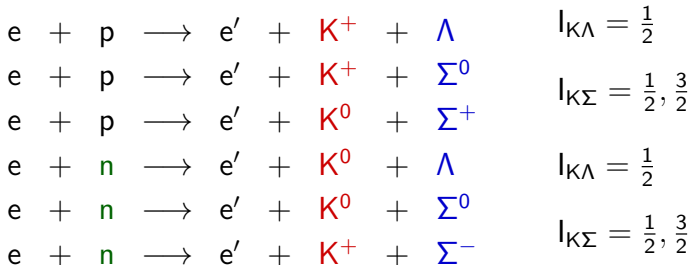


Electroproduction of the Λ hyperon on the proton

Workshop on Perspectives of high resolution hypernuclear spectroscopy at JLab

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electroproduction

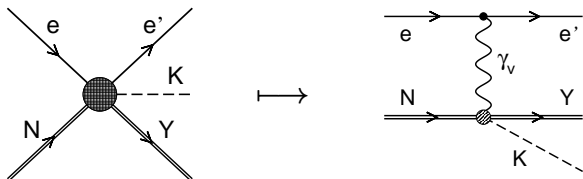


(virtual) photoproduction – a simpler process



Description of electroproduction

- the electromagnetic part of the process is well known
- the coupling constant is small $\alpha = \frac{e^2}{2\pi} \doteq \frac{1}{137} \ll 1$
→ the one-photon approximation:



- no Coulomb distortion of e and e'
– important for electroproduction on heavy nuclei
- leptonic and hadronic parts can be separated
- photoproduction is not a limit case of electroproduction ($q^2 \rightarrow 0$)

- kinematics

$$E_e, E'_e, \theta_e, \theta_K, \Phi_K$$

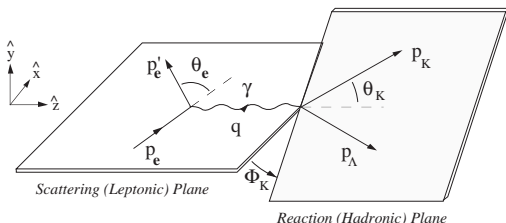
$$Q^2 = -q^2, \quad \epsilon = \left(1 + \frac{2\bar{q}^2}{-q^2} \tan^2 \frac{\theta_e}{2}\right)^{-1}, \quad s = (q + p_p)^2, \quad t = (q - p_K)^2, \quad \Phi_K$$

$$\sqrt{s} = W \in (W_{\text{thr}}, 2.6) \text{ GeV}, \quad W_{\text{thr}} = 1.609 \text{ for } K\Lambda$$

- the unpolarized cross section

$$\frac{d^5\sigma}{dE'_e d\Omega'_e d\Omega_K} = \Gamma \left[\frac{d\sigma_T}{d\Omega_K} + \epsilon \frac{d\sigma_L}{d\Omega_K} + \epsilon \frac{d\sigma_{TT}}{d\Omega_K} \cos 2\Phi_K + \sqrt{2\epsilon(1+\epsilon)} \frac{d\sigma_{TL}}{d\Omega_K} \cos \Phi_K \right]$$

- in Laboratory frame:



Motivation

- Which degrees of freedom are relevant? *partons* \leftrightarrow *hadrons*
- non perturbative regime of QCD: *quark models* \leftrightarrow *isobar models*
- many opened channels \rightarrow channel couplings, unitarity, final-state interaction
multi-channel \leftrightarrow *single-channel analysis*
- description of the background part: $f = f_{bgr} + f_{res}$
isobar \leftrightarrow *Regge approach*
- effective Lagrangian approach
 - properties of baryon resonances in the 3rd resonance region ($E_{\gamma}^{lab} \geq 1$ GeV)
 \rightarrow parameters, structure, existence (missing states)
 - SU(3)_f symmetry: $g_{\pi NN} \leftrightarrow g_{K\Lambda N}$
 - dynamics of the process – important resonances N^* , Y^* , K^*
 - a satisfactory description of the process at small angles
 \rightarrow an input in hypernucleus calculations
- plenty of new experimental data
- studying all channels is important – neutron target, production of K^0

