Hyperon equation of state for core-collapse simulations based on the variational many-body theory

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Collaborators: E. Hiyama (RIKEN), M. Takano (Waseda University)

Outline

HYP2018@ VA USA, June 28, 2018

1: Introduction

The era of multi-messenger astronomy has officially begun!

Selected for a Viewpoint in *Physics*
PRL 119, 161101 (2017) PHYSICAL REVIEW LETTERS week ending 20 OCTOBER 2017

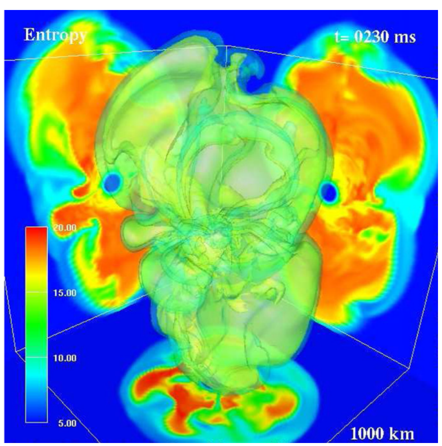
GW170817: Observation of Gravitational Waves from a Binary Neutron Star Inspiral

B. P. Abbott *et al.**
(LIGO Scientific Collaboration and Virgo Collaboration)

(Received 26 September 2017; revised manuscript received 2 October 2017; published 16 October 2017)

The nuclear equation of state (EOS) is important input for numerical simulations.

Core-Collapse Supernovae



(APJ 786 (2014) 83)

Black Hole Formation

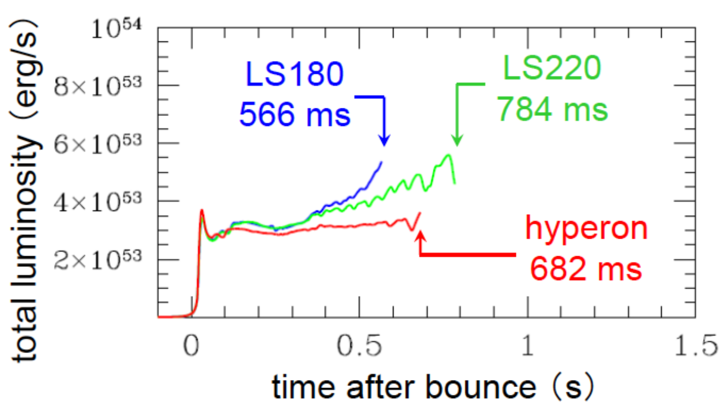
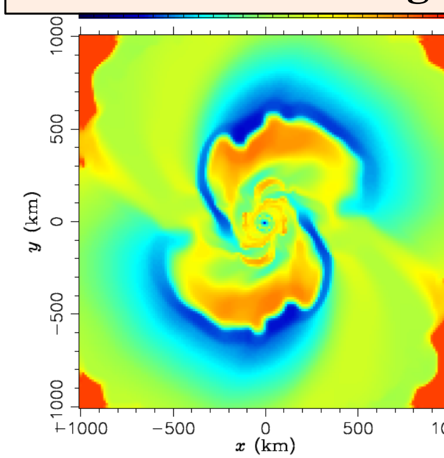


Figure by K. Nakazato

Neutron Star Merger



(APJL 789 (2014) L39)

Nuclear EOS for supernova simulations

Supernova equation of state (SN-EOS)

Model	Nuclear Interaction	Degrees of Freedom	$M_{\max}$ ($M_\odot$)	$R_{1.4M_\odot}$ (km)	Ξ	publ. avail.	References (M. Oertel et al., Rev. Mod. Phys. 89 (2017) 015007)
H&W	SKa	$n, p, \alpha, \{(A_i, Z_i)\}$	2.21[a]	13.9 [a]		n	El Eid and Hillebrandt (1980); Hillebrandt *et al.* (1984)
LS180	LS180	$n, p, \alpha, (A, Z)$	1.84	12.2	0.27	y	Lattimer and Swesty (1991)
LS220	LS220	$n, p, \alpha, (A, Z)$	2.06	12.7	0.28	y	Lattimer and Swesty (1991)
LS375	LS375	$n, p, \alpha, (A, Z)$	2.72	14.5	0.32	y	Lattimer and Swesty (1991)
STOS	TM1	$n, p, \alpha, (A, Z)$	2.23	14.5	0.26	y	Shen *et al.* (1998); Shen *et al.* (1998, 2011)
FYSS	TM1	$n, p, d, t, h, \alpha, \{(A_i, Z_i)\}$	2.22	14.4	0.26	n	Furusawa *et al.* (2013b)
HS(TM1)	TM1*	$n, p, d, t, h, \alpha, \{(A_i, Z_i)\}$	2.21	14.5	0.26	y	Hempel and Schaffner-Bielich (2010); Hempel *et al.* (2012)
HS(TMA)	TMA*	$n, p, d, t, h, \alpha, \{(A_i, Z_i)\}$	2.02	13.9	0.25	y	Hempel and Schaffner-Bielich (2010)
HS(FSU)	FSUgold*	$n, p, d, t, h, \alpha, \{(A_i, Z_i)\}$	1.74	12.6	0.23	y	Hempel and Schaffner-Bielich (2010); Hempel *et al.* (2012)
HS(NL3)	NL3*	$n, p, d, t, h, \alpha, \{(A_i, Z_i)\}$	2.79	14.8	0.31	y	Hempel and Schaffner-Bielich (2010); Fischer *et al.* (2014a)
HS(DD2)	DD2	$n, p, d, t, h, \alpha, \{(A_i, Z_i)\}$	2.42	13.2	0.30	y	Hempel and Schaffner-Bielich (2010); Fischer *et al.* (2014a)
HS(IUFSU)	IUFSU*	$n, p, d, t, h, \alpha, \{(A_i, Z_i)\}$	1.95	12.7	0.25	y	Hempel and Schaffner-Bielich (2010); Fischer *et al.* (2014a)
SFHo	SFHo	$n, p, d, t, h, \alpha, \{(A_i, Z_i)\}$	2.06	11.9	0.30	y	Steiner *et al.* (2013a)
SFHx	SFHx	$n, p, d, t, h, \alpha, \{(A_i, Z_i)\}$	2.13	12.0	0.29	y	Steiner *et al.* (2013a)
SHT(NL3)	NL3	$n, p, \alpha, \{(A_i, Z_i)\}$	2.78	14.9	0.31	y	Shen *et al.* (2011b)
SHO(FSU)	FSUgold	$n, p, \alpha, \{(A_i, Z_i)\}$	1.75	12.8	0.23	y	Shen *et al.* (2011a)
SHO(FSU2.1)	FSUgold2.1	$n, p, \alpha, \{(A_i, Z_i)\}$	2.12	13.6	0.26	y	Shen *et al.* (2011a)
TNTYST	AV18+UIX	n, p, α, (A, Z)	2.21	11.5	0.32	y	Togashi *et al.* (2017)

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HS(NL3)	NL3*					y	Hempel and Schaffner-Bielich (2010); Fischer *et al.* (2014a)
HS(DD2)	DD2					y	Hempel and Schaffner-Bielich (2010); Fischer *et al.* (2014a)
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Skyrme-type effective interaction

Relativistic Mean Field Theory

We have constructed a new SN-EOS with the variational method starting from bare nuclear forces.

