(Weak decay of hypernuclei) focusing on Pionic decay of hypernuclei

Toshio MOTOBA (Osaka E-C)

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Life of strangeness many-body systems
Hypernuclear weak decay: 2 modes

Mesonic mode (MWD) \( q(\pi) = 100 \text{ MeV/c} \)

\[
\Lambda \to p + \pi^- + 37.8 \text{ MeV} \ (64.2\%),
\]

\[
\Lambda \to n + \pi^0 + 41.1 \text{ MeV} \ (35.8\%).
\]

Nonmesonic mode (NMWD) \( q(\text{NM}) = 400 \text{ MeV/c} \)

\[
\Lambda + p \to n + p + 176 \text{ MeV},
\]

\[
\Lambda + n \to n + n + 176 \text{ MeV}.
\]
Mesonic Weak Decay

\[ \Lambda \rightarrow p + \pi^- \quad 2 \]
\[ \Lambda \rightarrow n + \pi^0 \quad 1 \]

\[ \Delta I = 1/2 \text{ rule} \]

\[ \Lambda \rightarrow p + \pi^- \quad (\sim 2/3) \]
\[ \Lambda \rightarrow n + \pi^0 \quad (\sim 1/3) \]

\[ 1/2^+ \quad 1/2^+ \quad 0^- \]

\[ L=0 \quad \text{s-wave; PV} \]
\[ L=1 \quad \text{p-wave; PC} \]

\[ I(\theta) \propto 1 - \alpha P \cos \theta \]

\[ \alpha = \frac{2a_s \Re a_p^*}{|a_s|^2 + |a_p|^2} \]

88\% s-wave
12\% p-wave
Theoretical framework

Pi-mesonic decay interaction is expressed as

$$\mathcal{H}_\pi = ig_w \bar{\psi}_N (1 + \lambda \gamma_5) \tau \begin{pmatrix} 0 \\ \psi_\Lambda \end{pmatrix} \phi_\pi,$$

$$g_w = 0.233 \times 10^{-6} \quad \text{and} \quad \lambda = -6.9$$

Non-relativistic expression is employed:

$$H_\pi = \left[ s_\pi X^{(s)}(r) + ip_\pi X^{(p)}(r) \frac{\sigma \cdot \nabla}{q_0} \right] \chi^{(-)*}_\pi(q; r),$$

$$s_{\pi^-} = -\frac{\sqrt{2}}{\sqrt{4\pi}} g_\omega = 0.96 \times 10^{-7}, \quad s_{\pi^0} = -s_{\pi^-}/\sqrt{2},$$

$$p_{\pi^-} = \frac{q_0}{2\sqrt{M_\Lambda M_N}} \lambda s_{\pi^-} = -0.35 \times 10^{-7}, \quad p_{\pi^0} = -p_{\pi^-}/\sqrt{2}.$$
Solve pion distorted waves with the optical potential

\[2\omega U_{\pi}^{\text{FULL}} = -4\pi [b(r) + B(r)] + 4\pi \nabla \cdot \mathcal{L}(r) [c(r) + C(r)] \nabla \]

\[-4\pi \left[ \frac{\rho_1 - 1}{2} \nabla^2 c(r) + \frac{\rho_2 - 1}{2} \nabla^2 C(r) \right],\]

\[\mathcal{L}(r) = \frac{1}{1 + \frac{4\pi}{3} \lambda [c(r) + C(r)]}\]

\[M_A + E(AZ; J_i T_i) = M_N + E(AZ'; J_f T_f) + \frac{q^2}{2M_A} + \omega_q,\]

\[\omega_q = \sqrt{m_{\pi}^2 + q^2}\]
Pion distorted waves $\chi_\pi^*(q;r)$ solve Klein-Gordon Eq.

