The nΛ scattering length and the nnΛ resonance

B. F. Gibson
Nuclear and Particle Physics, Astrophysics and Cosmology Group, T-2
Theoretical Division
Los Alamos National Laboratory

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Collaborator: Iraj R. Afnan, Flinders University

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Brief Outline

- What do we know about nΛ scattering?
- Experimental data in hand. (pΛ, CSB)
- An nnΛ bound state? A resonance?
- What is possible theoretically? Our model.
- nnΛ resonance properties.
- Summary
- JLab ^{3}H(e,e' K$^+$)nnΛ tritium experiment.

- Nothing about nΛ scattering has been measured.
- We have inferred something from pΛ scattering plus the binding energies of few-body Λ hypernuclei.
- Starting from the sparse pΛ data we have inferred a Charge Symmetry Breaking (CSB) difference between pΛ and nΛ interactions from the A=4 mirror hypernuclei binding. However, we do **not** know whether the observed CSB arises from the fundamental two-body NΛ interaction or from a possible NNΛ three-body force.
- The HypHI collaboration reported seeing a $^{3}_{\Lambda}$n bound state. Given our knowledge of the nn interaction, a $^{3}_{\Lambda}$n bound state would provide a strong nΛ interaction constraint. Moreover, JLab would be an ideal facility to check the claim using the ^{3}H(e,e$'$K^{+})$^{3}_{\Lambda}$n reaction.

Theoretically, the possibility of a $^{3}_{\Lambda}$n bound state seems remote. However, the coming tritium experiment by Tang and collaborators should provide a definitive answer.

Question: How can we model the nΛ interaction, when we have only limited data regarding pΛ scattering?

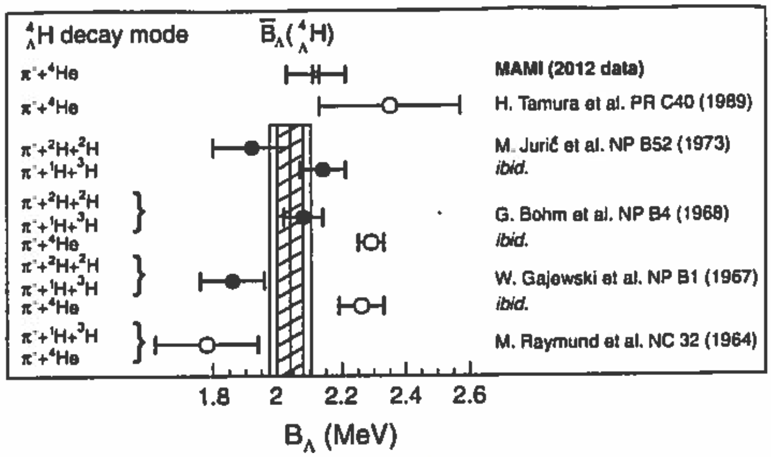
Few-body hypernuclei:

- Λ hypernuclei provide weak constraints
  - ${}^3_\Lambda$H is weakly bound [$B_\Lambda({}^3_\Lambda\text{H}) = 0.13 \pm 0.05$ MeV]; small separation energy implies that ${}^3_\Lambda$H is one of our largest halo nuclei.
  - The A=4 isodoublet seems to exhibit significant Charge Symmetry Breaking, some 2-3 times that in the ^{3}H-^{3}He isodoublet.
  - The uncertainty in the sparse pΛ data implies a potentially wide range of variation in the nΛ interaction.

- Recent experiments have suggested a decrease in the apparent size of the A=4 CSB.
  - Esser *et al.*, PRL **114**, 232501 (2015) report a value for $\text{B}_\Lambda({}^4_\Lambda\text{H})$ of 2.12 $\pm$ 0.01(stat.) $\pm$ 0.09(syst.).
  - Yamamoto *et al.*, PRL **115**, 222501 (2015) have measured the gamma decay in ${}^4_\Lambda$H and obtained a value of 1.406 $\pm$ 0.02 $\pm$ 0.02 MeV, indicating that CSB in the 1^+ excited states is quite small.