Nuclear EOS table with the variational method

http://www.np.phys.waseda.ac.jp/EOS/

Equation of state for nuclear matter with the variational method

Equation of state (EOS) based on the variational many-body theory with realistic nuclear forces is provided. For uniform matter, the EOS is constructed with the cluster variational method starting from the Argonne v18 two-body nuclear potential and the Urbana IX three-body nuclear potential. Non-uniform nuclear matter is treated in the Thomas-Fermi approximation. Alpha particle mixing is also taken into account. See Togashi et al., Nucl. Phys. A 961 (2017) 78 for details. This EOS table is open for general use in any studies for nuclear physics and astrophysics, provided that our paper is referred to in your publication.

User's Guide (read me first)

guide.pdf

EOS tables

eos.zip

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Table A.1: Ranges of temperature T, proton fraction Y_p, and baryon mass density ρ_B in the table of the variational EOS. At the top of the last column, "+1" represents the case at $T = 0$ MeV.

Parameter	Minimum	Maximum	Mesh	Number
$\log_{10}(T)$ [MeV]	-1.00	2.60	0.04	$91+1$
Y_p	0	0.65	0.01	66
$\log_{10}(\rho_B)$ [g/cm^3]	5.1	16.0	0.10	110

(HT *et al*., NPA961 (2017) 78)

Nuclear EOS table with the variational method

http://www.np.phys.waseda.ac.jp/EOS/

Equation of state for nuclear matter with the variational method

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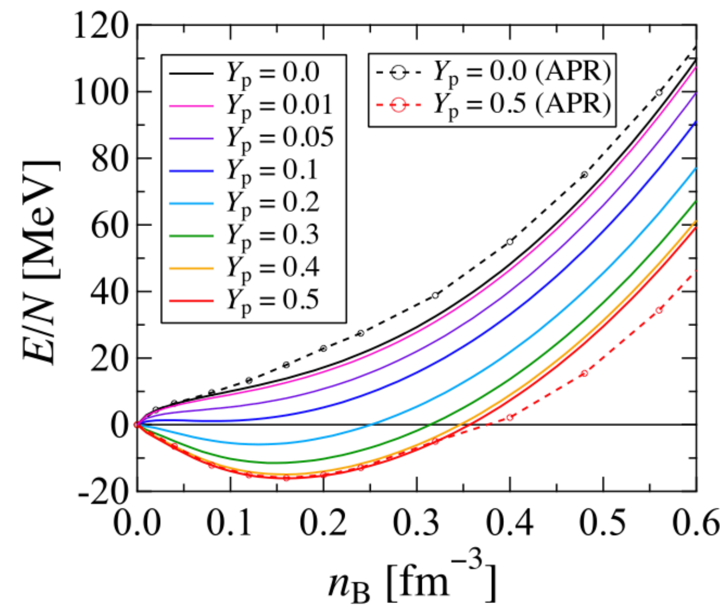

Uniform EOS: AV18 + UIX

eos.zip

perature T, proton fraction Y_p, and baryon mass density ρ_B in
EOS. At the top of the last column, "+1" represents the case

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(HT *et al.*, NPA961 (2017) 78)

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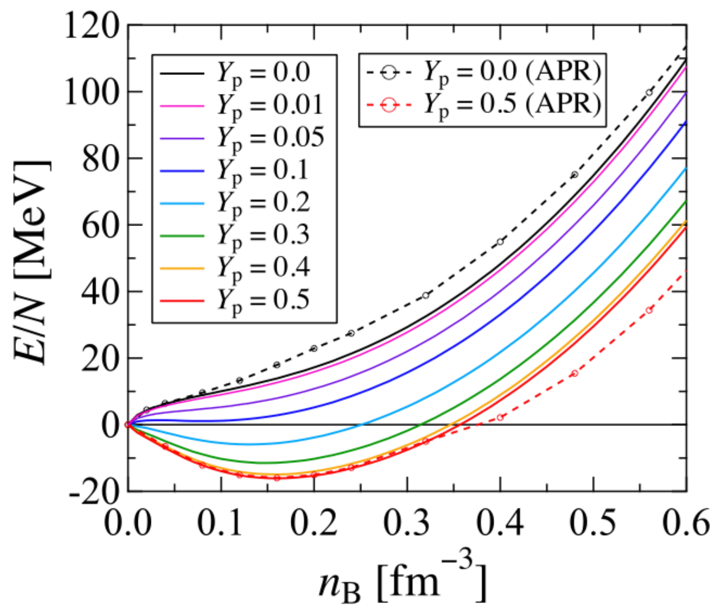
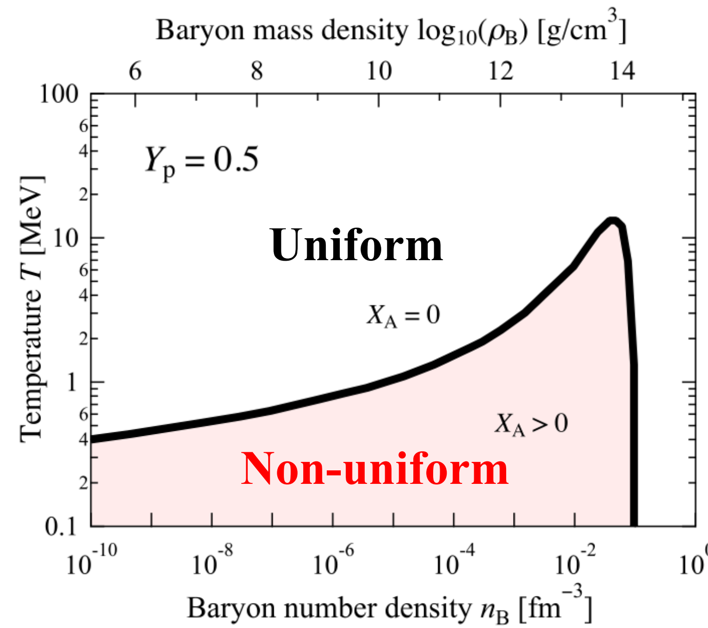
Equation of state for nuclear matter with the variational method



