Binding, Bonding and Charge Symmetry Breaking in Λ-Hypernuclei

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Outline of the Talk

- Introduction
- A single Mass formula for Normal nuclei, Strange & Multi-Strange Hypernuclei.
- The Λ, $\Lambda\Lambda$ and Ξ^- Binding Energy.
- Lambda-Lambda Bonding Energy.
- Charge Symmetry Breaking
- Summary

How to find the binding energy of a hypernucleus?

- Experiment
- Microscopic Theory
    - RMF
    - QMF
    - W-S Potential
- Phenomenology
    - Mass Formula
        - Provides a quick check of the microscopic calculations.
        - Can extrapolate to a wider mass region - beyond the domain of the microscopic calculations.
        - Can provide an estimation of the hyperon(s) binding energy to guide the experiments.
        - Can be used as an input to calculate hypernuclear production cross section(s).

Generalized Mass Formula For Non-Strange & Strange nuclei

C. Samanta et al., J. Phys. G 32 (2006) 363

A systematic search using experimental separation energy (S_Y) for Λ^o, $\Lambda\Lambda$, Σ^+ and Ξ^- -hypernuclei leads to a generalized mass formula that is valid for normal nuclei (n_Y=0) and Hypernuclei ($n_Y \neq 0$) of all kind, having different mass and Strangeness.

$$B(A, Z) = 15.777A - 18.34A^{2/3} - 0.71\frac{Z(Z-1)}{A^{1/3}} - \frac{23.21(N - Z_c)^2}{[(1 + e^{-A/17})A]} + (1 - e^{-A/30})\delta$$
$$+ n_Y[0.0335(m_Y) - 27.8 - 48.7|S|A^{-2/3}],$$

Explicit dependence on:

- n_Y = no. of hyperons in a nucleus,
- m_Y = mass of hyperon in MeV,
- S = strangeness of the hyperon.

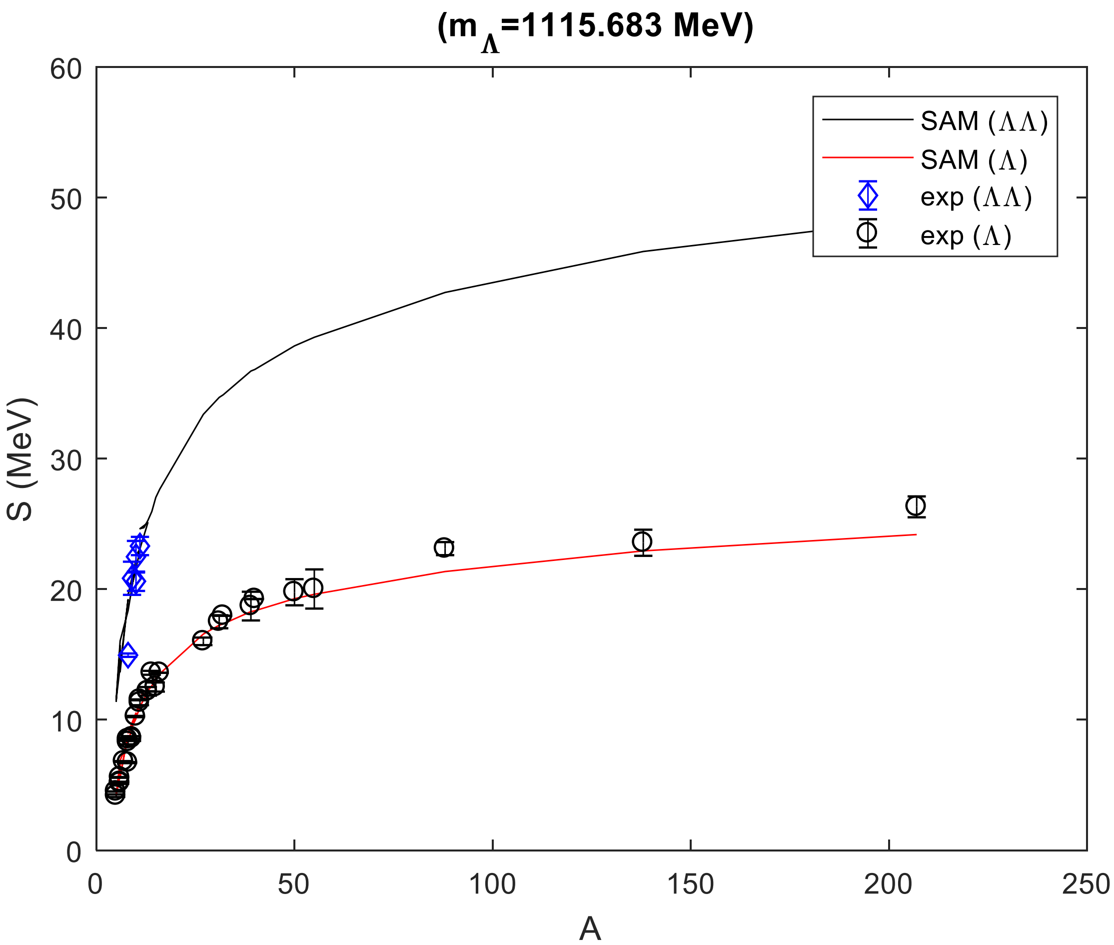
A = N + Z_c+ n_Y = total no. of baryons,
Z_c = number of protons,
Z = Z_c + n_Y q_Y = net charge number,
q_Y = charge no. of Hyperon with proper sign. (viz., q= -1, 0, 1).