2.12 ± 0.01 ± 0.09 MeV

PRL 114, 232501 (2015)



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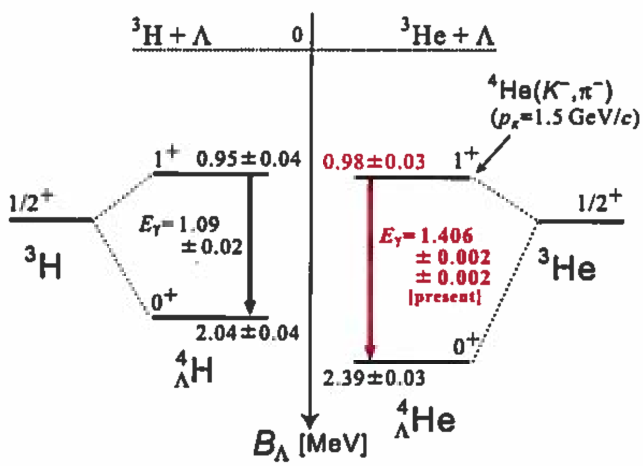

FIG. 1 (color online). Level schemes of the mirror hypernuclei, $^4_\Lambda$H and $^4_\Lambda$He. Λ binding energies (B_Λ) of $^4_\Lambda$H(0^+) and $^4_\Lambda$He(0^+) are taken from past emulsion experiments [2]. $B_\Lambda(^4_\Lambda\mathrm{He}(1^+))$ and $B_\Lambda(^4_\Lambda\mathrm{H}(1^+))$ are obtained using the present data and past γ-ray data [6–8], respectively. Recently, $B_\Lambda(^4_\Lambda\mathrm{H}(0^+)) = 2.12 \pm 0.01(\mathrm{stat}) \pm 0.09(\mathrm{syst})$ MeV was obtained with an independent technique [5].

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  - The A=4 isodoublet seems to exhibit significant Charge Symmetry Breaking, some 2-3 times that in the ^{3}H-^{3}He isodoublet.
  - The uncertainty in the sparse pΛ data implies a possible wide range of variation in the nΛ interaction.
- Recent experiments have decreased the apparent size of the A=4 CSB.
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Nevertheless, we do not understand CSB in the mirror Λ hypernuclei. Therefore, we cannot accurately model the nΛ interaction.

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A ${}^{3}_{\Lambda}$n hypernucleus?

- The HypHI collaboration reported a bound nnΛ system (${}^{3}_{\Lambda}$n).
  - C. Rappold *et al.*, Phys. Rev. C **88**, 041001(R) (2013).
  - They observed both two-body and three-body decay modes.
  - ${}^{3}_{\Lambda}$n would be the lightest neutron-rich hypernucleus observed.
  - Such a bound state would provide a significant constraint on the nΛ interaction, because the nn interaction is well known.
  - Such a bound state could be observed directly in a ^{3}H(e,e$'$K^{+})${}^{3}_{\Lambda}$n experiment at JLab, although a weakly bound system would imply a small cross section.
  - Alternative reactions at J-PARC would be ^{3}H(K^{-}, π^{0})${}^{3}_{\Lambda}$n and ^{3}He(K^{-}, π^{+})${}^{3}_{\Lambda}$n. The latter, being a double-charge-exchange reaction, suggests a very small cross section.

A ${}^{3}_{\Lambda}$n hypernucleus?

A ${}^{3}_{\Lambda}$n bound state has been strongly questioned:

- H. Garcilazo and A. Valcarce, Phys. Rev. C **89** 057001 (2014).
- E. Hiyama *et al.*, Phys. Rev. C **89** 061302 (2014).
- A. Gal and H. Garcilazo, Phys. Lett. **B736**, 93 (2014).

Simple physics suggests that one would not expect a bound state.

- The hypertriton is barely bound and its core is a deuteron.
- A ${}^{3}_{\Lambda}$n bound state would have as a core an unbound di-neutron.

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Could there instead exist an nnΛ three-body resonance?

If so, then one could still utilize the JLab electro-production reaction (or perhaps the J-PARC strangeness-exchange reaction) to constrain the nΛ interaction.

Where do we stand?

The Science

- *No published* nΛ data exist **!** Our numerous NΛ potential models have never been tested against nΛ data.
- Were a bound ${}^{3}_{\Lambda}$n hypernucleus to exist, our knowledge of the nn interaction would permit us to significantly constrain the low-energy properties of the nΛ system. *Existence of such a bound state has been thoroughly questioned theoretically* **!**
- However, either a strong sub-threshold resonance or an actual physical resonance in the nnΛ system should exist. The resonance position and the shape of the spectrum in a K^+ electro-production measurement from a tritium target at JLab leading to an nnΛ final state would provide a significant constraint upon the low-energy properties of the heretofore unmeasured nΛ interaction.