Uniform EOS: AV18 + UIX



Non-uniform EOS: Thomas-Fermi method

Parameter	Minimum	Maximum	Mesh	Number
$\log_{10}(T)$ [MeV]	-1.00	2.60	0.04	$91+1$
Y_p	0	0.65	0.01	66
$\log_{10}(\rho_B)$ [g/cm^3]	5.1	16.0	0.10	110

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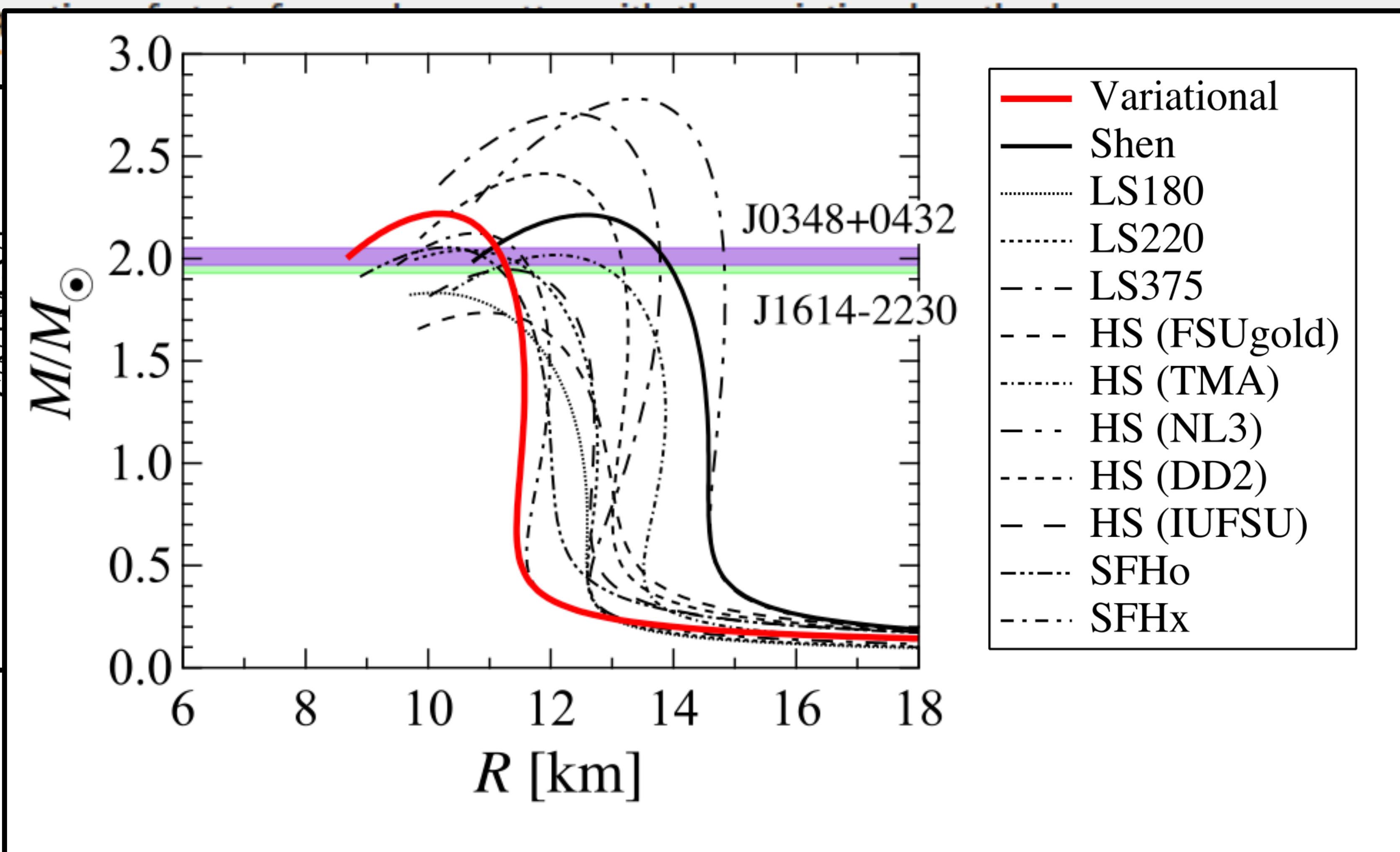

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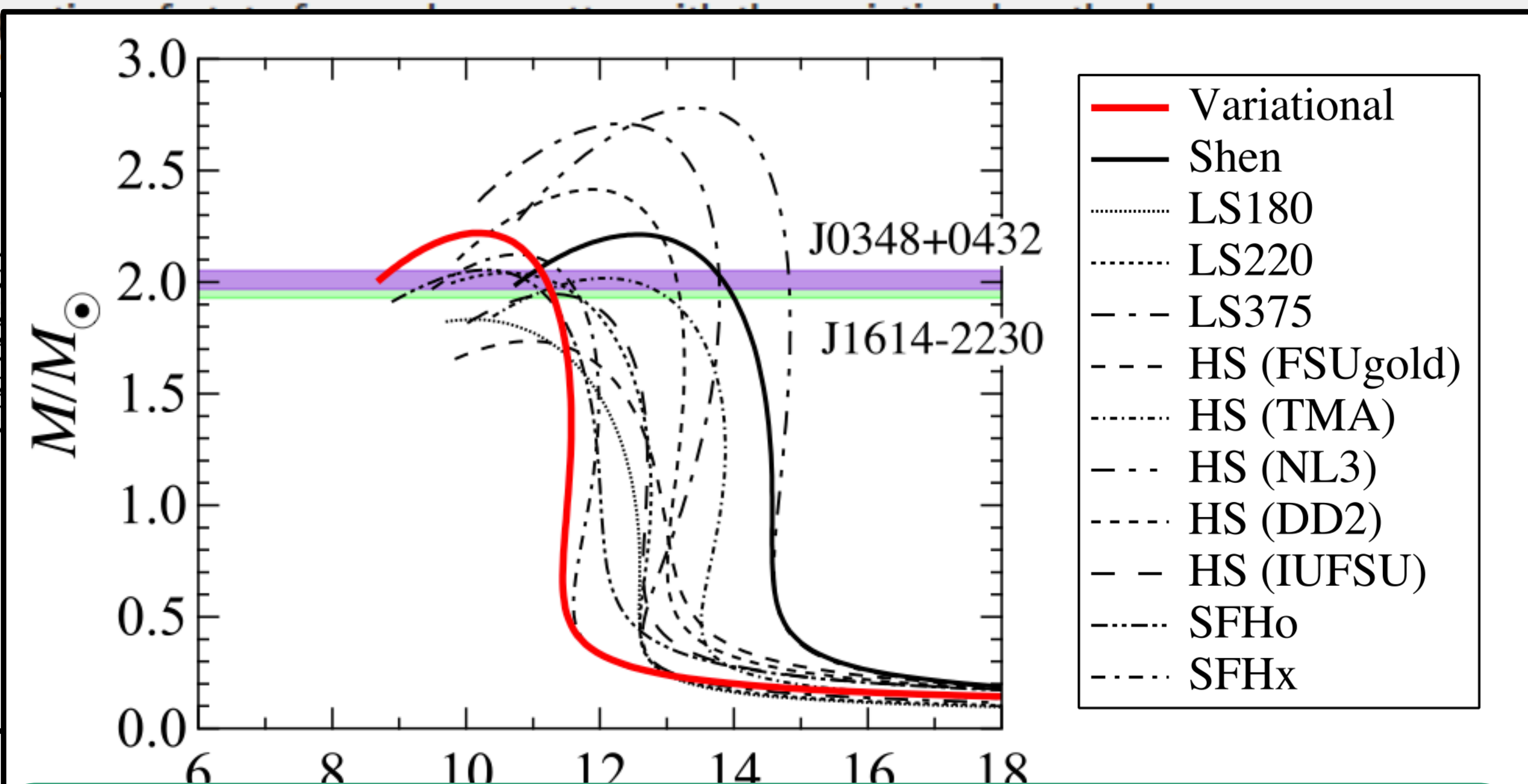