Hyperon	S	n_Y	q_Y
Λ^0	-1	1	0
$\Lambda\Lambda$	-2	2	0
Ξ^-	-2	1	-1
Normal	0	0	0

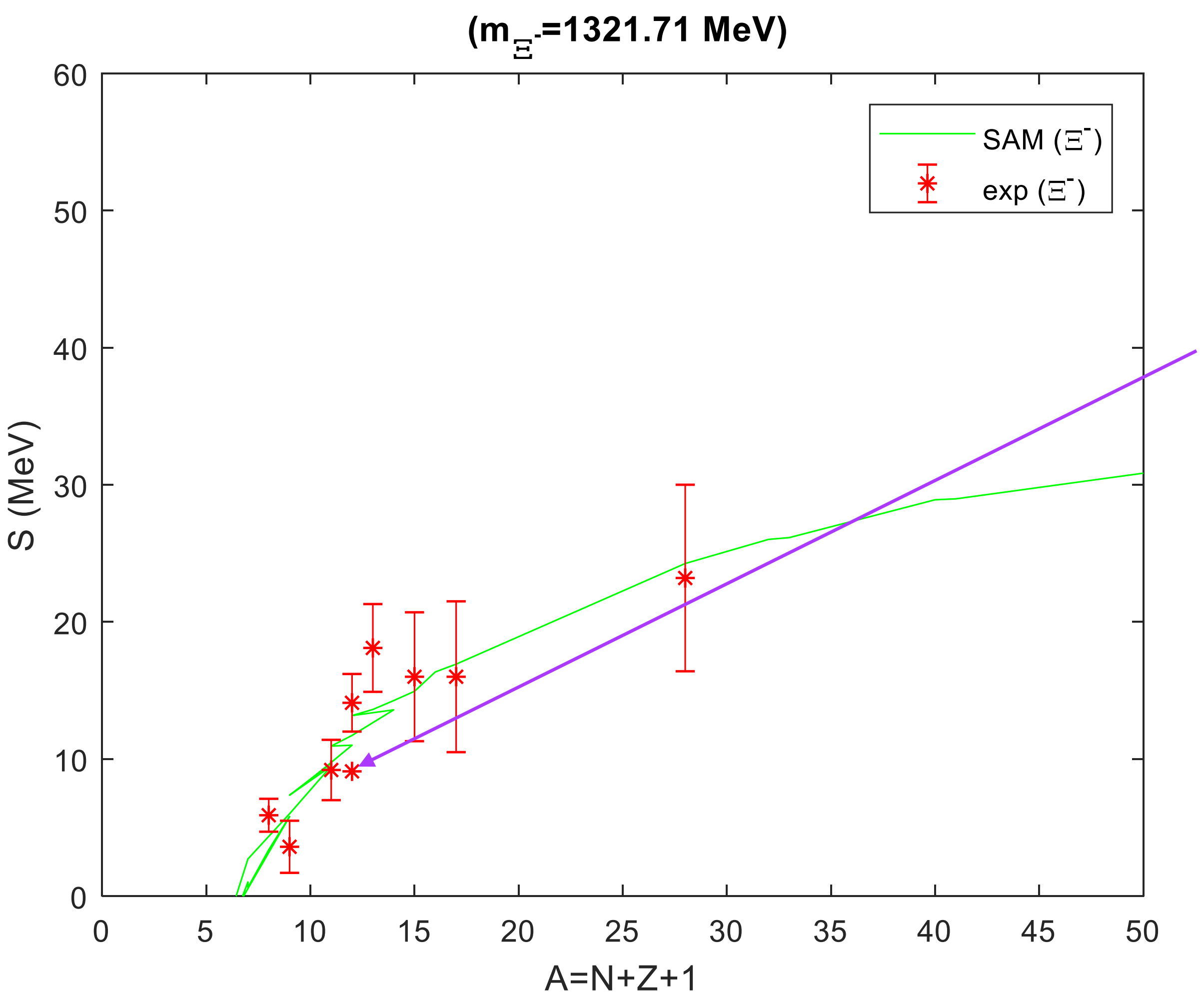
Hyperon Separation Energy:

S_Y = B (A, Z)$_{Hyper}$ – B (A - n_Y, Z_c)$_{Core}$

Variation of Single-Λ and Double-Λ Separation Energies with A=N+Z



Variation of Single Ξ^- Separation Energies with A=N+Z+1



Old Emulsion data

$^{12}{}_{\Xi^-}Be$:

BE(Exp) ≥ 9.1 MeV

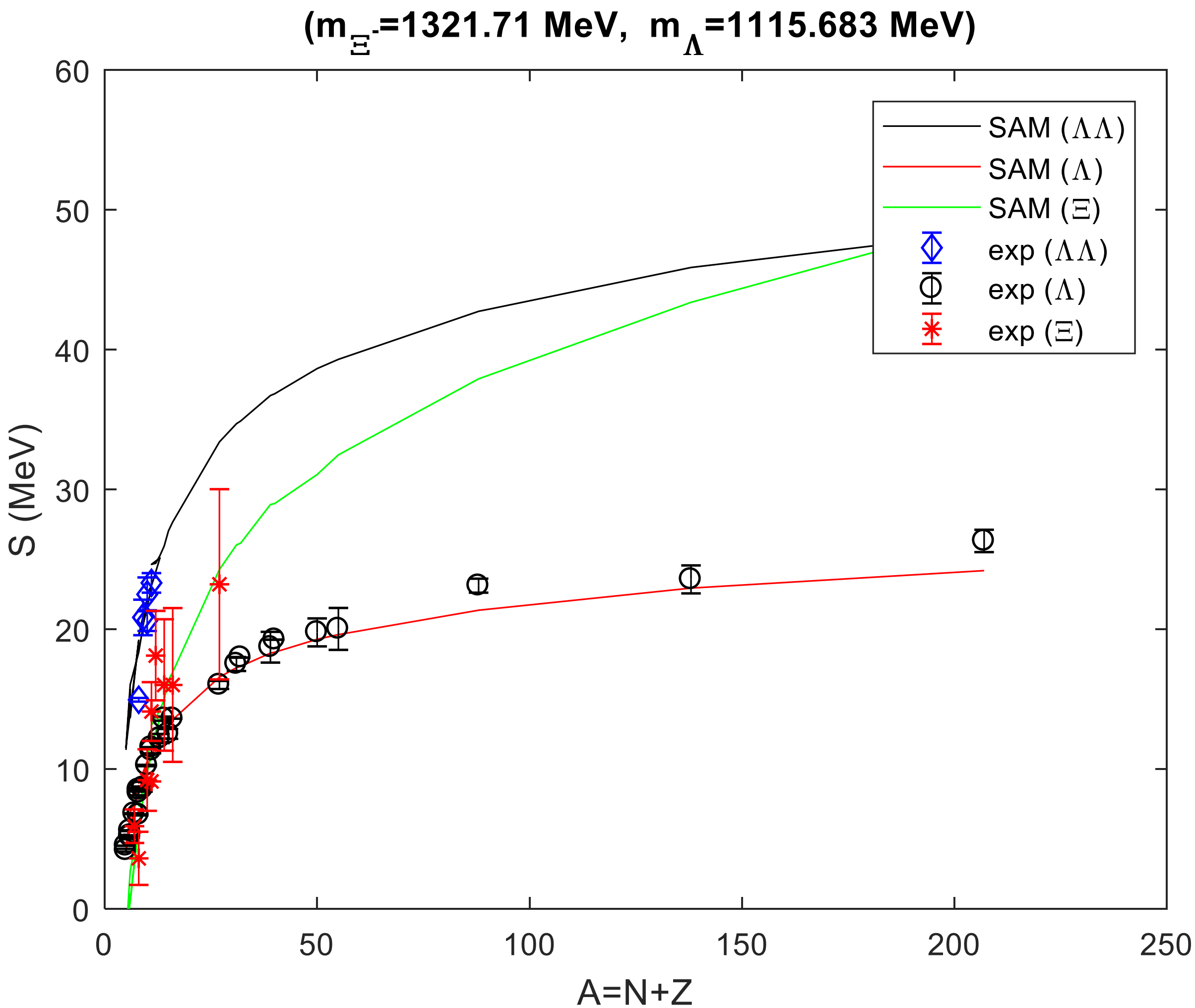
From the talk of

T. Nagae

BE(This work):
$^{12}{}_{\Xi^-}Be = {}^{11}B + 1\Xi^-$

=11.01 MeV

Variation of Λ, Ξ^- & $\Lambda\Lambda$ Separation Energies with A=N+Z



Effect of Addition of Lambda

A Lambda-particle makes a nucleus more bound, and can change the neutron-, proton-drip points by creating bound nucleus beyond the normal drip lines.