K^+ production in the small-angle region

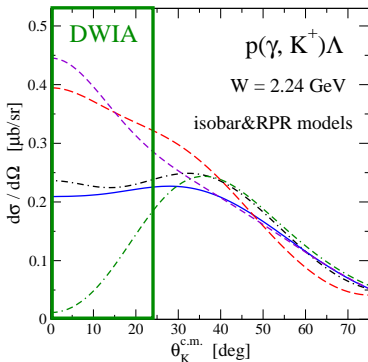
- the elementary amplitude is an input in DWIA \Rightarrow a good description of the elementary process is important for reliable predictions of the hypernucleus-production cross sections

DWIA (frozen-nucleon approx.):

$$\langle \Psi_H | \sum_i \chi_\gamma \chi_K^* \mathcal{J}^\mu(i) | \Psi_A \rangle$$

\mathcal{J}^μ – elementary hadron current in lab frame

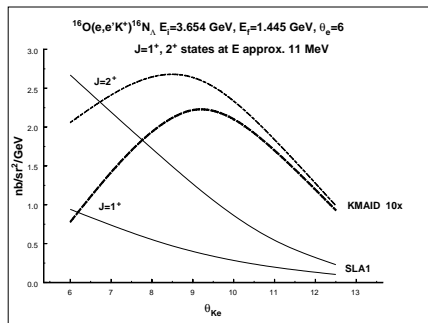
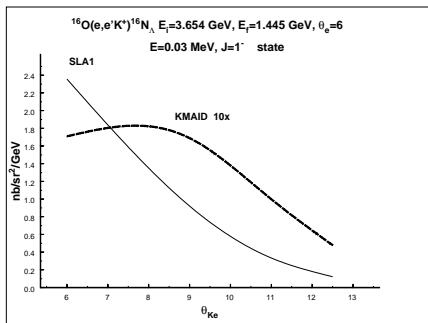
– the main contribution of \mathcal{J}^μ is from **a small θ_K region**



- vice versa: the hypernucleus-production cross sections can be used to test the elementary models in the small-angle region

Angular dependence of the hypernucleus cross section

- electroproduction of $^{16}\text{N}_\Lambda$ ($E_\gamma^{\text{lab}} = 2.21 \text{ GeV}$, $\theta_e^{\text{lab}} = 6^\circ$)
→ magnitudes differ by factor of 10
- generally a steeply decreasing dependence, the slope depends on the spin (J)
- the angular behaviour depends on the spin structure of the elementary amplitude at small θ_K → more detailed information about the amplitude



Methods of description

- single-channel analysis
 - ▶ **Isobar model**
 - effective hadronic Lagrangian description
 - Adelseck-Sanghai, Williams-Ji-Cotanch, Saclay-Lyon, Kaon-MAID, Gent isobar, Maxwell
 - ▶ **Regge-plus-resonance model**
 - hybrid isobar-Regge description
 - Gent group: RPR-2007, RPR-2011
 - ▶ multipole analysis – Mart and Sulaksono
- multi-channel analysis
 - ▶ unitary isobar approach (coupled channels)
 - Giessen, Bonn-Gatchina, Dubna-Mainz-Taipei, Julia-Diaz et al, Usov et al, Shyam et al
 - ▶ partial-wave analysis – SAID
 - ▶ chiral unitary framework
 - chiral Lagrangian, coupled channels, threshold region
 - Borasoy et al
- Quark model – resonances included; Close, Zhenping Li et al