We aim to extend the microscopic EOS table to consider Λ hyperon mixing.

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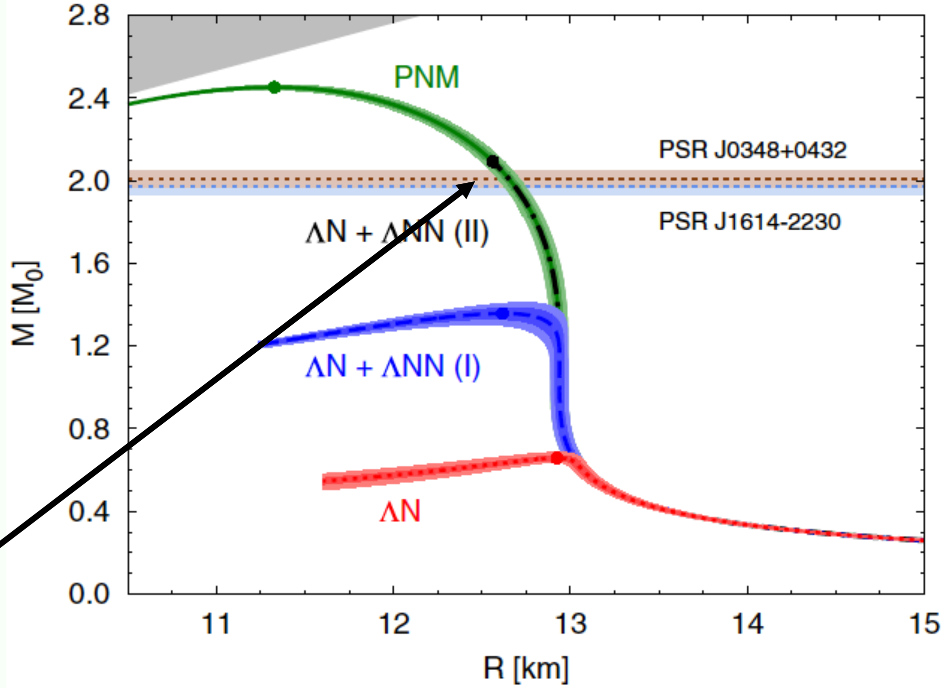
2-1 Hirosawa, Wako, Saitama 351-0198, Japan

Nuclear EOS with hyperons

HYPERON PUZZLE

- **EOS becomes softer due to hyperon mixing.**
- **Maximum mass tends to be lower than the observational data.**

Hyperon Three-Body Force (ΛNN TBF)



D. Lonardoni *et al.*, PRL 114 (2015) 092301

SN-EOS with Hyperons

- **Shen EOS with Λ, Σ, Ξ** [M_{max} = 1.67 $M_\odot$] (C. Ishizuka et al., JPG 35 (2008) 085201)
- **Shen EOS with Λ** [M_{max} = 1.75 $M_\odot$] (H. Shen et al., APJS 197 (2011) 20)
- **LS EOS with Λ** [M_{max} = 1.91 $M_\odot$] (M. Oertel et al., PRC 85 (2012) 055806)
- **HS EOS with Λ** [M_{max} = 2.11 $M_\odot$] (S. Banik et al., APJS 214 (2014) 22)
- **HS EOS with Λ, Σ, Ξ** [M_{max} = 2.04 $M_\odot$] (M. Marques et al., PRC 96 (2017) 045806)

Nuclear EOS with hyperons

HYPERON PUZZLE

- **EOS becomes softer due to hyperon mixing.**
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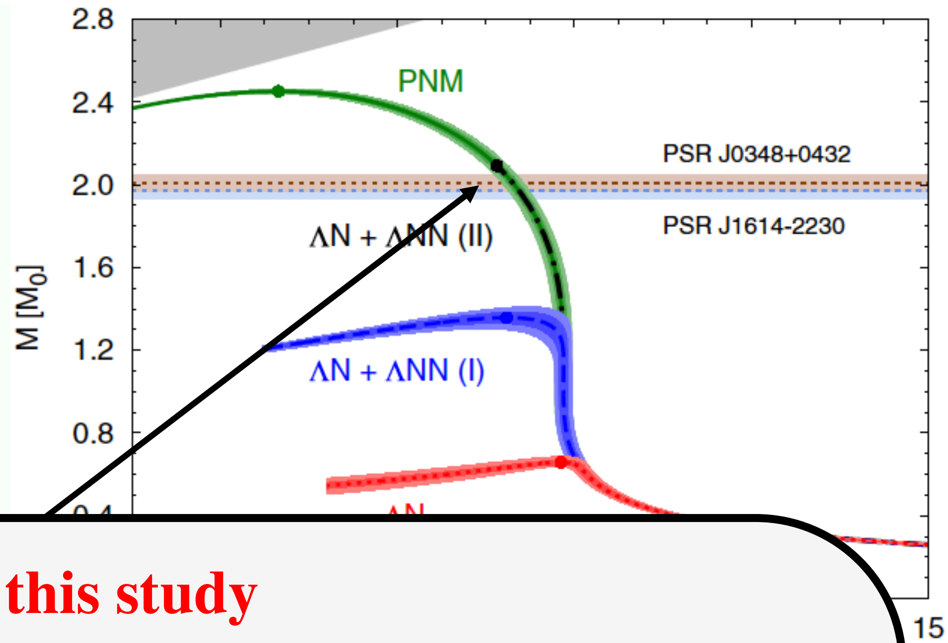

