(Weak decay of hypernuclei)
focusing on
Pionic decay of hypernuclei

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Hypernuclear Workshop

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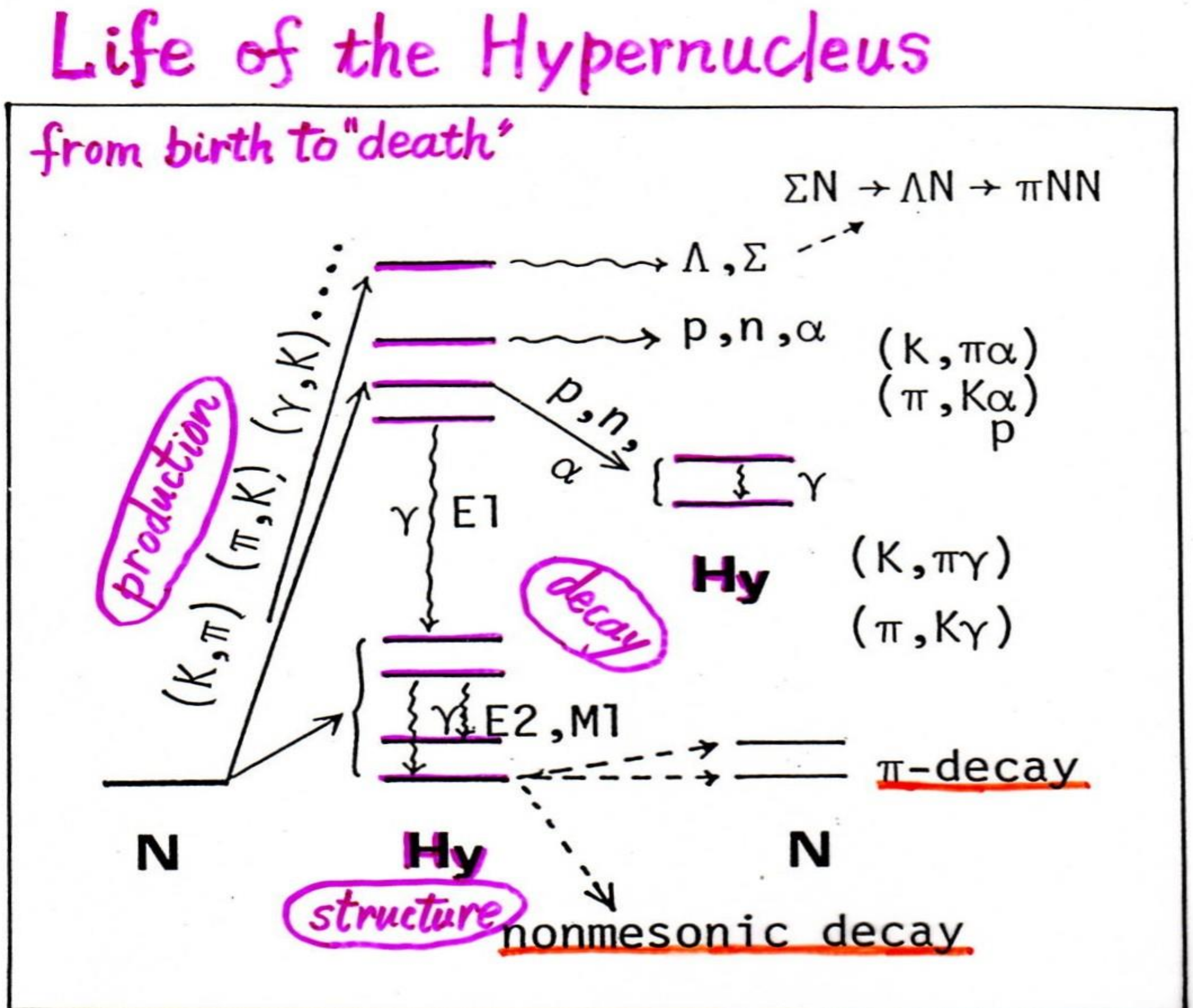
Life of strangeness many-body systems



Hypernuclear weak decay : 2 modes

Mesonic mode (MWD) q(π)= 100 MeV/c

$$\Lambda \to p + \pi^- + 37.8 \text{ MeV } (64.2\%),$$

$$\Lambda \to n + \pi^0 + 41.1 \text{ MeV } (35.8\%).$$

Nonmesonic mode (NMWD) q(NM)= 400 MeV/c

$$\Lambda + p \to n + p + 176 \quad \text{MeV} \,,$$

$$\Lambda + n \to n + n + 176 \quad \text{MeV}$$

(from Outa slide)

Mesonic Weak Decay

$\Lambda \rightarrow p + \pi^-$ 2

$\Lambda \rightarrow n + \pi^0$ 1

$\Delta I = 1/2$ rule

$1/2^+$ $1/2^+$ 0^-

Free Λ decay

$\Lambda \rightarrow p + \pi_0^- \ (\sim 2/3)$

$\Lambda \rightarrow n + \pi^0 \ (\sim 1/3)$

↑ ↑   L=0   s-wave; PV

↑ ↓   L=1   p-wave; PC

$I(\theta) \propto 1 - \alpha P \cos\theta$

$$\alpha = \frac{2 a_s \operatorname{Re} a_p^*}{|a_s|^2 + |a_p|^2}$$

88% s-wave

12% p-wave

Theoretical framework

Pi-mesonic decay interaction is expressed as

$$\mathcal{H}_\pi = i g_w \bar{\psi}_N (1+\lambda\gamma_5)\tau \begin{pmatrix} 0 \\ \psi_\Lambda \end{pmatrix} \phi_\pi ,$$

$g_w = 0.233 \times 10^{-6}$ and $\lambda = -6.9$

Non-relativistic expression is employed:

$$H_\pi = \left[s_\pi X^{(s)}(r) + i p_\pi X^{(p)}(r) \frac{\sigma \cdot \nabla}{q_0} \right] \chi_\pi^{(-)*}(\boldsymbol{q}; \boldsymbol{r}) ,$$

$$s_{\pi^-} = -\frac{\sqrt{2}}{\sqrt{4\pi}} g_\omega = 0.96 \times 10^{-7} , \quad s_{\pi^0} = -s_{\pi^-}/\sqrt{2} ,$$

$$p_{\pi^-} = \frac{q_0}{2\sqrt{M_\Lambda M_N}} \lambda s_{\pi^-} = -0.35 \times 10^{-7} , \quad p_{\pi^0} = -p_{\pi^-}/\sqrt{2} .$$

Solve pion distorted waves with the optical potential

$$2\omega U_\pi^{\mathrm{FULL}} = -4\pi[b(r)+B(r)] + 4\pi\nabla\cdot\mathcal{L}(r)[c(r)+C(r)]\nabla$$

$$-4\pi\left[\frac{p_1-1}{2}\nabla^2 c(r) + \frac{p_2-1}{2}\nabla^2 C(r)\right],$$

$$\mathcal{L}(r) = \frac{1}{1+\frac{4\pi}{3}\lambda[c(r)+C(r)]}$$

$$M_\Lambda + E({}^A_\Lambda Z; J_i T_i) \;=\; M_N + E({}^A Z'; J_f T_f) + \frac{q^2}{2M_A} + \omega_q \;,$$

$$\omega_q = \sqrt{m_\pi^2 + q^2}$$

Pion distorted waves $\chi_\pi^{(-)*}(q;r)$

solve Klein-Gordon Eq.

Optical potential (MSU group)

J.A. Carr et al. P.R. C25 (1982) 952

effective form:

$$2\omega U_\pi = -4\pi\left[b_{eff}\rho(r) - c_{eff}\nabla\rho(r)\nabla + c_{eff}\frac{\omega}{2M}\nabla^2\rho(r)\right]$$

adopted

general form: ⊕ Vertex renormalization

M. Ericson & H. Bandō, P.L. B273 (1990) 169

$$2\omega U_\pi = -4\pi\left[b(r)+B(r)\right] + 4\pi\nabla\cdot\{\mathcal{L}(r)[c(r)+C(r)]\}\nabla$$

LLEE

$$-4\pi\left\{\frac{p_1-1}{2}\nabla^2 c(r) + \frac{p_2-1}{2}\nabla^2 C(r)\right\}$$

$$b(r) = p_1[b_0\rho(r) - \epsilon_\pi b_1\delta\rho(r)$$

$$c(r) = \frac{1}{p_1}[c_0\rho(r) - \epsilon_\pi c_1\delta\rho(r)$$

$$B(r) = p_2 B_0\rho(r)^2$$

$$C(r) = \frac{1}{p_2}C_0\rho(r)^2$$

$$\mathcal{L}(r) = \left\{1 + \frac{4\pi}{3}\lambda[c(r)+C(r)]\right\}^{-1}$$

$$L^{(s)}(r) = 1$$

$$L^{(p)}(r) = \mathcal{L}(r)$$

Decay rate is expressed with suppression factors $S^{(s)}$ and $S^{(p)}$

$$\Gamma_\pi({}^A_\Lambda Z; J_i T_i \tau_i) = \sum_f \Gamma_\pi(J_i T_i \tau_i \to J_f T_f \tau_f) , \qquad \Gamma_\pi^{\text{free}} = \frac{2q_0}{1+(\omega_{q_0}/M_N)} (s_\pi{}^2 + p_\pi{}^2) ,$$

$$\Gamma_\pi(J_i T_i \tau_i \to J_f T_f \tau_f) = \frac{2q}{1+(\omega_q/M_A)} \left[s_\pi{}^2 \mathcal{S}_\pi{}^{(s)}(i, f; q) + p_\pi{}^2 \left(\frac{q}{q_0}\right)^2 \mathcal{S}_\pi{}^{(p)}(i, f; q) \right] ,$$

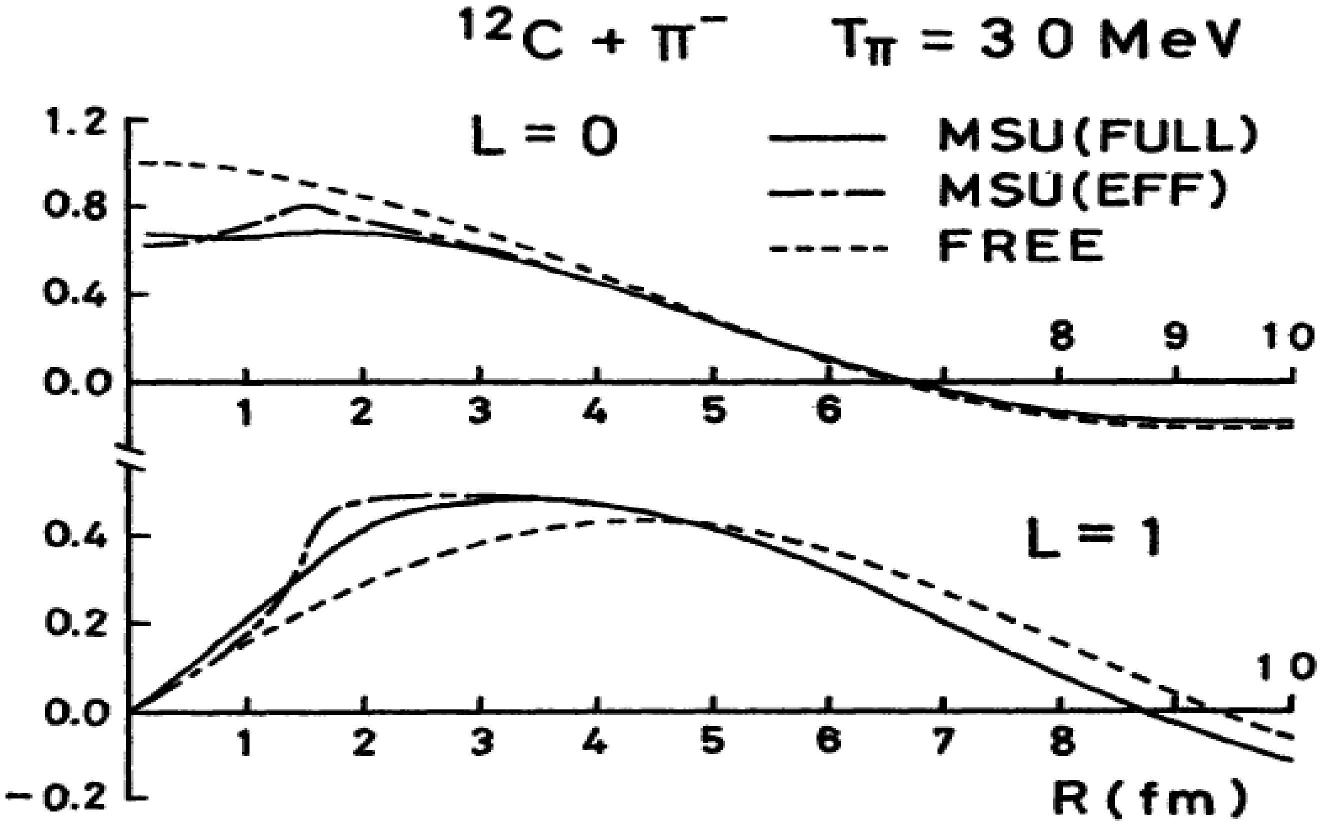
$$\mathcal{S}_\pi{}^{(s,p)}(i, f; q) = \frac{4\pi}{2J_i+1} \sum_K \left| \sqrt{A} \langle \Psi({}^A Z'; J_f T_f \tau_f) \| F_K{}^{(s,p)} \theta_\pi \| \Phi({}^A_\Lambda Z; J_i T_i \tau_i) \rangle \right|^2 .$$

$$F_K{}^{(s)} = \tilde{j}_K(q; r) X^{(s)}(r) Y_K(\hat{\boldsymbol{r}}) ,$$

$$F_K{}^{(p)} = -i \sum_{k=K\pm 1} (10K0|k0) f_k{}^K(q; r) X^{(p)}(r) [\sigma \times Y_k(\hat{\boldsymbol{r}})]_K$$

