Ab Initio Calculations of Charge Symmetry Breaking in Light Hypernuclei

Daniel Gazda

Nuclear Physics Institute
Prague, Czech Republic

Together with:
A. Gal (HU Jerusalem)

Outline

- Introduction
  - Charge symmetry in hadron physics
  - Charge symmetry breaking in light (hyper)nuclei
- Methodology
  - *Ab initio* hypernuclear calculations
- Results
  - *Ab initio* calculations of ${}^{4}_{\Lambda}\mathrm{He}$–${}^{4}_{\Lambda}\mathrm{H}$
- Conclusions & Outlook

Introduction

Introduction

Charge symmetry in hadron physics

- invariance of the strong interaction under the interchange of *up* and *down* quarks (protons and neutrons)
- **broken in QCD by the *up* and *down* light quark mass differences and their QED interactions**
- should break down at $(m_u - m_d)/M \sim 10^{-3}$

Charge symmetry breaking in nuclear physics

- manifest in *pp* and *nn* scattering lengths, well understood
- $^3\mathrm{He} - {}^3\mathrm{H}$: $\Delta E_{\mathrm{SI}}^{\mathrm{CSB}} \approx 70$ keV $\sim 10^{-3} E$

Charge symmetry breaking in hypernuclear physics

- poor $p\Lambda$ and no $n\Lambda$ scattering data
- highly suppressed in ${}^3_\Lambda\mathrm{H}$
- ${}^4_\Lambda\mathrm{He} - {}^4_\Lambda\mathrm{H}$ energy level splittings, $\Delta B_\Lambda^{\mathrm{CSB}} \approx 200$ keV $\sim 10^{-2} B_\Lambda$

Introduction

Charge symmetry in hadron physics

- invariance of the strong interaction under the interchange of *up* and *down* quarks (protons and neutrons)
- **broken in QCD by the *up* and *down* light quark mass differences and their QED interactions**
- should break down at $(m_u - m_d)/M \sim 10^{-3}$

Charge symmetry breaking in nuclear physics

- manifest in *pp* and *nn* scattering lengths, well understood
- $^3\mathrm{He} - {}^3\mathrm{H}$: $\Delta E_{\mathrm{SI}}^{\mathrm{CSB}} \approx 70$ keV $\sim 10^{-3} E$

Charge symmetry breaking in hypernuclear physics

- poor $p\Lambda$ and no $n\Lambda$ scattering data
- highly suppressed in ${}^3_\Lambda\mathrm{H}$
- ${}^4_\Lambda\mathrm{He} - {}^4_\Lambda\mathrm{H}$ energy level splittings, $\Delta B_\Lambda^{\mathrm{CSB}} \approx 200$ keV $\sim 10^{-2} B_\Lambda$

Introduction

Charge symmetry in hadron physics

- invariance of the strong interaction under the interchange of *up* and *down* quarks (protons and neutrons)
- **broken in QCD by the *up* and *down* light quark mass differences and their QED interactions**
- should break down at $(m_u - m_d)/M \sim 10^{-3}$

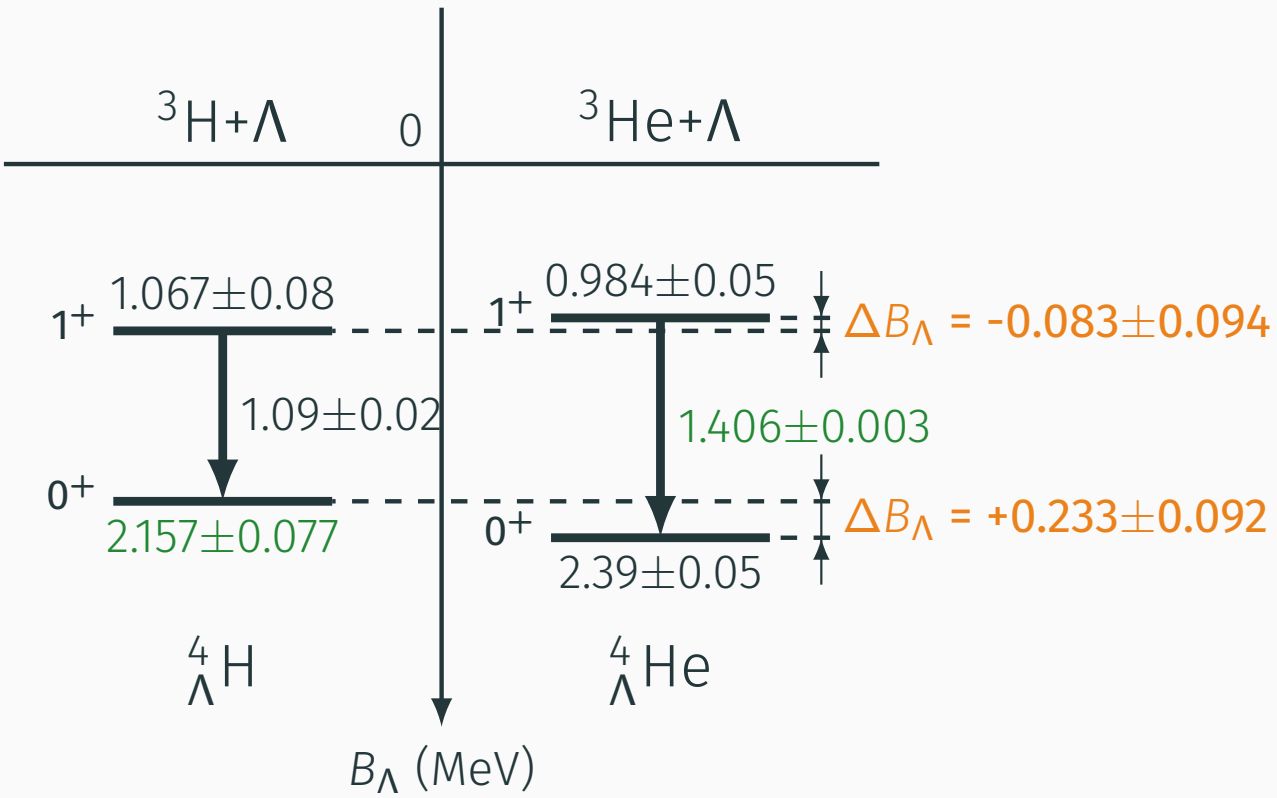
Charge symmetry breaking in nuclear physics

