Hypernuclear physics: past, present, and future

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Studies of Λ hypernuclei

- $n(K^-, \pi^-)\Lambda$ – stopped or in-flight; first emulsions $\Rightarrow B_\Lambda$, then CERN, BNL, KEK, Frascati
- $n(\pi^+, K^+)\Lambda$ – BNL, KEK   1985 – 2001
- Hypernuclear γ-ray spectroscopy with Hyperball $(\pi^+, K^+\gamma)$ at KEK and $(K^-, \pi^-\gamma)$ at BNL   1998 – 2005
- $p(e, e'K^+)\Lambda$ – JLab, Hall A and Hall C   Ongoing

Experimental Λ separation energies, B_Λ from emulsion studies

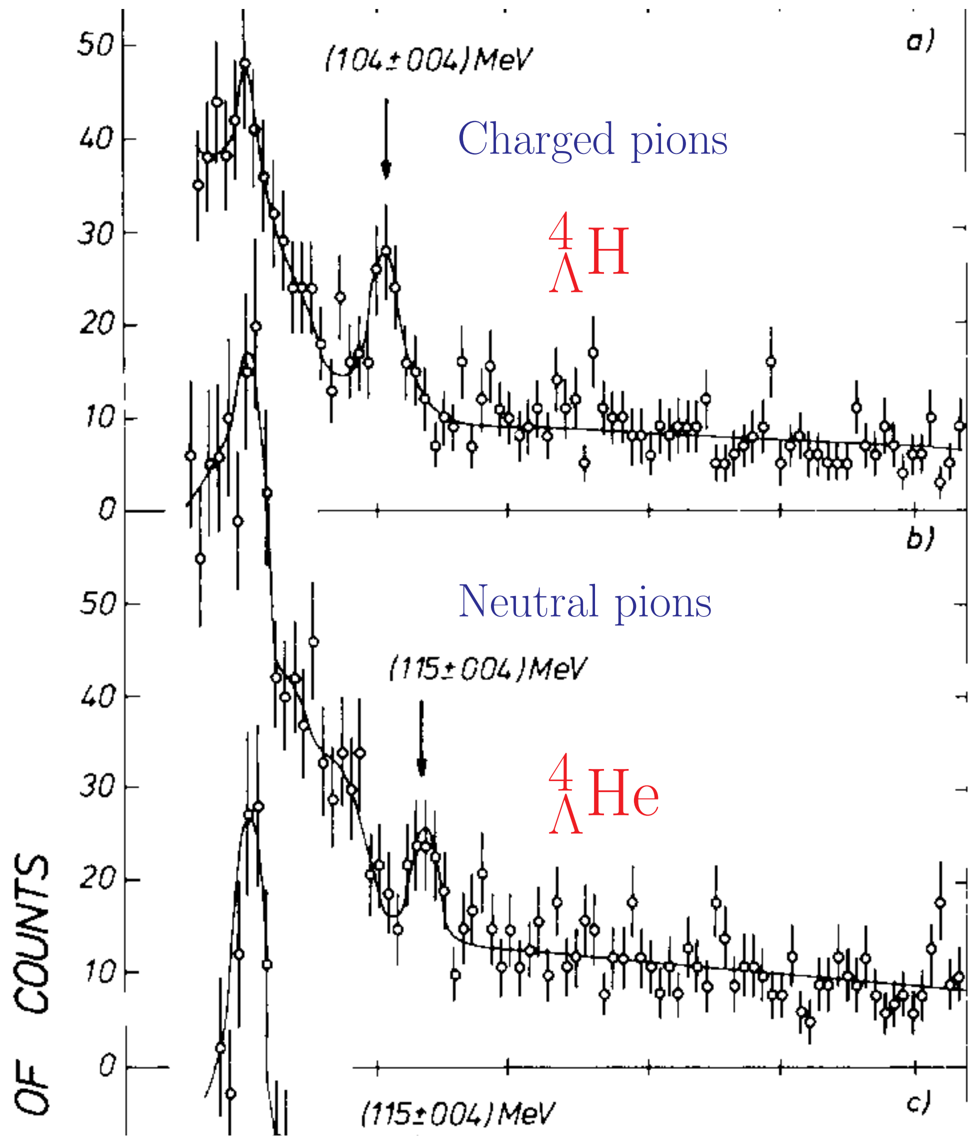
Hypernucleus	# events	$B_\Lambda \pm \Delta B_\Lambda$ MeV	Hypernucleus	# events	$B_\Lambda \pm \Delta B_\Lambda$ MeV
${}^{3}_{\Lambda}\mathrm{H}(1/2^+)$	204	0.13 ± 0.05	${}^{9}_{\Lambda}\mathrm{Li}$	8	8.50 ± 0.12
${}^{4}_{\Lambda}\mathrm{H}(0^+)$	155	2.04 ± 0.04	${}^{9}_{\Lambda}\mathrm{Be}$	222	6.71 ± 0.04
${}^{4}_{\Lambda}\mathrm{He}$	279	2.39 ± 0.03	${}^{9}_{\Lambda}\mathrm{B}$	4	8.29 ± 0.18
${}^{5}_{\Lambda}\mathrm{He}$	1784	3.12 ± 0.02	${}^{10}_{\Lambda}\mathrm{Be}$	3	9.11 ± 0.22
${}^{6}_{\Lambda}\mathrm{He}$	31	4.18 ± 0.10	${}^{10}_{\Lambda}\mathrm{B}$	10	8.89 ± 0.12
${}^{7}_{\Lambda}\mathrm{He}$	16	not averaged	${}^{11}_{\Lambda}\mathrm{B}(5/2^+)$	73	10.24 ± 0.05
${}^{7}_{\Lambda}\mathrm{Li}$	226	5.58 ± 0.03	${}^{12}_{\Lambda}\mathrm{B}(1^-)$	87	11.37 ± 0.06
${}^{7}_{\Lambda}\mathrm{Be}$	35	5.16 ± 0.08	${}^{12}_{\Lambda}\mathrm{C}$	6	10.80 ± 0.18
${}^{8}_{\Lambda}\mathrm{He}$	6	7.16 ± 0.70	${}^{13}_{\Lambda}\mathrm{C}$	6	11.69 ± 0.12
${}^{8}_{\Lambda}\mathrm{Li}(1^-)$	787	6.80 ± 0.03	${}^{14}_{\Lambda}\mathrm{C}$	3	12.17 ± 0.33
${}^{8}_{\Lambda}\mathrm{Be}$	68	6.84 ± 0.05	${}^{15}_{\Lambda}\mathrm{N}$	14	13.59 ± 0.15

