Many-body theory of the $A(e, e'K^+)_Y A$ process

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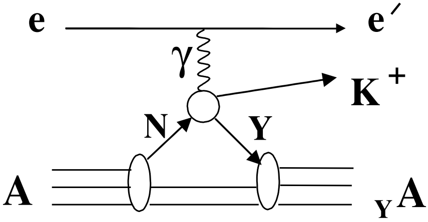
Outline

- ⋆ The $(e, e'K)$ cross section
- ⋆ The $(e, e'p)$ cross section and the nuclear spectral function
- ⋆ Generalization of the spectral function formalism to describe the $(e, e'K)$ process
- ⋆ The Λ self-energy and spectral function
- ⋆ Outlook

The $A(e, e'K^+)_Y A$ cross section

- Amplitude of the process

$$e(k) + A(p_A) \to e'(k') + K^+(p_K) +_Y A(p_{_YA})$$



- Cross section $(i, j = 1,\ 2,\ 3)$

$$d\sigma \propto L_{ij} W^{ij}$$

⋆ The lepton tensor $L_{\mu\nu}$, fully specified by the measured electron kinematical variables

$$L = \left(\begin{array}{ccc} \eta_+ & 0 & -\sqrt{\epsilon_L \eta_+} \\ 0 & \eta_- & 0 \\ -\sqrt{\epsilon_L \eta_+} & 0 & \epsilon_L \end{array} \right) ,$$

$$\eta_\pm = \frac{1}{2}\,(1 \pm \epsilon) \quad , \quad \epsilon = \left(1 - 2\frac{|\mathbf{q}|^2}{Q^2}\ \tan^2\frac{\theta_e}{2} \right)^{-1} \quad , \quad \epsilon_L = -\frac{Q^2}{\omega^2}\,\epsilon$$

⋆ Target response tensor

$$W^{ij} = \langle i|J_A^i(q)|f\rangle\langle f|J_A^j(q)|i\rangle\ \delta^{(4)}(q + p_i - p_f)$$

⋆ Building blocks

$$|i\rangle = |A\rangle \quad , \quad J_A^i = \sum_{n=1}^{A} j^i(n) \quad , \quad |f\rangle = |K^+,_Y A\rangle$$

⋆ The current j^i drives the elementary process

$$e + N \rightarrow Y + K^+$$

⋆ Impulse approximation: at momentum transfer $|\mathbf{q}|^{-1} \ll d$, d being the average nucleon-nucleon separation distance in the target nucleus, the beam particles interact with individual (bound, moving) nucleons

The $(e, e'p)$ cross section, as an example

- Transition amplitude of the process

$$e + A \to e' + p + (A-1)_n$$

- Factorization *ansatz*

$$\langle (A-1)_n \, p \, | j_\mu | A \rangle = \sum_k M_n(k) \, \langle p | j^\mu | k \rangle$$

$$M_n(k) = \{ \, \langle (A-1)_n | \otimes \langle k | \, \} | A \rangle$$

- Cross section

$$d\sigma_A \propto d\sigma_N P(p_m, E_m)$$

$$p_m = |\mathbf{p} - \mathbf{q}| \quad , \quad E_m = \omega - T_p \quad , \quad T_p = \sqrt{\mathbf{p}^2 + m^2} - m$$

Enter the spectral function

- ⋆ Definition

$$P(k,E) = \sum_n |M_n(k)|^2 \delta(E + E_A - E_{A-1})$$

probability of removing a nucleon of momentum k from the target nucleus, leaving the residual nucleus with excitation energy E

- ⋆ Relation to the nucleon self-enrgy (consider uniform matter, for simplicity)

$$P(k,E) = \frac{1}{\pi} \frac{\mathrm{Im}\Sigma(k,E)}{[E - k^2/2M - \mathrm{Re}\Sigma(k,E)]^2 + [\mathrm{Im}\Sigma(k,E)]^2}$$