Optical potential (MSU group)


effective form:

$$2\omega U_\pi = -4\pi \left[ b_{\text{eff}} P(r) - c_{\text{eff}} \nabla P(r) \nabla \right]$$

$$+ c_{\text{eff}} \frac{\omega}{2M} \nabla^2 P(r)$$

adopted

Vertex renormalization

M. Ericson & H. Bandô, P.L. B273 (1990) 69

general form:

$$2\omega U_\pi = -4\pi \left[ b(r) + B(r) \right] + 4\pi \nabla \cdot \left\{ \mathcal{L}(r) \left[ c(r) + \mathcal{C}(r) \right] \nabla \right\}$$

$$-4\pi \left\{ \frac{p-1}{2} \nabla^2 c(r) + \frac{p-3}{2} \nabla^2 C(r) \right\}$$

$$L_{(r)}^{(s)} = L_{(r)}^{(p)} = L_{(r)}^{(c)}$$

$$L_{(r)}^{(s)} = 1$$

$$L_{(r)}^{(p)} = L_{(r)}^{(c)}$$

$$L_{(r)} = \left\{ 1 + \frac{4\pi}{3} \lambda \left[ c(r) + \mathcal{C}(r) \right] \right\}^{-1}$$

$$b(r) = p_{\lambda} \left[ b_{\text{p}} P(r) - c_{\text{p}} \nabla P(r) \nabla \right]$$

$$c(r) = \frac{1}{p_{\lambda}} \left[ c_{\text{p}} P(r) - c_{\text{p}} \nabla P(r) \nabla \right]$$

$$B(r) = p_{\lambda} B_{\text{p}} P(r)^2$$

$$C(r) = \frac{1}{p_{\lambda}} C_{\text{p}} P(r)^2$$

$$\mathcal{L}(r) = \left\{ 1 + \frac{4\pi}{3} \lambda \left[ c(r) + \mathcal{C}(r) \right] \right\}^{-1}$$
Decay rate is expressed with suppression factors $S^{(s)}$ and $S^{(p)}$

$$
\Gamma_{\pi}(A^{A}Z; J_{i} T_{i} \tau_{i}) = \sum_{f} \Gamma_{\pi}(J_{i} T_{i} \tau_{i} \rightarrow J_{f} T_{f} \tau_{f}), \quad \Gamma_{\pi}^{\text{free}} = \frac{2q_{0}}{1+(\omega_{q_{0}}/M_{\pi})} (s_{\pi}^{2} + p_{\pi}^{2}),
$$

$$
\Gamma_{\pi}(J_{i} T_{i} \tau_{i} \rightarrow J_{f} T_{f} \tau_{f}) = \frac{2q}{1+(\omega_{q}/M_{A})} \left[ s_{\pi}^{2} S^{(s)}(i, f; q) + p_{\pi}^{2} \left( \frac{q}{q_{0}} \right)^{2} S^{(p)}(i, f; q) \right],
$$

$$
S^{(s, p)}(i, f; q) = \frac{4\pi}{2J_{i} + 1} \sum_{K} \left| \sqrt{A} \langle \Psi^{(A)Z'; J_{f} T_{f} \tau_{f}} | F_{K}^{(s, p)} \theta_{\pi} | \Phi^{(A)Z; J_{i} T_{i} \tau_{i}} \rangle \right|^{2}.
$$

$$
F_{K}^{(s)} = \tilde{j}_{K}(q; r) X^{(s)}(r) Y_{K}(\mathbf{r}),
$$

$$
F_{K}^{(p)} = -i \sum_{k=K \pm 1} (10K0|k0)f_{k}^{K}(q; r) X^{(p)}(r)[\sigma \times Y_{k}(\mathbf{r})]_{K}
$$

$$
f_{k}^{K}(q; r) = \frac{\partial \tilde{j}_{K}(q; r)}{q \partial r} + \left\{ 1 - \frac{k(k+1) - K(K+1)}{2} \right\} \frac{\tilde{j}_{K}(q; r)}{qr}.
$$
Effect of pion wave distortion