- manifest in *pp* and *nn* scattering lengths, well understood
- $^3\mathrm{He} - {}^3\mathrm{H}$: $\Delta E_{\mathrm{SI}}^{\mathrm{CSB}} \approx 70$ keV $\sim 10^{-3} E$

Charge symmetry breaking in hypernuclear physics

- poor $p\Lambda$ and no $n\Lambda$ scattering data
- highly suppressed in ${}^3_\Lambda\mathrm{H}$
- ${}^4_\Lambda\mathrm{He} - {}^4_\Lambda\mathrm{H}$ energy level splittings, $\Delta B_\Lambda^{\mathrm{CSB}} \approx 200$ keV $\sim 10^{-2} B_\Lambda$

Charge symmetry breaking in A=4 mirror hypernuclei



Recently reaffirmed by:

- J-PARC E13 observation of $^4_\Lambda\mathrm{He}(1^+_{\mathrm{exc.}} \rightarrow 0^+_{\mathrm{g.s.}})$ γ-ray transition [Yamamoto *et al.*, PRL 115, 222501 (2015)]
- MAMI-A1 determination of $B_\Lambda(^4_\Lambda\mathrm{H})$ [Esser *et al.*, PRL 114, 232501 (2015)]

Charge symmetry breaking in A=4 mirror hypernuclei

- Until recently, no microscopic calculation was able to consistently reproduce the large value of ΔB_Λ.

NY model	ΔB_Λ (keV)	
NSC97$_e$	75	[Nogga *et al.*, PRL 88, 172501 (2002)]
NSC97$_f$	100	[Haidenbauer *et al.*, LNP 724, 113 (2007)]
LO χEFT	-10 ± 30	[Gazda, Gal, NPA 954, 161 (2016)]
NLO χEFT	46	[Nogga, NPA 914, 140 (2013)]
LO χEFT+Λ-Σ^0 mixing	180$\pm$130	[Gazda, Gal, PRL 116, 122501 (2016)]
exp.	233 ± 92	

Residual CSB

- increased Coulomb repulsive energy in ${}^4_\Lambda$He
- Σ hyperon mass differences in intermediate $N\Sigma$ states

Electromagnetic $\Lambda - \Sigma^0$ mixing

- Physical Λ and Σ^0 hyperons have mixed isospin composition in terms of the SU(3) pure-isospin Λ (I=0) and Σ (I=1) hyperons:

$$\Lambda_{\text{phys.}} = \Lambda \cos\alpha - \Sigma \sin\alpha$$
$$\Sigma^0_{\text{phys.}} = \Lambda \sin\alpha + \Sigma \cos\alpha$$

with mixing angle:

$$\tan\alpha = \frac{\langle \Sigma | \delta M | \Lambda \rangle}{M_\Sigma - M_\Lambda}$$

proportional to electromagnetic mass matrix element, related to isospin-breaking mass differences within the baryon octet:

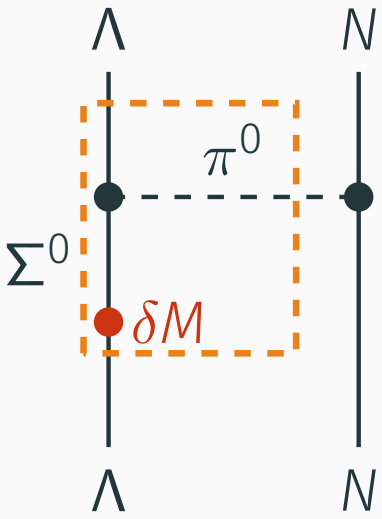
$$\langle \Sigma | \delta M | \Lambda \rangle = \frac{1}{\sqrt{3}} [(M_{\Sigma^0} - M_{\Sigma^+}) - (M_n - M_p)] = 1.14(05) \text{ MeV}$$

[Dalitz, von Hippel, Phys. Lett. 10, 153 (1964)]

- Recent LQCD calculation $\langle \Sigma | \delta M | \Lambda \rangle = 0.52(23)$ MeV
  [R. Hosley *et al.*, PRD 91, 074512 (2015)]
  $\leftarrow$ **no QED effects**, baryon mass differences are not reproduced

Electromagnetic $\Lambda - \Sigma^0$ mixing

- Direct $\pi\Lambda\Lambda$ coupling is forbidden by isospin conservation
- $\Lambda - \Sigma^0$ mixing results in effective $\pi\Lambda\Lambda$ coupling, giving rise to CSB $N\Lambda$ interaction:



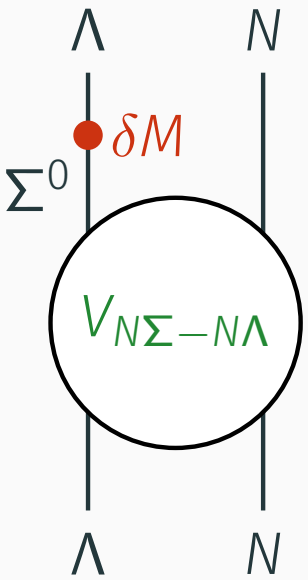
$$V_{N\Lambda}^{\mathrm{CSB}} = g_{\pi\Lambda\Lambda}\tau_{N3}[\boldsymbol{\sigma}_\Lambda \cdot \boldsymbol{\sigma}_N + T(r)S(\hat{r}, \boldsymbol{\sigma}_\Lambda, \boldsymbol{\sigma}_N)]Y(r)$$

$$g_{\pi\Lambda\Lambda} = -2\frac{\langle\Sigma^0|\delta M|\Lambda\rangle}{M_{\Sigma^0} - M_\Lambda} g_{\pi\Lambda\Sigma}$$

- contributions from heavier shorter-range isovector meson exchanges, e.g. ρ – (replaced by contact terms in χEFT)
- expected $\Delta B_\Lambda({}^4_\Lambda\mathrm{He} - {}^4_\Lambda\mathrm{H}) \approx 170$ keV [Gazda, Gal, NPA 954, 161 (2016)]

Electromagnetic $\Lambda - \Sigma^0$ mixing

- For NY interaction models with explicit $N\Sigma - N\Lambda$ coupling, the electromagnetic $\Lambda - \Sigma^0$ mixing relates matrix elements of $V^{\rm CSB}_{N\Lambda}$ with $V_{N\Sigma-N\Lambda}$:



$$\langle N\Lambda|V^{\rm CSB}_{N\Lambda}|N\Lambda\rangle = -2\frac{\langle\Sigma^0|\delta M|\Lambda\rangle}{M_{\Sigma^0}-M_\Lambda}\tau_{N3}\frac{1}{\sqrt{3}}\langle N\Sigma|V_{N\Sigma-N\Lambda}|N\Lambda\rangle$$

[Gal, PLB 744, 352 (2015)]

- The first microscopic model which generates large $\Delta B_\Lambda(0^+_{\rm g.s.}) \approx 200$ keV in A = 4 hypernuclei!
  [Gazda, Gal, PRL 116, 122501 (2016)]

Methodology

Ab initio hypernuclear calculations

Given a Hamiltonian solve the eigenvalue problem of A nucleons

$$\left[\sum_{i\leq A}\frac{\hat{\mathbf{p}}_i^2}{2m}+\sum_{i<j\leq A}\hat{V}_{BB}(i,j)+\sum_{i<j<k\leq A}\hat{V}_{BBB}(i,j,k)\right]\Psi=E\,\Psi$$

- realistic interparticle interactions
- controllable approximations

Ab initio no-core shell model

- Hamiltonian is diagonalized in a *finite* A-particle harmonic oscillator basis

$$\Psi(\mathbf{r}_1,\ldots,\mathbf{r}_A)=\sum_{n\leq N_{\text{tot}}}\phi_n^{\text{HO}}(\mathbf{r}_1,\ldots,\mathbf{r}_A)$$

  (dimensions up to $\sim 10^{10}$ with $\sim 10^{14}$ nonzero matrix elements)
- all particles are active (no core)
- NCSM results converge to exact results, $N_{\text{tot}}\to\infty$