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Aim of this study

- ***We extend the microscopic EOS table to consider Λ hyperon mixing in a self-consistent manner.***
- ***We investigate the effects of hyperon three-body forces (TBF) on the structure of compact stars.***

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- 06)
-)
- **HS EOS with Λ, Σ, □ [M_{max} = 2.04 $M_\odot$]** (M. Marques et al., PRC 96 (2017) 045806)

2. Variational Method for Hyperon EOS

Two-body Hamiltonian

$$H_2 = -\sum_i \frac{\hbar^2}{2m_i}\nabla_i^2 + \sum_{i<j}\left[V_{ij}^{\mathrm{NN}} + V_{ij}^{\Lambda\mathrm{N}} + V_{ij}^{\Lambda\Lambda}\right]$$

V_{ij}^{NN} **: Argonne v18 (AV18) potential**

Hyperon Two-Body Central Potentials

$V_{ij}^{\Lambda\mathrm{N}}$ **: Λ-Nucleon (N) potential** (E. Hiyama et al., PRC 74 (2006) 054312)

- Constructed so as to reproduce the experimental binding energies of light Λ hypernuclei with the Gaussian expansion method.

$V_{ij}^{\Lambda\Lambda}$ **: Λ-Λ potential** (E. Hiyama et al., PRC 66 (2002) 024007)

- the experimental double-Λ binding energy from ${}_{\Lambda\Lambda}^{\;6}He$ *(NAGARA event)*

E_2: Expectation value of H_2 in ***the two-body cluster approximation***

Expectation value of the Hamiltonian

Jastrow wave function

$$\Psi = \mathrm{Sym}\left[\prod_{i<j} f_{ij}\right] \Phi_{\mathrm{F}}$$

Φ_{F}: The Fermi-gas wave function

Correlation function:

$$f_{ij} = \sum_{\mu,p,s} [f^{\mu}_{\mathrm{C}ps}(r_{ij}) + s f^{\mu}_{\mathrm{T}p}(r_{ij}) S_{\mathrm{T}ij} + s f^{\mu}_{\mathrm{SO}p}(r_{ij}) (\boldsymbol{L}_{ij} \cdot \boldsymbol{s})] P^{\mu}_{psij}$$

p: parity s: two-particle total spin μ: particle pair

Cluster-expansion

$$\boxed{\frac{\langle H_2 \rangle}{N} = \frac{1}{N} \frac{\langle \Psi \mid H_2 \mid \Psi \rangle}{\langle \Psi \mid \Psi \rangle} = \frac{\langle H_2 \rangle_2}{N} + \frac{\langle H_2 \rangle_3}{N} + \cdots}$$

E_2 is the expectation value of H_2 in *the two-body cluster approximation* with the Jastrow wave function.

Three-Body Energy

Nuclear EOS (previous study)

$$\frac{E_3}{N} = \underbrace{\frac{1}{N}\langle \sum_{i<j<k}^{N} \left[\alpha V_{ijk}^{\mathrm{R}} + \beta V_{ijk}^{2\pi}\right]\rangle_{\mathrm{F}}}_{\text{Modified expectation value of } H_3 \text{ with } \Phi_{\mathrm{F}}} + \underbrace{\gamma n_{\mathrm{B}}^2 e^{-\delta n_{\mathrm{B}}}\left[1-(1-2Y_{\mathrm{p}})^2\right]}_{\text{Correction term}}$$

Repulsive part of UIX pot. is extended to the potential for Hyperon TBF

$$V_{ijk}^{\mathrm{R}} = \sum_{\mu} \alpha^{\mu} V_{ijk}^{\mathrm{R}} P_{ijk}^{\mu} \quad (\mu = \mathrm{NNN},\ \Lambda\mathrm{NN},\ \Lambda\Lambda\mathrm{N},\ \Lambda\Lambda\Lambda)$$

P_{ijk}^{μ}: Three-particle projection operator

α^{NNN}: we use the value in the nuclear EOS
(Saturation properties + Thomas-Fermi calculation for atomic nuclei)

$\alpha^{\Lambda\mathrm{NN}}$, $\alpha^{\Lambda\Lambda\mathrm{N}}$, $\alpha^{\Lambda\Lambda\Lambda}$: We assume "$0 \le \alpha^{\mu} \le \alpha^{\mathrm{NNN}}$"