S-Shell Λ Hypernuclei

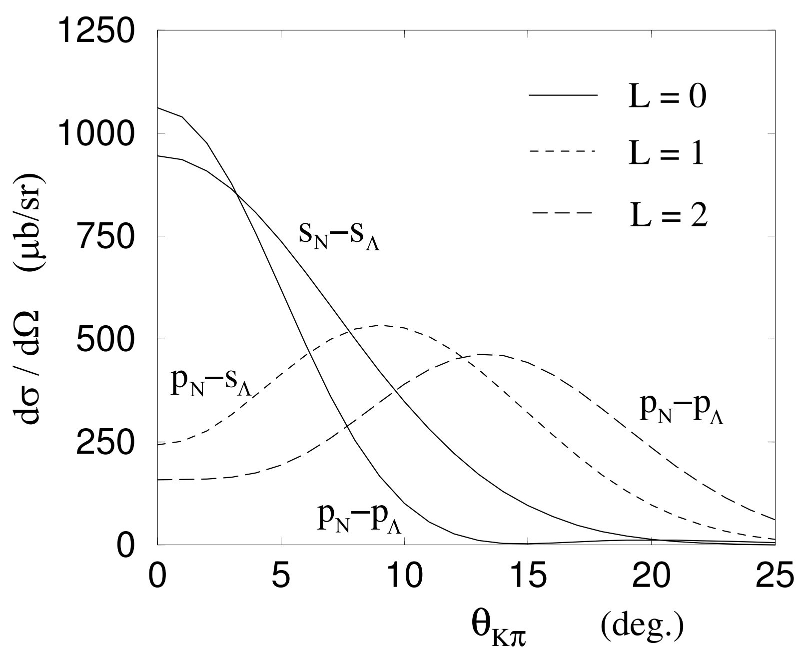
Hypernucleus	$J^\pi(gs)$	B_Λ MeV	J^π	E_x MeV
${}^{3}_{\Lambda}H$	$1/2^+$	0.13(5)		
${}^{4}_{\Lambda}H$	0^+	2.04(4)	1^+	1.04(5)
${}^{4}_{\Lambda}He$	0^+	2.39(3)	1^+	1.15(4)
${}^{5}_{\Lambda}He$	$1/2^+$	3.12(2)		

Ab Initio Calculations

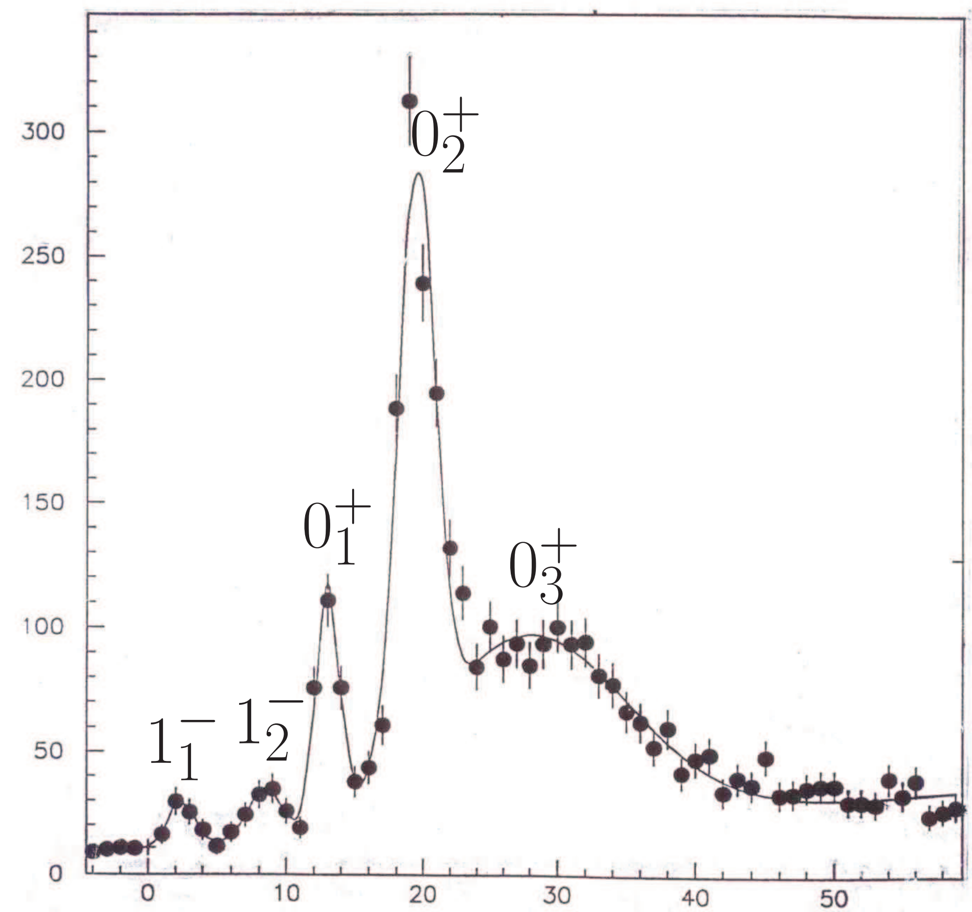
- $A = 3, 4$ A. Nogga et al., PRL 88 (2002) 172501
  Faddeev and Faddeev-Yakubovsky
- $A = 4$ E. Hiyama et al., PRC 65 (2002) 011301(R)
  Jacobi-coordinate Gaussian basis
- $A = 3, 4, 5$ H. Nemura et al., PRL 89 (2002) 142504
  Stochastic variation with correlated Gaussians