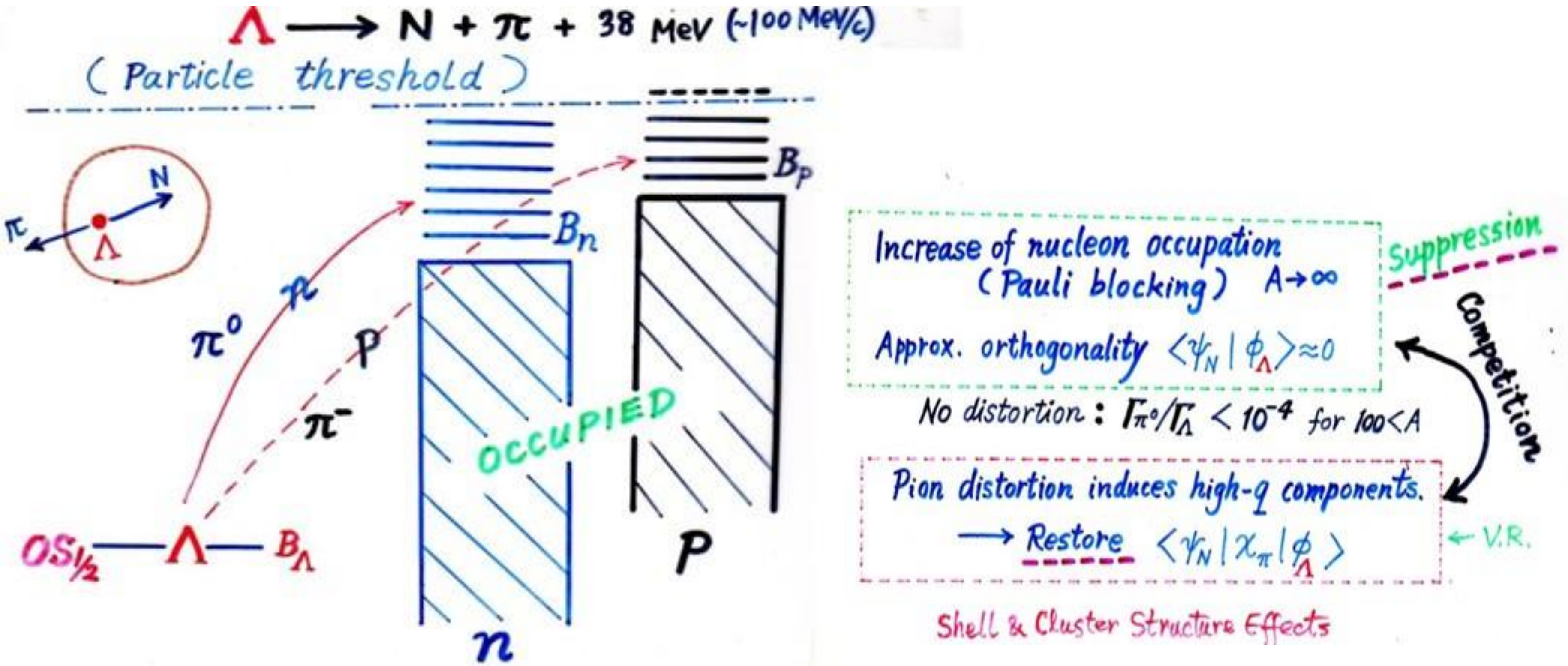
$$f_k{}^K(q; r) = \frac{\partial \tilde{j}_K(q; r)}{q \partial r} + \left\{ 1 - \frac{k(k+1) - K(K+1)}{2} \right\} \frac{\tilde{j}_K(q; r)}{qr} .$$

Effect of pion wave distortion



DW vs. PW: $e^{-i\boldsymbol{q}\cdot\boldsymbol{r}}$ if there is no distortion

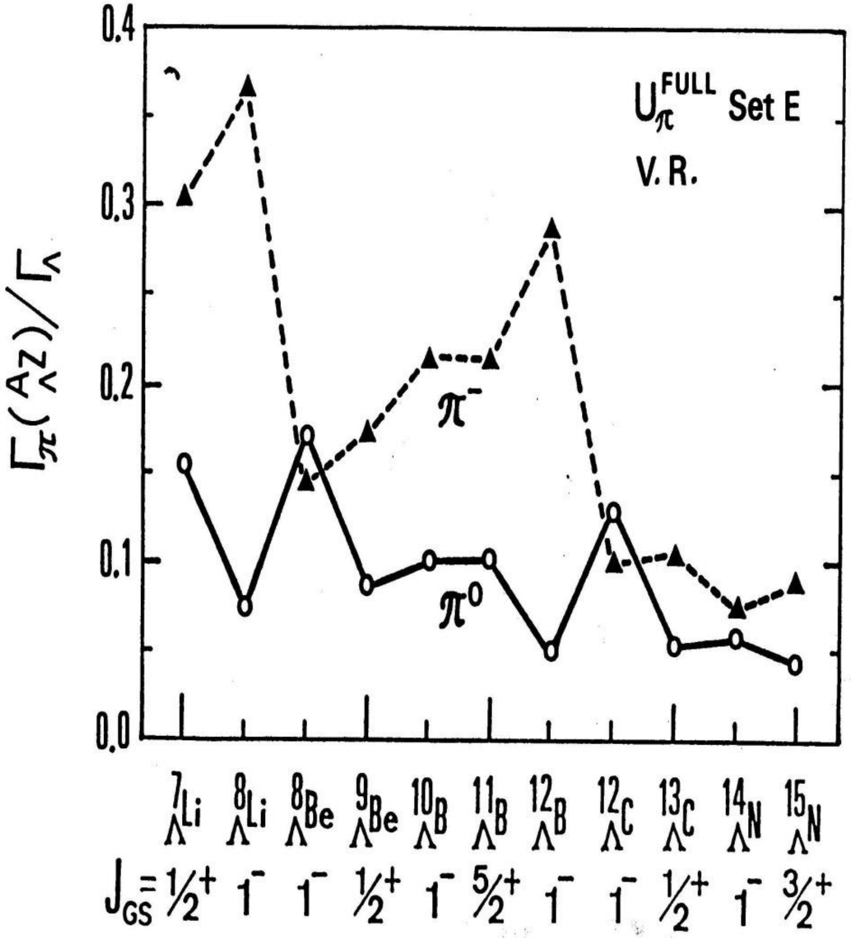
Actual process (illustrative)

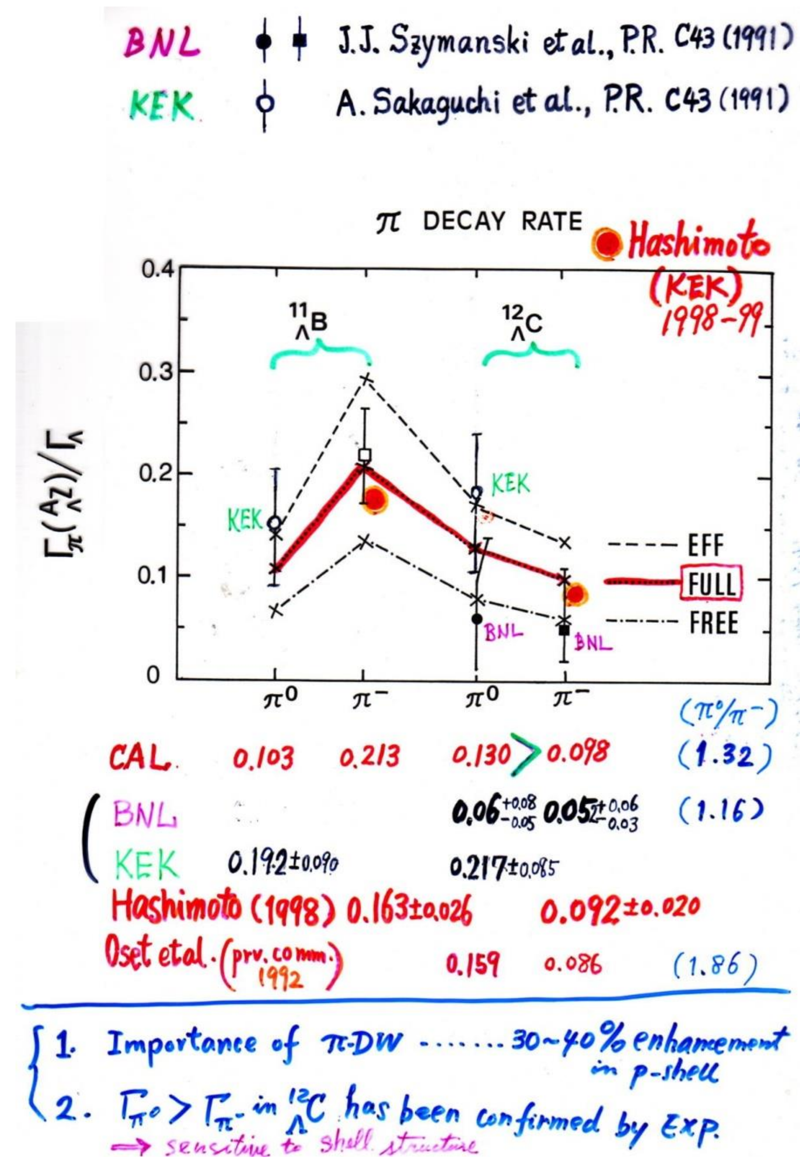


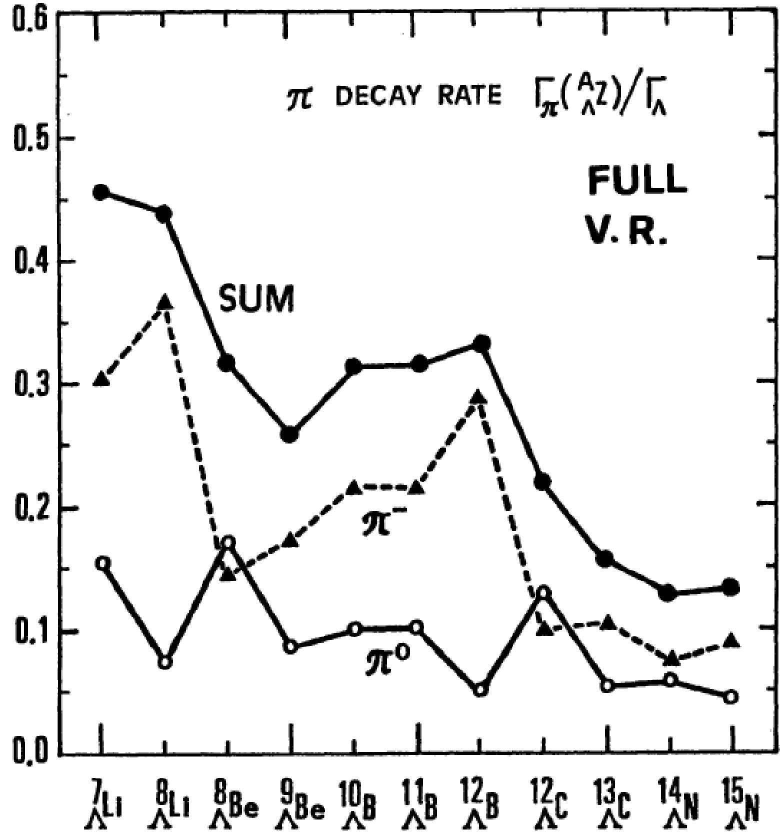
Decay rate and decay pattern are very sensitive to shell structure (spin and binding energy)

--→ Useful tool to identify the hypernucleus

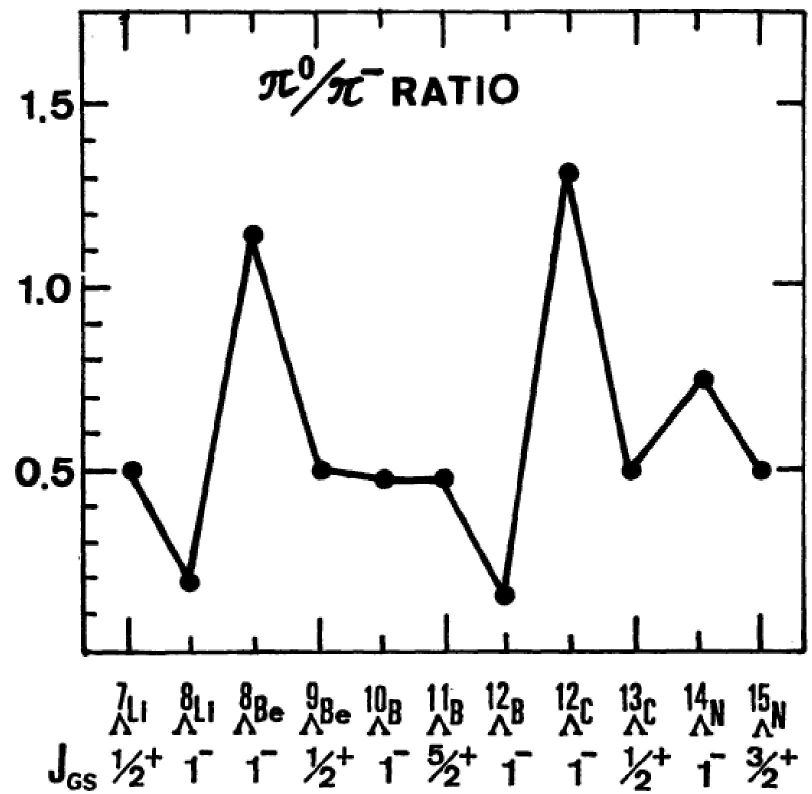
Very sensitive shell dependence predicted.