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The Impact

- Understanding Λ hypernuclei (ground states and single-particle spectra) requires knowledge of the nΛ interaction. Scattering data exist only for pΛ.
- Data on an nnΛ (sub-threshold or physical) resonance would constrain the critical low-energy properties of the nΛ interaction.
- nΛ knowledge would elucidate Charge Symmetry Breaking in the NΛ interaction, provide a realistic basis for understanding the Λ-hypernuclei data, enhance our calculations of neutron-rich hypernuclei, and constrain our modeling of neutron stars.

Our nnΛ Three-Body Model

- pairwise s-wave interactions of rank one separable form

$$V(k,k') = g(k)Cg(k') \; ; \qquad g(k) = 1/(k^2 + \beta^2)$$

- nn potential strength and range $\sim$ effective range parameters:

  a_{nn} = -18.9 ± 0.4 fm and r_{nn} = 2.75 ± 0.11 fm

- $n\Lambda$ strength and range fitted to the Nijmegen model D values:

  a_s = -2.03 ± 0.32 fm and r_s = 3.66 ± 0.32 fm

  a_t = -1.84 ± 0.10 fm and r_t = 3.32 ± 0.11 fm

  M. M. Nagels, T. A. Rijken, & J. J. deSwart, PRD **15**, 2547 (1977)

- Separable potentials allow us to (simply) analytically continue onto the second sheet of the energy plane.
- We search for resonance poles by examining the eigenvalue spectrum of the kernel of the Faddeev equations for the $nn\Lambda$ system.
- We previously used such a technique to explore $\Lambda - d$ scattering:

  I. R. Afnan and B. F. Gibson, PRC **47**, 1000 (1993).

Searching for Resonances in the nnΛ System

We must analytically continue the Faddeev equations onto the second energy sheet.

- For two identical Fermions interacting via Yamaguchi pairwise potentials, the homogeneous integral equation is of the form

$$\lambda_n(E)\,\phi_{n,k_\alpha}(q,E) = \sum_{k_\beta}\int_0^\infty Z^{JT}_{k_\alpha,k_\beta}(q,q';E)\;\tau_{k_\beta}[E-\epsilon_\beta(q')] \times \phi_{n;k_\beta}(q',E)\;q'^2\,dq'\;.$$

- We analytically continue onto the second energy sheet by utilizing the transformation

$$q \to q\,e^{-i\theta} \qquad q' \to q'\,e^{-i\theta} \qquad \text{with} \qquad \theta > 0\;. \tag{1}$$

- The rotation angle θ is limited by kernel singularities . The Born amplitude $Z^{JT}_{k_\alpha,k_\beta}$ requires that $\theta < \frac{\pi}{2}$, which gives us the region $\Im(E) < 0$ on the second Riemann sheet. The other source of singularities is the quasi-particle propagator $\tau_{k_\beta}[E-\epsilon_\beta(q')]$; because there are no two-body bound states, the rotation is not limited.

Results of the Eigenvalue Search

Consider the specific example utilizing the nn and the $^1S_0$ and $^3S_1$ nΛ potentials defined previously.

- nn potential strength and range $\sim$ effective range parameters:

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$$a_s = \text{-}2.03 \pm 0.32 \text{ fm and } r_s = 3.66 \pm 0.32 \text{ fm}$$
$$a_t = \text{-}1.84 \pm 0.10 \text{ fm and } r_t = 3.32 \pm 0.11 \text{ fm}$$

1. We searched the complex energy plane for the largest eigenvalue of the kernel $= \mathbf{1}$ and found a pole at:

$$E = -0.154 - 0.753\,i \text{ MeV} \quad \text{with eigenvalue} \quad \lambda(E) = 1.0000 - 0.0001\,i\ .$$

2. Because $\Re(E) < 0$, this pole corresponds to a sub-threshold resonance, one that lies below the breakup threshold in a region inaccessible by experiment.