$^{12}\text{C} + \pi^-$  \hspace{1cm} $T_{\pi} = 30 \text{ MeV}$

$L = 0$
- MSU(FULL)
- MSU(EFF)
- FREE

$L = 1$

DW vs. PW: $e^{-i\mathbf{q} \cdot \mathbf{r}}$ if there is no distortion
Actual process (illustrative)

Decay rate and decay pattern are very sensitive to shell structure (spin and binding energy)

→ Useful tool to identify the hypernucleus
Very sensitive shell dependence predicted.
Shell dependence comes from energy-momentum available for $\pi$
Potential dependence of decay rate

H. Noumi et al. PRC52, 2936 (1995)
J. J. Szymanski et al. PRC43, 849 (1991)
A. Sakaguchi et al. PRC43, 73 (1991)
T. Motoba et al., PTP Suppl. No.117, 477 (1994)
sd-shell hypernuclear $\pi$-decays
Ratio for Mdecay in sd-shell
**Very recent assignment of $^7_\Lambda$Li with Ge ball & SKS**

- KEK-PS E419 experiment


Formation of $^7_\Lambda$Li was identified by $(\pi^+, K^+)$ reaction

Coincidence measurement

- M1 429 keV
Fig. 6. Calculated $\pi$-decay spectrum as a function of $^7$Be excitation energy.

Fig. 7. $\pi^-$ spectrum as a function of the kinetic energy. The data are taken from Ref. 37 and clipped for comparison.
Central repulsion in $\alpha$-Λ potential?

Can be directly checked by the mesonic decay widths

Mesonic decay rate

$\Gamma_\pi(YNG) > \Gamma_\pi(ORG)$

Non-mesonic decay rate

$\Gamma_{nm} \propto \int \frac{\psi_N^2}{\rho_0} \cdot \psi_\Lambda^2 \, d\vec{r}$

$\Gamma_{nm}(ORG) > \Gamma_{nm}(YNG)$
<table>
<thead>
<tr>
<th>Refs.</th>
<th>$\Gamma_{tot}/\Gamma_A$</th>
<th>$\Gamma_{\pi^-}/\Gamma_A$</th>
<th>$\Gamma_{\pi^0}/\Gamma_A$</th>
<th>$\Gamma_{nm}/\Gamma_A$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$^5\text{He (exp.)}$</td>
<td>1.03±0.08</td>
<td>0.44±0.11</td>
<td>0.18±0.20</td>
<td>0.41±0.14</td>
</tr>
<tr>
<td>$^5\text{He (ORG,SG)}$</td>
<td>0.321(ORG), 0.271(SG)</td>
<td>0.177(ORG), 0.158(SG)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$^5\text{He (YN,Isle)}$</td>
<td>0.393(YNG), 0.354(Isle)</td>
<td>0.215(YNG), 0.205(Isle)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$^{12}\text{C (exp.)}$</td>
<td>1.14±0.08</td>
<td>0.113±0.015</td>
<td>0.200±0.068</td>
<td>0.828±0.087</td>
</tr>
<tr>
<td>$^5\text{He (exp.)}$</td>
<td>present</td>
<td>0.940 ± 0.040 ± 0.007</td>
<td>0.322 ± 0.018 ± 0.003</td>
<td>0.207 ± 0.012 ± 0.005</td>
</tr>
<tr>
<td>$^{12}\text{C (exp.)}$</td>
<td>present</td>
<td>1.213 ± 0.034 ± 0.009</td>
<td>0.120 ± 0.014 ± 0.005</td>
<td>0.164 ± 0.008 ± 0.004</td>
</tr>
</tbody>
</table>
Selected comparison to confirm the theory

Results of $\Gamma_{\pi^0}$

$\frac{\Gamma_{\pi^0}}{\Gamma_\Lambda}$ of $^5 \Lambda$He: $0.207 \pm 0.013$

$\frac{\Gamma_{\pi^0}}{\Gamma_\Lambda}$ of $^{12} \Lambda$C: $0.164 \pm 0.009$
Results of $\pi^-$ Mesonic decay width