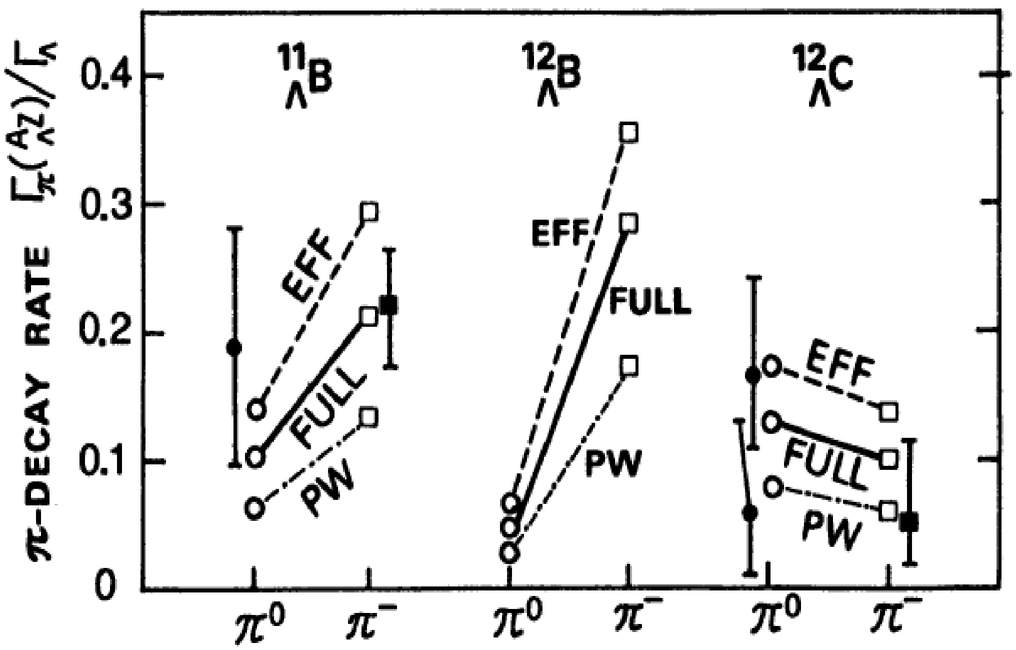


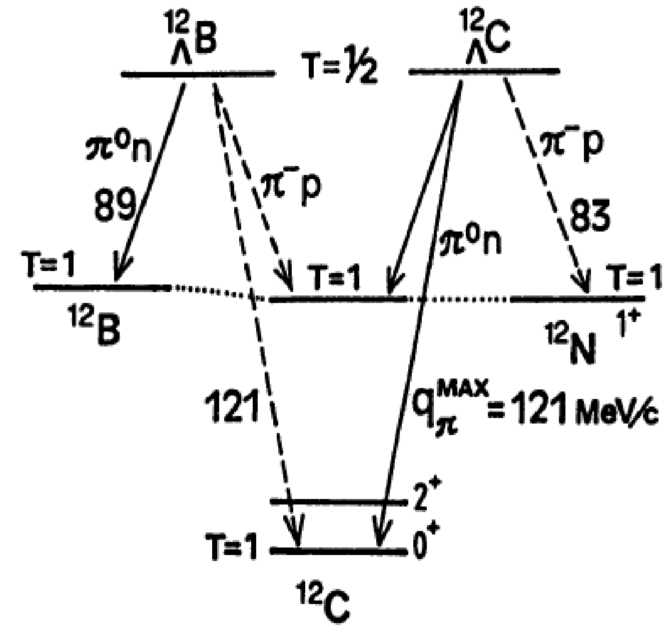
0.6
0.5
0.4
0.3
0.2
0.1
0.0
π DECAY RATE Γπ(ΛAZ)/ΓΛ
FULL
V.R.
SUM
π⁻
π⁰
7ΛLi 8ΛLi 8ΛBe 9ΛBe 10ΛB 11ΛB 12ΛB 12ΛC 13ΛC 14ΛN 15ΛN



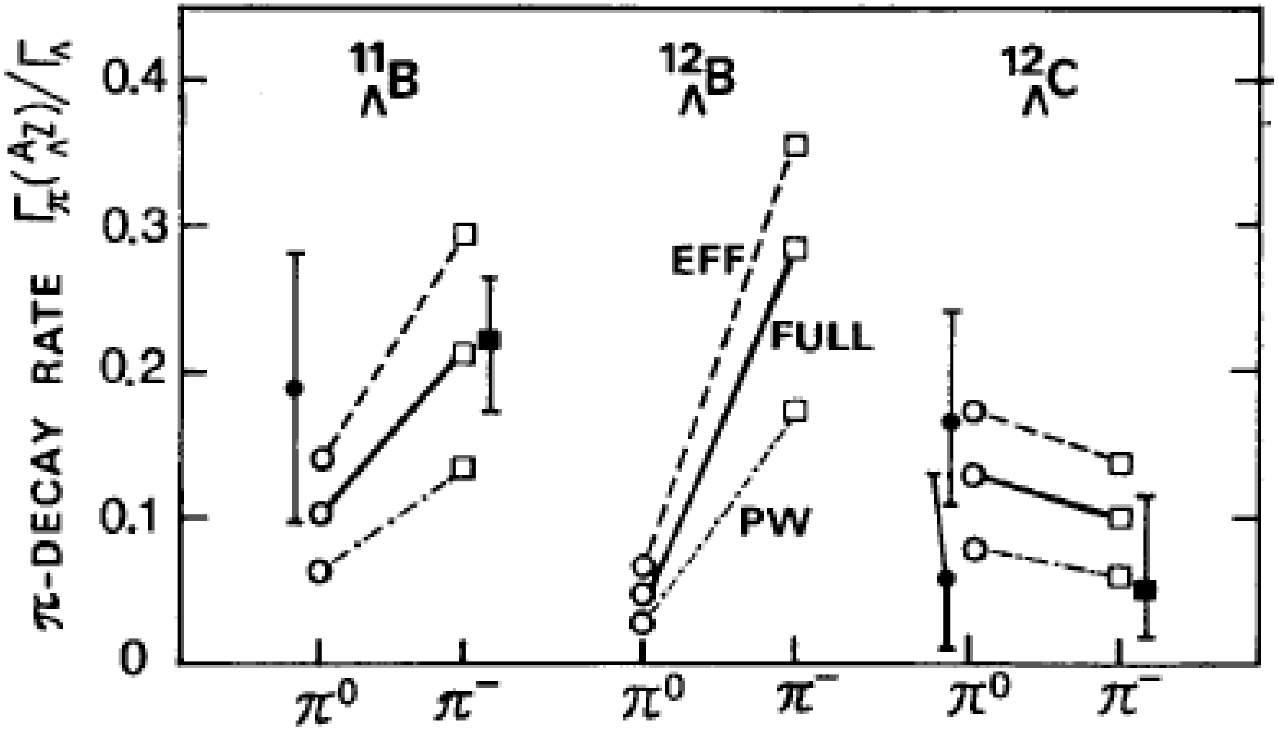
π⁰/π⁻ RATIO
1.5
1.0
0.5
7ΛLi 8ΛLi 8ΛBe 9ΛBe 10ΛB 11ΛB 12ΛB 12ΛC 13ΛC 14ΛN 15ΛN
JGS 1/2+ 1- 1- 1/2+ 1- 5/2+ 1- 1- 1/2+ 1- 3/2+

Shell dependence comes from energy-momentum available for π





Potential dependence of decay rate



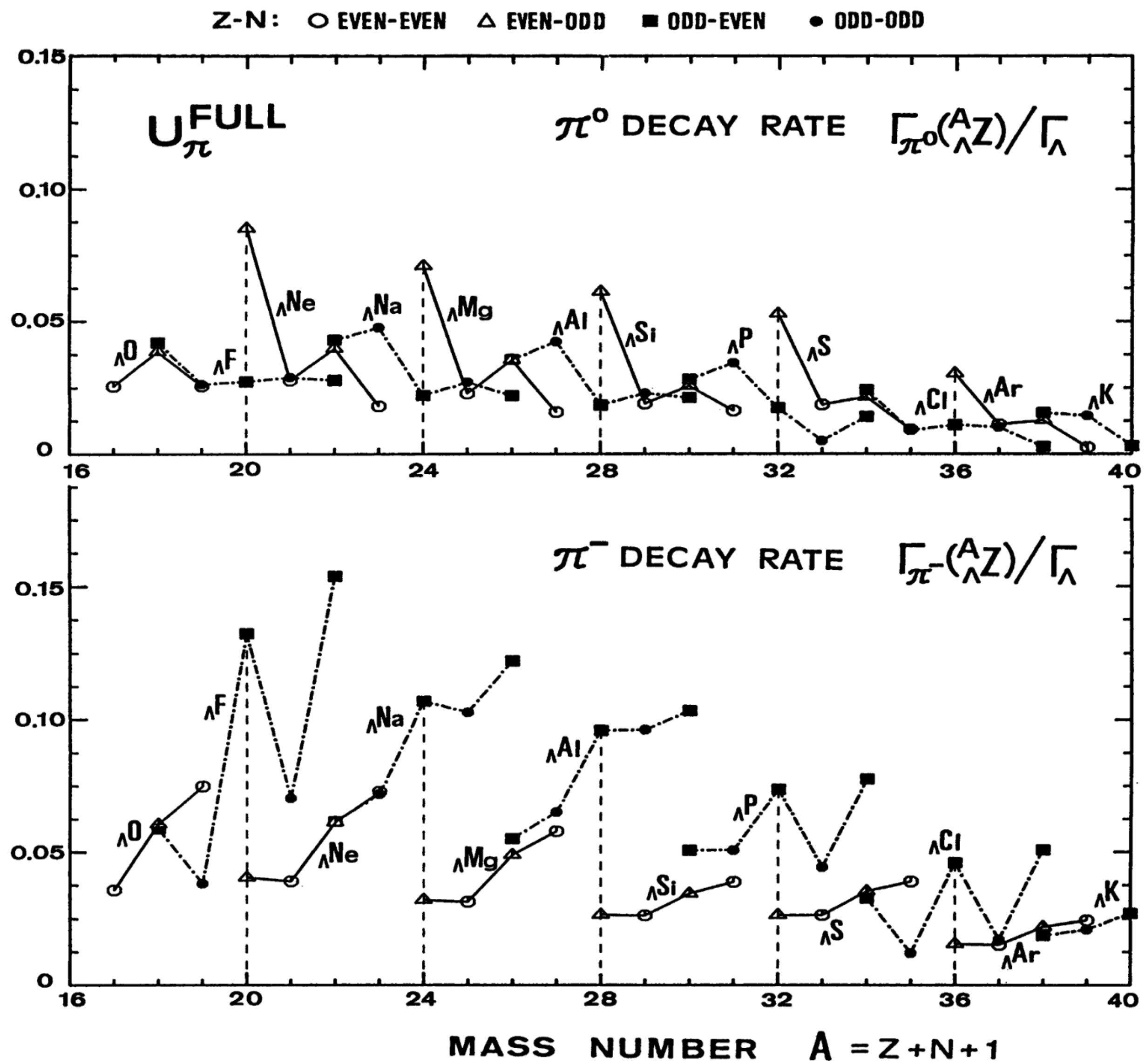
H. Noumi et al. PRC52, 2936 (1995)

J. J. Szymanski et al. PRC43, 849 (1991)

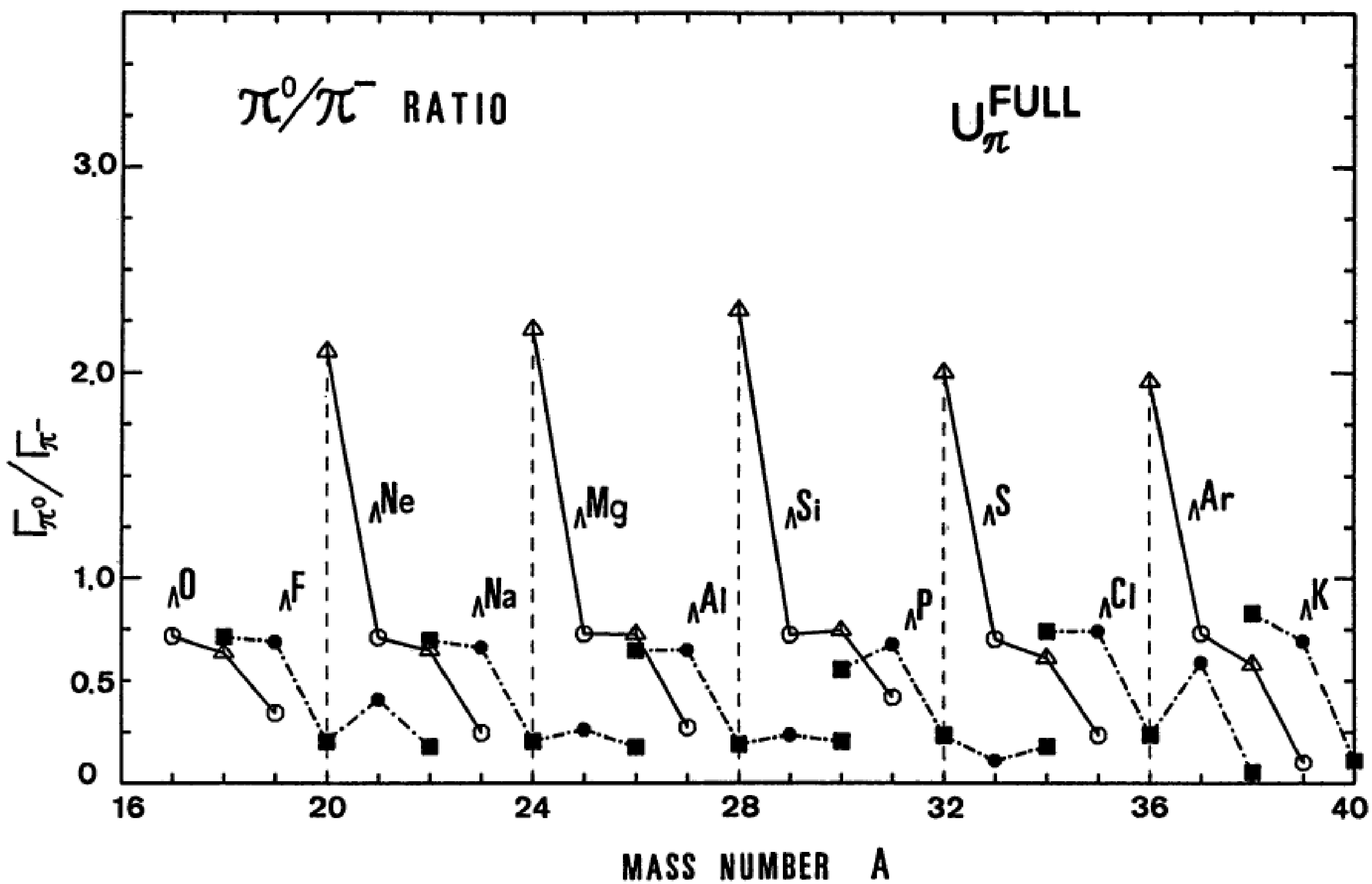
A.Sakaguchi et al. PRC43, 73 (1991)