M. Bedjidian et al., Phys. Lett. B 83, 252 (1979)



$^{16}\mathrm{O}(\mathrm{K}^-,\pi^-)^{16}_{\Lambda}\mathrm{O}$



1_2^- $p_{3/2}^{-1} s_{1/2\Lambda}$

0_1^+ $p_{1/2}^{-1} p_{1/2\Lambda}$

2^+ $p_{3/2}^{-1} p_{3/2\Lambda} + p_{3/2}^{-1} p_{1/2\Lambda}$

1_1^- $p_{1/2}^{-1} s_{1/2\Lambda}$

0_2^+ $p_{3/2}^{-1} p_{3/2\Lambda}$

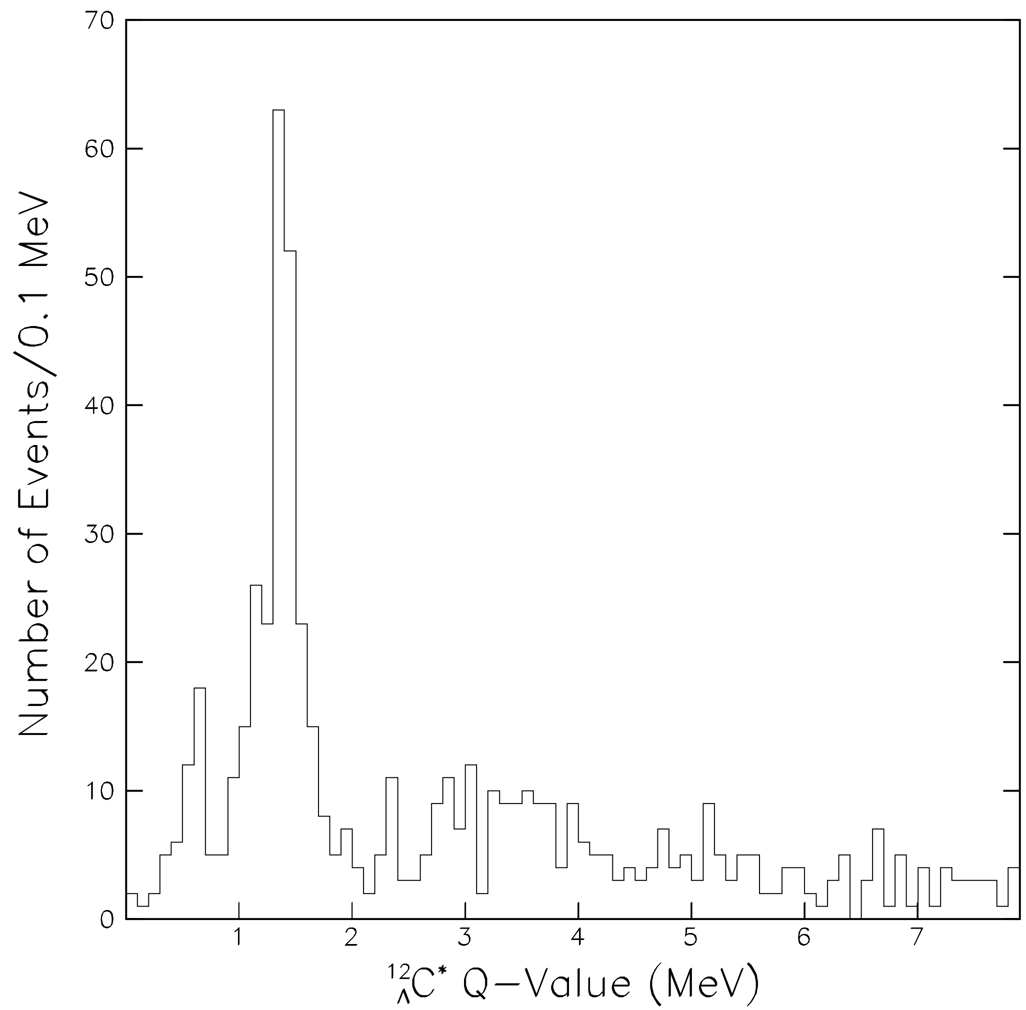
0_3^+ $s_{1/2}^{-1} s_{1/2\Lambda}$

J^π	# events	Γ keV	B_Λ MeV
0^+	64	< 100	0.14(5)
2^+	193	~ 600	0.20(5)
2^+	48	~ 150	0.95(5)

Upper 2^+ strong in $(e, e'K^+)$; 3^+ predicted 0.07 MeV higher $\Rightarrow$ B_Λ for p_Λ peak is 0.16 MeV

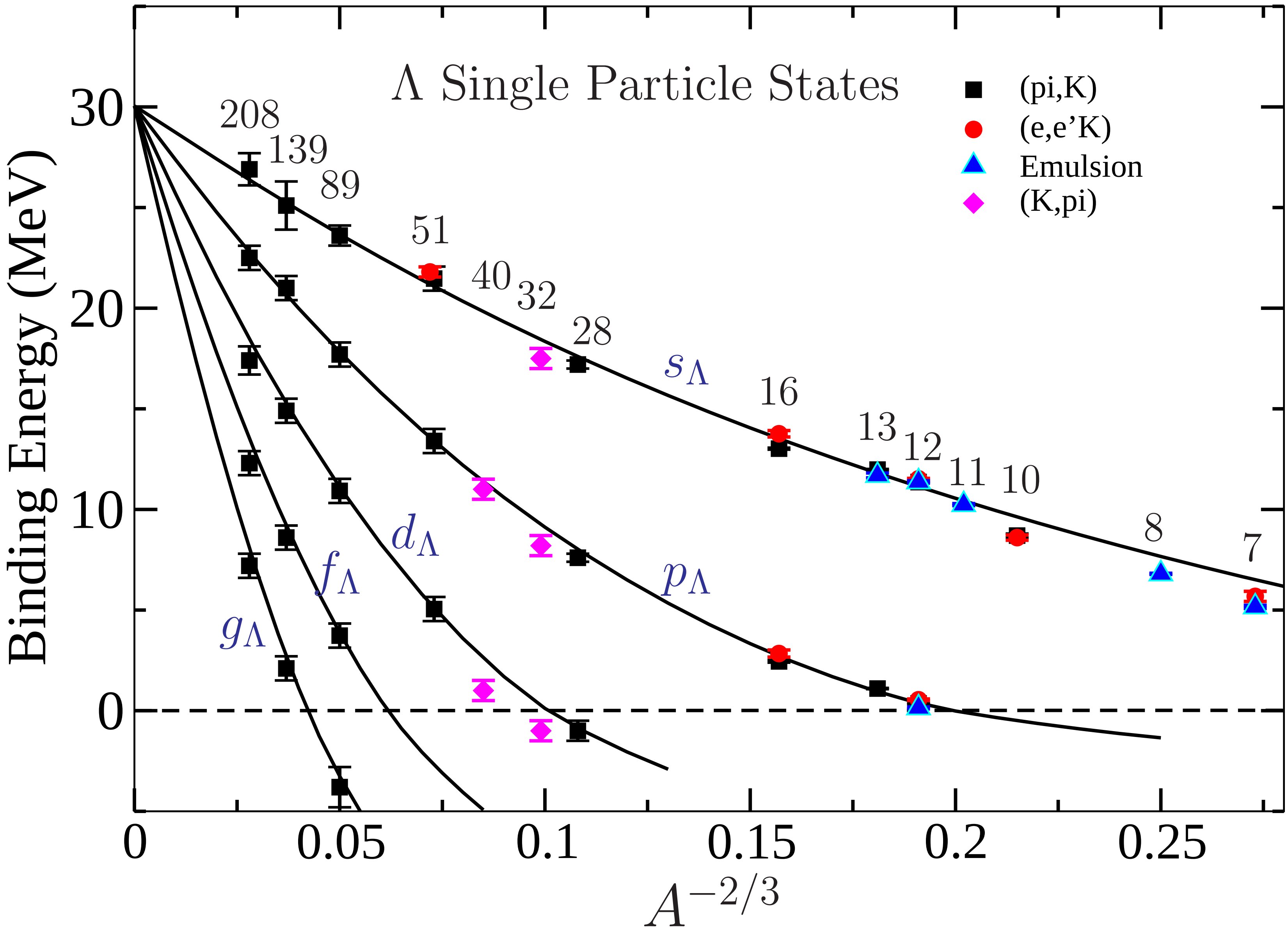
Iodice PRL 99, 052501 (2007) $\Rightarrow$ 10.93 MeV between s_Λ and p_Λ peaks; centroid of gs doublet at 0.13 MeV $\Rightarrow$ B_Λ for ${}^{12}_{\Lambda}$C $= 11.22$ MeV