Mean field *vs* correlation effects

- Consider uniform nuclear matter, as an example

$$P(k,E) = P_{MF}(k,E) + P_{\rm corr}(k,E)$$

- Mean field, or "pole" contribution

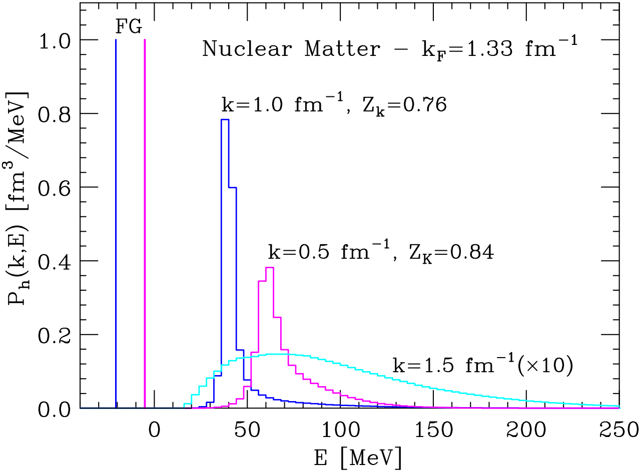
$$P_{MF}(k,E) = \frac{1}{\pi} \frac{Z_k^2 \, {\rm Im}\Sigma(k,\epsilon_k)}{[E-\epsilon_k]^2 + [Z_k \, {\rm Im}\, \Sigma(\mathbf{k},\epsilon_k)]^2}$$

$$\epsilon_k = \frac{k^2}{2m} + {\rm Re}\, \Sigma(k,\epsilon_k) \quad , \quad Z_k = |\langle -\mathbf{k}|a_{\mathbf{k}}|0\rangle|^2$$

The spectroscopic factor Z_k yields the normalization of the single-particle state

- The correlation contribution is a broad background, extending to large values of momentum and energy

Uniform nuclear matter at equilibrium density



Local Density Approximation (LDA)

$$P(k,E) = P_{MF}(k,E) + P_{\rm corr}(k,E)$$

- ⋆ $P_{MF}(k,E) \rightarrow$ from $(e,e'p)$ data
- ⋆ $P_{\rm corr}(k,E) \rightarrow$ from uniform nuclear matter calculations at different densities

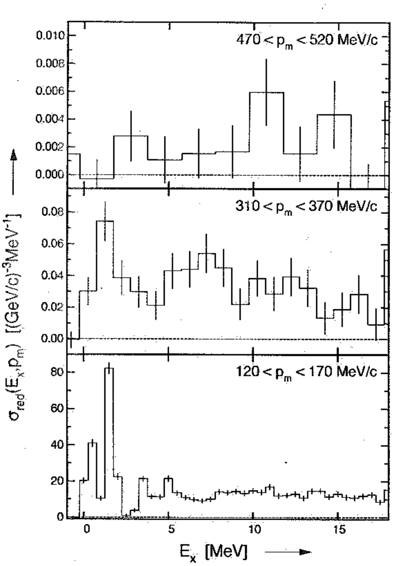
$$P_{MF}(k,E) = \sum_n Z_n |\phi_n(k)|^2 \, F_n(E-E_n)$$

$$P_{\rm corr}(k,E) = \int d^3r \, \rho_A(r) \, P^{NM}_{\rm corr}(k,E;\rho=\rho_A(r))$$

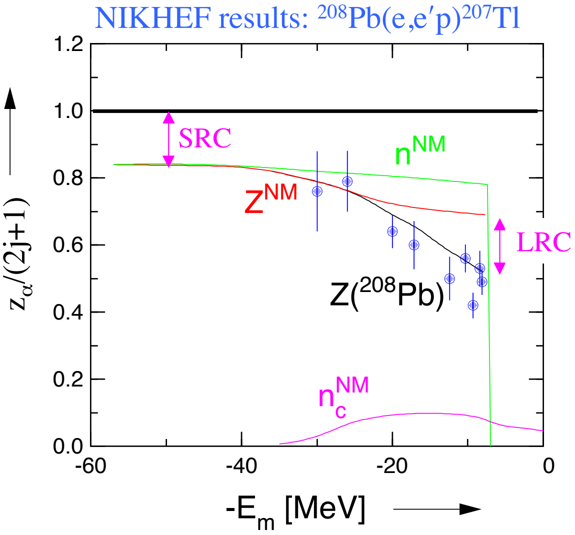
- ⋆ Pure mean field

$$Z_n \rightarrow 1 \quad , \quad F_n(E-E_n) \rightarrow \delta(E-E_n) \quad , \quad P_{\rm corr}(k,E) \rightarrow 0$$