T. Motoba et al., PTP Suppl. No.117, 477 (1994)

sd-shell hypernuclear π-decays



Ratio for Mdecay in sd-shell



Very recent assignment of $^{7}_{\Lambda}$Li w/ Ge ball & SKS

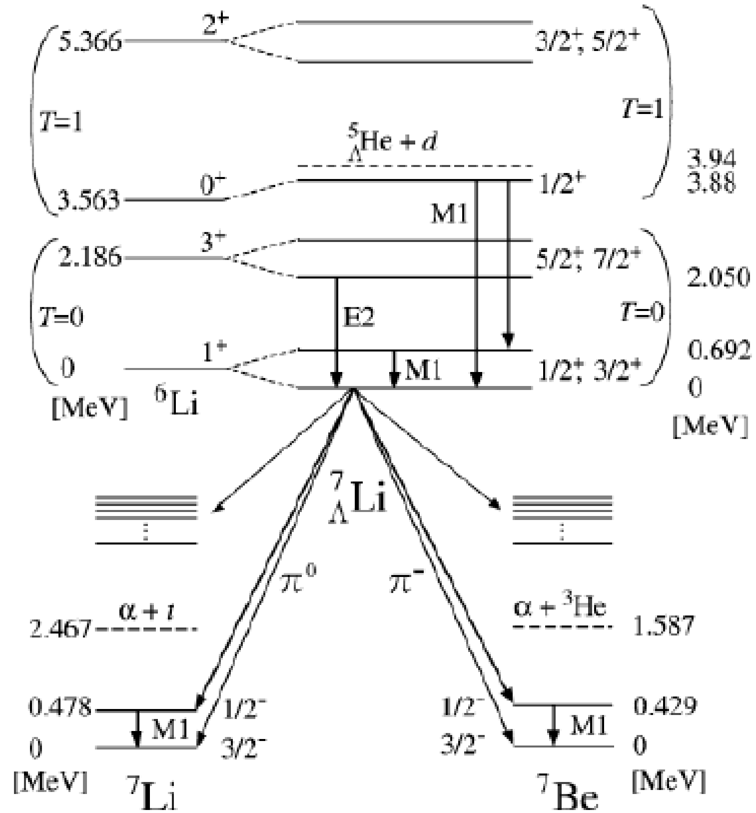
- KEK-PS E419 experiment

Phys. Lett. B578 (2004) 258-264

Formation of $^{7}_{\Lambda}$Li was identified by (π+,K+) reaction

Coincidence measurement

M1 429keV



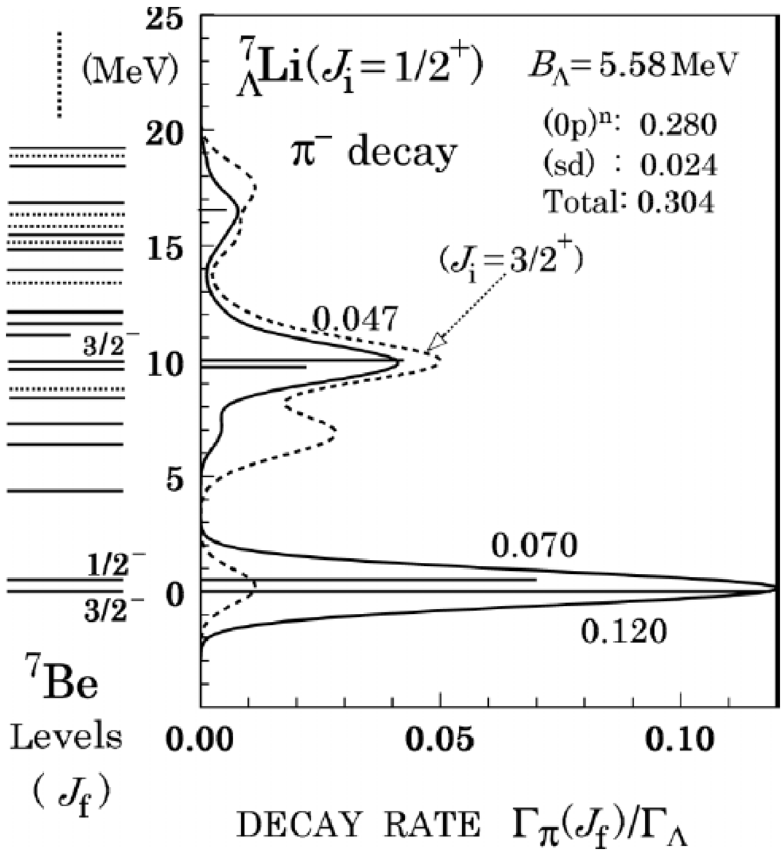


Fig. 6. Calculated π-decay spectrum as a function of ^{7}Be excitation energy.

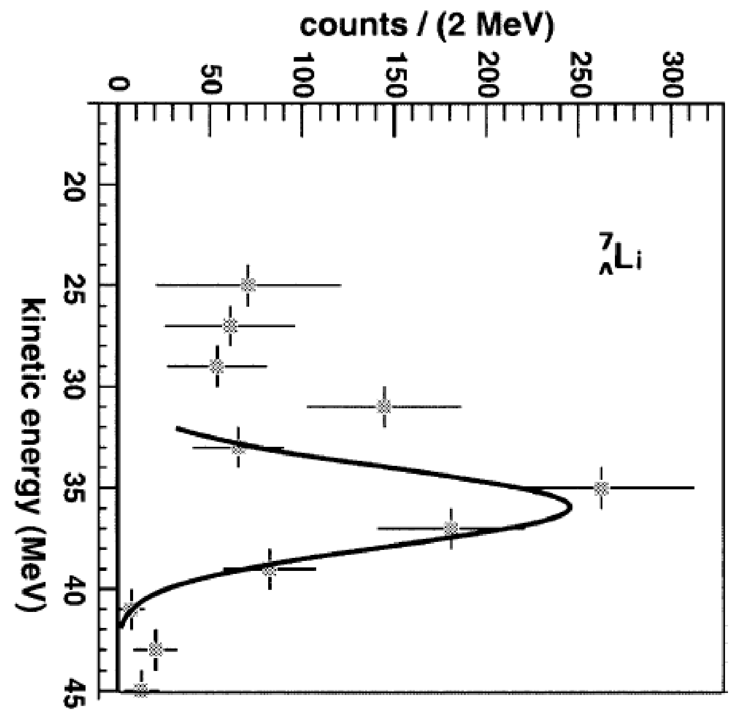


Fig. 7. π^- spectrum as a function of the kinetic energy. The data are taken from Ref. 37) and clipped for comparison.

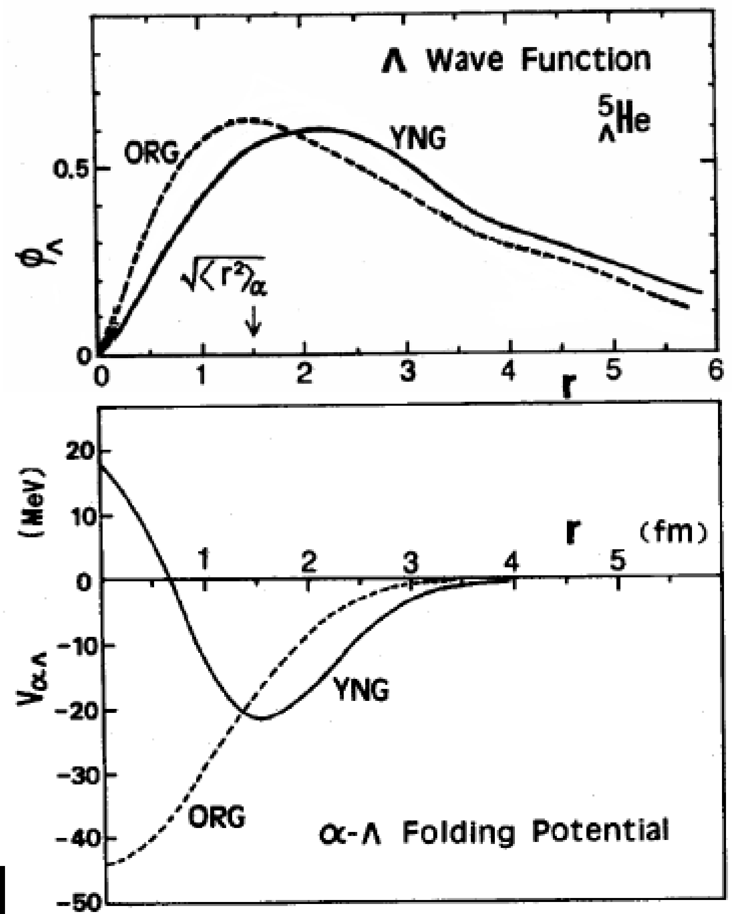
Outa slide)

Γπ/Γnm and Λ-Nucleus Potential

Central repulsion in α-Λ potential ?

Can be directly checked by the mesonic decay widths

Calculation using two types of α-Λ potential ORG and YNG (Motoba *et al.* NPA577)



Mesonic decay rate

Γπ(YNG) > Γπ(ORG)

Non-mesonic decay rate

$$\Gamma_{nm} \propto \int \frac{\psi_N^2}{\rho_0} \cdot \psi_\Lambda^2 d\vec{r} \quad ??$$

Γnm(ORG) >Γnm(YNG)

5

	Refs.	$\Gamma_{tot}/\Gamma_{\Lambda}$	$\Gamma_{\pi^-}/\Gamma_{\Lambda}$	$\Gamma_{\pi^0}/\Gamma_{\Lambda}$	$\Gamma_{nm}/\Gamma_{\Lambda}$
$^{5}_{\Lambda}$He (exp.)	[2]	1.03±0.08	0.44±0.11	0.18±0.20	0.41±0.14
$^{5}_{\Lambda}$He (ORG,SG)	[1][5]		0.321(ORG), 0.271(SG)	0.177(ORG), 0.158(SG)	
$^{5}_{\Lambda}$He (YNG,Isle)	[1][5]		0.393(YNG), 0.354(Isle)	0.215(YNG), 0.205(Isle)	
$^{12}_{\Lambda}$C (exp.)	[3][4]	1.14±0.08	0.113±0.015	0.200±0.068	0.828±0.087
$^{5}_{\Lambda}$He (exp.)	present	0.940 ± 0.040 ± 0.007	0.322 ± 0.018 ± 0.003	0.207 ± 0.012 ± 0.005	0.411 ± 0.023 ± 0.006
$^{12}_{\Lambda}$C (exp.)	present	1.213 ± 0.034 ± 0.009	0.120 ± 0.014 ± 0.005 [3]	0.164 ± 0.008 ± 0.004	0.929 ± 0.027 ± 0.016

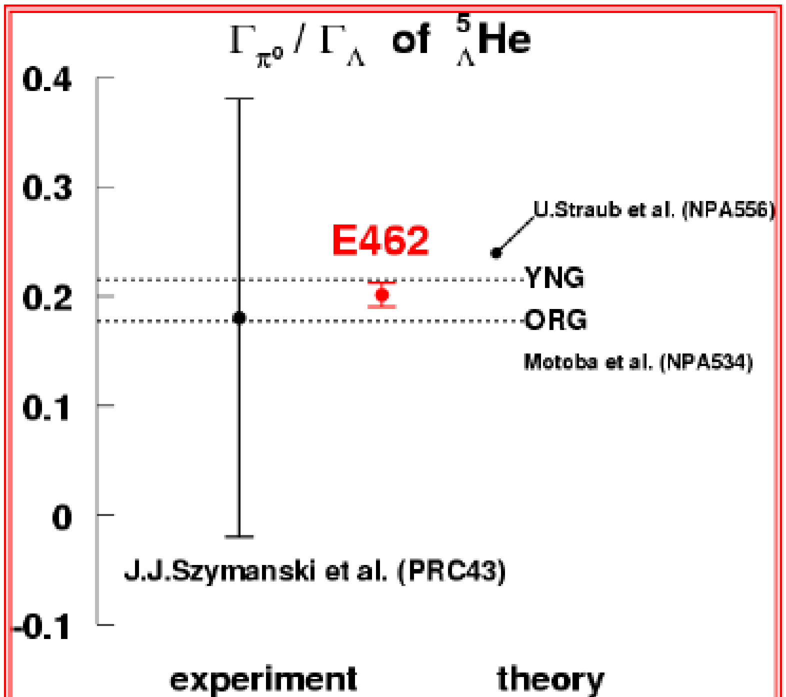
Selected comparison

to confirm the theory

Results of Γ_{π^0}

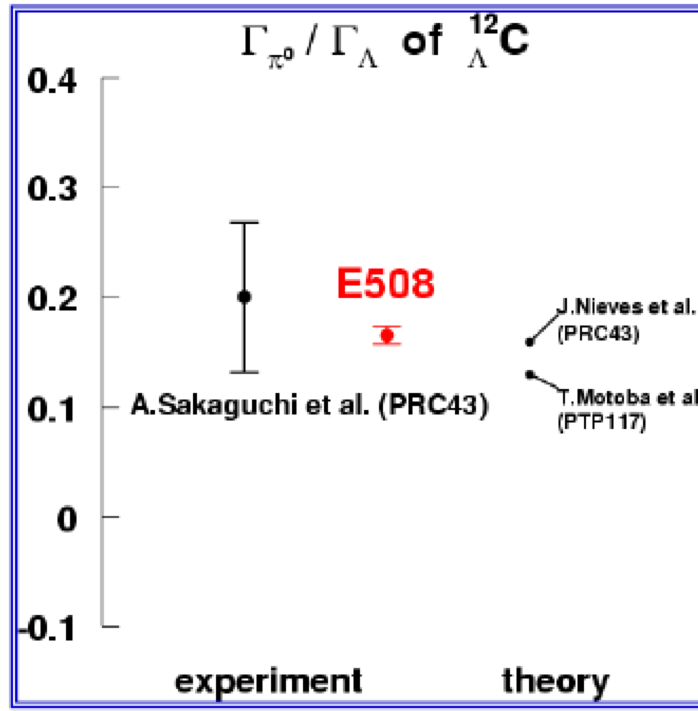
Lifetime : 278^{+11}_{-10} ps (E462)