Input V_{NN}, V_{NNN} and V_{NY} potentials

Potentials derived from chiral EFT

- long-range part (π, K, η-exchange) predicted by χPT
- short-range part parametrized by contact interactions, LECs fitted to experimental data

NN+*NNN* interaction

- chiral N^{3}LO *NN* potential [Entem, Machleidt, PRC 68, 041001 (2003)]
- chiral N^{2}LO *NNN* potential [Navrátil, FBS 41, 14 (2007)]

NY interaction

- chiral LO potential [Polinder *et al.*, NPA 779, 244 (2006)]
- $\Lambda N - \Sigma N$ mixing explicitly taken into account:

$$V_{NY} = \begin{pmatrix} V_{\Lambda N-\Lambda N} & V_{\Lambda N-\Sigma N} \\ V_{\Sigma N-\Lambda N} & V_{\Sigma N-\Sigma N} \end{pmatrix} + \begin{pmatrix} 0 & 0 \\ 0 & m_\Sigma - m_\Lambda \end{pmatrix}$$

- supplemented by $V_{N\Lambda}^{\mathrm{CSB}}$

Results

Charge symmetry breaking in ${}^{4}_{\Lambda}$He–${}^{4}_{\Lambda}$H hypernuclei

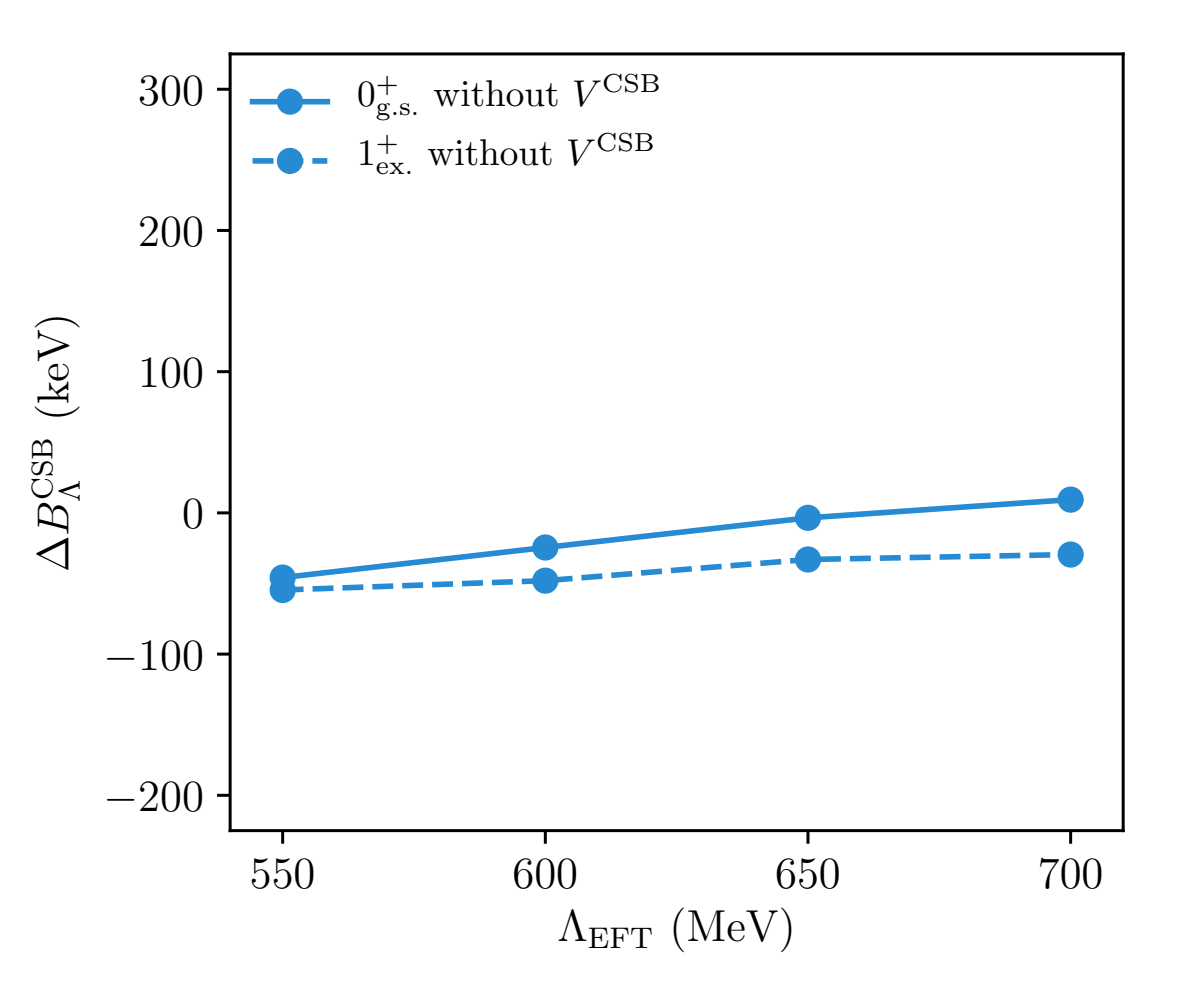