Branching ratio

$B_{\pi^-}(^{12}_\Lambda C) = 0.099 \pm 0.011 \pm 0.004$

$B_{\pi^-}(^{11}_\Lambda B) = 0.170 \pm 0.027 \pm 0.036$

$B_{\pi^-}(^{28}_\Lambda Si) = 0.036 \pm 0.008 \pm 0.002$

$B_{\pi^-}(^{27}_\Lambda Al) = 0.032 \pm 0.008 \pm 0.015$

$B_{\pi^-}(^{\Lambda}_{\Lambda} Fe) < 0.012 \quad (90\% \, CL)$

$\pi^-$ Mesonic decay width

$\Gamma_{\pi^-}(^{12}_\Lambda C) = 0.113 \pm 0.014 \pm 0.005 \Gamma_{\Lambda}$

$\Gamma_{\pi^-}(^{11}_\Lambda B) = 0.212 \pm 0.036 \pm 0.045 \Gamma_{\Lambda}$

$\Gamma_{\pi^-}(^{28}_\Lambda Si) = 0.046 \pm 0.011 \pm 0.002 \Gamma_{\Lambda}$

$\Gamma_{\pi^-}(^{27}_\Lambda Al) = 0.041 \pm 0.010 \pm 0.019 \Gamma_{\Lambda}$

$\Gamma_{\pi^-}(^{\Lambda}_{\Lambda} Fe) < 0.015 \Gamma_{\Lambda} \quad (90\% \, CL)$
Gross behavior

Gross behavior of hypernuclear $\pi$-mesonic decay rate

Mesonic decay scheme of $^4_\Lambda H$ and $^4_\Lambda He$ — "Mirror" hypernuclei

**Free $\Lambda$ decay**

$$\Lambda \rightarrow p + \pi^- \ (\sim 2/3)$$
$$\Lambda \rightarrow n + \pi^0 \ (\sim 1/3)$$
Test of $\Lambda$-nucleus potential for $^4_\Lambda\text{H}$ and $^4_\Lambda\text{He}$

- Isle wf
- SG wf
- Isle pot
- SG pot

Repulsive-core in $\Lambda$-nucleus potential?
Table II. Calculated decay rates, $\Gamma_n/\Gamma_p$ ratios and intrinsic asymmetry parameters of s- and p-shell and medium-to-heavy hypernuclei. The interaction $V_\pi + V_{2\pi/\rho} + V_{2\pi/\sigma} + V_\omega + V_K + V_{\rho\pi/a_1} + V_{\sigma\pi/a_1}$ is adopted. For p-shell hypernuclei, calculations with configuration-mixed nuclear SM + $\Lambda$ particle wave functions are shown without parentheses and those with simple nuclear SM + $\Lambda$ particle wave functions are shown in parentheses. Calculations are done with restriction of $L_0 = 0$ only. The decay rates are given in units of the free $\Lambda$ decay rate $\Gamma_\Lambda$. See the text.