Results of the Eigenvalue Search

We considered the example utilizing the nn and the $^1\mathrm{S}_0$ and $^3\mathrm{S}_1$ $\mathrm{n}\Lambda$ potentials previously defined.

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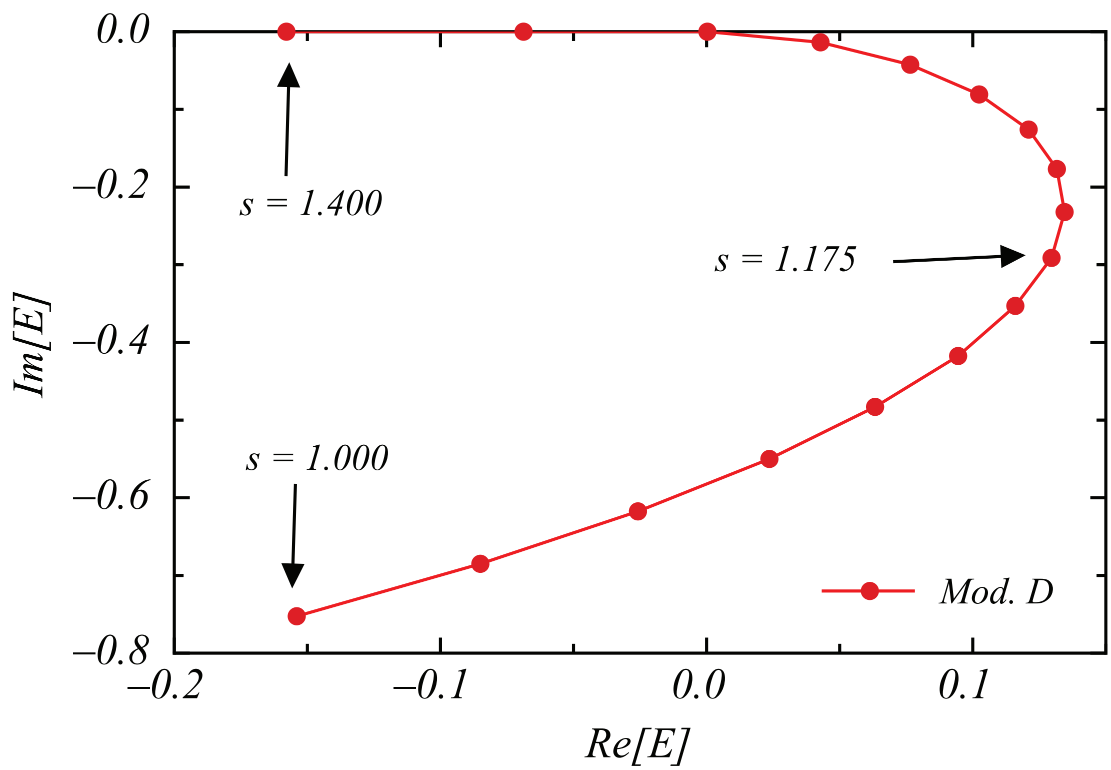
Because the pole lies below the breakup threshold, we ask how easily the pole can be converted into a physical resonance (or bound state).

- We scale the strength of the $^1\mathrm{S}_0$ and $^3\mathrm{S}_1$ $\mathrm{n}\Lambda$ potentials by s.
- We follow the path of the pole as it turns into a "resonance" and then into a bound state.
- An increase in strength of $\sim 7\%$ produces a physical resonance, one above the three-body breakup threshold.
- An increase of 35% produces a $^3_\Lambda\mathrm{n}$ bound state.

Trajectory of the nnΛ "Resonance" Pole



Trajectory of the nnΛ "Resonance" Pole

In the figure one follows the trajectory of the "resonance" pole as the strength s of the nΛ interaction is increased from a value of 1.0 in increments (Δs) of 0.025. One starts from a sub-threshold resonance at values of $s = 1.000$ up to $s = 1.050$. For $s = 1.075$ up to $s = 1.350$ we obtain a physical resonance; in particular, we obtain a resonance with $E = 0.129 - 0.291i$ MeV at $s = 1.175$. As s is further increased, we obtain a bound state with energy $E = -0.069$ MeV at $s = 1.375$ and $E = -0.158$ at $s = 1.400$. Thus, for this particular model one can see that an nΛ potential whose parameters lie within the uncertainty of the observed low energy pΛ scattering parameters could produce a physical resonance in the nnΛ system.