J. Phys. G: Nucl. Part. Phys. **35** (2008) 065101 C Samanta *et al*

Table 1. One-nucleon separation energies (in MeV) on drip lines for each element with the lowest and highest numbers of bound neutrons in normal and Λ-hypernuclei.

Symbol Z	Normal p-drip $^{A}Z, S_p$	Normal n-drip $^{A}Z, S_n$	Hyper p-drip $^{A}Z, S_p$	Hyper n-drip $^{A}Z, S_n$
Li	^{5}Li, 3.36	^{11}Li, 0.58	$^{5}_{\Lambda}$Li, 1.22	$^{12}_{\Lambda}$Li, 1.87
Be	^{6}Be, 1.11	^{14}Be, 0.90	$^{7}_{\Lambda}$Be, 3.79	$^{15}_{\Lambda}$Be, 1.84
B	^{8}B, 0.74	^{17}B, 0.99	$^{9}_{\Lambda}$B, 2.39	$^{18}_{\Lambda}$B, 1.73
C	^{9}C, 0.17	^{20}C, 1.01	$^{10}_{\Lambda}$C, 1.80	$^{21}_{\Lambda}$C, 1.63
N	^{12}N, 1.98	^{23}N, 0.97	$^{12}_{\Lambda}$N, 0.24	$^{24}_{\Lambda}$N, 1.50
O	^{13}O, 1.98	^{26}O, 0.94	$^{13}_{\Lambda}$O, 0.44	$^{27}_{\Lambda}$O, 1.40
F	^{15}F, 0.09	^{29}F, 0.89	$^{16}_{\Lambda}$F, 0.81	$^{32}_{\Lambda}$F, 0.01
Ne	^{16}Ne, 0.64	^{32}Ne, 0.87	$^{17}_{\Lambda}$Ne, 1.39	$^{35}_{\Lambda}$Ne, 0.04
Na	^{19}Na, 0.54	^{35}Na, 0.84	$^{20}_{\Lambda}$Na, 1.05	$^{38}_{\Lambda}$Na, 0.08
Mg	^{20}Mg, 1.37	^{38}Mg, 0.84	$^{20}_{\Lambda}$Mg, 0.05	$^{41}_{\Lambda}$Mg, 0.13

Normal drip nucleus is ^{35}Na.

Neutron-drip nucleus is $^{38}{}_{\Lambda}$Na instead of $^{36}{}_{\Lambda}$Na i.e., 2 extra neutrons can be accommodated due to the addition of a Λ.

The exact drip-point can change due to the deformation and other parameters near the drip line. Need microscopic calculations.

Generalized Mass Formula for Non-strange-, Strange- and Multiply-strange Nuclear Systems

C. Samanta, JPG 37 (2010) 075104

A = n + z_c+ n_Y : **n**= no. of neutrons, **z_c** = no. of protons, **n_Y** = no. of hyperons

Binding Energy: B(A,Z) = m_A- $z_c.m_p - n.m_n - n_Y.m_Y$ = Negative

$$
\begin{aligned}
- B(A,Z) = \quad & 15.777A - 18.34A^{2/3} - 0.71Z(Z-1)/A^{1/3} - 23.21(n-z_c)^2/[(1+e^{-A/17})A] + (1-e^{-A/30})\delta \\
& + \sum_Y n_Y [0.0335(m_Y) - 26.7 - 48.7 \mid S \mid /A^{2/3} \\
& - a_Y\{(n_\Lambda + n_{\Xi^o} + n_{\Xi^-} - z_c)^2 + (n_\Lambda + n_{\Xi^o} + n_{\Xi^-} - n)^2\}/\{(1+e^{-A/17})A\}].
\end{aligned}
$$

m_Y = mass of hyperon in MeV

S = strangeness no. of the hyperon

$$Z = \left| z_c + \sum_Y n_Y q_Y \right|$$

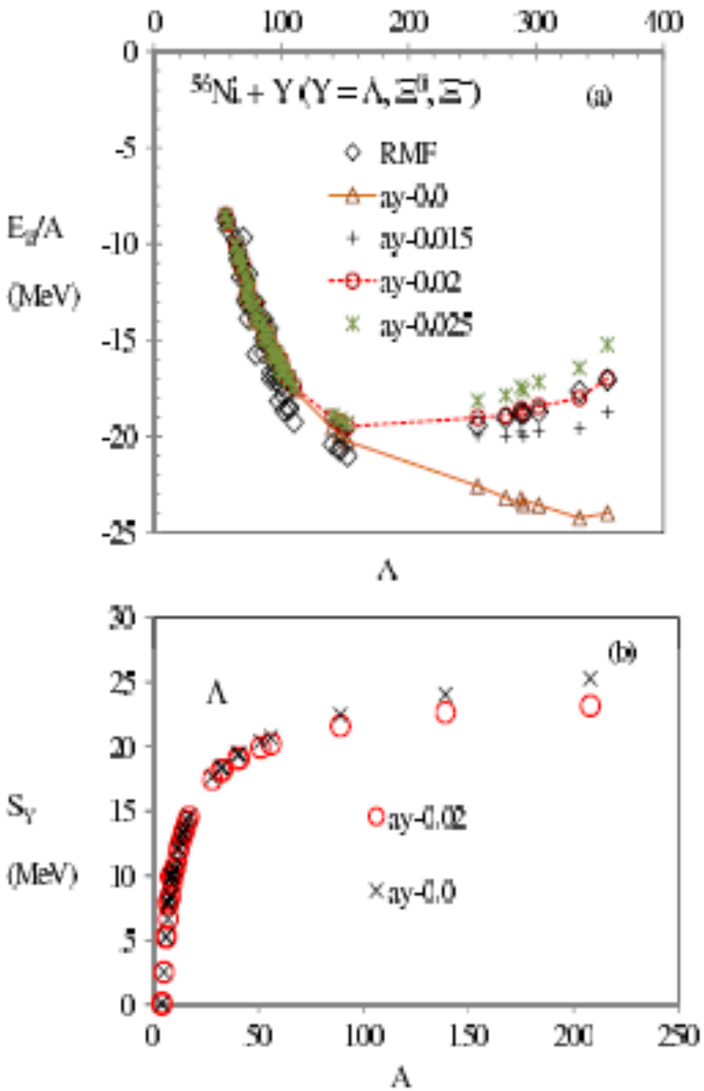
Z = Net Charge

= Proton charges + Total hyperon charges

q = -1, 0, 1 depending on the hyperon type

Note: the net charge of a nucleus can be **negative** if the hyperon number is larger than the proton number and the hyperon has a negative charge.