| $^A$H (0$^+$) | $^A$He (0$^+$) | $^A$He (1/2$^+$) | $^7$Li (1/2$^+$) | $^8$Li (1$^-$) | $^9$Be (1/2$^+$) | $^{10}$B (1$^-$) | $^{10}$B$^*$ (2$^-$) | $^{11}$B (5/2$^+$) | $^{12}$B (1$^-$) | $^{12}$C (1$^-$) | $^{12}$C$^*$ (2$^-$) | $^{13}$C (1/2$^+$) | $^{14}$N (1$^-$) | $^{15}$N (3/2$^+$) | $^{16}$N (1$^-$) | $^{16}$O (0$^-$) | $^{16}$O$^*$ (1$^-$) | $^{28}$Si (2$^+$) | $^{51}$V (11/2$^+$) | $^{56}$Fe (1$^-$) | $^{80}$Y (7/2$^-$) | $^{139}$La (9/2$^+$) | $^{209}$Bi (9/2$^+$) |
| $\Gamma_p$ | 0.006 | 0.185 | 0.237 | 0.297 | 0.297 | 0.401 | 0.442 | 0.492 | 0.444 | 0.446 | 0.535 | 0.536 | 0.495 | 0.551 | 0.555 | 0.519 | 0.586 | 0.586 | 0.735 | 0.792 | 0.764 | 0.774 | 0.757 | 0.733 |
| $\Gamma_n$ | 0.099 | 0.012 | 0.121 | 0.152 | 0.200 | 0.200 | 0.198 | 0.198 | 0.223 | 0.266 | 0.223 | 0.237 | 0.247 | 0.246 | 0.278 | 0.298 | 0.275 | 0.264 | 0.336 | 0.412 | 0.392 | 0.418 | 0.455 | 0.477 |
| $\Gamma_{nm}$ | 0.105 | 0.198 | 0.358 | 0.449 [0.481] | 0.498 [0.501] | 0.601 [0.601] | 0.640 [0.635] | 0.690 [0.688] | 0.667 [0.660] | 0.711 [0.697] | 0.758 [0.754] | 0.773 [0.778] | 0.741 [0.740] | 0.797 [0.803] | 0.833 [0.832] | 0.816 [0.818] | 0.860 [0.861] | 0.850 [0.850] | 1.071 | 1.203 | 1.156 | 1.192 | 1.212 | 1.210 |
| $\Gamma_n/\Gamma_p$ | 16.02 | 0.066 | 0.508 | 0.512 [0.505] | 0.673 [0.657] | 0.499 [0.499] | 0.448 [0.452] | 0.403 [0.412] | 0.502 [0.503] | 0.596 [0.617] | 0.418 [0.407] | 0.443 [0.451] | 0.499 [0.498] | 0.446 [0.451] | 0.502 [0.503] | 0.574 [0.571] | 0.469 [0.470] | 0.451 [0.452] | 0.456 | 0.520 | 0.513 | 0.540 | 0.601 | 0.651 |
| $\alpha_\Lambda$ | — | — | 0.083 | 0.138 [0.093] | 0.120 [0.113] | 0.053 [0.053] | 0.083 [0.086] | 0.024 [0.029] | 0.072 [0.079] | 0.061 [0.082] | 0.044 [0.045] | 0.044 [0.045] | 0.023 [0.019] | 0.034 [0.045] | 0.037 [0.049] | 0.038 [0.038] | — | 0.037 [0.035] | — | — | — | — | — | — |
Calculated MWD and NMWD rates

Table VIII. Calculated hypernuclear weak decay rates and lifetimes for \( s \)-shell, \( p \)-shell, medium and heavy hypernuclei. The decay rates are given in units of the free \( \Lambda \) decay rate \( \Gamma_\Lambda \). The lifetimes are given in units of picosecond. Experimental data of nonmesonic decay rates and lifetimes are listed for comparison. See the text.