$^{5}_{\Lambda}$He : $\Gamma_{\pi^0} / \Gamma_{\Lambda}$ = **0.207±0.013**



Lifetime : 212^{+7}_{-6} ps (E508)

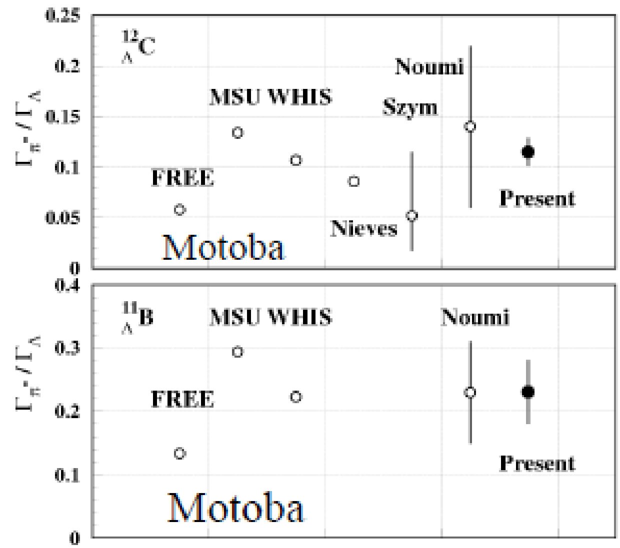
$^{12}_{\Lambda}$C : $\Gamma_{\pi^0} / \Gamma_{\Lambda}$ = **0.164±0.009**

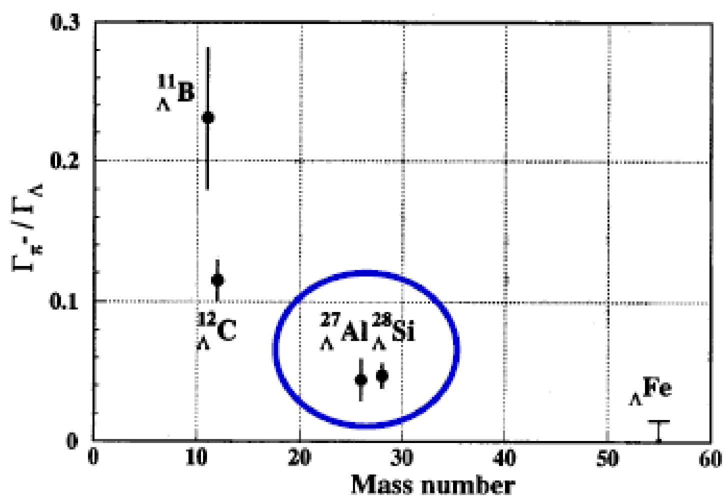


Results of π⁻ Mesonic decay width

(Outa slide)

Y.Sato *et al.*
PRC





Branching ratio

$$B_{\pi^-}(^{12}_{\Lambda}C) = 0.099 \pm 0.011 \pm 0.004$$
$$B_{\pi^-}(^{11}_{\Lambda}B) = 0.170 \pm 0.027 \pm 0.036$$
$$B_{\pi^-}(^{28}_{\Lambda}Si) = 0.036 \pm 0.008 \pm 0.002$$
$$B_{\pi^-}(^{27}_{\Lambda}Al) = 0.032 \pm 0.008 \pm 0.015$$
$$B_{\pi^-}(_{\Lambda}Fe) < 0.012 \quad (90\%\ CL)$$

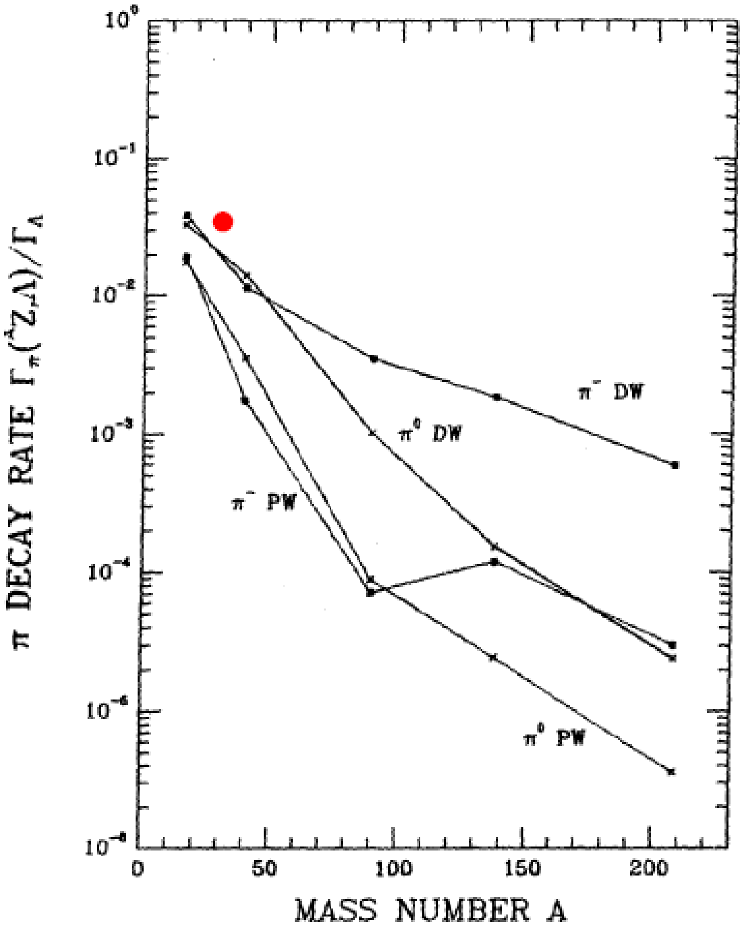
π⁻ Mesonic decay width

$$\Gamma_{\pi^-}(^{12}_{\Lambda}C) = 0.113 \pm 0.014 \pm 0.005\Gamma_\Lambda$$
$$\Gamma_{\pi^-}(^{11}_{\Lambda}B) = 0.212 \pm 0.036 \pm 0.045\Gamma_\Lambda$$
$$\Gamma_{\pi^-}(^{28}_{\Lambda}Si) = 0.046 \pm 0.011 \pm 0.002\Gamma_\Lambda$$
$$\Gamma_{\pi^-}(^{27}_{\Lambda}Al) = 0.041 \pm 0.010 \pm 0.019\Gamma_\Lambda$$
$$\Gamma_{\pi^-}(_{\Lambda}Fe) < 0.015\Gamma_\Lambda \quad (90\%\ CL)$$

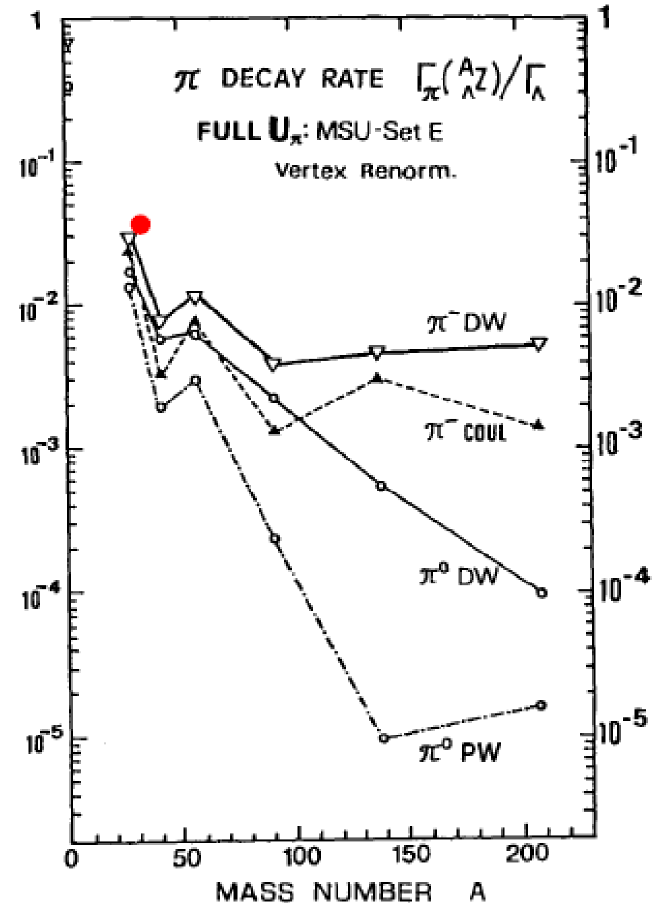
Gross behavior

Gross behavior of hypernuclear π-mesonic decay rate

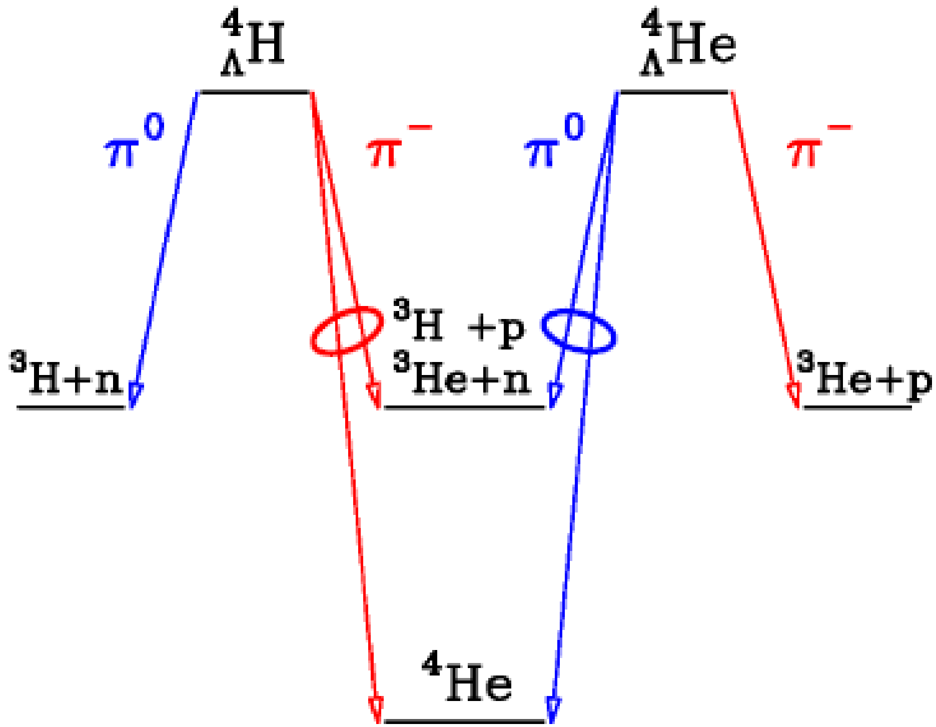
E.Oset et al. Prog. Theo. Phys. Suppl. No.117,461 (1994)



T. Motoba et al. Nucl. Phys. A547, 115c (1992)



Mesonic decay scheme of ${}^{4}_{\Lambda}H$ and ${}^{4}_{\Lambda}He$ —"*Mirror*" hypernuclei

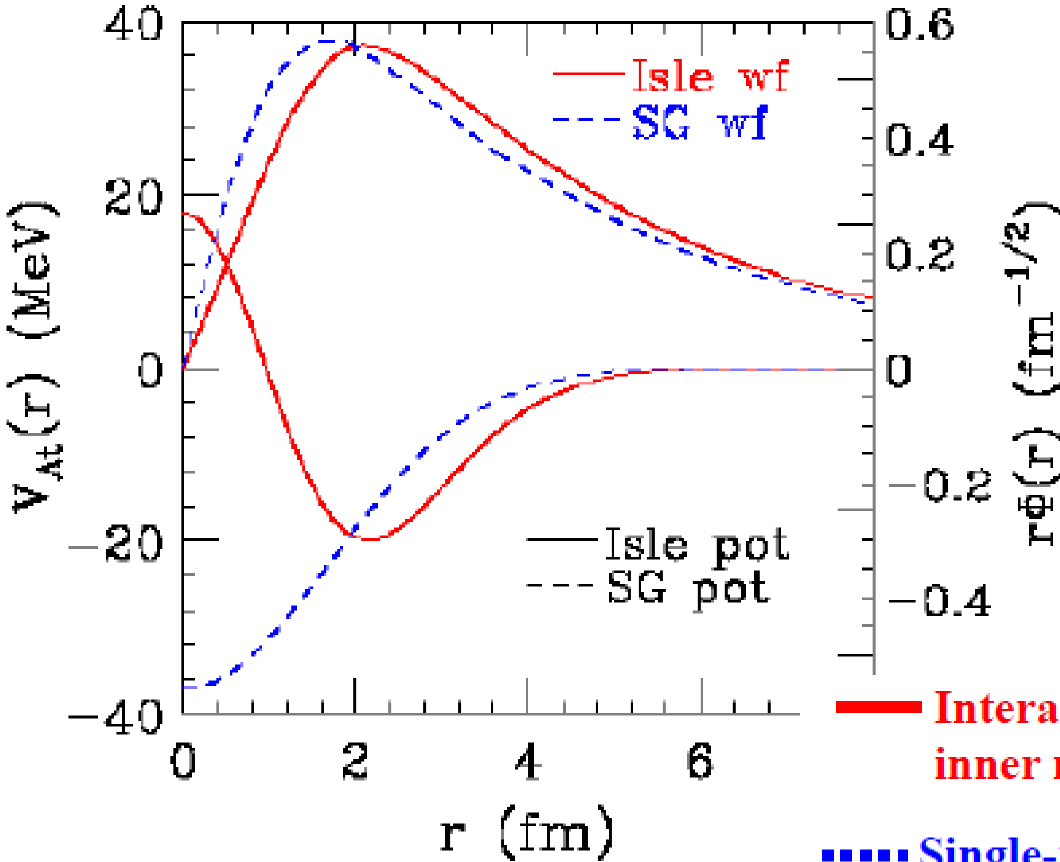


Free Λ decay

$\Lambda \rightarrow p + \pi^- \ (\sim 2/3)$

$\Lambda \rightarrow n + \pi^0 \ (\sim 1/3)$

Test of Λ - nucleus potential for ${}^{4}_{\Lambda}H$ and ${}^{4}_{\Lambda}He$



Interation with inner repulsion

Single-range gaussian pottential

Repulsive-core in Λ-nucleus potential?