$^{208}Pb(e, e'p)$ missing energy spectra (NIKHEF data)



Spectroscopic factors of ^{208}Pb (NIKHEF data)



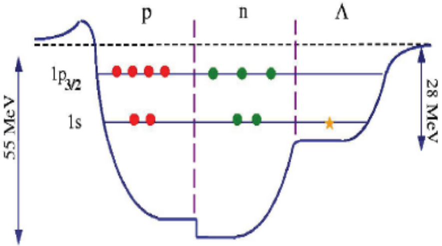
Back to $(e, e'K)$

- ⋆ The approach underlying the spectral function formalism can be generalized to describe the nuclear $(e, e'K)$ cross section
- ⋆ Introducing some additional simplifying assumptions one gets the simple expression

$$d\sigma_A \propto \int d^3p dE \, d\sigma_N \, P(p, E) P_Y(|\mathbf{p}_m + \mathbf{p}|, E_m + E)$$

$$\mathbf{p}_m = \mathbf{q} - \mathbf{p}_K \quad , \quad E_m = \omega - T_K$$

- ⋆ The spectral function $P_Y(p_Y, E_Y)$ describes the probability of adding the hyperon Y , with momentum p_Y and energy E_Y to a nucleus



Λ spectral function in nuclear matter

- ⋆ In the absence of ΛN interactions

$$P_\Lambda(p_Y, E_Y) = \delta[E_\Lambda - \epsilon_\Lambda(p_Y)]$$

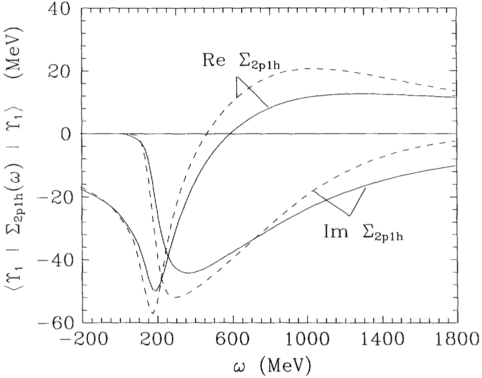
- ⋆ The general expression in terms of the self-energy still holds

$$P_\Lambda(k,E) = \frac{1}{\pi} \frac{\mathrm{Im}\Sigma_\Lambda(k,E)}{[E - T_\Lambda - \mathrm{Re}\Sigma_\Lambda(k,E)]^2 + [\mathrm{Im}\Sigma_\Lambda(k,E)]^2}$$

- ⋆ Calculation of the self-energy Σ_Λ in oxygen and of the full spectral function in nuclear matter are available

Λ self-energy in oxygen

- ⋆ The Λ self-energy in nuclei has been studied by Hjorth-Jensen *et al*, NPA 605, 458 (1996), and Vidaña *et al*, NPA 644, 201 (1998).



- ⋆ Ground state expectation value of the Λ self-energy in ^{17}O . Solid lines: Julich potential; dashed lines: Nijmegen potential

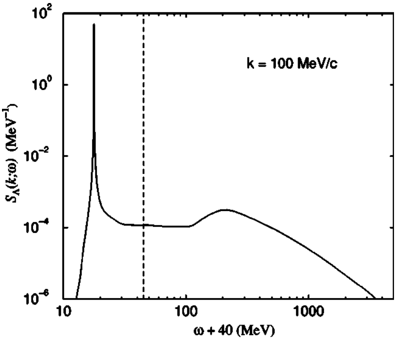
Λ binding energies in ^{208}Pb

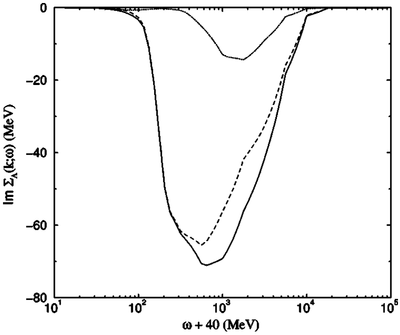
⋆ Courtesy I. Vidaña. Columns correspond to different *YN* potentials