Multi-Strange Nuclei: Model-2 with Y-Y interaction



C. Samanta, JPG 37 (2010) 075104

Effect of the new hyperon-asymmetry term: choice of different a_y value

Figures: E_B/A vs. A plots for stable multi-strange systems.

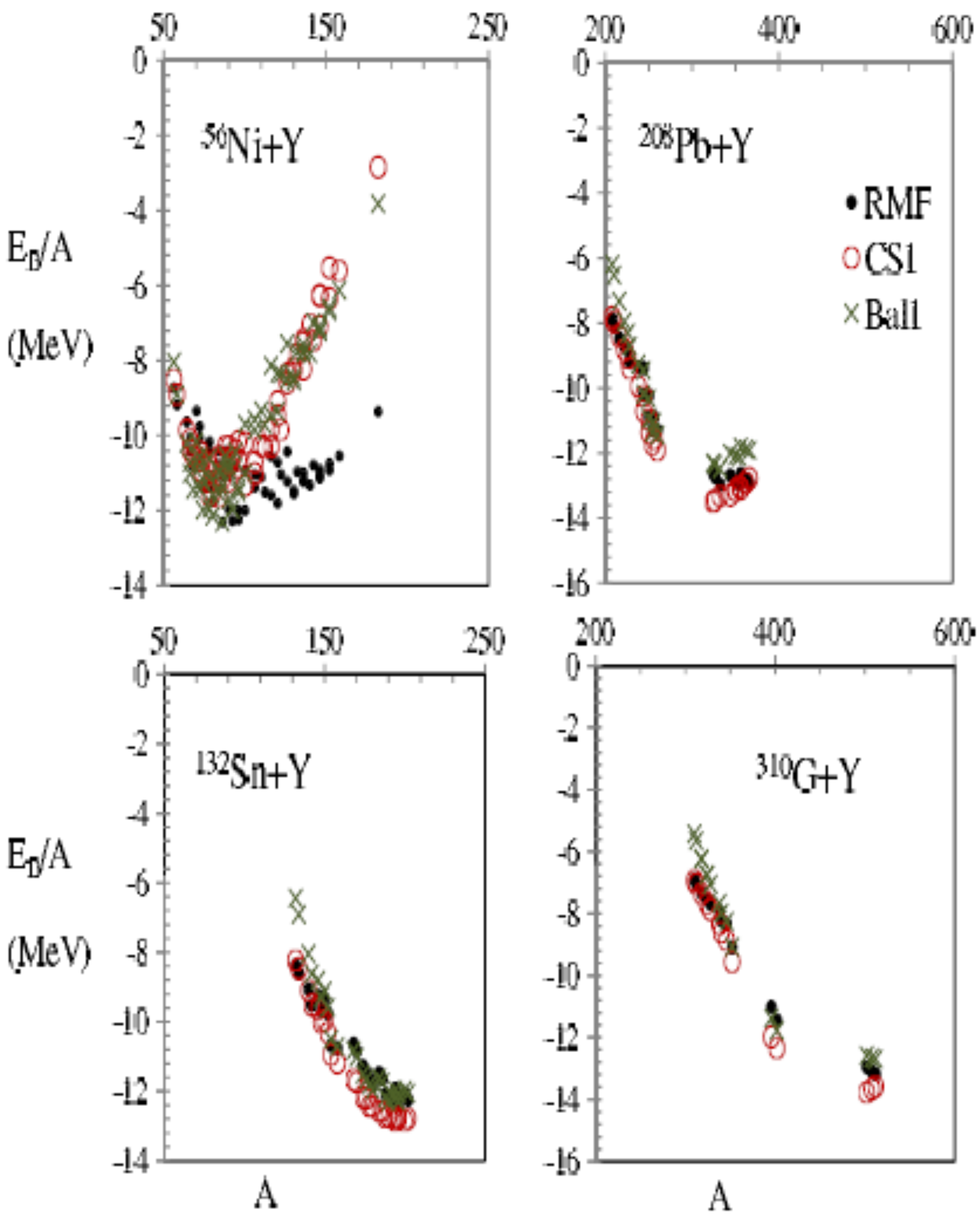
(a) The a_y = 0.02 gives the best fit to the relativistic mean field (RMF) calculations of Schaffner et al. (Ref.1) that is based on ^{56}Ni nuclear cores with YY interaction (model 2).

(b) Single lambda-hyperon separation energy S_Y vs. A for different elements. With a_Y=0.0 and 0.2.

Ref.1: J. Schaffner J, C.B. Dover, A. Gal , C. Greiner, D.J. Millener and H. Stöcker, Ann. Phys. NY 235 (1994) 35

Multi-Strange Nuclei: Model-1 without Y-Y interaction

C.Samanta, JPG 37 (2010) 075104



(Current wisdom: Y-Y interaction is weak.)

Model-1=No Y-Y interaction

Model-2=Strong Y-Y interaction

CS1(BE,Model1)=CS2(BE,Model2) -Cr

$$C_r = 12.0 A f_s (f_\Lambda + f_{\Xi^0} + f_{\Xi^-})$$

$$f_s = \sum_Y n_Y |S| / A.$$

Stable-multiply-strange systems in RMF calculations, based on ^{56}Ni, ^{132}Sn, ^{208}Pb and ^{310}G(Zc=126, n=184) core, by:

1. Model-1 of Schaffner et al.,
2. SET-I of Balberg et al.,
3. this work CS1.

Lightest Bound Nuclei without any Neutrons and Protons!

C. Samanta, Jour. Phys. G: Nucl. Part. Phys. 37 (2010)075104

6Λ

$10\Xi^0$

$11\Xi^-$

$1\Lambda+7\Xi^0$

$1\Lambda + 8\Xi^-$

$1\Xi^0 +$
$9\Xi^-$

$1\Xi^- +$
$8\Xi^o$

$2\Lambda + 3\Xi^0$
$+3\Xi^-$

Pure Hyperonic Systems can be bound (predicted by CS2, with Y-Y interaction)

No bound pure-hyperonic matter is possible by Model-1 (no Y-Y interaction)

Production of hypernuclei

in multifragmentation in H.I. collision

In Statistical Multifragmentation model (SMM), mass formula has been used as an input to calculate the relative yield of hypernuclei.