Iso-bar model

- meson-baryon FSI is neglected (channel couplings, re-scattering)
→ one-channel approximation: $T = V + V G t$
 - ▶ violation of unitarity
 - ▶ effective coupling constants include a part of the FSI effects
- driving term V :
 - ▶ effective hadron Lagrangian (fields, couplings, resonances, form factors)
 - ▶ tree-level perturbation expansion \Rightarrow s -, t -, and u -channel exchanges; contact term (gauge invariance)
 - ▶ a set of relevant resonances in the intermediate states has to be selected (3rd resonance region: heavy, high-spin, or missing resonances)
 - ▶ free parameters are fitted to data ($\approx 10 - 20$ parameters)
 - ▶ many sets of resonances with a good χ^2 can be chosen
 \Rightarrow large number of possible models \Rightarrow constraints or better data analysis
- constraints on the models:
 - ▶ SU(3) symmetry: $-4.4 \leq g_{KN\Lambda}/\sqrt{4\pi} \leq -3.0$ $0.8 \leq g_{KN\Sigma}/\sqrt{4\pi} \leq 1.3$
 - ▶ crossing symmetry: description of $\Gamma(K^- p \rightarrow \gamma \Lambda)$

Iso-bar model

- large contributions to the background part of the amplitude from the Born diagrams are reduced:
 - ▶ hadronic form factors – gauge invariance → a contact term
 - inclusion of a hadron structure
 - dipole, Gauss, multidipole-Gauss (high spin) form
 - which cut-off parameter: hard × soft?
 - ▶ hyperon resonances in u channel – which are relevant?
 - ▶ hadronic form factors + hyperon exchanges
 - ▶ this problem is avoided in the [Regge-plus-resonance model](#)
- the resonant part: s -channel exchanges of the nucleon resonances
(resonance width in the Feynman propagator → partial restoration of unitarity)
- electroproduction (Q^2 dependence)
 - ▶ electromagnetic form factors (gauge invariance)
 - EVDM (electron scattering data) for γNN and QM for the γNN^* transitions
 - ▶ couplings of baryon fields to the photon transversal modes
 - ▶ couplings to the longitudinal mode of the virtual photon
$$\bar{\psi}_{N^*} \Gamma \gamma^\nu \psi_N \partial^\mu F_{\mu\nu} \quad (\text{spin } 1/2) \quad \bar{\psi}_{N^*}^\nu \Gamma \psi_N \partial^\mu F_{\mu\nu} \quad (\text{spin } 3/2)$$
$$\Rightarrow \text{the coupling constants have to be determined in electroproduction}$$

Regge-plus-resonance model

Invariant amplitude: $\mathcal{M} = \mathcal{M}_{bgr}(Regge) + \mathcal{M}_{res}(isobar)$

- the background part - exchanges of degenerate K and K^* trajectories

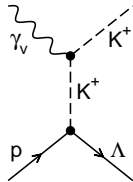
$$\mathcal{M}_{bgr}(s, t) = \mathcal{P}_{Regge}^K(s, t) \times \beta_K(s, t) + \mathcal{P}_{Regge}^{K^*}(s, t) \times \beta_{K^*}(s, t) \\ + \mathcal{M}_{Feyn}^{p,elec} \times \mathcal{P}_{Regge}^K(s, t) \times (t - m_K^2)$$

- the Regge propagator

$$\mathcal{P}_{Regge}^x(s, t) = \frac{(s/s_0)^{\alpha_x(t)}}{\sin \pi \alpha_x(t)} \frac{\pi \alpha'_x}{\Gamma(1 + \alpha_x(t))} \left\{ \begin{array}{c} 1 \\ e^{-i\pi \alpha_x(t)} \end{array} \right\}$$

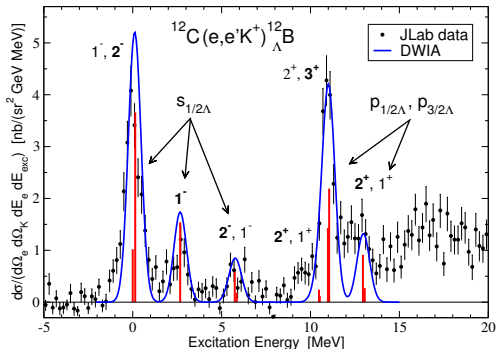
- the Regge trajectories: $\alpha_x(t) = \alpha'_x(t - m^2)$, $x = K, K^*$
- the residue of the lowest pole

$$\text{for } t \rightarrow m_K^2 \quad \mathcal{P}_{Regge}^K(s, t) \times \beta_K(s, t) \longrightarrow \frac{\beta_K(s, t)}{t - m_K^2}$$