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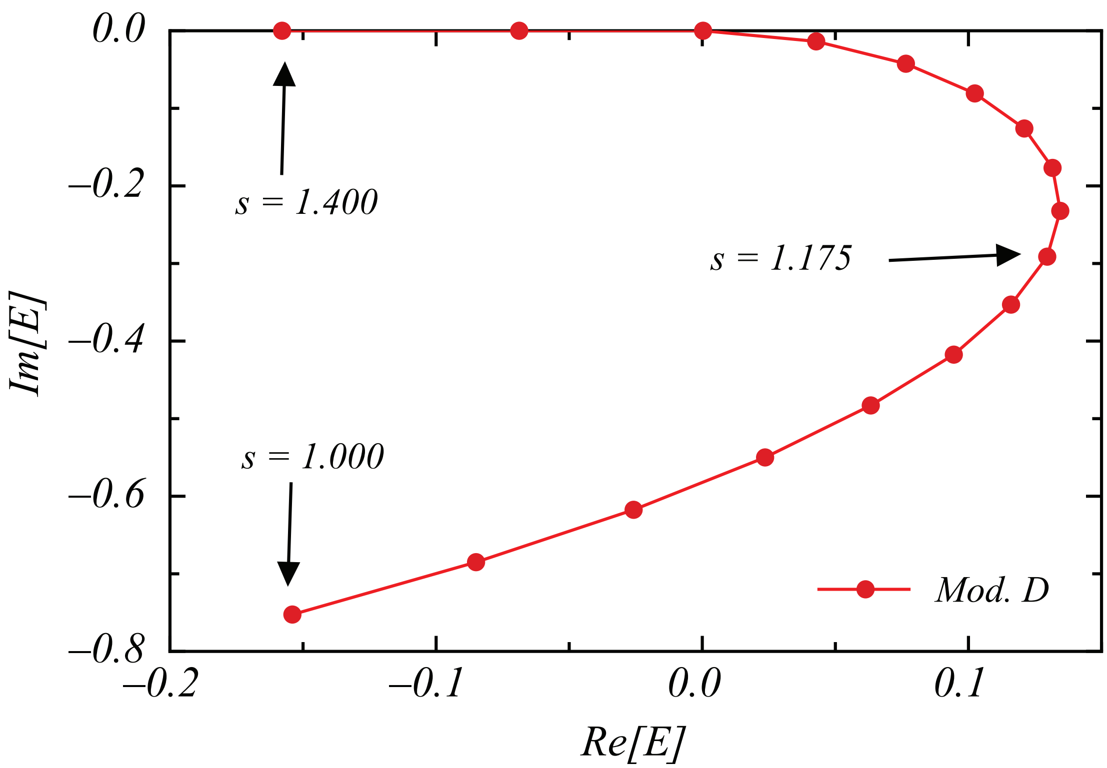

Trajectory of the nnΛ "Resonance" Pole for Four Contemporary Models

To explore the sensitivity to four rather different NΛ potential models

- M. M. Nagels, T. A. Rijken, and J. J. de Swart, "Baryon-baryon scattering in a one-boson-exchange-potential approach, II. Hyperon-nucleon scattering", Phys, Rev. D **15**, 2547 (1977). (*ModD*)
- T. A. Rijken, V. G. J. Stoks, Y. Yamamoto, "Soft-core hyperon-nucleon potentials", Phys. Rev. C **59**, 21 (1999). ($NSC97f$)
- J. Haidenbauer and U.-G. Meißner, "Jülich hyperon-nucleon model revisited", Phys. Rev. C **72**, 044005 (2005). (*Jülich*04)
- J. Haidenbauer, *et al.*, "Hyperon-nucleon interaction at next-to-leading order in chiral effective field theory", Nucl. Phys. A **915**, 24 (2013). (*Chiral*)

we repeated the eigenvalue search for each of the potential models. None of the four models produces an observable resonance; each produces a sub-threshold resonance, which lies below the nnΛ threshold. A change in s of as little as 5% produces a resonance above the three-body threshold, but a change of at least 25% is required to produce a bound ${}^{3}_{\Lambda}$n.