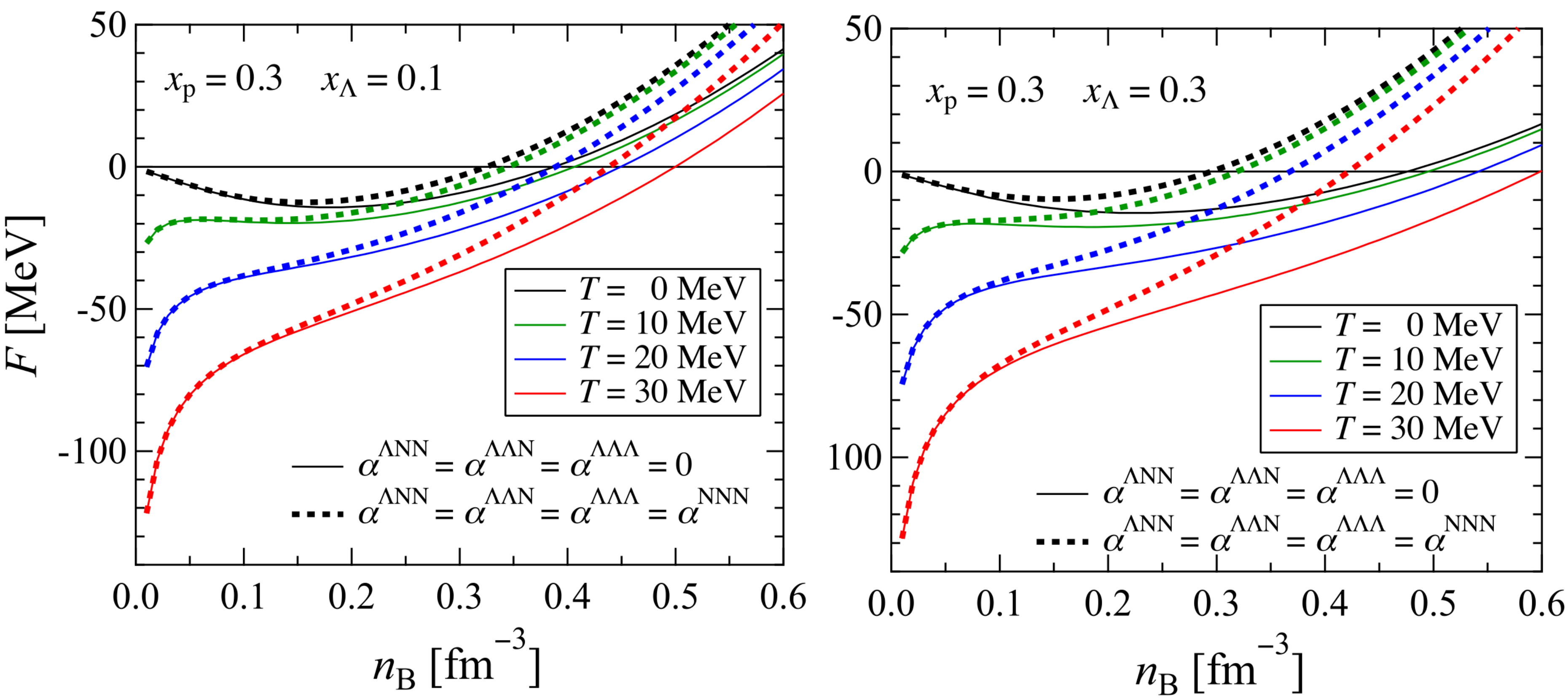
E_3 is the expectation value with the Fermi-gas wave function.

Free energy for Λ hyperon matter

Total energy per baryon: $E\,(n_B, x_p, x_\Lambda) = E_2 + E_3$

Baryon number density : $n_B = n_p + n_n + n_\Lambda$ Particle fraction: $x_i = n_i / n_B \quad (i = \mathrm{p}, \Lambda)$

The prescription by Schmidt and Pandharipande is employed to obtain the free energy ***at finite temperature***.



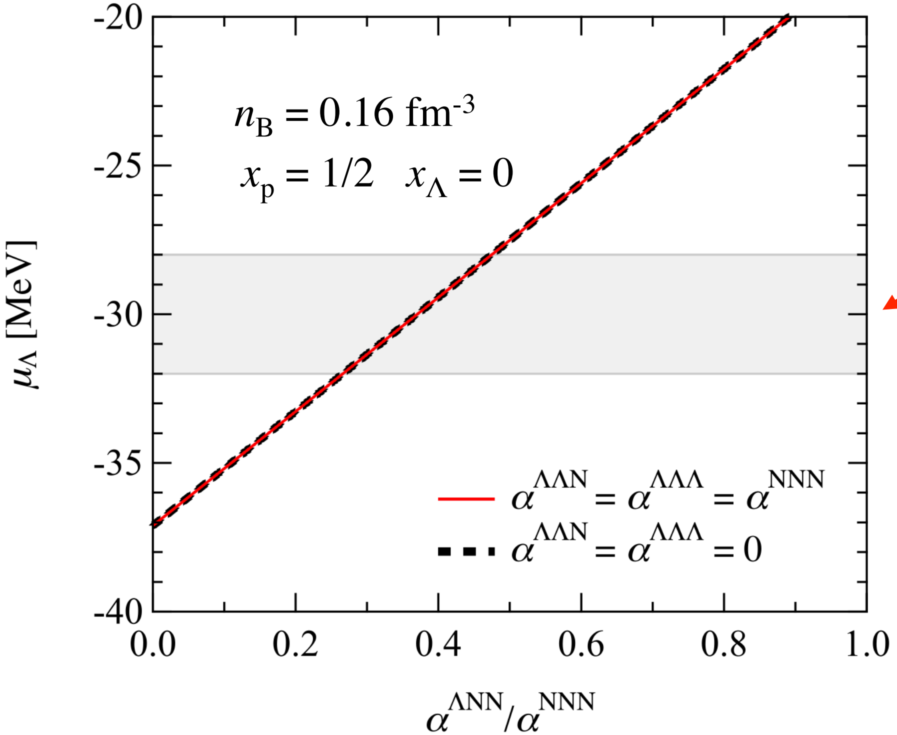
Single particle potential for Λ particle

Total energy per baryon: $E\,(n_B, x_p, x_\Lambda) = E_2 + E_3$

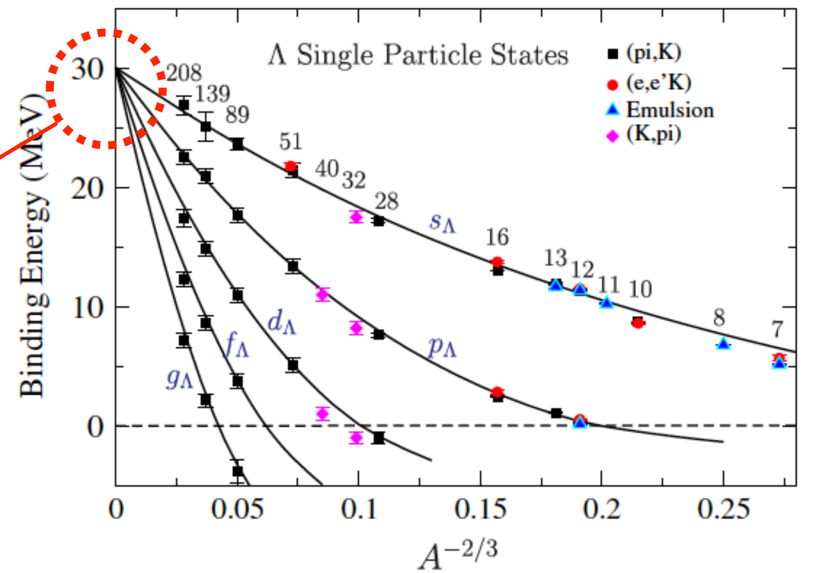
Baryon number density : $n_B = n_p + n_n + n_\Lambda$

Particle fraction: $x_i = n_i / n_B \quad (i = p, \Lambda)$

$\mu_\Lambda(n_B, x_p, x_\Lambda) = [\partial(n_B E)/\partial n_\Lambda]_{n_n, n_p}$