<table>
<thead>
<tr>
<th>(^4\Lambda)H</th>
<th>( \Gamma_\pi )</th>
<th>( \Gamma_{nm} )</th>
<th>( \Gamma_\pi + \Gamma_{nm} )</th>
<th>( \tau )</th>
<th>( \Gamma^\text{exp}_{nm} )</th>
<th>( \tau^\text{exp} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>(^4\Lambda)He</td>
<td>0.891</td>
<td>0.105</td>
<td>0.996</td>
<td>264.1</td>
<td>0.17 ± 0.11 [Ref. 62]</td>
<td>194(^\pm)24(_{26}^{+} ) [Ref. 62]</td>
</tr>
<tr>
<td>(^4\Lambda)He</td>
<td>0.658</td>
<td>0.198</td>
<td>0.856</td>
<td>307.2</td>
<td>0.17 ± 0.05 [Ref. 62]</td>
<td>256 ± 27 [Ref. 62]</td>
</tr>
<tr>
<td>(^5\Lambda)He</td>
<td>0.308</td>
<td>0.358</td>
<td>0.666</td>
<td>272.3</td>
<td>0.20 ± 0.03 [Ref. 63]</td>
<td></td>
</tr>
<tr>
<td>(^1\Lambda)B</td>
<td>0.316</td>
<td>0.667</td>
<td>0.983</td>
<td>267.5</td>
<td>0.95 ± 0.13 ± 0.04 [Ref. 67]</td>
<td>192 ± 22 [Ref. 65]</td>
</tr>
<tr>
<td>(^1\Lambda)B</td>
<td>0.861 ± 0.063 ± 0.073 [Ref. 51]</td>
<td>211 ± 13 [Ref. 66]</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>(^1\Lambda)C</td>
<td>0.228</td>
<td>0.758</td>
<td>0.986</td>
<td>266.7</td>
<td>211 ± 15 [Ref. 66]</td>
<td></td>
</tr>
<tr>
<td>(^1\Lambda)C</td>
<td>231 ± 15 [Ref. 66]</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>(^16\Lambda)O (0(^-))</td>
<td>0.074*</td>
<td>0.860</td>
<td>0.934</td>
<td>281.6</td>
<td>86(^\pm)33(_{-26}^{+} ) [Ref. 68]</td>
<td></td>
</tr>
<tr>
<td>(^28\Lambda)Si</td>
<td>0.088</td>
<td>1.071</td>
<td>1.159</td>
<td>226.9</td>
<td>1.125 ± 0.067 ± 0.106 [Ref. 51]</td>
<td>206 ± 12 [Ref. 66]</td>
</tr>
<tr>
<td>(^51\Lambda)V</td>
<td>0.02*</td>
<td>1.203</td>
<td>1.223</td>
<td>215.0</td>
<td></td>
<td></td>
</tr>
<tr>
<td>(^56\Lambda)Fe</td>
<td>0.02*</td>
<td>1.156</td>
<td>1.176</td>
<td>223.6</td>
<td>1.21 ± 0.08 [Ref. 51]</td>
<td>215 ± 14** [Ref. 66]</td>
</tr>
<tr>
<td>(^80\Lambda)Y</td>
<td>0.005*</td>
<td>1.192</td>
<td>1.197</td>
<td>219.7</td>
<td></td>
<td></td>
</tr>
<tr>
<td>(^139\Lambda)La</td>
<td>0.005*</td>
<td>1.212</td>
<td>1.217</td>
<td>216.1</td>
<td></td>
<td></td>
</tr>
<tr>
<td>(^209\Lambda)Bi</td>
<td>0.005*</td>
<td>1.210</td>
<td>1.215</td>
<td>216.5</td>
<td>250(^\pm)250(_{100}^{+} ) [Ref. 69]</td>
<td></td>
</tr>
</tbody>
</table>
Lifetimes (light to heavy hypernuclei)
MWD rates for p-shell hypernuclei

Table IX. $\pi^-$-decay rates of $p$-shell hypernuclei in units of the free $\Lambda$ decay rate $\Gamma_\Lambda$.