Can play similar games with FINUDA, KEK E336, and Hall C data

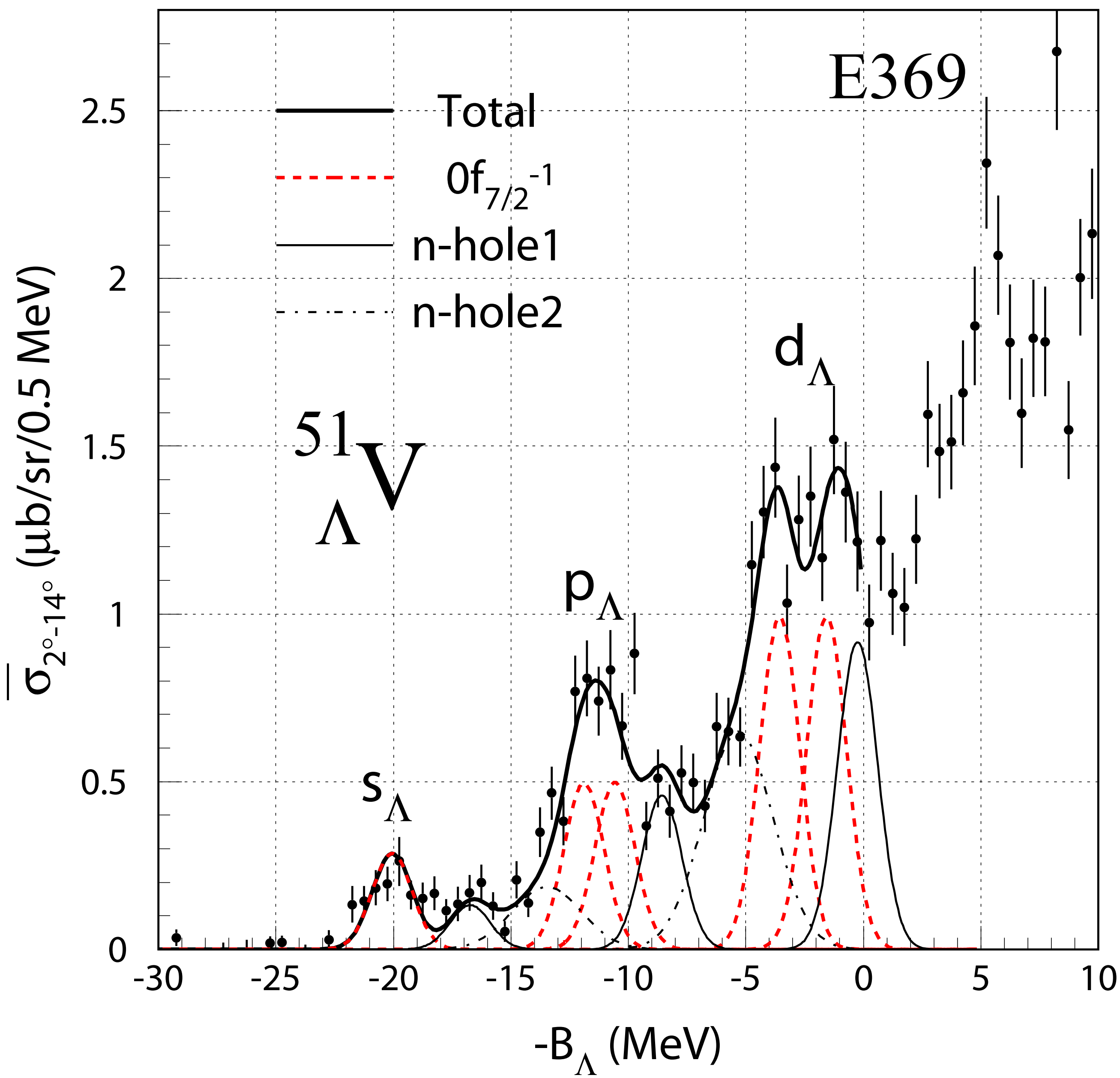


D. H. Davis NPA 804 (2008) 5

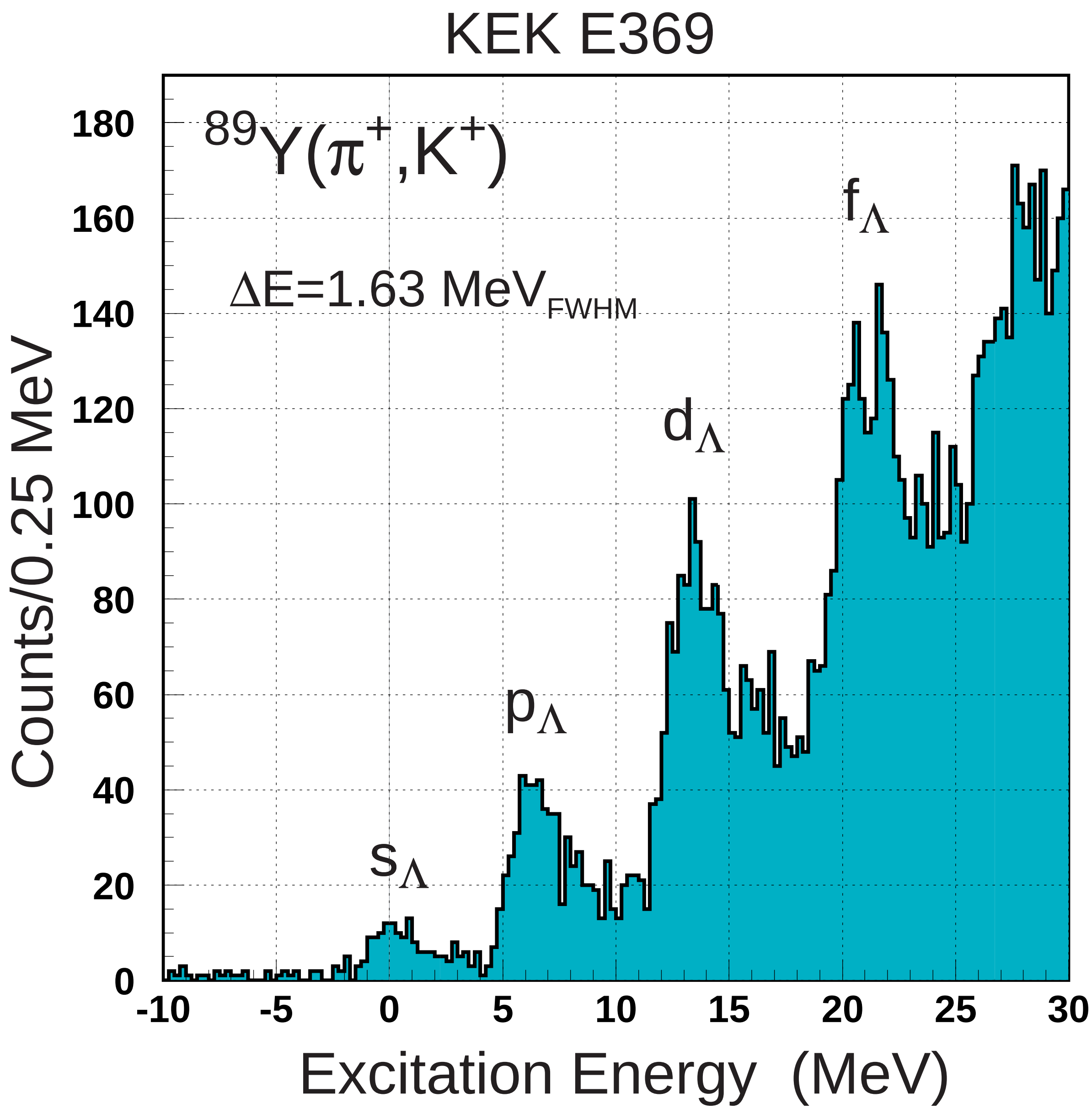
Update: Millener, Dover, Gal PRC 38, 2700 (1988)



