Ab Initio Calculations of Charge Symmetry Breaking in Light Hypernuclei

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Together with:
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• Introduction
  • Charge symmetry in hadron physics
  • Charge symmetry breaking in light (hyper)nuclei

• Methodology
  • *Ab initio* hypernuclear calculations

• Results
  • *Ab initio* calculations of $^4\Lambda\text{He}^4\Lambda\text{H}$

• Conclusions & Outlook
Introduction
### Charge symmetry in hadron physics

- Invariance of the strong interaction under the interchange of *up* and *down* quarks (protons and neutrons)
- Broken in QCD by the *up* and *down* light quark mass differences and their QED interactions
- Should break down at \((m_u - m_d)/M \approx 10^{-3}\)

### Charge symmetry breaking in nuclear physics

- Manifest in *pp* and *nn* scattering lengths, well understood
- \(^3\text{He} - ^3\text{H}: \Delta E_{\text{SI}}^{\text{CSB}} \approx 70 \text{ keV} \approx 10^{-3}E\)

### Charge symmetry breaking in hypernuclear physics

- Poor *pΛ* and no *nΛ* scattering data
- Highly suppressed in \(^3\Lambda\text{H}\)
- \(^4\Lambda\text{He} - ^4\Lambda\text{H} energy level splittings, \(\Delta B_\Lambda^{\text{CSB}} \approx 200 \text{ keV} \approx 10^{-2}B_\Lambda\)
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Charge symmetry breaking in $A=4$ mirror hypernuclei

Recently reaffirmed by:

- J-PARC E13 observation of $^4_\Lambda$He($1^{+}_{\text{exc.}} \rightarrow 0^{+}_{\text{g.s.}}$) $\gamma$-ray transition [Yamamoto et al., PRL 115, 222501 (2015)]
- MAMI-A1 determination of $B_\Lambda(^4_\Lambda H)$ [Esser et al., PRL 114, 232501 (2015)]
Charge symmetry breaking in $A=4$ mirror hypernuclei

- Until recently, no microscopic calculation was able to consistently reproduce the large value of $\Delta B_\Lambda$.

<table>
<thead>
<tr>
<th>NY model</th>
<th>$\Delta B_\Lambda$ (keV)</th>
<th>Reference</th>
</tr>
</thead>
<tbody>
<tr>
<td>NSC97$_e$</td>
<td>75</td>
<td>[Nogga et al., PRL 88, 172501 (2002)]</td>
</tr>
<tr>
<td>NSC97$_f$</td>
<td>100</td>
<td>[Haidenbauer et al., LNP 724, 113 (2007)]</td>
</tr>
<tr>
<td>LO $\chi$EFT</td>
<td>$-10 \pm 30$</td>
<td>[Gazda, Gal, NPA 954, 161 (2016)]</td>
</tr>
<tr>
<td>NLO $\chi$EFT</td>
<td>46</td>
<td>[Nogga, NPA 914, 140 (2013)]</td>
</tr>
<tr>
<td>LO $\chi$EFT+$\Lambda-\Sigma^0$ mixing</td>
<td>$180\pm130$</td>
<td>[Gazda, Gal, PRL 116, 122501 (2016)]</td>
</tr>
<tr>
<td>exp.</td>
<td>233 $\pm$ 92</td>
<td></td>
</tr>
</tbody>
</table>

Residual CSB

- increased Coulomb repulsive energy in $^4\Lambda$He
- $\Sigma$ hyperon mass differences in intermediate $N\Sigma$ states
Electromagnetic $\Lambda - \Sigma^0$ mixing

- Physical $\Lambda$ and $\Sigma^0$ hyperons have mixed isospin composition in terms of the SU(3) pure-isospin $\Lambda (I=0)$ and $\Sigma (I=1)$ hyperons:

$$\Lambda_{\text{phys.}} = \Lambda \cos \alpha - \Sigma \sin \alpha$$
$$\Sigma^0_{\text{phys.}} = \Lambda \sin \alpha + \Sigma \cos \alpha$$

with mixing angle:

$$\tan \alpha = \frac{\langle \Sigma | \delta M | \Lambda \rangle}{M_\Sigma - M_\Lambda}$$

proportional to electromagnetic mass matrix element, related to isospin-breaking mass differences within the baryon octet:

$$\langle \Sigma | \delta M | \Lambda \rangle = \frac{1}{\sqrt{3}} \left[ (M_{\Sigma^0} - M_{\Sigma^+}) - (M_n - M_p) \right] = 1.14(05) \text{ MeV}$$

[Dalitz, von Hippel, Phys. Lett. 10, 153 (1964)]

- Recent LQCD calculation $\langle \Sigma | \delta M | \Lambda \rangle = 0.52(23) \text{ MeV}$
  [R. Hosley et al., PRD 91, 074512 (2015)]

← no QED effects, baryon mass differences are not reproduced
Electromagnetic $\Lambda - \Sigma^0$ mixing

- Direct $\pi\Lambda\Lambda$ coupling is forbidden by isospin conservation
- $\Lambda - \Sigma^0$ mixing results in effective $\pi\Lambda\Lambda$ coupling, giving rise to CSB $N\Lambda$ interaction:

\[
V_{N\Lambda}^{\text{CSB}} = g_{\pi\Lambda\Lambda} T_{N3} \left[ \sigma_\Lambda \cdot \sigma_N + T(r) S(\hat{r}, \sigma_\Lambda, \sigma_N) \right] Y(r)
\]

\[
g_{\pi\Lambda\Lambda} = -2 \frac{\langle \Sigma^0 | \delta M | \Lambda \rangle}{M_{\Sigma^0} - M_{\Lambda}} g_{\pi\Lambda\Sigma}
\]