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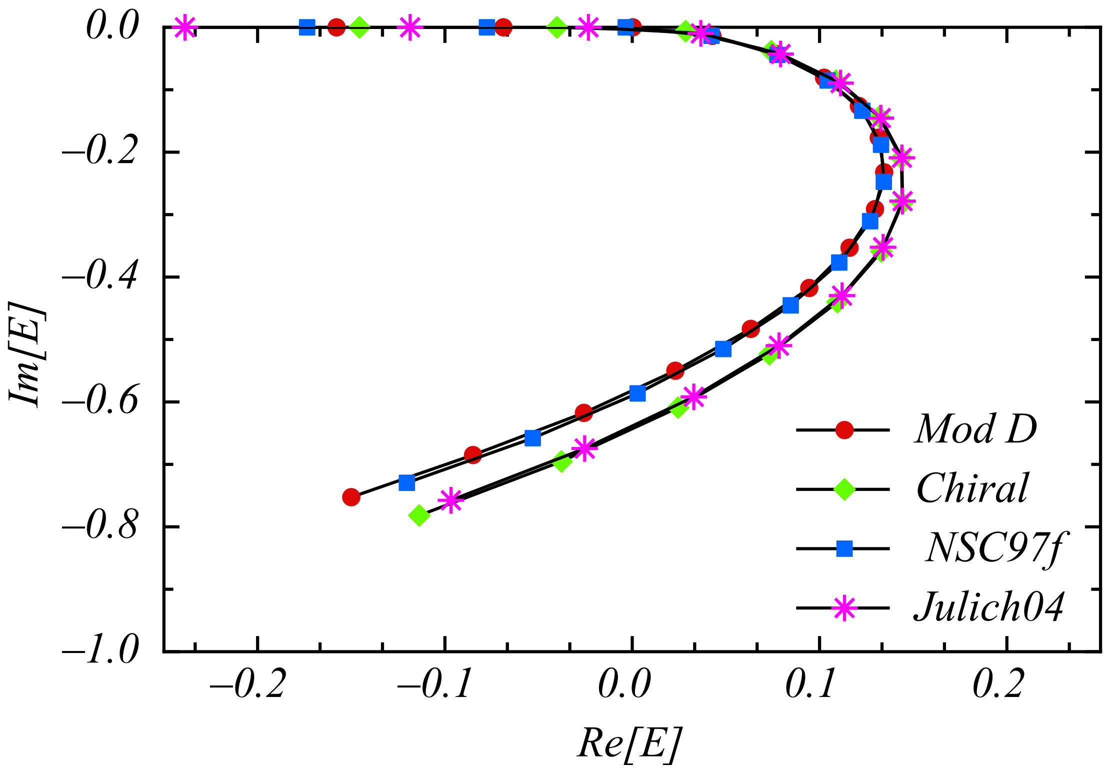

Summary

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The Impact

Understanding Λ hypernuclei (ground states and single-particle spectra) measured at facilities around the world (JLab, RHIC, Mainz, J-PARC, DaΦne) requires knowledge of the nΛ interaction. Scattering data exist only for pΛ. Data on an nnΛ (sub-threshold or physical) resonance would enhance our knowledge of the critical low-energy properties of the nΛ interaction. Such knowledge would elucidate the important Charge Symmetry Breaking relative to the measured pΛ interaction, provide a realistic basis for understanding the long existing Λ hypernuclei data, enhance our calculations of neutron-rich hypernuclei, and constrain our modeling of neutron stars.

Immediate Future: JLab Tritium Experiment

Determining the Unknown Λ-n Interaction by Investigating the Λnn Resonance

Spokespersons: L. Tang, F. Garibaldi, P.E.C. Markowitz, S.N. Nakamura, J. Reinhold

The availability of a T_2 gas target in Hall A provides a unique opportunity to measure the n-n-Λ three-body resonance. Only at JLab with the (e,e' K^+) reaction can the resonance position (excitation energy) and width be measured with the required precision. It is anticipated that using a 4.4 GeV beam one can achieve an energy resolution of $\sim$2 MeV FWHM and an absolute missing mass precision of $\sim \pm 0.20$ MeV. This data will permit use of a theoretical pole search technique to determine experimentally for the first time the nΛ interaction utilizing together the JLab improved pΛ scattering data and the Λ hypernuclei Charge Symmetry Breaking data. Scheduled run begining in mid October.