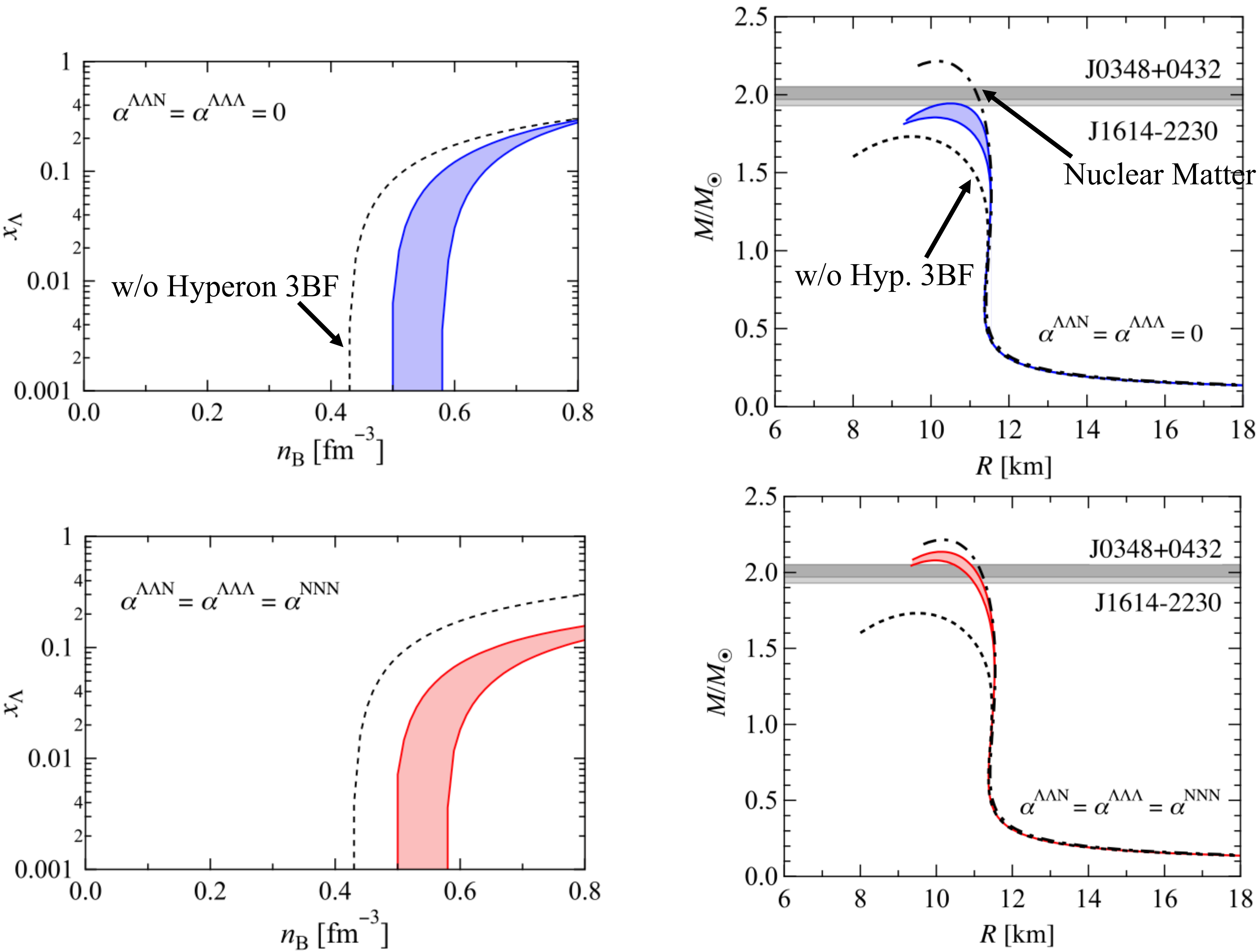
Chemical potential of Λ: μ_Λ



Energy levels of the Λ single-particle major shells (Rev. Mod. Phys. 88 (2016) 035004)

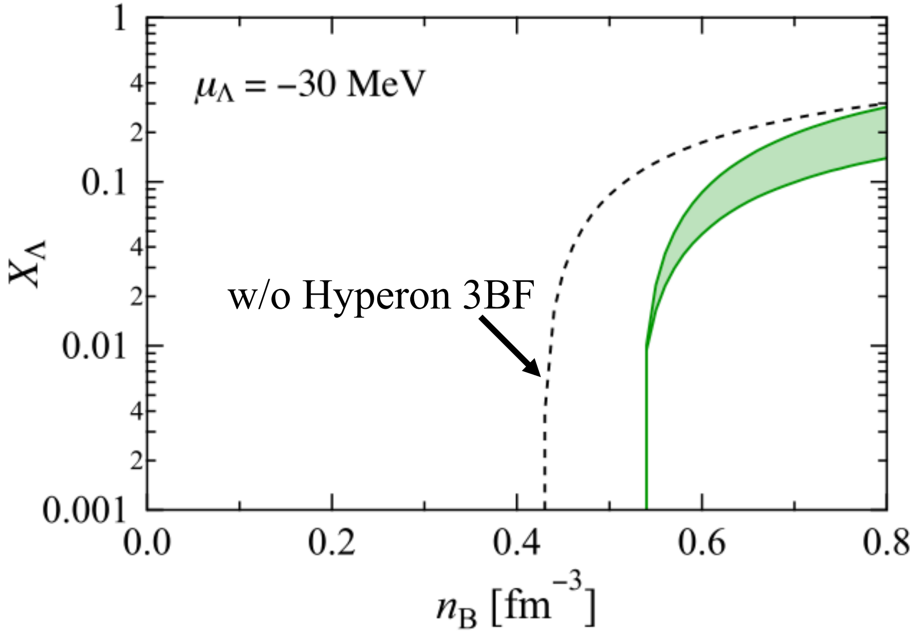
$-32\ \text{MeV} \le \mu_\Lambda \le -28\ \text{MeV} \quad \rightarrow \quad 0.266 \le \alpha^{\Lambda NN}/\alpha^{NNN} \le 0.475$