Table II. Calculated decay rates, Γ_n/Γ_p ratios and intrinsic asymmetry parameters of s- and p-shell and medium-to-heavy hypernuclei. The interaction $V_\pi + V_{2\pi/\rho} + V_{2\pi/\sigma} + V_\omega + V_K + V_{\rho\pi/a_1} + V_{\sigma\pi/a_1}$ is adopted. For p-shell hypernuclei, calculations with configuration-mixed nuclear SM + Λ particle wave functions are shown without parentheses and those with simple nuclear SM + Λ particle wave functions are shown in parentheses. Calculations are done with restriction of $L_0 = 0$ only. The decay rates are given in units of the free Λ decay rate Γ_Λ. See the text.

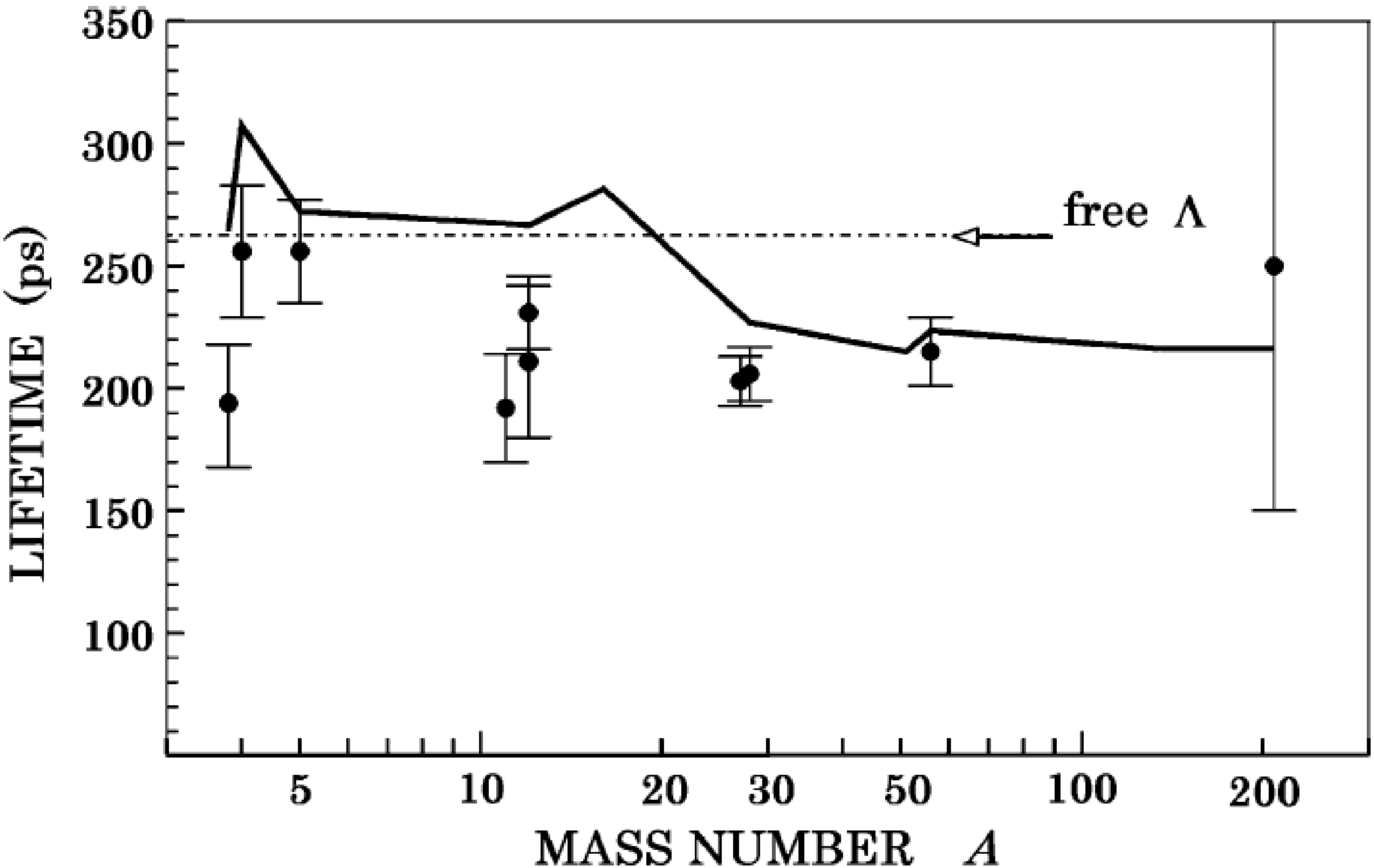
	Γ_p	Γ_n	Γ_{nm}	Γ_n/Γ_p	α_Λ
${}^{4}_{\Lambda}$H (0^+)	0.006	0.099	0.105	16.02	—
${}^{4}_{\Lambda}$He (0^+)	0.185	0.012	0.198	0.066	—
${}^{5}_{\Lambda}$He $(1/2^+)$	0.237	0.121	0.358	0.508	0.083
${}^{7}_{\Lambda}$Li $(1/2^+)$	0.297	0.152	0.449 [0.481]	0.512 [0.505]	0.138 [0.093]
${}^{8}_{\Lambda}$Li (1^-)	0.297	0.200	0.498 [0.501]	0.673 [0.657]	0.120 [0.113]
${}^{9}_{\Lambda}$Be $(1/2^+)$	0.401	0.200	0.601 [0.601]	0.499 [0.499]	0.053 [0.053]
${}^{10}_{\Lambda}$B (1^-)	0.442	0.198	0.640 [0.635]	0.448 [0.452]	0.083 [0.086]
${}^{10}_{\Lambda}$B* (2^-)	0.492	0.198	0.690 [0.688]	0.403 [0.412]	0.024 [0.029]
${}^{11}_{\Lambda}$B $(5/2^+)$	0.444	0.223	0.667 [0.660]	0.502 [0.503]	0.072 [0.079]
${}^{12}_{\Lambda}$B (1^-)	0.446	0.266	0.711 [0.697]	0.596 [0.617]	0.061 [0.082]
${}^{12}_{\Lambda}$C (1^-)	0.535	0.223	0.758 [0.754]	0.418 [0.407]	0.044 [0.045]
${}^{12}_{\Lambda}$C* (2^-)	0.536	0.237	0.773 [0.778]	0.443 [0.451]	0.044 [0.045]
${}^{13}_{\Lambda}$C $(1/2^+)$	0.495	0.247	0.741 [0.740]	0.499 [0.498]	0.023 [0.019]
${}^{14}_{\Lambda}$N (1^-)	0.551	0.246	0.797 [0.803]	0.446 [0.451]	0.034 [0.045]
${}^{15}_{\Lambda}$N $(3/2^+)$	0.555	0.278	0.833 [0.832]	0.502 [0.503]	0.037 [0.049]
${}^{16}_{\Lambda}$N (1^-)	0.519	0.298	0.816 [0.818]	0.574 [0.571]	0.038 [0.038]
${}^{16}_{\Lambda}$O (0^-)	0.586	0.275	0.860 [0.861]	0.469 [0.470]	—
${}^{16}_{\Lambda}$O* (1^-)	0.586	0.264	0.850 [0.850]	0.451 [0.452]	0.037 [0.035]
${}^{28}_{\Lambda}$Si (2^+)	0.735	0.336	1.071	0.456	0.024
${}^{51}_{\Lambda}$V $(11/2^+)$	0.792	0.412	1.203	0.520	0.017
${}^{56}_{\Lambda}$Fe (1^-)	0.764	0.392	1.156	0.513	0.008
${}^{89}_{\Lambda}$Y $(7/2^-)$	0.774	0.418	1.192	0.540	−0.003
${}^{139}_{\Lambda}$La $(9/2^+)$	0.757	0.455	1.212	0.601	−0.007
${}^{209}_{\Lambda}$Bi $(9/2^+)$	0.733	0.477	1.210	0.651	−0.016

Calculated MWD and NMWD rates

Table VIII. Calculated hypernuclear weak decay rates and lifetimes for *s*-shell, *p*-shell, medium and heavy hypernuclei. The decay rates are given in units of the free Λ decay rate Γ_Λ. The lifetimes are given in units of picosecond. Experimental data of nonmesonic decay rates and lifetimes are listed for comparison. See the text.

	Γ_π	Γ_{nm}	$\Gamma_\pi + \Gamma_{nm}$	τ	Γ_{nm}^{exp}	τ^{exp}
${}^{4}_{\Lambda}$H	0.891	0.105	0.996	264.1	0.17 ± 0.11 [Ref. 62)]	194^{+24}_{-26} [Ref. 62)]
${}^{4}_{\Lambda}$He	0.658	0.198	0.856	307.2	0.17 ± 0.05 [Ref. 62)]	256 ± 27 [Ref. 62)]
					0.20 ± 0.03 [Ref. 63)]	
${}^{5}_{\Lambda}$He	0.608	0.358	0.966	272.3		256 ± 21 [Ref. 64)]
${}^{11}_{\Lambda}$B	0.316	0.667	0.983	267.5	$0.95 \pm 0.13 \pm 0.04$ [Ref. 67)]	192 ± 22 [Ref. 65)]
					$0.861 \pm 0.063 \pm 0.073$ [Ref. 51)]	211 ± 13 [Ref. 66)]
${}^{12}_{\Lambda}$C	0.228	0.758	0.986	266.7		211 ± 31 [Ref. 65)]
						231 ± 15 [Ref. 66)]
${}^{16}_{\Lambda}$O (0^-)	0.074*	0.860	0.934	281.6		86^{+33}_{-26} [Ref. 68)]
${}^{28}_{\Lambda}$Si	0.088	1.071	1.159	226.9	$1.125 \pm 0.067 \pm 0.106$ [Ref. 51)]	206 ± 12 [Ref. 66)]
${}^{51}_{\Lambda}$V	0.02*	1.203	1.223	215.0		
${}^{56}_{\Lambda}$Fe	0.02*	1.156	1.176	223.6	1.21 ± 0.08 [Ref. 51)]	$215 \pm 14^{**}$ [Ref. 66)]
${}^{89}_{\Lambda}$Y	0.005*	1.192	1.197	219.7		
${}^{139}_{\Lambda}$La	0.005*	1.212	1.217	216.1		
${}^{209}_{\Lambda}$Bi	0.005*	1.210	1.215	216.5		250^{+250}_{-100} [Ref. 69)]

Lifetimes (light to heavy hypernuclei)



MWD rates for p-shell hypernuclei

Table IX. π^--decay rates of p-shell hypernuclei in units of the free Λ decay rate Γ_Λ.

	${}^{7}_{\Lambda}$Li	${}^{8}_{\Lambda}$Li	${}^{8}_{\Lambda}$Be	${}^{9}_{\Lambda}$Be	${}^{10}_{\Lambda}$B	${}^{11}_{\Lambda}$B	${}^{12}_{\Lambda}$B	${}^{12}_{\Lambda}$C	${}^{13}_{\Lambda}$C	${}^{14}_{\Lambda}$N	${}^{15}_{\Lambda}$N
J_{gs}	$1/2^+$	1^-	1^-	$1/2^+$	1^-	$5/2^+$	1^-	1^-	$1/2^+$	1^-	$3/2^+$
$(0p)^n$	0.280	0.344	0.130	0.144	0.197	0.195	0.256	0.077	0.079	0.053	0.067
$(1s0d)$	0.024	0.024	0.019	0.029	0.018	0.018	0.030	0.022	0.025	0.020	0.023
$\Gamma_{\pi^-}^{(\mathrm{Sum})}$	0.304	0.368	0.149	0.172	0.215	0.213	0.286	0.099	0.105	0.073	0.090
$\Gamma_{\pi^-}^{(\mathrm{AG})}$	0.332			0.157		0.178					0.066
Exp[37)]	0.353			0.178		0.249					0.108
±error	±0.059			±0.050		±0.051					±0.038
Exp[65),82)]						0.218		0.05			
±error						±0.046		$^{+0.06}_{-0.03}$			

Comparison with FINUDA data

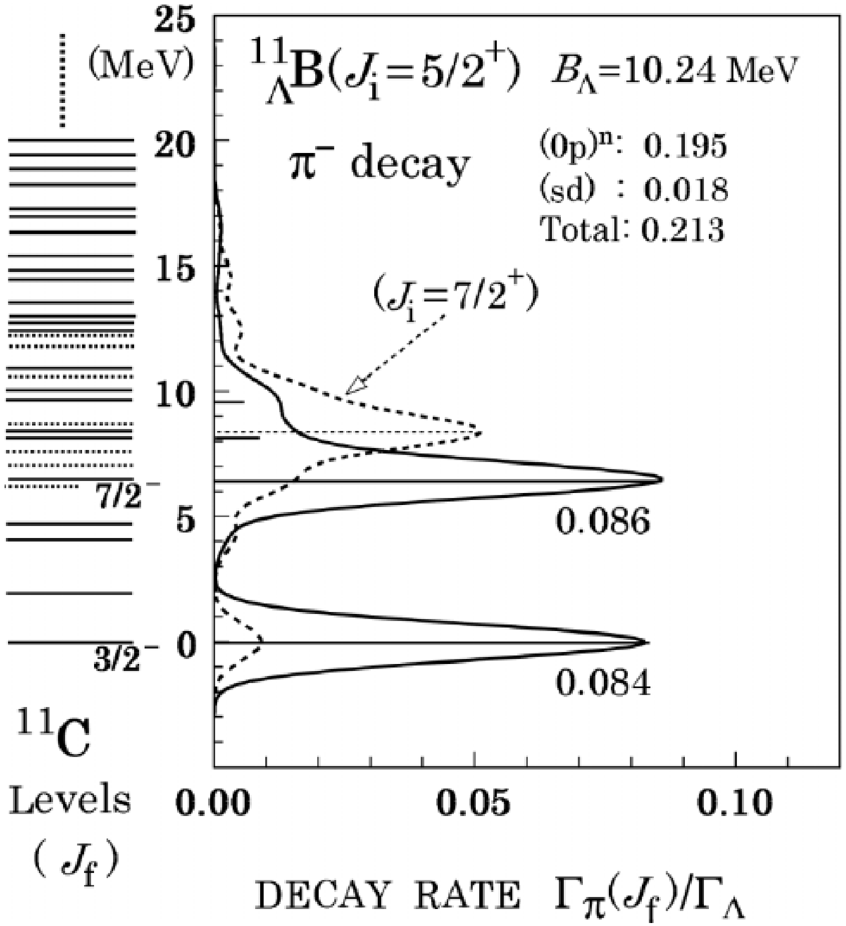


Fig. 10. Calculated π-decay spectrum as a function of ^{11}C excitation energy.