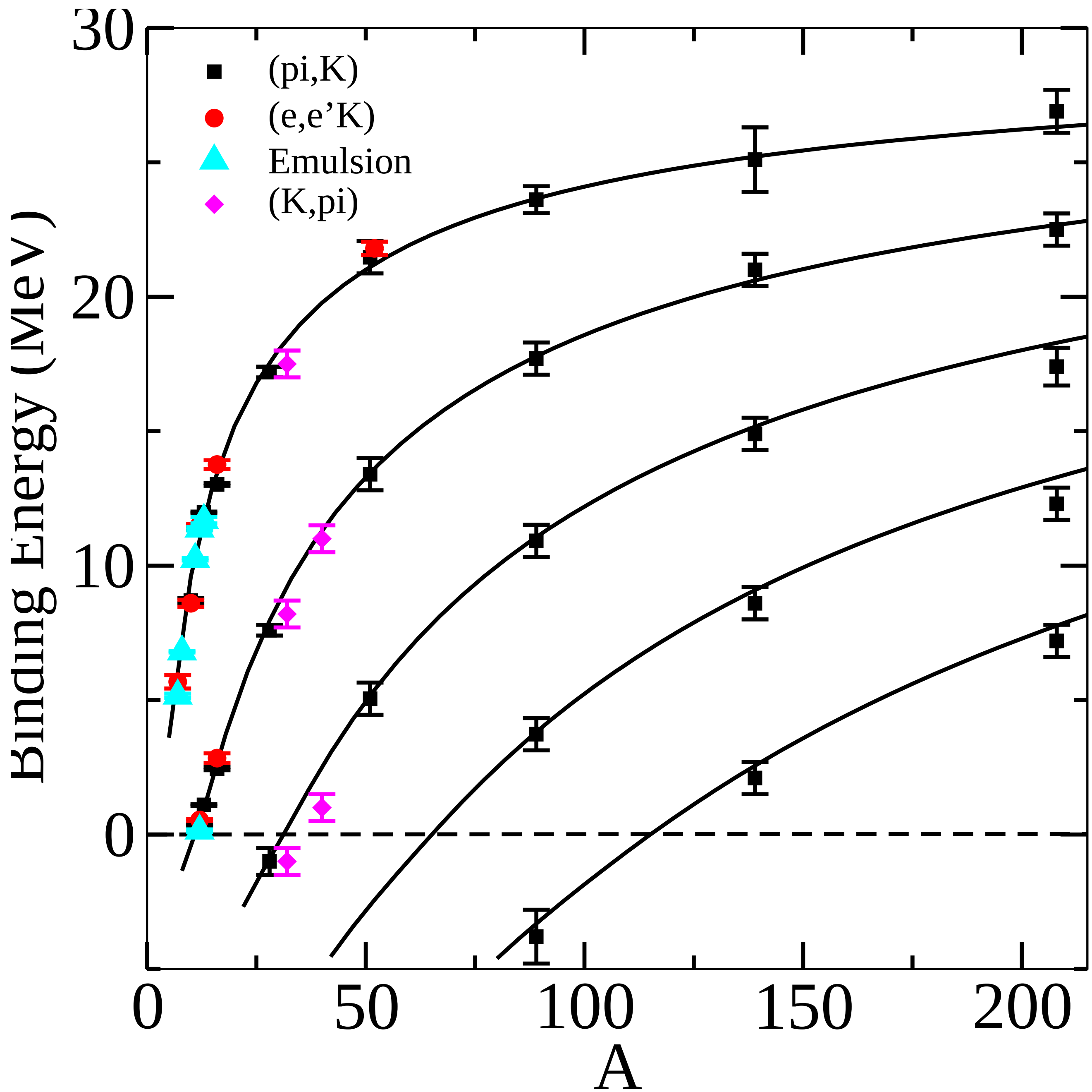
Woods-Saxon $V = 30.05$ MeV, $r = 1.165$ fm, $a = 0.6$ fm



Hotchi et al., Phys. Rev. C 64 (2001) 044302 $B_\Lambda = 19.97 \pm 0.13$ MeV



Hotchi et al., Phys. Rev. C 64 (2001) 044302 $B_\Lambda = 23.11 \pm 0.10$ MeV



(pi,K)
(e,e'K)
Emulsion
(K,pi)
Binding Energy (MeV)
A
0
10
20
30
0
50
100
150
200

$(e, e'K^+)$ vs. (π^+, K^+)

- Large and similar momentum transfers; can get to inner Λ orbits; past maximum of transition form factor; small cross sections falling with increasing q.
- (π^+, K^+) more selective because of strong pion absorption; advantage or disadvantage for $(e, e'K^+)$?
- Can use Λ and Σ^0 peaks from $p(e, e'K^+)Y$ to calibrate B_Λ; have to normalize (π^+, K^+) to known results - e.g. from emulsion.

- Have the intuitive result that if one sums over all final states $(J_f j_\Lambda)$ in the weak-coupling limit, the cross section is proportional to the nucleon C^2S for target to core state.
- No chance of resolving these multiplets; have to pick target to give a clean spectrum of Λ single-particle states.