Fig. 1: Cutoff momentum Λ_{EFT} dependence of the difference $\Delta B_\Lambda^{\text{CSB}} = B_\Lambda({}^{4}_{\Lambda}\text{He}) - B_\Lambda({}^{4}_{\Lambda}\text{H})$ of Λ separation energies in ${}^{4}_{\Lambda}$He and ${}^{4}_{\Lambda}$H in *ab initio* NCSM calculations without $V^{\text{CSB}}_{\Lambda N}$ generated by $\Lambda N - \Sigma N$ conversion from LO chiral *NY* interactions.

Charge symmetry breaking in ${}^{4}_{\Lambda}$He–${}^{4}_{\Lambda}$H hypernuclei

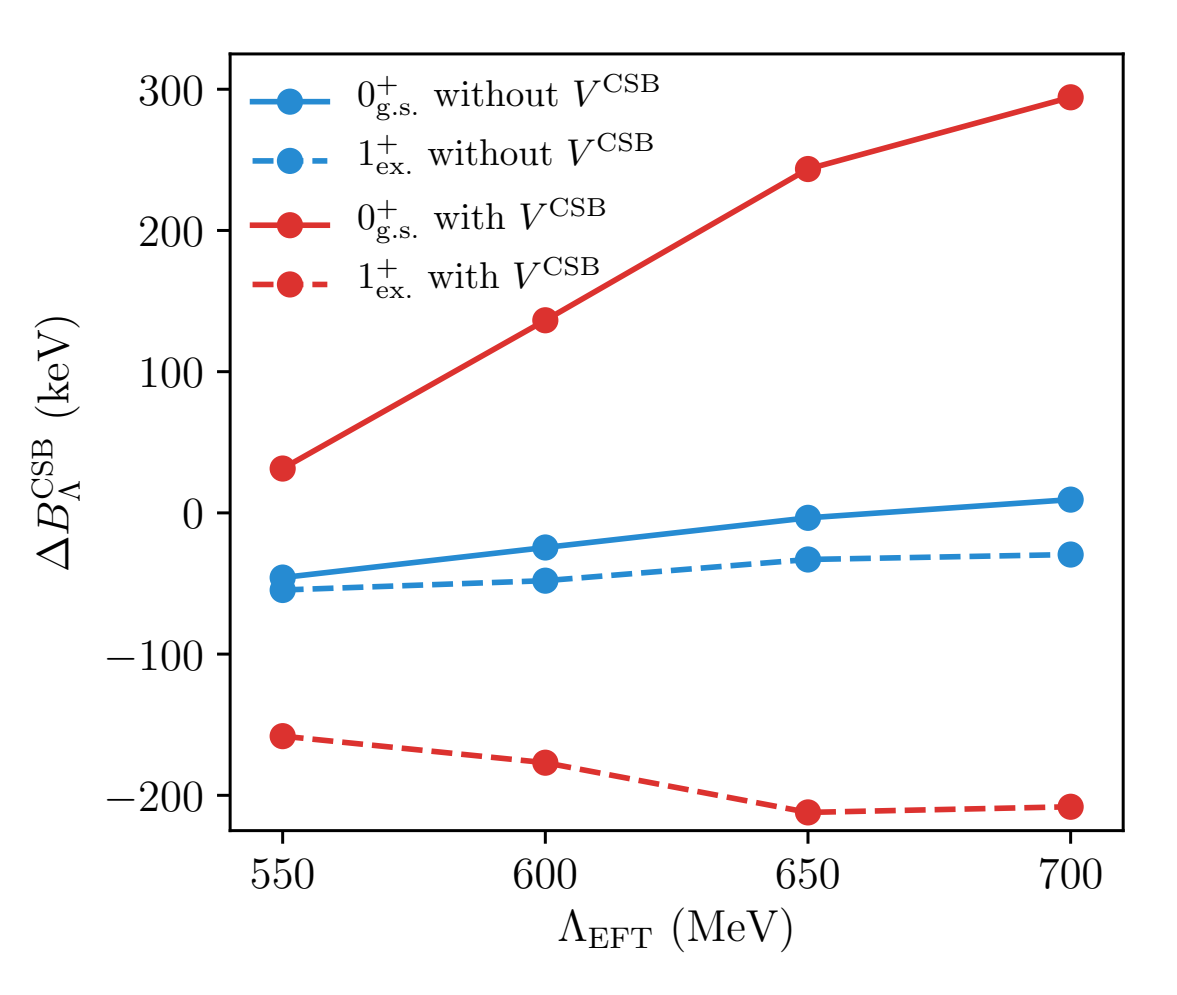


Fig. 1: Cutoff momentum Λ_{EFT} dependence of the difference $\Delta B^{\text{CSB}}_{\Lambda} = B_{\Lambda}({}^{4}_{\Lambda}\text{He}) - B_{\Lambda}({}^{4}_{\Lambda}\text{H})$ of Λ separation energies in ${}^{4}_{\Lambda}$He and ${}^{4}_{\Lambda}$H in *ab initio* NCSM calculations without and with $V^{\text{CSB}}_{\Lambda N}$ generated by $\Lambda N - \Sigma N$ conversion from LO chiral NY interactions.