Two different mass formulae have been used:

1. Liquid drop model (applicable for Λ hypernuclei only).
2. Generalized mass formula (applicable for all hypernuclei)

A. Botvina and J. Pochodzalla PRC 76 (2007) 024909

Production of hypernuclei in multifragmentation of nuclear spectator matter

A.Botvina and J. Pochodzalla PRC 76 (2007) 024909

PHYSICAL REVIEW C **76**, 024909 (2007)

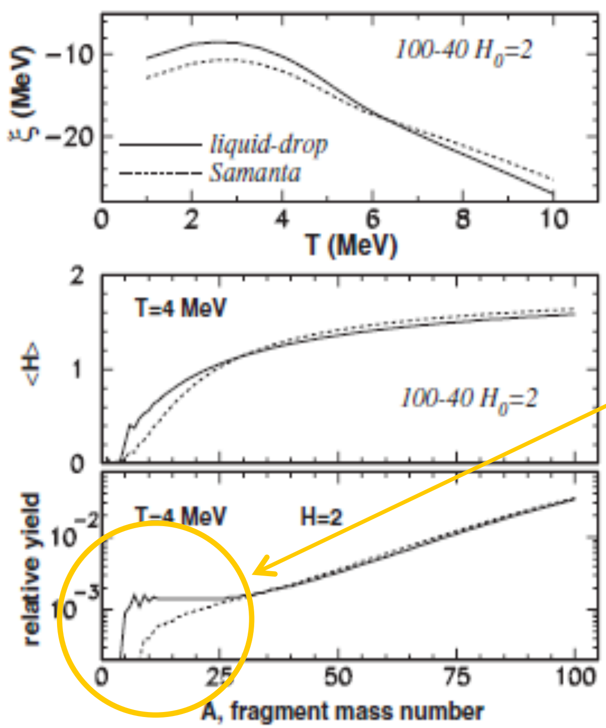


FIG. 3. Comparison of SMM calculations with the liquid-drop and Samanta descriptions of hyper terms in the mass formula, for the same sources as in Fig. 2. Top panel – the strangeness chemical potential ξ versus temperature T. Middle panel – average number of Λ hyperons in fragments, and bottom panel – yields of fragments with two Λ, at $T = 4$ MeV.

The SMM calculations show a discrepancy between the generalized mass formula and the Liquid drop mass formula at the low fragment mass number (A) region.

Liquid-drop Mass formula

(valid for Λ-hyernuclei only)

Binding Energy: $B(A,Z) = m_A - z_c.m_p - n.m_n - n_Y.m_Y = $ Negative

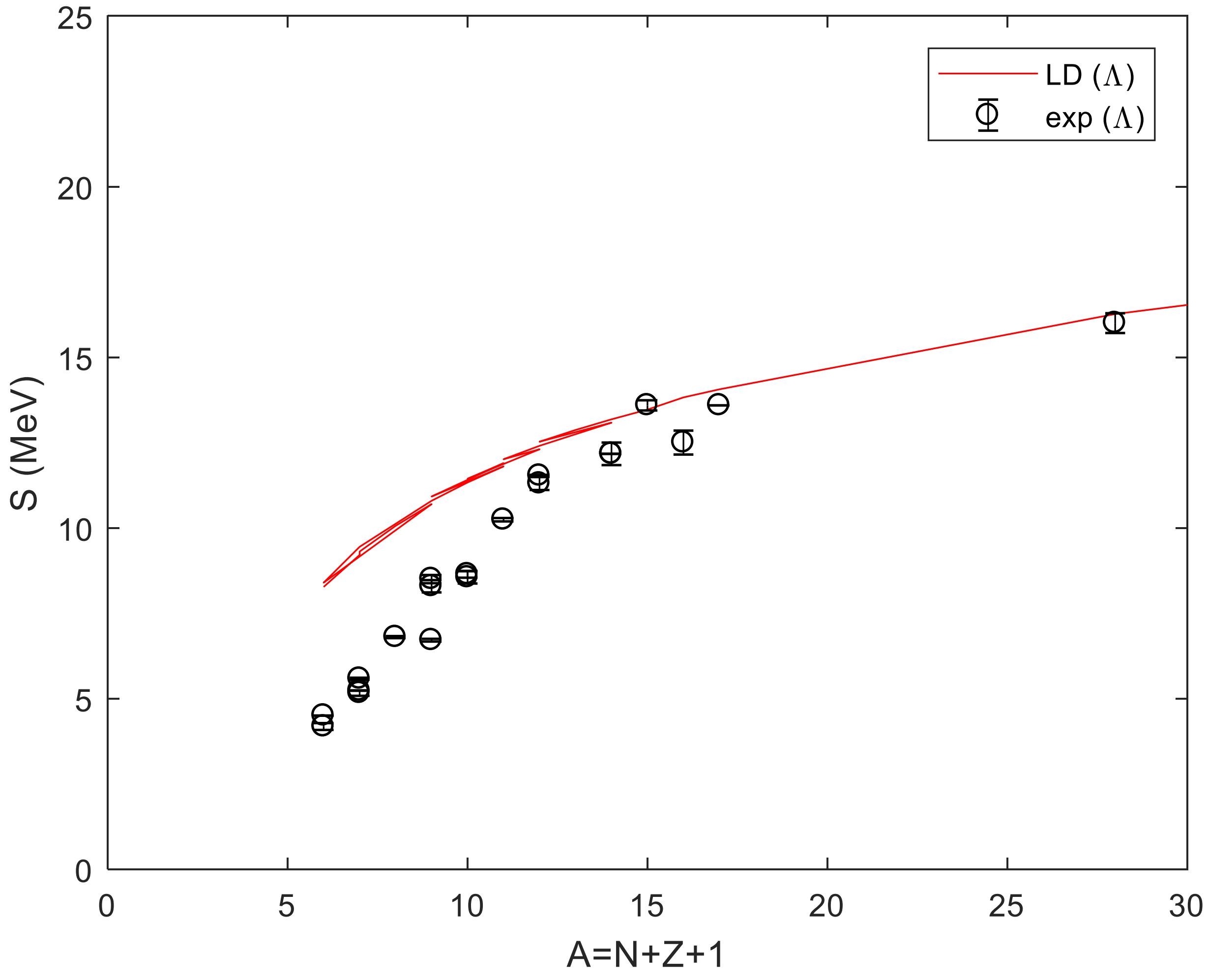
n= no. of neutrons, $\mathbf{z_c}$ = no. of protons,
$\mathbf{A = n + z_c + n_Y}$