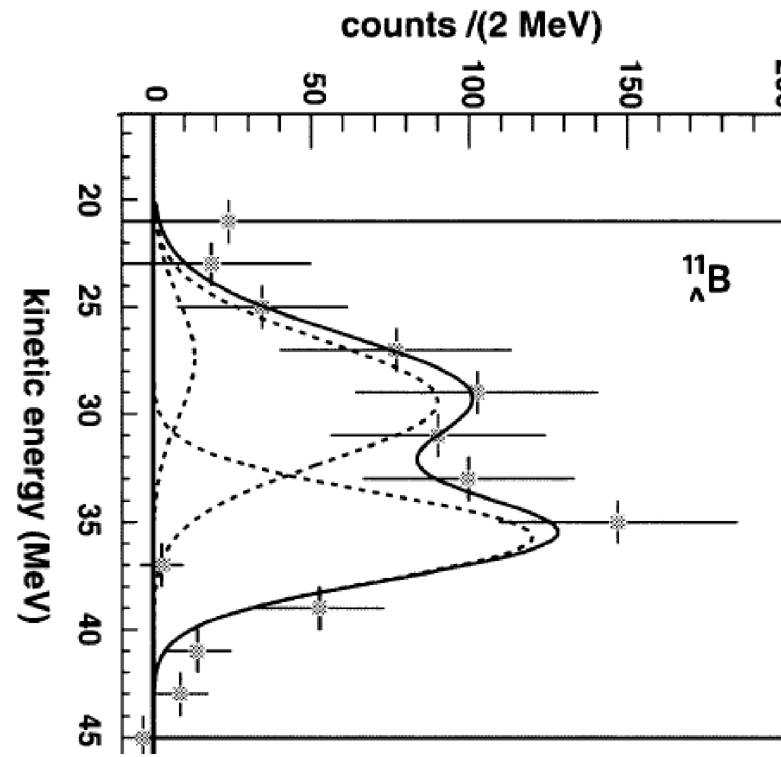


Fig. 11. π^- spectrum as a function of the kinetic energy. The data are taken from Ref. 37) and clipped for comparison.

Comparison with recent FINUDA exp.

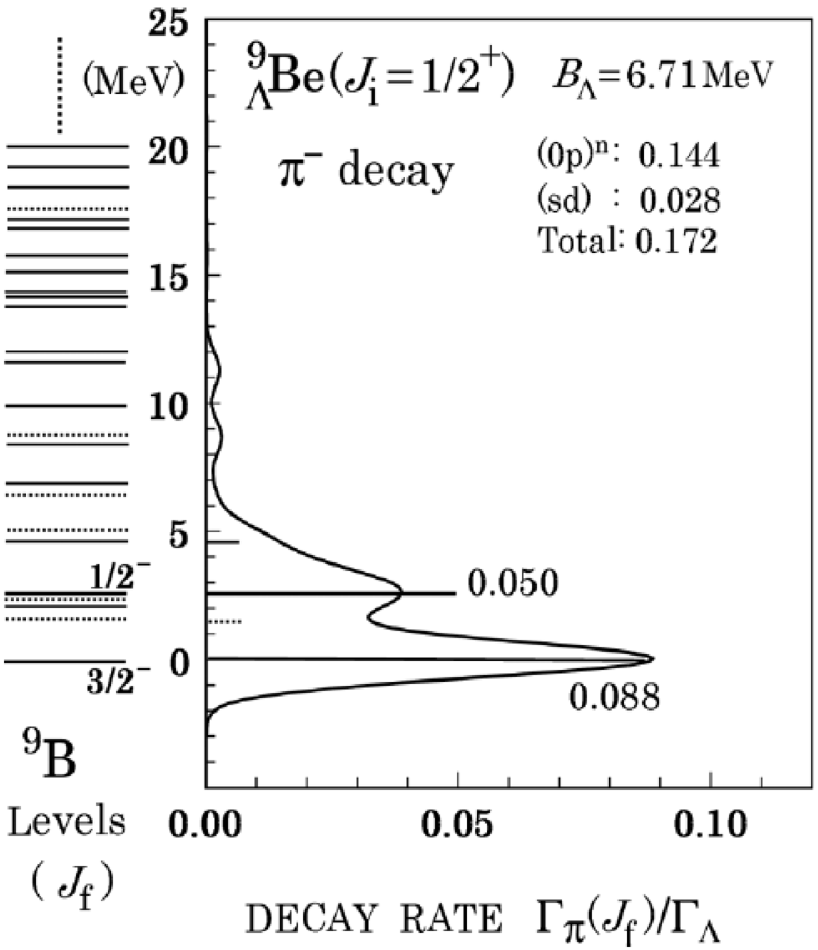


Fig. 8. Calculated π-decay spectrum as a function of ^{9}B excitation energy.

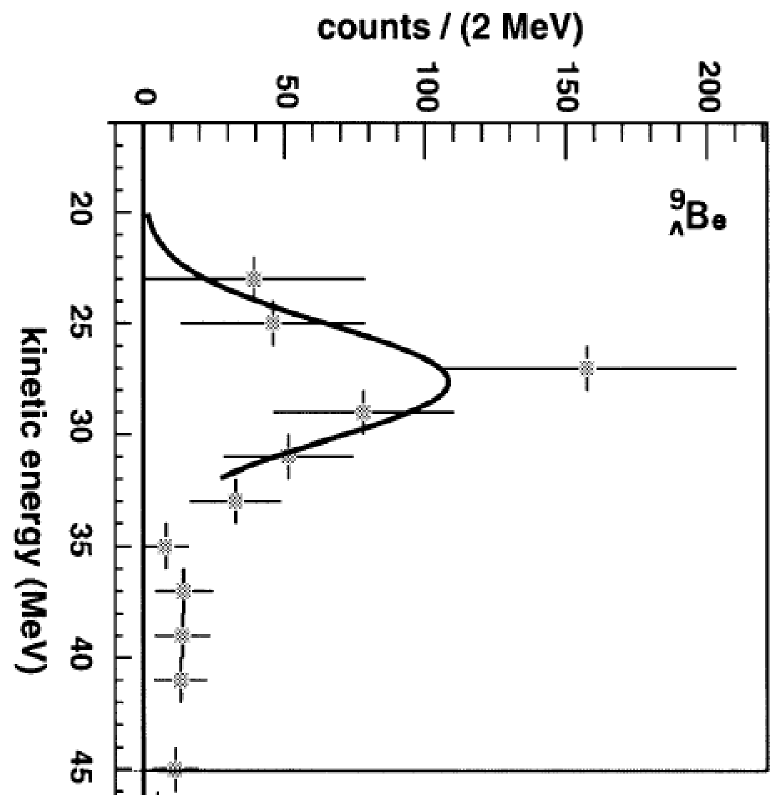
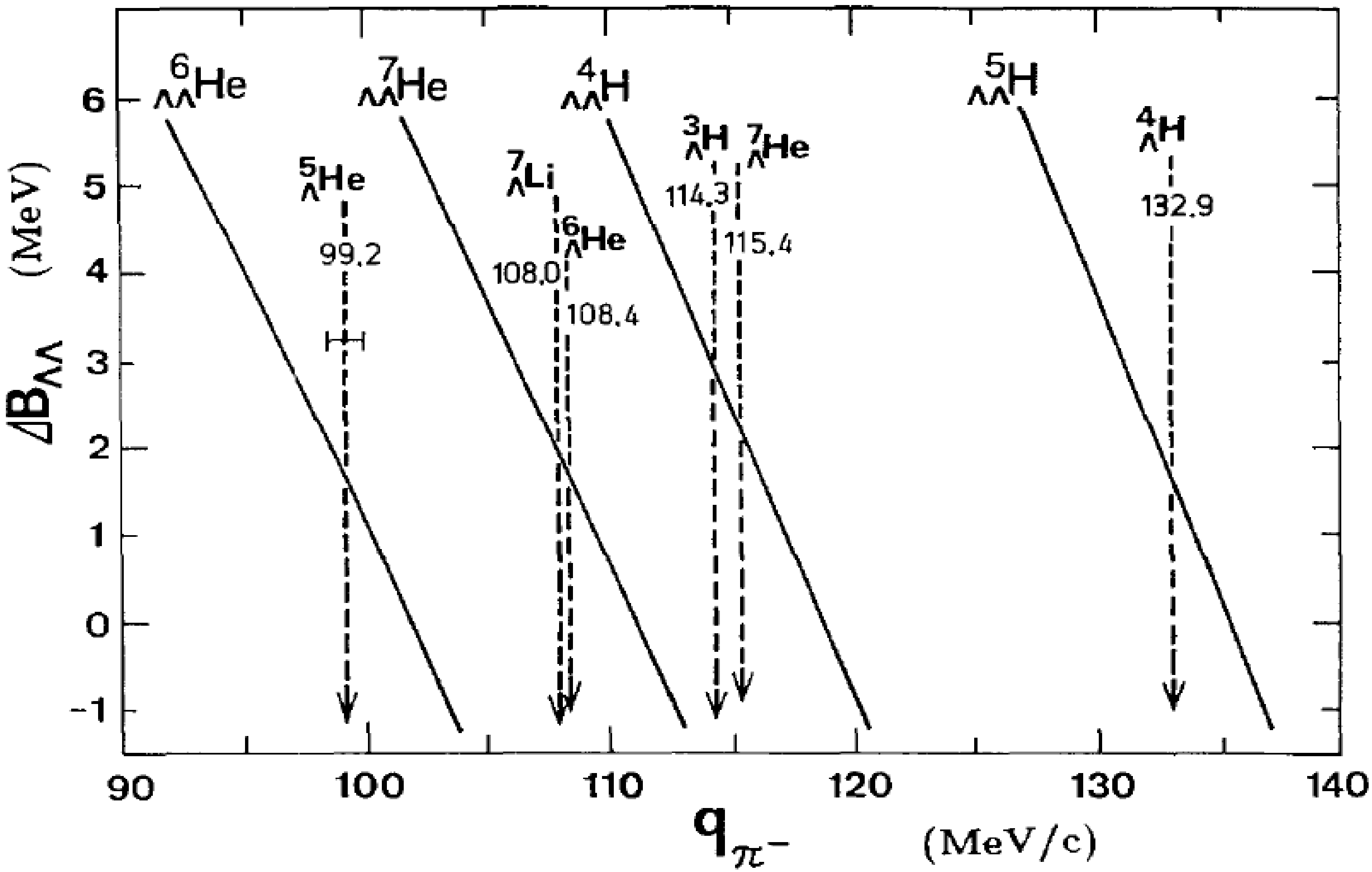


Fig. 9. π^- spectrum as a function of the kinetic energy. The data are taken from Ref. 37) and clipped for comparison.

Characteristic pion momentum helps to identify the hypernucleus



Case of 2-body and 3-body deays

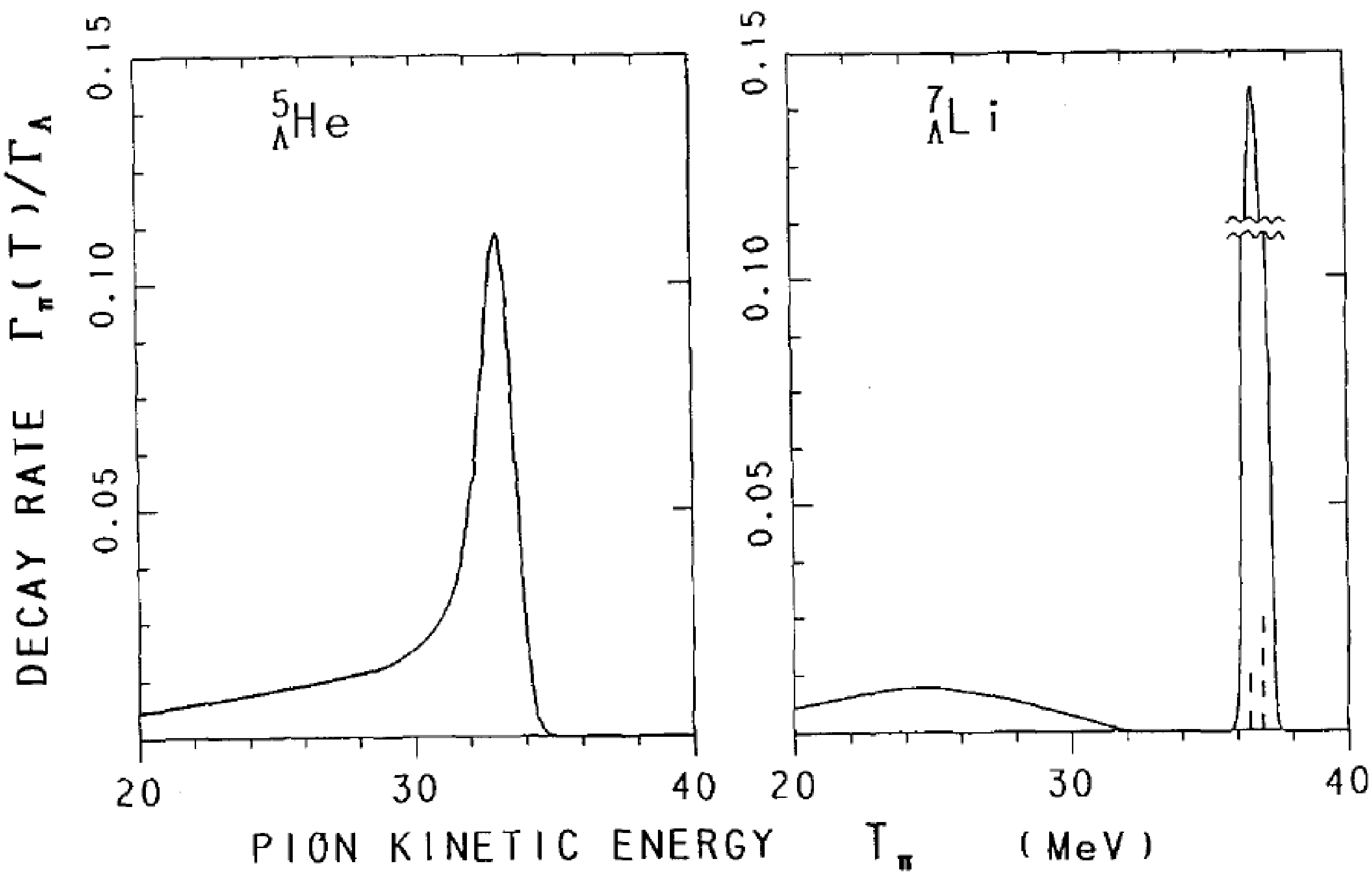


Fig. 10. The continuum pion spectrum calculated for the weak decay $^{5}_{\Lambda}\mathrm{He} \rightarrow {}^{4}\mathrm{He} + p + \pi^{-}$ (left) and that for the two-body and three-body π^{-} decay of $^{7}_{\Lambda}\mathrm{Li}$ (right).