Charge symmetry breaking in ${}^{4}_{\Lambda}$He–${}^{4}_{\Lambda}$H hypernuclei

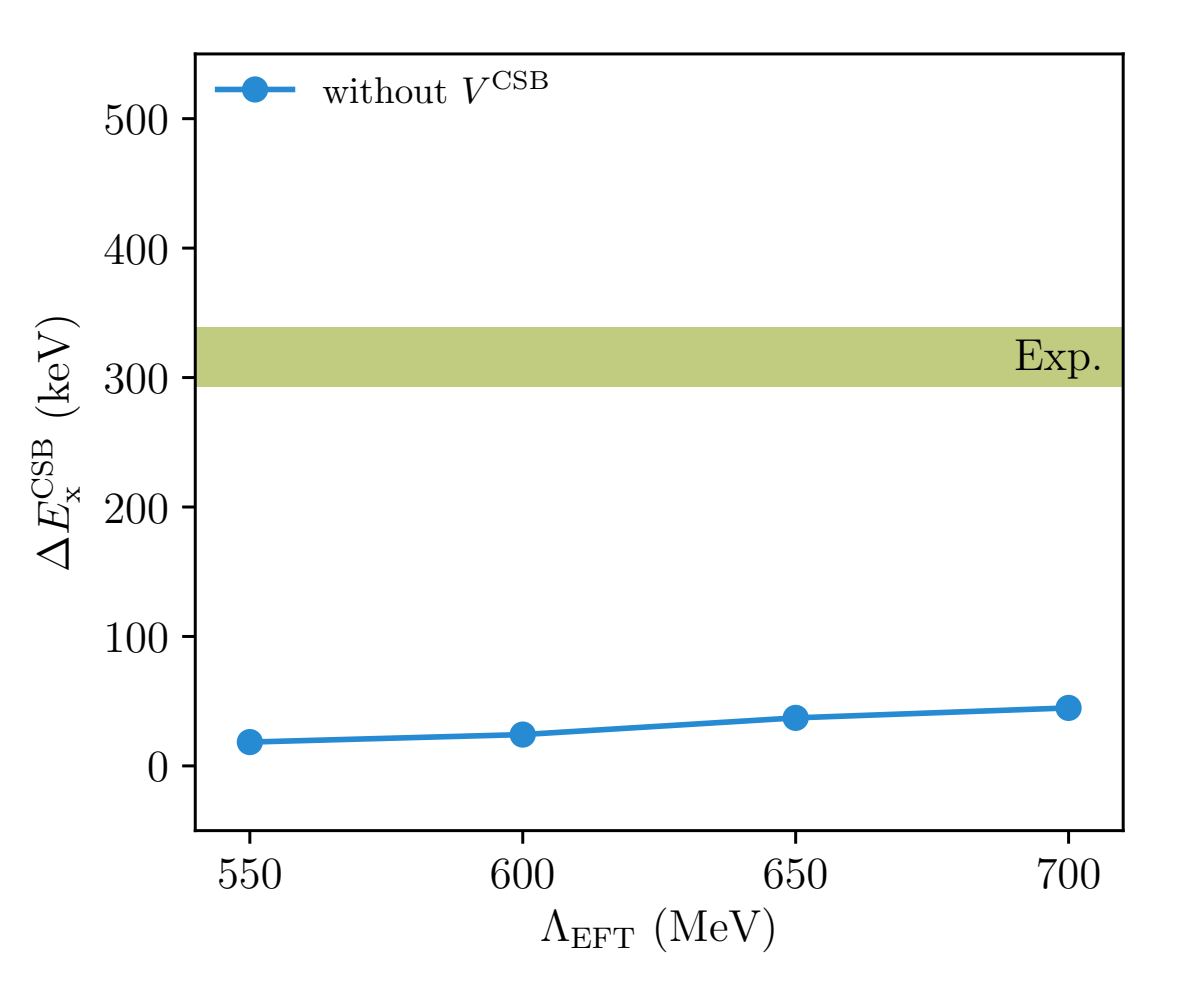


Fig. 2: Cutoff momentum (Λ_{EFT}) dependence of the difference $\Delta E_{\mathrm{x}}^{\mathrm{CSB}}$ of the excitation energies $E_x(0^+_{\mathrm{g.s.}} \to 1^+)$ in ${}^{4}_{\Lambda}$He and ${}^{4}_{\Lambda}$H in *ab initio* NCSM calculations without $V_{\Lambda N}^{\mathrm{CSB}}$ generated by ΛN–ΣN conversion from LO chiral *NY* interactions.