<table>
<thead>
<tr>
<th>$J_{gs}$</th>
<th>$^7\Lambda Li$</th>
<th>$^8\Lambda Li$</th>
<th>$^8\Lambda Be$</th>
<th>$^9\Lambda Be$</th>
<th>$^{10}\Lambda B$</th>
<th>$^{11}\Lambda B$</th>
<th>$^{12}\Lambda B$</th>
<th>$^{12}\Lambda C$</th>
<th>$^{13}\Lambda C$</th>
<th>$^{14}\Lambda N$</th>
<th>$^{15}\Lambda N$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$(0p)^n$</td>
<td>0.280</td>
<td>0.344</td>
<td>0.130</td>
<td>0.144</td>
<td>0.197</td>
<td>0.195</td>
<td>0.256</td>
<td>0.077</td>
<td>0.079</td>
<td>0.053</td>
<td>0.067</td>
</tr>
<tr>
<td>$(1s0d)$</td>
<td>0.024</td>
<td>0.024</td>
<td>0.019</td>
<td>0.029</td>
<td>0.018</td>
<td>0.018</td>
<td>0.030</td>
<td>0.022</td>
<td>0.025</td>
<td>0.020</td>
<td>0.023</td>
</tr>
<tr>
<td>$\Gamma^{(Sum)}_{\pi^-}$</td>
<td>0.304</td>
<td>0.368</td>
<td>0.149</td>
<td>0.172</td>
<td>0.215</td>
<td>0.213</td>
<td>0.286</td>
<td>0.099</td>
<td>0.105</td>
<td>0.073</td>
<td>0.090</td>
</tr>
<tr>
<td>$\Gamma^{(AG)}_{\pi^-}$</td>
<td>0.332</td>
<td>0.157</td>
<td>0.178</td>
<td>0.157</td>
<td>0.178</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>0.066</td>
</tr>
<tr>
<td>Exp$^{37)}$</td>
<td>0.353</td>
<td>0.178</td>
<td>0.249</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>0.108</td>
</tr>
<tr>
<td>$\pm$ error</td>
<td>±0.059</td>
<td>±0.050</td>
<td>±0.051</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>±0.038</td>
</tr>
<tr>
<td>Exp$^{65),82)}$</td>
<td></td>
<td></td>
<td></td>
<td>0.218</td>
<td>0.05</td>
<td>0.05</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\pm$ error</td>
<td></td>
<td></td>
<td></td>
<td>±0.046</td>
<td>$^{+0.06}_{-0.03}$</td>
<td></td>
<td></td>
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<td></td>
</tr>
</tbody>
</table>
Comparison with FINUDA data

\[ ^{11}_\Lambda B(J_f = 5/2^+) \quad B_\Lambda = 10.24 \text{ MeV} \]

\[ \pi^- \text{ decay} \]

(0p): 0.195
(sd): 0.018
Total: 0.213

\( (J_f = 7/2^+) \)

\[ ^{11}_\Lambda B \]

Levels

(\( J_f \))

DECAY RATE \( \Gamma_\pi(J_f)/\Gamma_\Lambda \)

Fig. 10. Calculated \( \pi^- \) decay spectrum as a function of \(^{11}\text{C} \) excitation energy.

Fig. 11. \( \pi^- \) spectrum as a function of the kinetic energy. The data are taken from Ref. 37) and clipped for comparison.
Comparison with recent FINUDA exp.

Fig. 8. Calculated $\pi$-decay spectrum as a function of $^9\Lambda$B excitation energy.

Fig. 9. $\pi^-$ spectrum as a function of the kinetic energy. The data are taken from Ref. 37) and clipped for comparison.
Characteristic pion momentum helps to identify the hypernucleus.
Case of 2-body and 3-body decays

Fig. 10. The continuum pion spectrum calculated for the weak decay $^5_\Lambda$He$\rightarrow ^4_\Lambda$He$+\rho + \pi^-$ (left) and that for the two-body and three-body $\pi^-$ decay of $^7_\Lambda$Li (right).
2-body and 3-body decays

Fig. 11. The calculated $\pi^-$ spectrum from the weak decay of $^4_A\text{H}$, which consists of the monochromatic peak for the two-body decay ($^4\text{He}+\pi^-$) and the continuum part for the three-body decays ($^3\text{H}+p+\pi^-$).
Pion spectrum helps

Fig. 18. The calculated $\pi^-$ spectrum from the weak decay of $^{10}_\Lambda$Be($1^-, 2^-$). Comment as for Fig. 13.
Summary

1) Pionic decay rates and spectra have been extensively calculated, using pion optical potential which reproduces nucleus-pion scattering at low energies.

2) Sensitivity of pi-decay momentum helps to identify the hypernucleus unambiguously.

3) Pionic decay spectrum also helps to determine the Ground-state spin.

4) Coincidence measurements are promising
Three-dimensional nuclear chart

(from H. Tamura)