2-body and 3-body decays

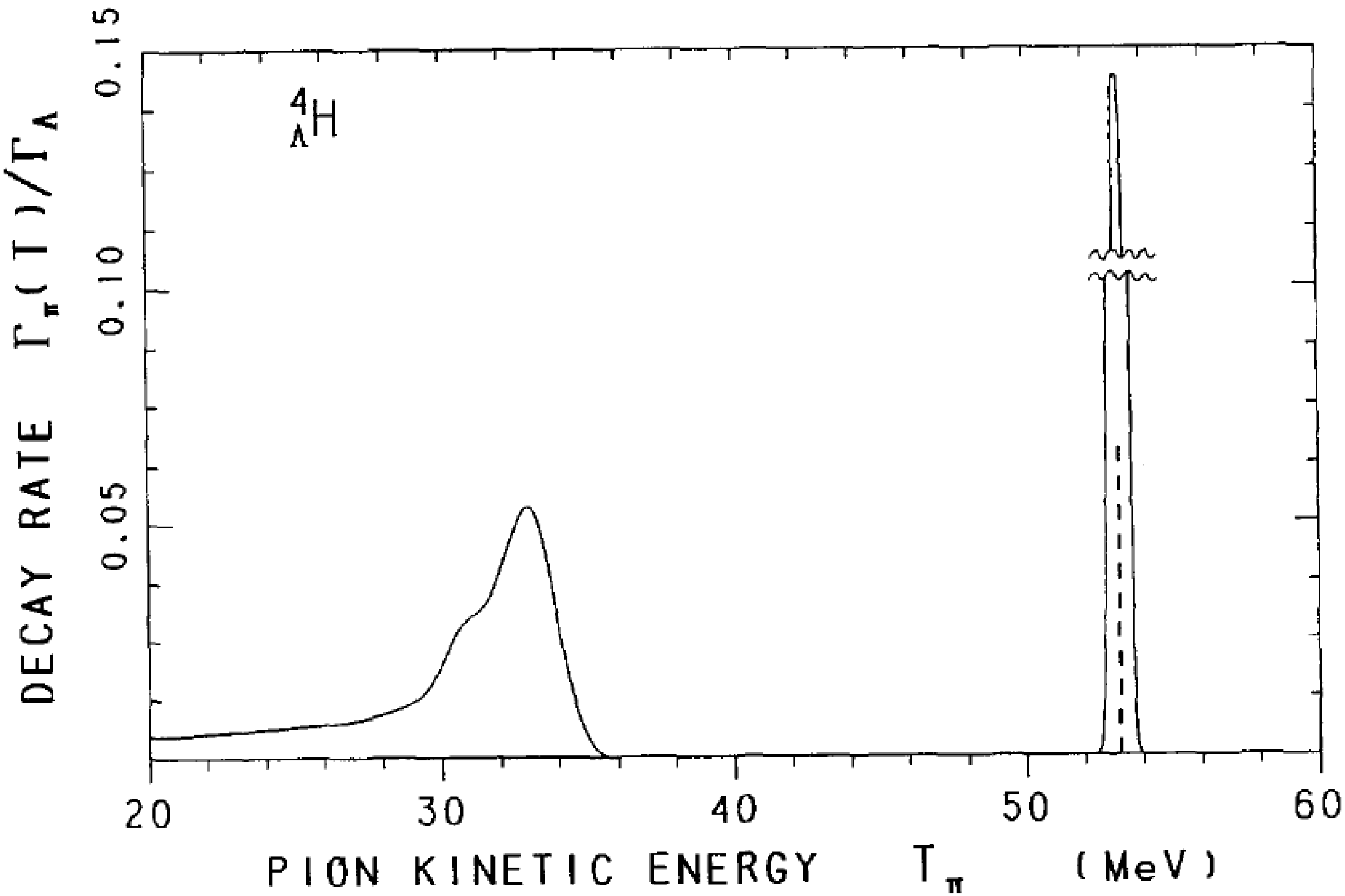


Fig. 11. The calculated π^- spectrum from the weak decay of $^4_\Lambda$H, which consists of the monochromatic peak for the two-body decay (^{4}He+π^-) and the continuum part for the three-body decays (^{3}H+p+π^-).

Pion spectrum helps

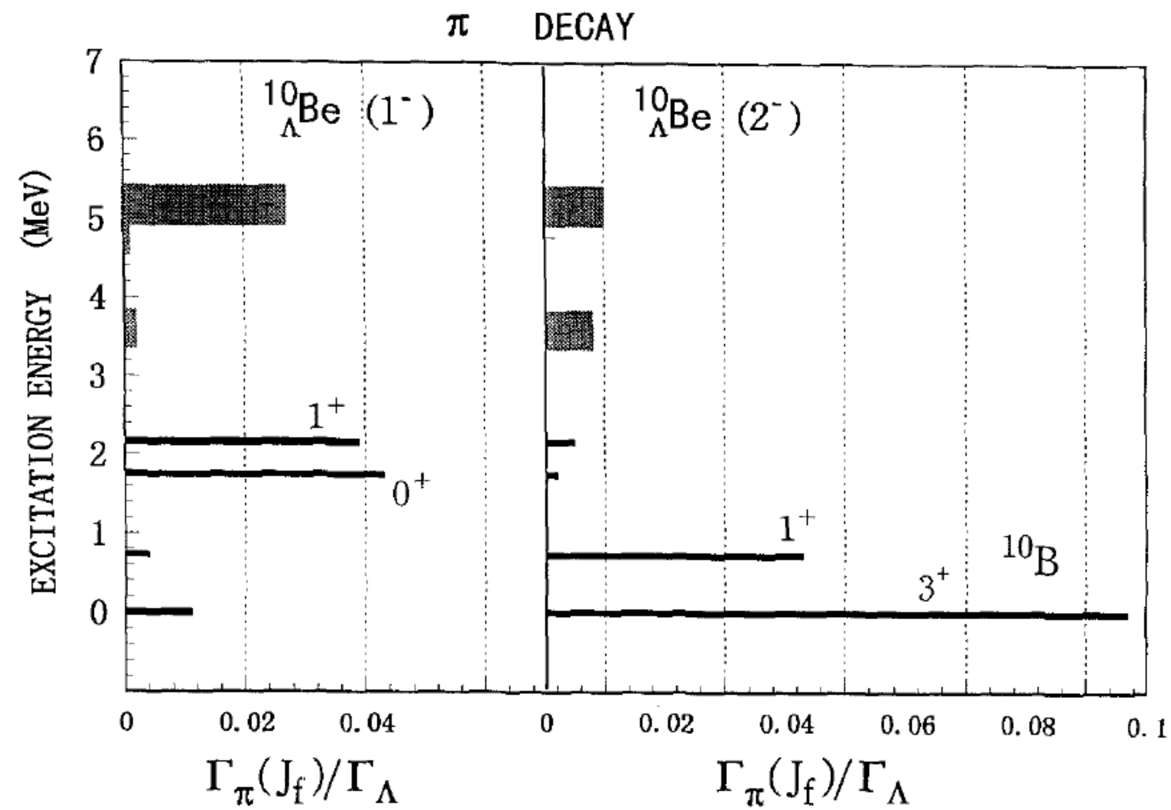


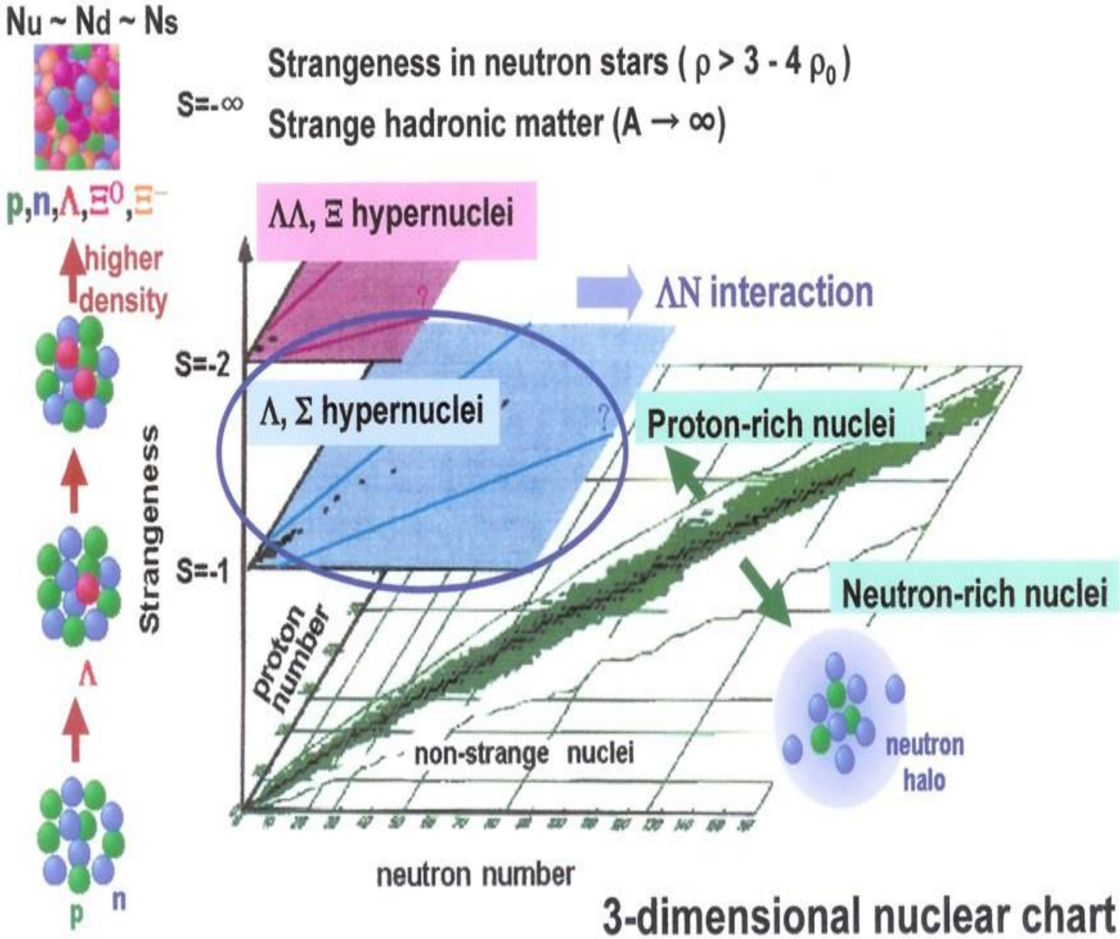
Fig. 18. The calculated π^- spectrum from the weak decay of $^{10}_{\Lambda}$Be(1^-, 2^-). Comment as for Fig. 13.

Summary

1) Pionic decay rates and spectra have been extensively calculated, using pion optical potential which reproduces nucleus-pion scattering at low energies.
2) Sensitivity of pi-decay momentum helps to identify the hypernucleus unambiguously.
3) Pionic decay spectrum also helps to determine the Ground-state spin.
4) Coincidence measurements are promising

Three-dimensional nuclear chart

(from H. Tamura)



(**First 3D illustration**
appeared In literature:
Genshikaku Kenkyu **32** (1987) 97.
See P.T.P. S.185 (2010) 1.)

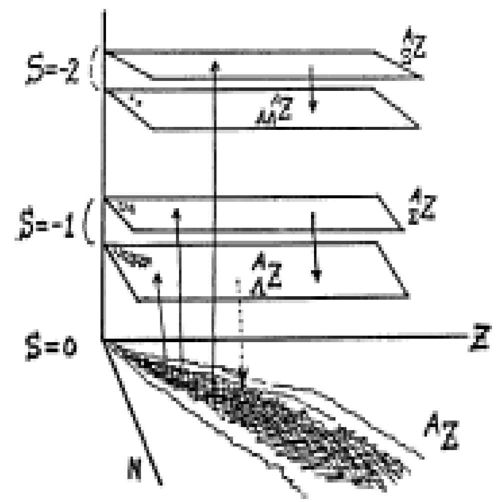


Fig. 3. 3D-illustration of hypernuclear chart drawn for the first time with the strangeness axis added (1986).[61)]