$\mathbf{n_Y}$ = no. of $\mathbf{\Lambda}$-hyperons

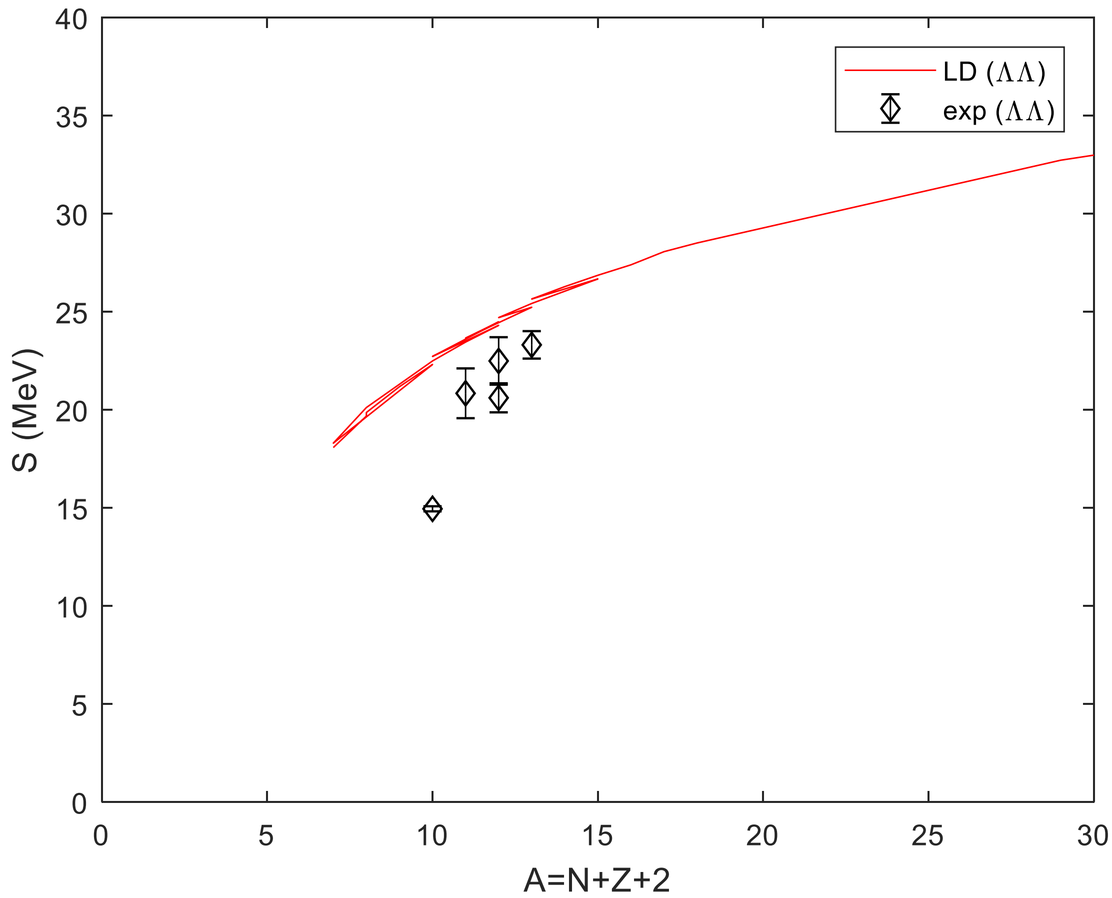
$$B(A,Z) = -16A + 18A^{2/3} + 0.72z^2/A^{1/3} + 25(N - z)^2/(A - n_Y) - (n_Y/A)\,[10.68A + 21.27\,A^{2/3}]$$

A.S. Botvina, J. Pochodzalla, PRC 76 (2007) 024909

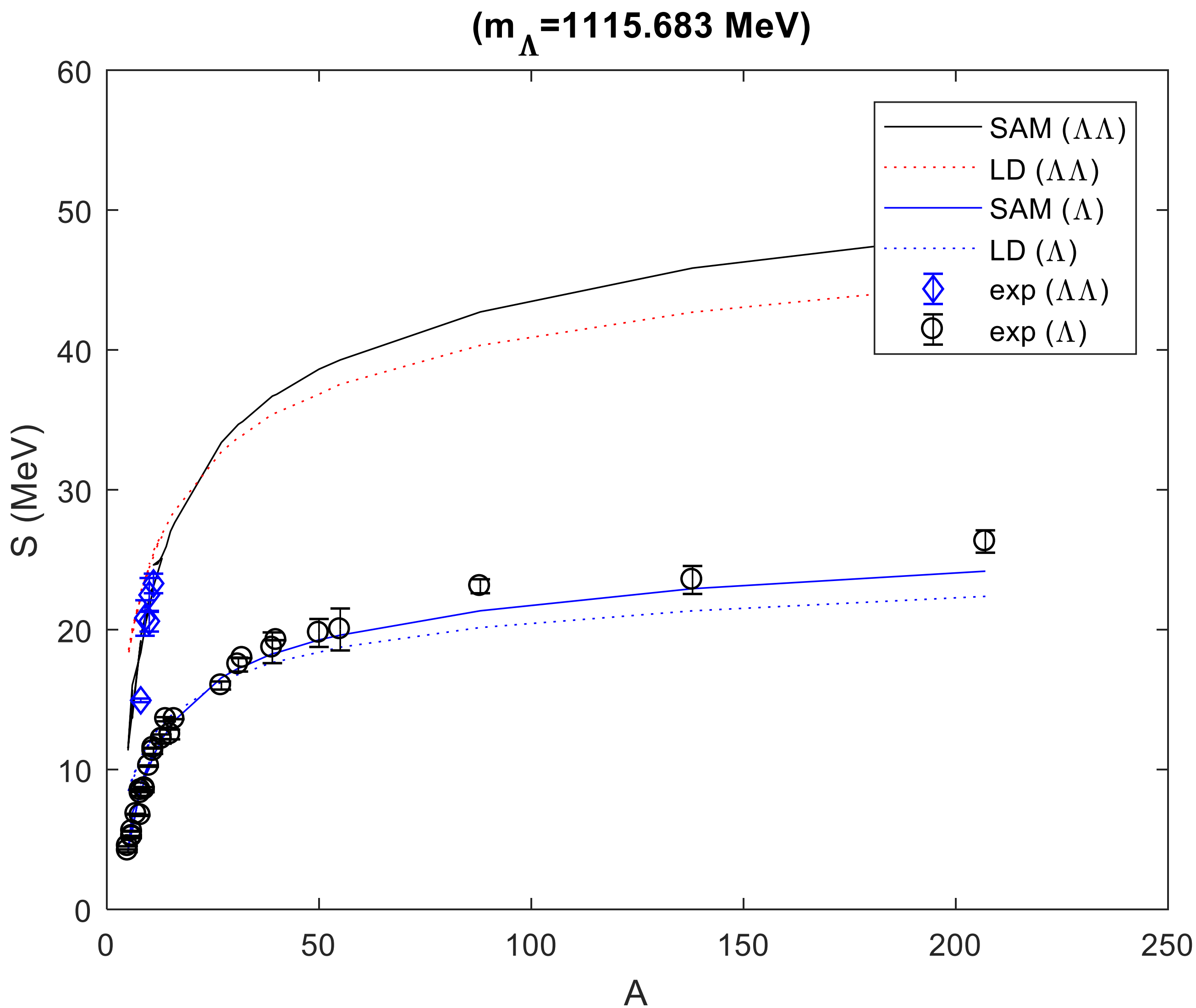
Variation of Single-Λ Separation Energies with Mass Number (A)