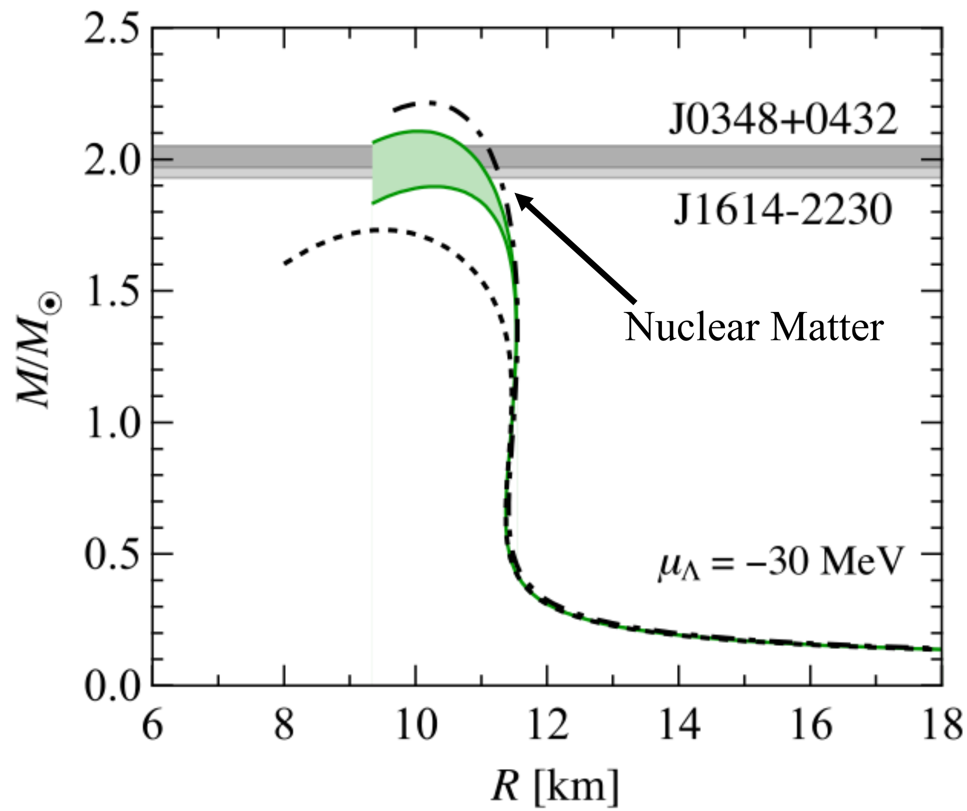
3. Application to Compact Stars



3. Application to Compact Stars

- $\mu_\Lambda = -30$ MeV $\quad (\alpha^{\Lambda NN} = 0.370\ \alpha^{NNN})$
- $0 \le \alpha^{\Lambda\Lambda N} = \alpha^{\Lambda\Lambda\Lambda} \le \alpha^{NNN}$



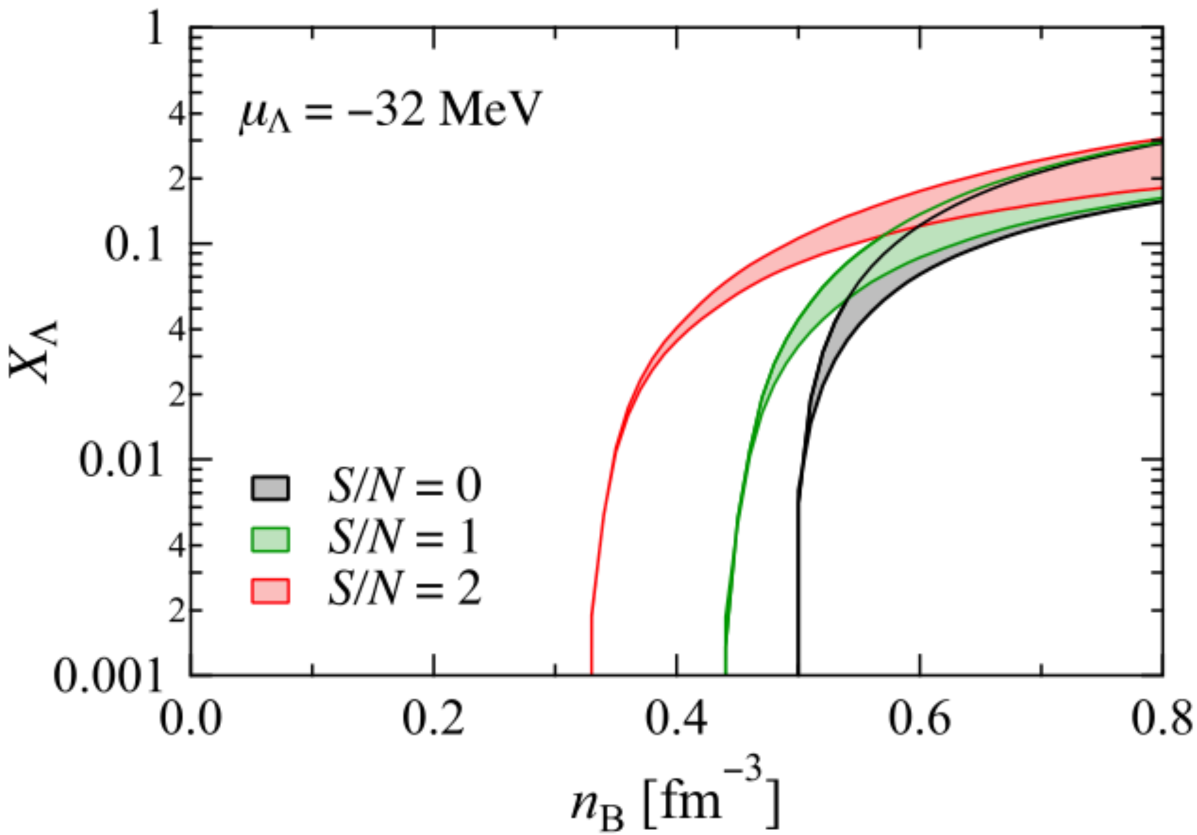


Application to Supernova matter

Supernova matter

- Charge neutral and Isentropic matter (The entropy per baryon $S \sim$ 1-2)
- Neutrino-free β-stable matter

$(0 \leq \alpha^{\Lambda\Lambda N} = \alpha^{\Lambda\Lambda\Lambda} \leq \alpha^{NNN})$



Summary

We construct the EOS for nuclear matter including Λ hyperons at zero and finite temperatures by the variational method.

Variational method for hyperon matter

- The obtained thermodynamic quantities are reasonable.

Application of the EOS to compact stars

- ΛNN three-body force:

affects on the single-particle potential and the onset density of Λ hyperon mixing

- $\Lambda\Lambda$N and $\Lambda\Lambda\Lambda$ three-body force:

affects on the maximum mass of neutron stars (Important for HYPERON PUZZLE!?)

- The effect of Λ hyperons on neutrino-free matter becomes larger at higher entropies.

The era of multi-messenger astronomy has officially begun.

→ It is necessary to construct the reliable EOS for numerical simulations of astrophysical compact phenomena!