We searched in the complex energy plane for the largest eigenvalue of the kernel $= 1$ and found a pole at:

$$E = -0.154 - 0.753 \, i \, \text{MeV} \quad \text{with eigenvalue} \quad \lambda(E) = 1.0000 - 0.0001 \, i \, .$$

- Because $\Re (E) < 0$, this pole corresponds to a sub-threshold resonance, one that lies below the breakup threshold in a region inaccessible to experiment.

Because the pole lies below the breakup threshold, we ask how easily the pole can be converted into a physical resonance (or bound state).

- We scale the strength of the $^1S_0$ and $^3S_1$ nΛ potentials by $s$.

- We follow the path of the pole as it turns into a ”resonance” and then into a bound state.

- An increase in strength of $\sim 7\%$ produces a physical resonance, one above the three-body breakup threshold.

- An increase of $35\%$ produces a $^3\Lambda n$ bound state.
Trajectory of the nnΛ ”Resonance” Pole

\[ \text{Re}[E] \]
\[ \text{Im}[E] \]

\( s = 1.000 \)
\( s = 1.400 \)
\( s = 1.175 \)

\( \text{Mod. D} \)
Trajectory of the nnΛ ”Resonance” Pole

In the figure one follows the trajectory of the ”resonance” pole as the strength \( s \) of the nΛ interaction is increased from a value of 1.0 in increments (\( \Delta s \)) of 0.025. One starts from a sub-threshold resonance at values of \( s = 1.000 \) up to \( s = 1.050 \). For \( s = 1.075 \) up to \( s = 1.350 \) we obtain a physical resonance; in particular, we obtain a resonance with \( E = 0.129 - 0.291i \) MeV at \( s = 1.175 \). As \( s \) is further increased, we obtain a bound state with energy \( E = -0.069 \) MeV at \( s = 1.375 \) and \( E = -0.158 \) at \( s = 1.400 \). Thus, for this particular model one can see that an nΛ potential whose parameters lie within the uncertainty of the observed low energy pΛ scattering parameters could produce a physical resonance in the nnΛ system.
Trajectory of the nnΛ ”Resonance” Pole

In the figure one follows the trajectory of the ”resonance” pole as the strength $s$ of the nΛ interaction is increased from a value of 1.0 in increments ($\Delta s$) of 0.025. One starts from a sub-threshold resonance at values of $s = 1.000$ up to $s = 1.050$. For $s = 1.075$ up to $s = 1.350$ we obtain a physical resonance; in particular, we obtain a resonance with $E = 0.129 - 0.291i$ MeV at $s = 1.175$. As $s$ is further increased, we obtain a bound state with energy $E = -0.069$ MeV at $s = 1.375$ and $E = -0.158$ at $s = 1.400$. Thus, for this particular model one can see that an nΛ potential whose parameters lie within the uncertainty of the observed low energy pΛ scattering parameters could produce a physical resonance in the nnΛ system.
To explore the sensitivity to four rather different NΛ potential models:


we repeated the eigenvalue search for each of the potential models. None of the four models produces an observable resonance; each produces a sub-threshold resonance, which lies below the nnΛ threshold. A change in $s$ of as little as 5% produces a resonance above the three-body threshold, but a change of at least 25% is required to produce a bound $^3\Lambda n$. 

Trajectory of the $nn\Lambda$ ”Resonance” Pole for Four Contemporary Models
Summary

The Science

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The Impact

Understanding Λ hypernuclei (ground states and single-particle spectra) measured at facilities around the world (JLab, RHIC, Mainz, J-PARC, DaΦne) requires knowledge of the nΛ interaction. Scattering data exist only for pΛ. Data on an nnΛ (sub-threshold or physical) resonance would enhance our knowledge of the critical low-energy properties of the nΛ interaction. Such knowledge would elucidate the important Charge Symmetry Breaking relative to the measured pΛ interaction, provide a realistic basis for understanding the long existing Λ hypernuclei data, enhance our calculations of neutron-rich hypernuclei, and constrain our modeling of neutron stars.
Immediate Future: JLab Tritium Experiment

Determining the Unknown \( \Lambda \)-n Interaction by Investigating the \( \Lambda nn \) Resonance


The availability of a \( T_2 \) gas target in Hall A provides a unique opportunity to measure the n-n-\( \Lambda \) three-body resonance. Only at JLab with the (e,e’ K\(^+\)) reaction can the resonance position (excitation energy) and width be measured with the required precision. It is anticipated that using a 4.4 GeV beam one can achieve an energy resolution of \( \sim 2 \) MeV FWHM and an absolute missing mass precision of \( \sim \pm 0.20 \) MeV. This data will permit use of a theoretical pole search technique to determine experimentally for the first time the n\( \Lambda \) interaction utilizing together the JLab improved p\( \Lambda \) scattering data and the \( \Lambda \) hypernuclei Charge Symmetry Breaking data. Scheduled run beginning in mid October.