Shell-model calculations

- Both $|p^n \alpha_c J_c T \times s_\Lambda\rangle$ and $|p^n \alpha_c J_c T_c \times s_\Sigma\rangle$ configurations included. In general, T_c can take three values.
- Supermultiplet basis $|p^n [f_c] \beta_c (L_c S_c) J_c T_c\rangle$ is very good for p shell $\Rightarrow$ states with different $[f_c]$ (often T_c) well separated.
- p-shell interactions fitted with tensor interaction constrained to give cancellation in ^{14}C β decay; single-particle LS spacing constrained by data at the beginning of the shell.
- Need $N\Lambda$-$N\Lambda$ (parametrized, Δ,..), $N\Lambda$-$N\Sigma$ (see following slides), and $N\Sigma$-$N\Sigma$ (for T=1/2 and T=3/2; from YNG-type interaction) two-body matrix elements. All can be represented in the same way.
- Diagonal energies of Λ and Σ states differ by $\sim$ 80MeV, plus core energy differences, plus contributions from YN interactions.

$$V = \sum_\alpha C(\alpha) \left[\left[a^+_{j_{\rm N}} \tilde{a}_{j'_{\rm N}} \right]^{J_\alpha T_\alpha} \left[a^+_{j_{\rm Y}} \tilde{a}_{j'_{\rm Y}} \right]^{J_\alpha T_\alpha} \right]^{00}$$

Need full set of one-body density-matrix elements for the core nucleus.

$$V_{\Lambda N} = V_0(r) + V_\sigma(r)\, s_N \cdot s_\Lambda + V_{LS}(r)\, l_{N\Lambda} \cdot (s_\Lambda + s_N) + V_{ALS}(r)\, l_{N\Lambda} \cdot (s_\Lambda - s_N) + V_T(r)\, S_{12}$$

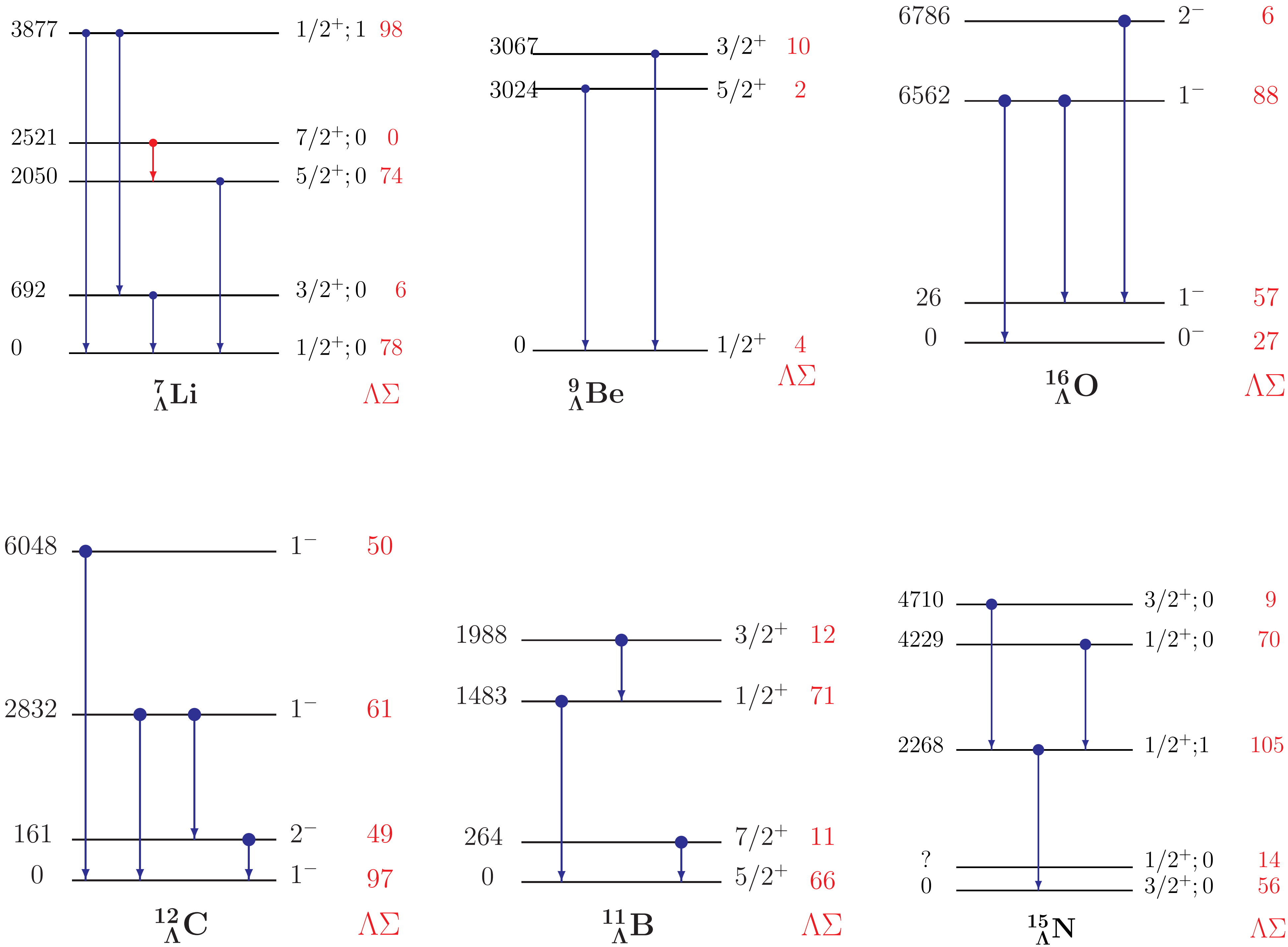
$$V_0 = 1/4\, {}^1V_C + 3/4\, {}^3V_C \qquad V_\sigma = {}^3V_C - {}^1V_C$$

For $p_N s_Y$ $\quad V_{\Lambda N} = \bar{V} + \Delta\, s_N \cdot s_\Lambda + S_\Lambda\, l_N \cdot s_\Lambda + S_N\, l_N \cdot s_N + T\, S_{12}$

Summed over nucleons, the operators with Δ and S_Λ are $S_{core} \cdot s_\Lambda$ and $L_{core} \cdot s_\Lambda$

Parameters in MeV

	$\bar{V}$	Δ	S_Λ	S_N	T
$N\Lambda$-$N\Lambda$ $A = 7-$?		0.430	-0.015	-0.390	0.030
$A = 11 - 16$		0.330	-0.015	-0.350	0.024
$N\Lambda$-$N\Sigma$	1.45	3.04	-0.085	-0.085	0.157