Charge symmetry breaking in ${}^{4}_{\Lambda}$He–${}^{4}_{\Lambda}$H hypernuclei

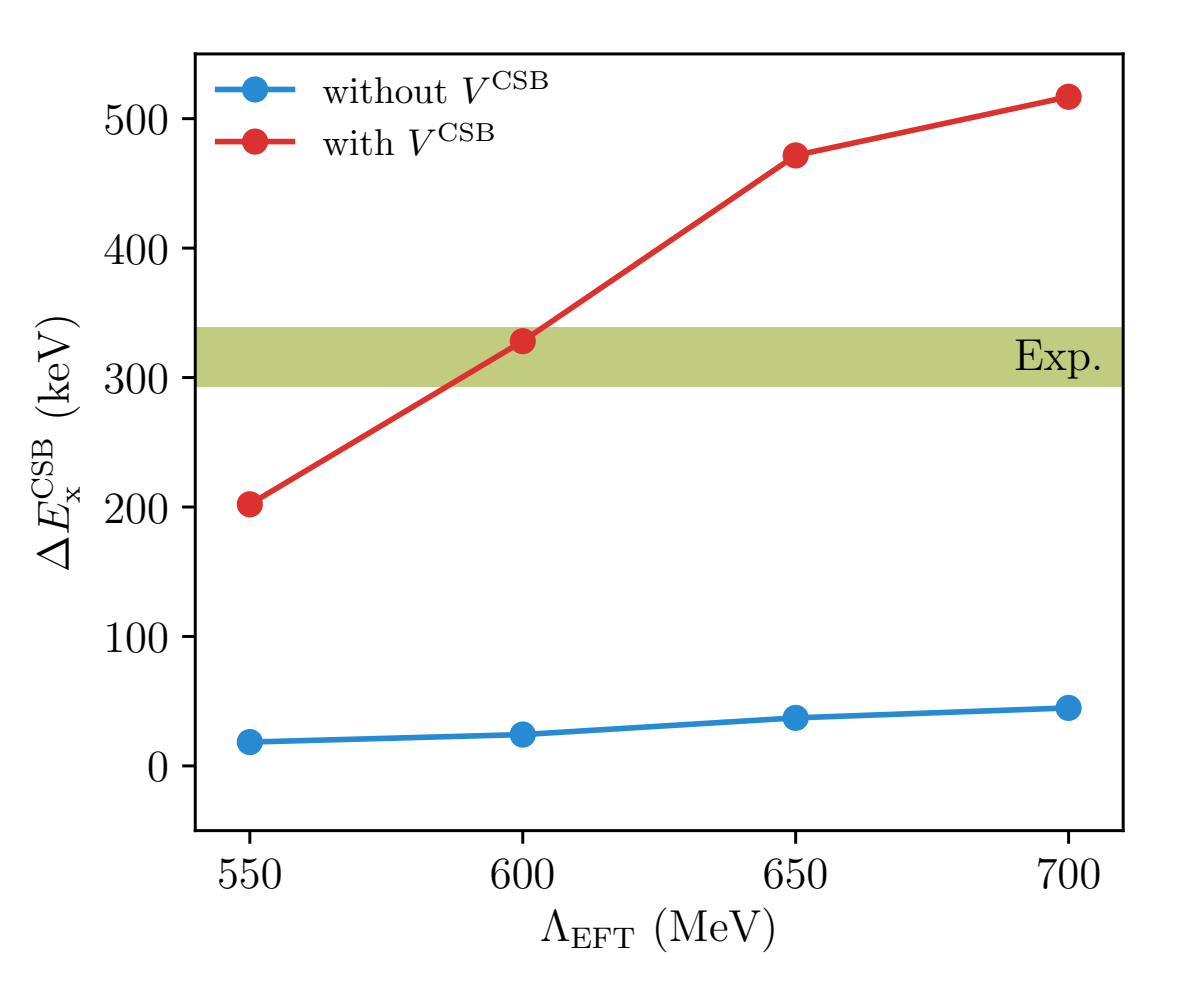


Fig. 2: Cutoff momentum (Λ_{EFT}) dependence of the difference $\Delta E_{\mathrm{x}}^{\mathrm{CSB}}$ of the excitation energies $E_x(0^+_{\mathrm{g.s.}} \to 1^+)$ in ${}^{4}_{\Lambda}$He and ${}^{4}_{\Lambda}$H in *ab initio* NCSM calculations without and with $V_{\Lambda N}^{\mathrm{CSB}}$ generated by ΛN–ΣN conversion from LO chiral NY interactions.