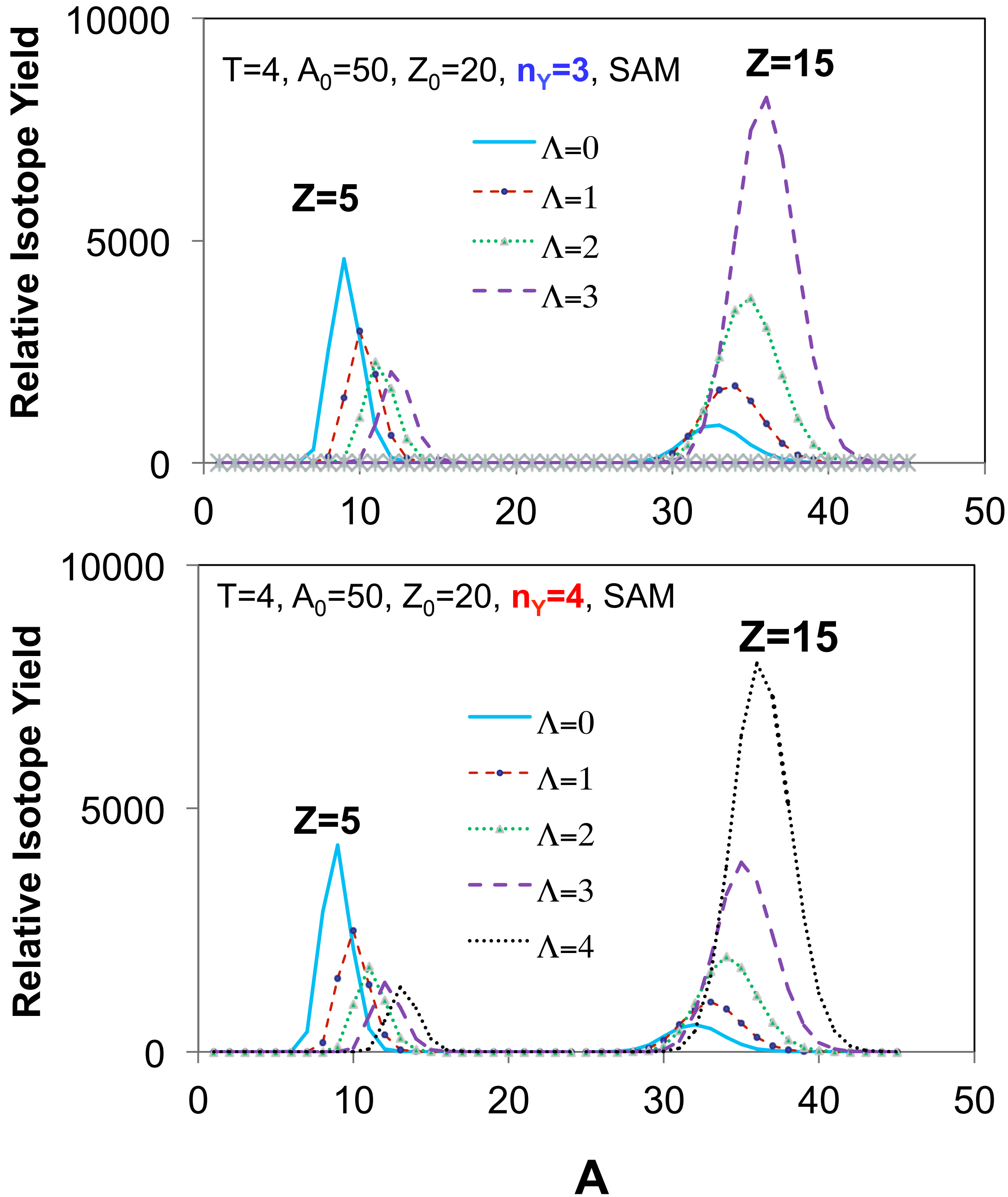


Variation of Double-Λ Separation Energies with Mass Number (A)



Variation of Single-Λ and Double-Λ Separation Energies with A = N + Z





C. Samanta,
A. Botvina,
I. Mishustin,
& W. Greiner

Relative yields of 1Λ, 2Λ, 3Λ and 4Λ was calculated using Statistical Multifragmentation Model (SMM) and the generalized mass formula.

It indicates production yield of 2Λ, 3Λ and 4Λ-hypernuclei to be higher than normal nuclei (Λ=0) at high temperature for fragments with larger mass number.

$\Lambda\Lambda$-Bond Energy

$$B_{\Lambda}({}^{A}_{\Lambda}Z) = M({}^{A-1}Z) + M(\Lambda) - M({}^{A}_{\Lambda}Z) \qquad (1)$$

$$B_{\Lambda\Lambda}({}^{A}_{\Lambda\Lambda}Z) = M({}^{(A-2}Z) + 2M(\Lambda) - M({}^{A}_{\Lambda\Lambda}Z) \qquad (2)$$

$$\Delta B_{\Lambda\Lambda}({}^{A}_{\Lambda\Lambda}Z) = B_{\Lambda\Lambda}({}^{A}_{\Lambda\Lambda}Z) - 2B_{\Lambda}({}^{A-1}_{\Lambda}Z) \qquad (3)$$

where $M(\Lambda)$ is the mass of the Λ hyperon, $M({}^{A-1}Z)$ is the mass of the core nucleus of the single-Λ nucleus of mass $M({}^{A}_{\Lambda}Z)$, and $M({}^{(A-2}Z)$ is the mass of the core nucleus of the double-Λ nucleus of mass $M({}^{A}_{\Lambda\Lambda}Z)$. Here the mass number (A) of the hypernuclei is the total number of baryons (neutrons, protons and hyperons) in the nuclei. The bond energies ($\Delta B_{\Lambda\Lambda}$) are the measures of the energy released when the $\Lambda\Lambda$ bond is broken. These energies assist in understanding the nature of the in-medium strength of the Λ-Λ interaction.