$^{209}_{\Lambda}$Pb											($^{208}_{\Lambda}$Pb)
	$1s_{1/2}$	−31.36	−59.88	−29.52	−38.85	−39.14	−40.52	−40.40	−39.55	−39.30	−27.0
	$1p_{3/2}$	−27.13	−55.18	−26.01	−33.49	−33.47	−34.63	−34.30	−33.39	−31.03	−22.0 (1p)
	$1p_{1/2}$	−27.18	−55.37	−25.72	−33.38	−33.33	−34.44	−34.04	−33.09	−30.72	
	$1d_{5/2}$	−21.52	−45.00	−18.12	−22.44	−22.77	−23.54	−22.91	−21.98	−19.86	−17.0 (1d)
	$1d_{3/2}$	−21.58	−44.99	−17.96	−22.39	−22.69	−23.45	−22.83	−21.91	−19.81	
	$1f_{7/2}$	−12.77	−37.11	−9.73	−14.63	−14.68	−15.24	−14.17	−12.89	−10.10	−12.0 (1f)
	$1f_{5/2}$	−12.90	−37.10	−9.40	−14.61	−14.60	−15.12	−14.02	−12.73	−9.91	
	$1g_{9/2}$	−9.87	−33.75	−6.86	−11.22	−11.30	−11.82	−10.78	−9.56	−6.96	−7.0 (1g)
	$1g_{7/2}$	−10.01	−34.03	−6.36	−11.17	−11.17	−11.60	-10.47	−9.20	−6.57	

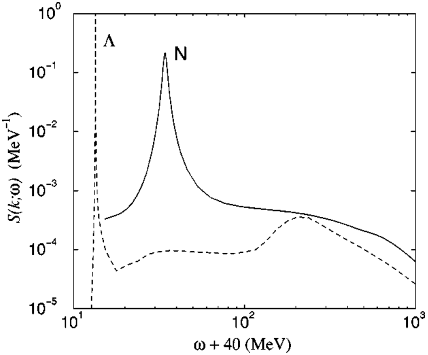
Λ spectral function in nuclear matter

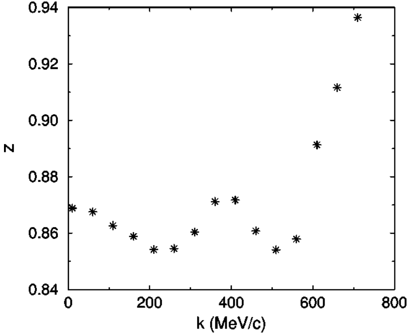
- ⋆ The Λ spectral function in nuclear matter has been computed by Robertson and Dickhoff, PRC 70, 044301 (2004), using the Nijmegen soft core (NSC89) *YN* potential





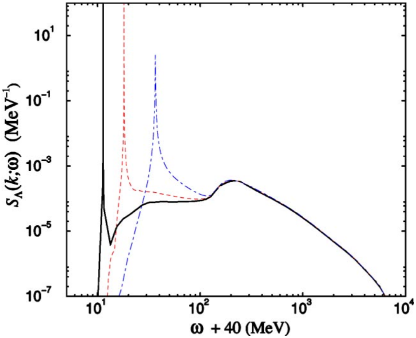
Compare N and Λ spectral functions in nuclear matter





Correlation strength in the Λ spectral function

- ⋆ The approach underlying the spectral function formalism can be generalized to describe the nuclear $(e, e'K)$ cross section



Outlook

- ⋆ Extracting information on the hyperon-nucleon interaction from $(e, e'K)$ data requires a state-of-the-art treatment of nucleon-nucleon sector
- ⋆ Electron scattering studies have provided ample evidence that the formalism based on spectral functions is best suited to capture the prominent features of nuclear dynamics: from single nucleon properties to the effects of short- and long-range correlations
- ⋆ The extension of the spectral function approach to the calculation of the $(e, e'K)$ cross section, while being non trivial, appears to be feasible
- ⋆ Thanks to the stunning progress of powerful and accurate many-body approaches, such as the G-matrix based techniques, the self consistent Green's function scheme and the Monte Carlo method, the input required to develop realistic models of the hyperon spectral functions may soon be available.