- only 3 parameters fixed by high-energy data ($W > 2.6$ GeV)

Electroproduction of hypernuclei



data: JLab Hall A
exp. E94-107

$$E_{\gamma}^{\text{lab}} = 2.21 \text{ GeV}$$

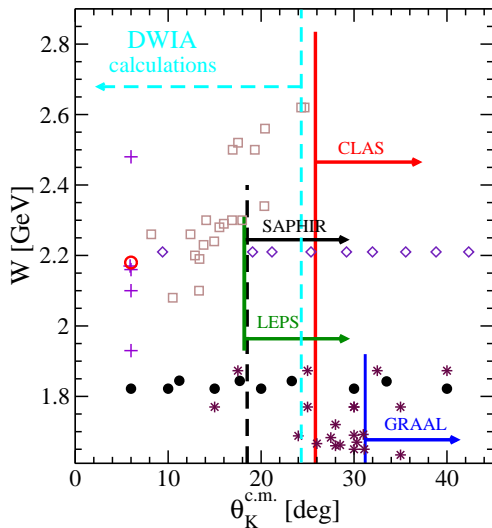
$$\theta_e^{\text{lab}} = \theta_K^{\text{lab}} = 6^{\circ}$$

$$Q^2 = 0.018 \text{ (GeV/c)}^2$$

calculations with
the Saclay-Lyon model

- in DWIA the elementary hadron current significantly contributes only for small θ_K and experiments are done at very small Q^2
 \Rightarrow **predictions of isobar models have to be reliable for small θ_K and Q^2**
- $\sigma_{TT} \sim \sin^2 \theta_K$, $\sigma_{TL} \sim \sqrt{Q^2} \sin \theta_K$, and $\sigma_L \sim Q^2$
 \Rightarrow σ_T (\Leftrightarrow photoproduction cross section) **should dominate** (unpolarized case)
 \Rightarrow **the models could be well determined by the photoproduction data**

World data for small θ_K and Q^2 kinematics



$0 < Q^2 < 0.3 \text{ (GeV/c)}^2$

CLAS 2005 - 2010

SAPHIR 2003

LEPS 2003-2007

GRAAL 2007

● Bleckmann 1970

□ Azemoon 1975

◇ Brauel 1979

+ Brown 1972

* old data 1959--72

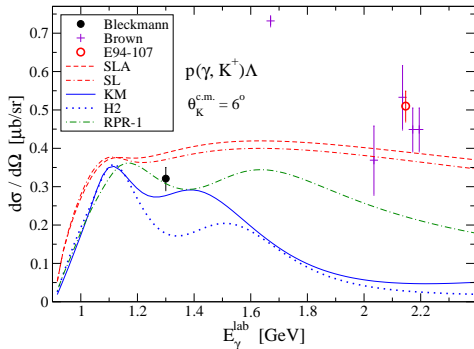
○ E94-107 2008

Predictions of isobar and RPR models differ at very small θ_K and Q^2

- photoproduction (σ_T) dominates the electroproduction cross section

$$\sigma = \sigma_T + \epsilon\sigma_L + \epsilon\sigma_{TT} + \sqrt{2\epsilon(1+\epsilon)}\sigma_{TL}, \quad \Phi_K = 0$$

$$\text{SL: } 0.41 = 0.35 + 0.004 - 0.001 + 0.06 \quad (\epsilon=0.7, \theta_K=6^\circ, Q^2=0.07 \text{ (GeV/c)}^2)$$



Electroproduction data:

Brown72 [PRL28(1972)1086]

$$\sigma = \sigma_T + \epsilon\sigma_L$$

$$W = 1.93 \div 2.17 \text{ GeV}$$

$$Q^2 = 0.18 \text{ and } 0.29 \text{ (GeV/c)}^2$$

$$\theta_K^{\text{c.m.}} = 5.9^\circ \text{ and } 6^\circ$$

E94-107 (JLab, Hall A)

$$W = 2.2 \text{ GeV}$$

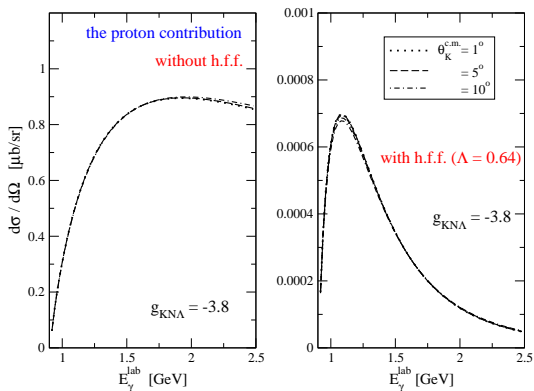
$$Q^2 = 0.07 \text{ (GeV/c)}^2$$

$$\theta_K^{\text{c.m.}} = 6^\circ$$

- **Isobar model H2:** $N^* - S_{11}(1650), P_{11}(1710), P_{13}(1720), D_{13}(1895); Y^* - S_{01}(1670), S_{01}(1800)$; hadronic form factors; fit to CLAS and LEPS data
- **Regge-plus-resonance model RPR-1:** $N^* - S_{11}(1535) S_{11}(1650), P_{11}(1710), P_{13}(1720), D_{13}(1895)$; multidipole-Gauss hadronic f.f.; fit to CLAS and LEPS data.

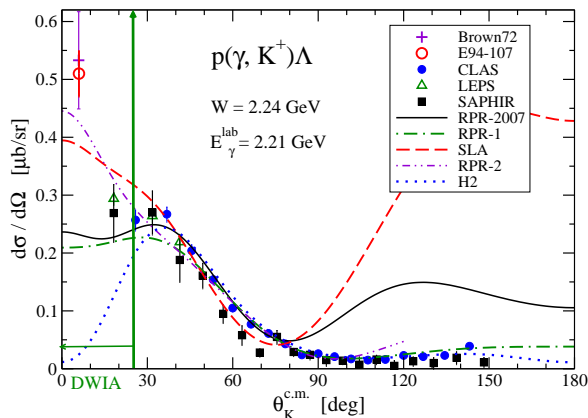
Suppression of $\sigma(\theta)$ at small θ_K due to the hadronic form factors

- The proton exchange:
 - ▶ contributes to the background part of the amplitude
 - ▶ is important contribution at small θ_K
 - ▶ is suppressed by **hadronic form factors**, e.g. in KM and H2 models
 - ▶ is not suppressed in SL and RPR models



The angular dependence by isobar and RPR models

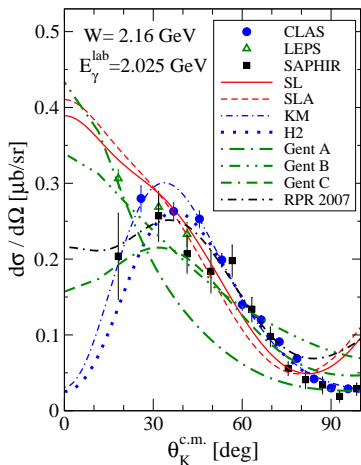
- H2 and RPR-1 fitted to CLAS and LEPS data \Rightarrow a good agreement for $30^\circ < \theta_K < 150^\circ$
- for $\theta_K < 30^\circ$ H2 suppressed by the hadronic form factor
- no suppression of the background part for the RPR-1 and SLA models
- SLA preferred by the hypernucleus data: DWIA calculations of ${}^{12}_\Lambda\text{B}$, ${}^{16}_\Lambda\text{N}$, and ${}^9_\Lambda\text{Li}$