ΛΛ-Bond Energy

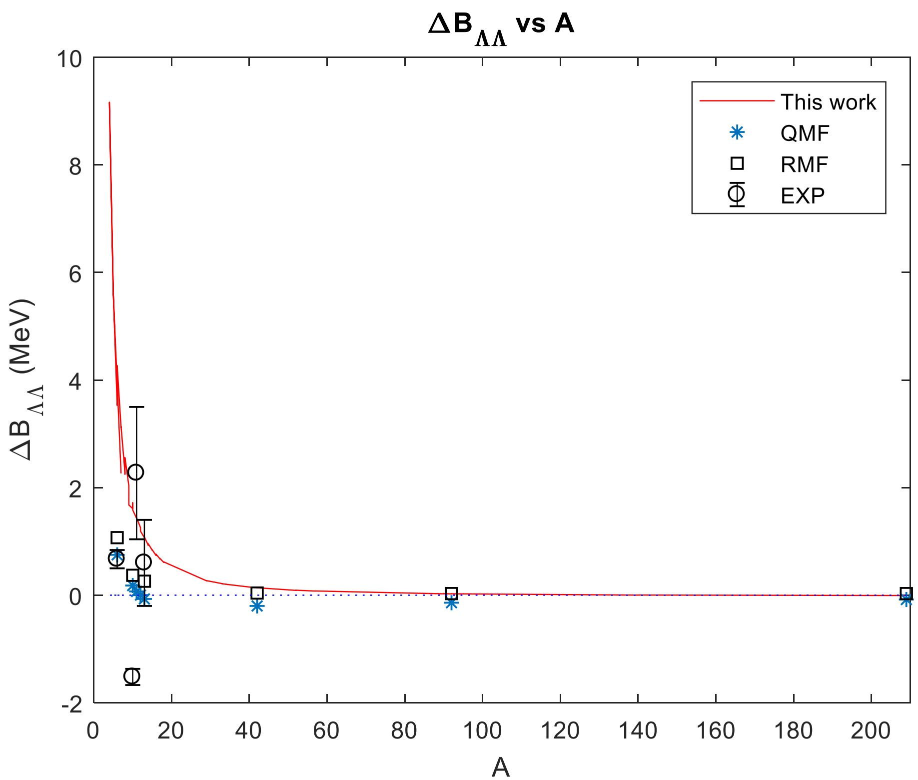


Table 2. ΛΛ binding and bond energies, $B_{\Lambda\Lambda}$ and $\Delta B_{\Lambda\Lambda}$ (in MeV) respectively for double-Λ hypernuclei for the QMF model, the RMF model, and the available experimental data.

	$B_{\Lambda\Lambda}$			$\Delta B_{\Lambda\Lambda}$		
${}^{A}_{\Lambda\Lambda}Z$	Exp.	QMF	RMF	Exp.	QMF	RMF
${}^{6}_{\Lambda\Lambda}$He	6.91 ± 0.16	7.94	5.52	0.67 ± 0.17	0.75	1.07
${}^{10}_{\Lambda\Lambda}$Be	11.90 ± 0.13	17.61	16.34	-1.52 ± 0.15	0.18	0.37
${}^{11}_{\Lambda\Lambda}$Be	20.49 ± 1.15	19.46		2.27 ± 1.23	0.07	
${}^{12}_{\Lambda\Lambda}$Be	22.23 ± 1.15	21.00			0.002	
${}^{13}_{\Lambda\Lambda}$B	23.30 ± 0.70	22.41	22.14	0.60 ± 0.80	-0.07	0.26
${}^{42}_{\Lambda\Lambda}$Ca		37.25	38.15		-0.20	0.04
${}^{92}_{\Lambda\Lambda}$Zr		45.24	47.11		-0.14	0.03
${}^{210}_{\Lambda\Lambda}$Pb		50.73	52.19		-0.08	0.03

PTEP

Prog. Theor. Exp. Phys. 2014, 013D02 (8 pages)
DOI: 10.1093/ptep/ptt119

Quark mean-field model for single and double Λ and Ξ hypernuclei

J. N. Hu[1], A. Li[2,3,*], H. Shen[4], and H. Toki[5]

Charge Symmetry Breaking (CSB)
(Binding energy difference in mirror nuclei)

Mirror pair	ΔB_Λ Exp	ΔB_Λ T.W.	CSB Ref.
${}^{40}_{\Lambda}$Ca - ${}^{40}_{\Lambda}$K	-	0.07	
${}^{32}_{\Lambda}$S - ${}^{32}_{\Lambda}$P	-	0.08	
${}^{28}_{\Lambda}$Si - ${}^{28}_{\Lambda}$A	-	0.09	
${}^{16}_{\Lambda}$O - ${}^{16}_{\Lambda}$N	-0.74 ± 0.43	0.09	-0.36 ± 0.43
${}^{14}_{\Lambda}$N - ${}^{14}_{\Lambda}$C	0 ± 0.33	0.09	
${}^{12}_{\Lambda}$C - ${}^{12}_{\Lambda}$B	-0.764 ± 0.20	0.09	-0.23 ± 0.19
${}^{10}_{\Lambda}$B - ${}^{10}_{\Lambda}$Be	-0.60 ± 0.36	0.09	-0.22 ± 0.250
${}^{8}_{\Lambda}$Be - ${}^{8}_{\Lambda}$Li	0.04 ± 0.08	0.10	0.04 ± 0.060

Ref: E. Botta et al, NPA960(2017)165

The ΔB_Λ values obtained in this work (T.W.) is basically the Coulomb energy difference of the mirror nuclei, not the CSB effect.

Since the binding energies of this mass formula well reproduces the experimental binding energies, the ΔB_Λ values of this work can be taken out from the experimental ΔB_Λ values to estimate the Coulomb deducted CSB effect.

Summary

❖ A single generalized mass formula is prescribed for all hypernuclei and normal nuclei.

❖ It reproduces binding energies obtained from the experimental data, and microscopic (RMF, QMF) calculations available on limited nuclei.

❖ It can guide future experiments by giving an estimation of the possible binding energy location.

For example, the Ξ^- - separation energy of $^{12}{}_{\Xi}$-Be, is predicted to be ~11.01 MeV.

❖ This mass formula indicates shifts in n, p drip lines, and predicts possible existence of bound nuclei without neutrons and protons.

❖ Relative yields of 1Λ, 2Λ, 3Λ and 4Λ calculated using Statistical Multifragmentation Model (SMM) indicate production yield of 2Λ, 3Λ and 4Λ-hypernuclei to be higher than normal nuclei at high temperature for fragments with larger mass number.

❖ The $\Lambda\Lambda$ bond energy rapidly drops off with the increasing mass number in agreement with RMF & QMF.

❖ The Coulomb energy difference of mirror nuclei can be deduced from this mass formula to extract the CSB effect.

❖ More Experimental data are needed.

Thank You