Conclusions & Outlook

Conclusions

Ab initio calculations of light hypernuclei

- Electromagnetic $\Lambda - \Sigma^0$ conversion is able to produce sizeable value of CSB in A=4 mirror hypernuclei.
- Within a model based on chiral LO NY $\Lambda N - \Sigma N$ interactions it is possible to resolve the CSB puzzle in the ground states of ${}^{4}_{\Lambda}$He and ${}^{4}_{\Lambda}$H.

[Gazda, Gal, PRL 116, 122501 (2016); NPA 954, 161 (2016)]

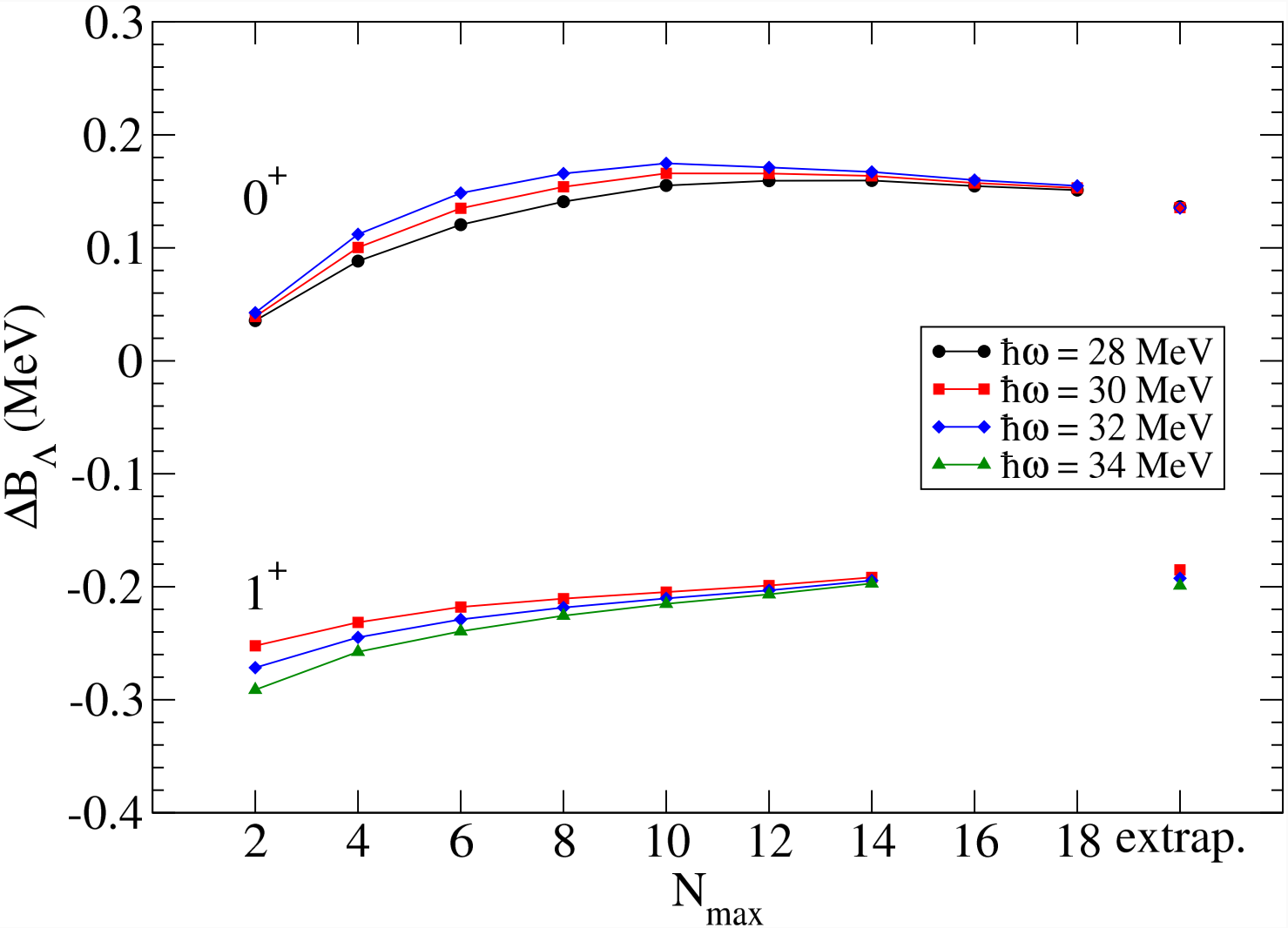
Outlook

- Sizeable cutoff dependence $\rightarrow$ need to study NLO V_{NY}.
- p-shell hypernuclei

Thank you!

Backup slides

Backup slides



Backup slides