3877
2521
2050
692
0
1/2+; 1 98
7/2+; 0 0
5/2+; 0 74
3/2+; 0 6
1/2+; 0 78
7 Λ Li
ΛΣ
3067
3024
0
3/2+ 10
5/2+ 2
1/2+ 4
9 Λ Be
ΛΣ
6786
6562
26
0
2− 6
1− 88
1− 57
0− 27
16 Λ O
ΛΣ
6048
2832
161
0
1− 50
1− 61
2− 49
1− 97
12 Λ C
ΛΣ
1988
1483
264
0
3/2+ 12
1/2+ 71
7/2+ 11
5/2+ 66
11 Λ B
ΛΣ
4710
4229
2268
?
0
3/2+; 0 9
1/2+; 0 70
1/2+;1 105
1/2+; 0 14
3/2+; 0 56
15 Λ N
ΛΣ

Doublet spacings in p-shell hypernuclei

	J^π_u	J^π_l	$\Lambda\Sigma$	Δ	S_Λ	S_N	T	ΔE^{th}	ΔE^{exp}
${}^{7}_{\Lambda}$Li	$3/2^+$	$1/2^+$	72	628	−1	−4	−9	693	692
${}^{7}_{\Lambda}$Li	$7/2^+$	$5/2^+$	74	557	−32	−8	−71	494	471
${}^{8}_{\Lambda}$Li	2^-	1^-	151	396	−14	−16	−24	450	(442)
${}^{9}_{\Lambda}$Li	$5/2^+$	$3/2^+$	116	530	−17	−18	−1	589	
${}^{9}_{\Lambda}$Li	$3/2^+_2$	$1/2^+$	−80	231	−13	−13	−93	−9	
${}^{9}_{\Lambda}$Be	$3/2^+$	$5/2^+$	−8	−14	37	0	28	44	43
${}^{11}_{\Lambda}$B	$7/2^+$	$5/2^+$	56	339	−37	−10	−80	267	264
${}^{11}_{\Lambda}$B	$3/2^+$	$1/2^+$	61	424	−3	−44	−10	475	505
${}^{12}_{\Lambda}$C	2^-	1^-	61	175	−12	−13	−42	153	161
${}^{15}_{\Lambda}$N	$3/2^+_2$	$1/2^+_2$	65	451	−2	−16	−10	507	481
${}^{16}_{\Lambda}$O	1^-	0^-	−33	−123	−20	1	188	23	26
${}^{16}_{\Lambda}$O	2^-	1^-_2	92	207	−21	1	−41	248	224

Λ-Σ and spin-dependent contributions to ground-state binding energies

	$^{7}_{\Lambda}$Li	$^{8}_{\Lambda}$Li	$^{9}_{\Lambda}$Li	$^{9}_{\Lambda}$Be	$^{10}_{\Lambda}$B	$^{11}_{\Lambda}$B	$^{12}_{\Lambda}$B	$^{13}_{\Lambda}$C	$^{15}_{\Lambda}$N	$^{16}_{\Lambda}$N
	$1/2^+$	1^-	$3/2^+$	$1/2^+$	1^-	$5/2^+$	1^-	$1/2^+$	$3/2^+$	1^-
Λ-Σ	78	160	183	4	35	66	103	28	59	62
Δ	419	288	350	0	125	203	108	-4	40	94
S_Λ	0	-6	-10	0	-13	-20	-14	0	12	6
S_N	94	192	434	207	386	652	704	841	630	349
T	-2	-9	-6	0	-15	-43	-29	-1	-69	-45
Sum	589	625	952	211	518	858	869	864	726	412
Expt	5.58	6.80	8.50	6.71	8.89	10.24	11.37	11.69		13.76
		6.84	8.29		9.11					*
$\overline{V}$	-0.94	-1.02	-1.06		-1.05	-1.04	-1.05	-0.96		-0.93

* $B_\Lambda = 13.76(16)$ MeV: F. Cusanno et al. PRL 103, 202501 (2009)

To get a rough $\overline{V}$, take $B_\Lambda(^{5}_{\Lambda}\mathrm{He}) = 3.12$ MeV as s_Λ single-particle energy, and subtract sum from experimental B_Λ value.

Double one-pion exchange ΛNN interaction

Gal, Soper, and Dalitz: Ann. Phys. (N.Y.) 63, 53 (1971)

Independent of Λ spin. Averaged over s_Λ wave function gives

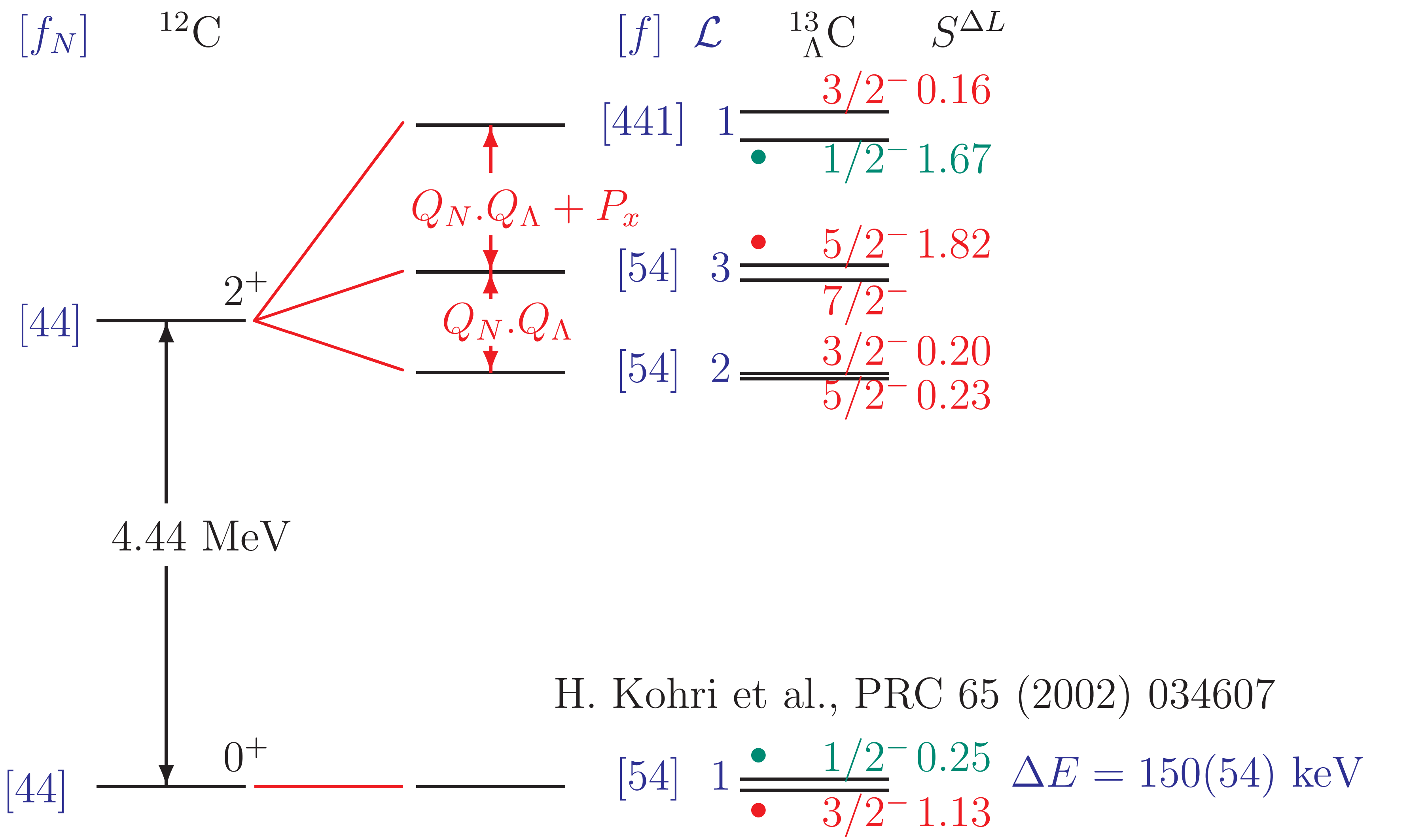
$$V_{NN}^{eff} = \sum_{klm} Q_{lm}^{k}(r_1, r_2)\,[\sigma_1, \sigma_2]^k \cdot [C_l(\hat{r_1}), C_m(\hat{r_2})]^k \;\; \tau_1 \cdot \tau_2$$