RPR-2 was fitted only to $0^\circ < \theta_K < 90^\circ$

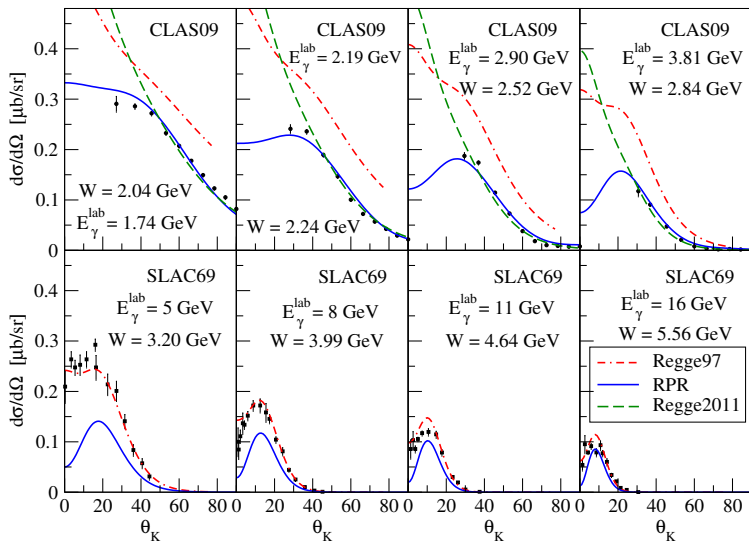
	RPR-1	RPR-2
$g_{K\Lambda p}$	-1.4	-3.0
$G_{K^*}^V$	-0.30	0.19
$G_{K^*}^T$	-1.80	0.31

The angular dependence by isobar and RPR models



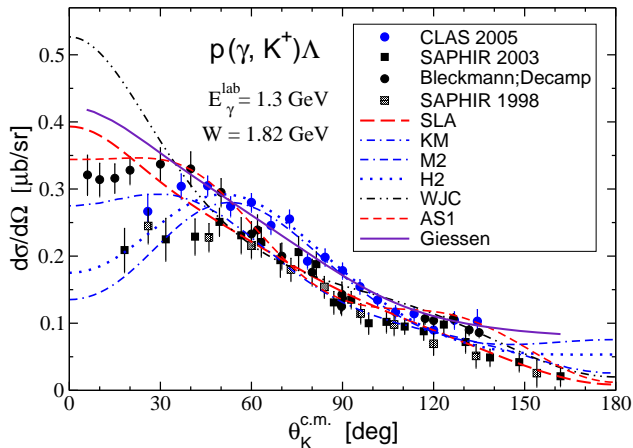
- note inconsistency of CLAS and SAPHIR data at $\theta \approx 20^\circ$
- H2 fits very well the CLAS data but it fails for $\theta_K < 30^\circ$
 \Rightarrow the CLAS data alone cannot fully determine the angular dependence of models
- Gent A, B, and C differ in the $\Lambda_{\text{cut}}^{\text{bgr}}$: 0.413, 1.538, and 1.856, respectively
- the hypernucleus electroproduction cross sections suggest that SL gives right predictions of the elementary cross section \rightarrow a forward-peaked cross section?

The very forward angular dependence above the resonance region



The angular dependence near threshold by isobar and Giessen models

- data inconsistency for $\theta_K < 50^\circ$
- the Giessen model is consistent rather with SL than with H2, M2 or KM for $\theta_K < 30^\circ$



Summary

- the calculations of hypernucleus electroproduction cross sections in DWIA require a proper understanding of the elementary process in the small θ_K and Q^2 kinematical region;
- predictions of the isobar and Regge-plus-resonance models for the cross sections still significantly differ in this kinematical region due to lack of data;
- the ample world data (CLAS, SAPHIR, LEPS,..) on the process cover mainly the region $\theta_K > 20^\circ$ which is not enough to fully determine the angular dependence of the elementary cross section;
- **new good quality data at very small θ_K and Q^2 are needed** to test the models which then will be able to provide more reliable predictions of the cross sections for electroproduction of hypernuclei;
- the small-angle data can contribute to revealing dynamics of the models, e.g. the background description, a role of the hyperon resonances, hadronic form factors (Λ_{cut}).