- Contributions from heavier shorter-range isovector meson exchanges, e.g. $\rho$ – (replaced by contact terms in $\chi$EFT)
- Expected $\Delta B_\Lambda(^4_{\Lambda}\text{He} - ^4_{\Lambda}\text{H}) \approx 170$ keV [Gazda, Gal, NPA 954, 161 (2016)]
Electromagnetic $\Lambda - \Sigma^0$ mixing

• For $NY$ interaction models with explicit $N\Sigma - N\Lambda$ coupling, the electromagnetic $\Lambda - \Sigma^0$ mixing relates matrix elements of $V^{\text{CSB}}_{N\Lambda}$ with $V_{N\Sigma - N\Lambda}$:

$$
\langle N\Lambda| V^{\text{CSB}}_{N\Lambda}| N\Lambda \rangle = -2 \frac{\langle \Sigma^0 |\delta M| \Lambda \rangle}{M_{\Sigma^0} - M_\Lambda} \tau_{N3} \frac{1}{\sqrt{3}} \langle N\Sigma| V_{N\Sigma - N\Lambda}| N\Lambda \rangle
$$

[Gal, PLB 744, 352 (2015)]

• The first microscopic model which generates large $\Delta B_\Lambda(0^+_{\text{g.s.}}) \approx 200$ keV in $A = 4$ hypernuclei!

[Gazda, Gal, PRL 116, 122501 (2016)]
Methodology
Ab initio hypernuclear calculations

Given a Hamiltonian solve the eigenvalue problem of A nucleons

\[
\left[ \sum_{i \leq A} \frac{\hat{p}_i^2}{2m} + \sum_{i<j \leq A} \hat{V}_{BB}(i,j) + \sum_{i<j<k \leq A} \hat{V}_{BBB}(i,j,k) \right] \Psi = E \Psi
\]

- realistic interparticle interactions
- controllable approximations

Ab initio no-core shell model

- Hamiltonian is diagonalized in a finite A-particle harmonic oscillator basis

\[
\Psi(r_1, \ldots, r_A) = \sum_{n \leq N_{tot}} \phi^\text{HO}_n(r_1, \ldots, r_A)
\]

(dimensions up to $\sim 10^{10}$ with $\sim 10^{14}$ nonzero matrix elements)

- all particles are active (no core)
- NCSM results converge to exact results, $N_{tot} \rightarrow \infty$
Input $V_{NN}$, $V_{NNN}$ and $V_{NY}$ potentials

Potentials derived from chiral EFT

- long-range part ($\pi, K, \eta$-exchange) predicted by $\chi$PT
- short-range part parametrized by contact interactions, LECs fitted to experimental data

**NN+NNN interaction**

- chiral $N^3$LO $NN$ potential [Entem, Machleidt, PRC 68, 041001 (2003)]
- chiral $N^2$LO $NNN$ potential [Navrátil, FBS 41, 14 (2007)]

**NY interaction**

- chiral LO potential [Polinder et al., NPA 779, 244 (2006)]
- $\Lambda N - \Sigma N$ mixing explicitly taken into account:

$$V_{NY} = \begin{pmatrix} V_{\Lambda N - \Lambda N} & V_{\Lambda N - \Sigma N} \\ V_{\Sigma N - \Lambda N} & V_{\Sigma N - \Sigma N} \end{pmatrix} + \begin{pmatrix} 0 & 0 \\ 0 & m_\Sigma - m_\Lambda \end{pmatrix}$$

- supplemented by $V_{CSB}^{\Lambda N}$
Results
Charge symmetry breaking in $^{4}_\Lambda$He–$^{4}_\Lambda$H hypernuclei

Fig. 1: Cutoff momentum $\Lambda_{EFT}$ dependence of the difference $\Delta B_{\Lambda}^{\text{CSB}} = B_{\Lambda}(^{4}_\Lambda\text{He}) - B_{\Lambda}(^{4}_\Lambda\text{H})$ of $\Lambda$ separation energies in $^{4}_\Lambda$He and $^{4}_\Lambda$H in \textit{ab initio} NCSM calculations \textbf{without} $V_{\Lambda N}^{\text{CSB}}$ generated by $\Lambda N - \Sigma N$ conversion from LO chiral $NY$ interactions.
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Fig. 2: Cutoff momentum ($\Lambda_{\text{EFT}}$) dependence of the difference $\Delta E_{x}^{\text{CSB}}$ of the excitation energies $E_{x}(0_{g.s.}^{+} \rightarrow 1^{+})$ in $^{4}_{\Lambda}\text{He}$ and $^{4}_{\Lambda}\text{H}$ in \textit{ab initio} NCSM calculations \textbf{without} $V_{\Lambda N}^{\text{CSB}}$ generated by $\Lambda N - \Sigma N$ conversion from LO chiral $NY$ interactions.
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Conclusions & Outlook
Conclusions

**Ab initio** calculations of light hypernuclei

- Electromagnetic $\Lambda - \Sigma^0$ conversion is able to produce sizeable value of CSB in $A=4$ mirror hypernuclei.
- Within a model based on chiral LO NY $\Lambda N - \Sigma N$ interactions it is possible to resolve the CSB puzzle in the ground states of $^4_\Lambda$He and $^4_\Lambda$H.

[Gazda, Gal, PRL 116, 122501 (2016); NPA 954, 161 (2016)]

Outlook

- Sizeable cutoff dependence $\rightarrow$ need to study NLO $V_{NY}$.
- $p$-shell hypernuclei
Thank you!
Backup slides