Parameters in MeV

Q_{00}^0	Q_{22}^0	Q_{22}^1	$Q_{02}^2 = Q_{20}^2$	Q_{22}^2
0.026	1.037	−0.531	−0.049	0.245

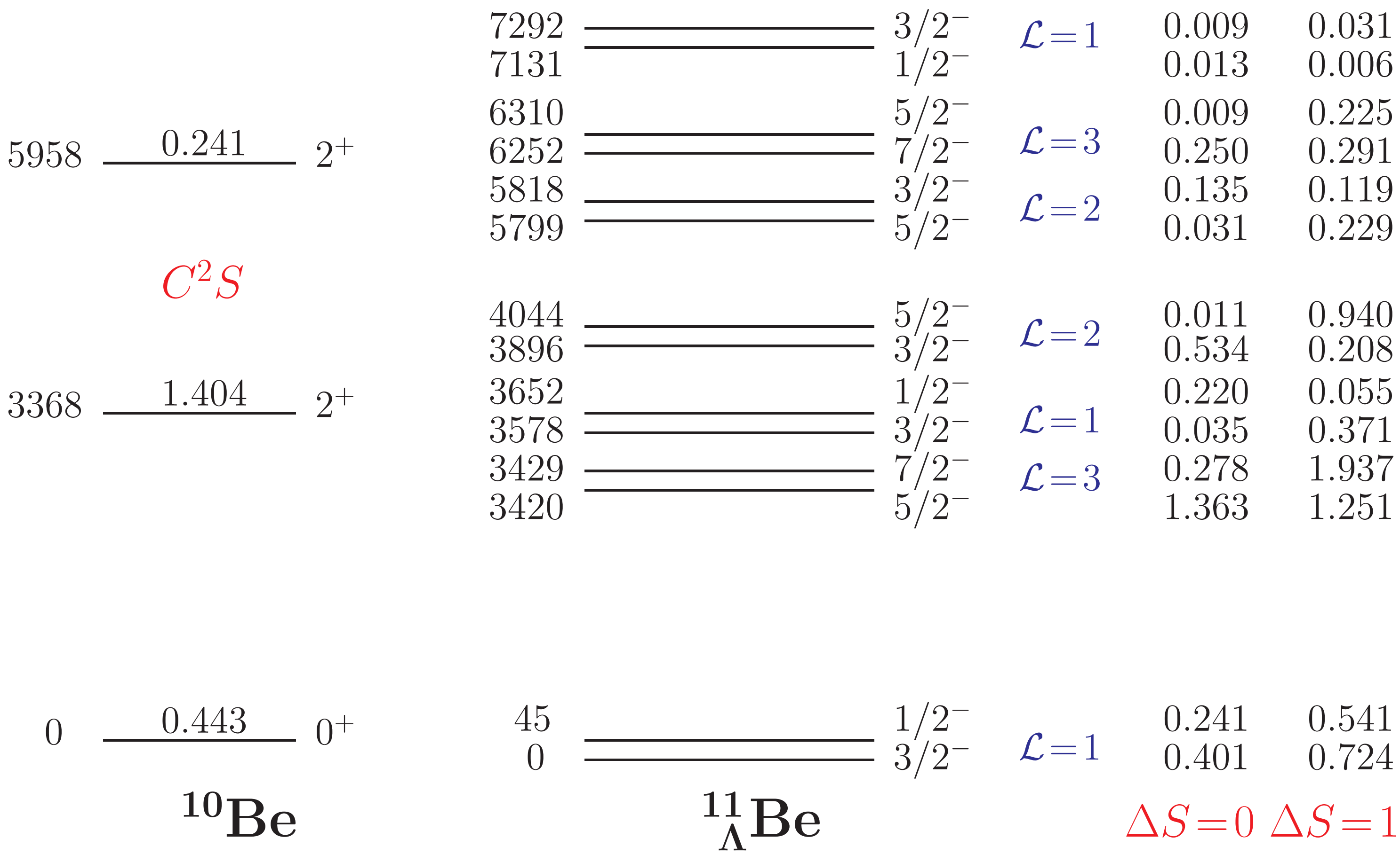
- Q_{00}^0 and Q_{22}^0 give repulsive contributions to B_Λ that depend quadratically on the number of p-shell nucleons in the core.
- Q_{22}^1 represents an anti-symmetric spin-orbit interaction that behaves rather like S_N

$^{12}\mathrm{C}(0^+, 2^+) \times p_\Lambda$ states of $^{13}_{\Lambda}\mathrm{C}$



$B_\Lambda = 11.69 \pm 0.12$ MeV $\quad 3/2^-/1/2^- \quad E_x = 10.83/10.98$ MeV

^{11}B(e,e′K$^+$)$^{11}_{\Lambda}$Be



Future Hypernuclear γ-ray Spectroscopy

- Hyperball-J at J-PARC (Tamura)
- $(K^-, \pi^-\gamma)$ $\quad p_K = 1.1$ or 1.5 GeV/c
- Larger spin-flip amplitudes - test case ${}^{4}_{\Lambda}\mathrm{He}\ 1^+ \longrightarrow 0^+$
- p-shell and light sd-shell nuclei as targets. First ^{19}F
- $(K^-, \pi^0\gamma)$ also possible

$(e, e'K^+)$ at JLab and Mainz

- This worshop.

J-PARC and GSI

- Many other aspects of Strangeness Nuclear Physics