Few-Body Hypernuclei

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“Perspectives of high resolution hypernuclear spectroscopy at Jefferson Lab“, Newport News, VA

- Motivation
- Numerical technique
- Light Hypernuclei
  - dependence on NN and 3N force
  - separation energies based on chiral interactions
  - CSB of four-body hypernuclei
- Conclusions & Outlook

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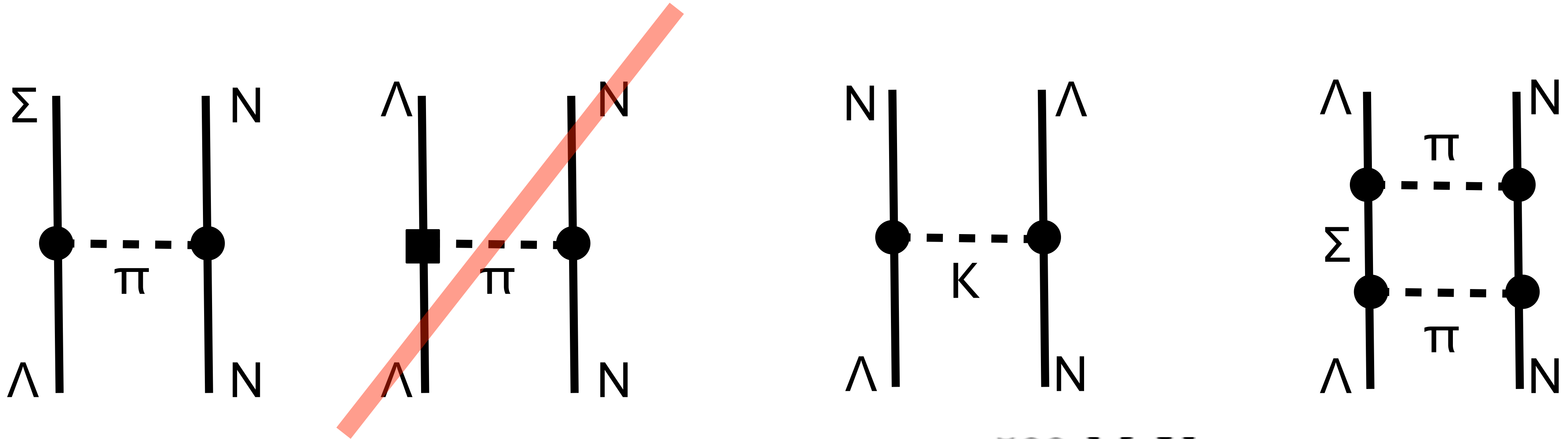
Hypernuclear interactions

Why is understanding hypernuclear interactions interesting?

(SN1987a)

- „phenomenologically“
  - *hyperon contribution to the EOS, neutron stars, supernovae*
  - *Λ as probe to nuclear structure*
- conceptually
  - ***Λ-Σ** conversion process*
  - *experimental access to explicit chiral symmetry breaking*



$m_K \approx 500$ MeV

suppressed by
isospin symmetry (CSB!)

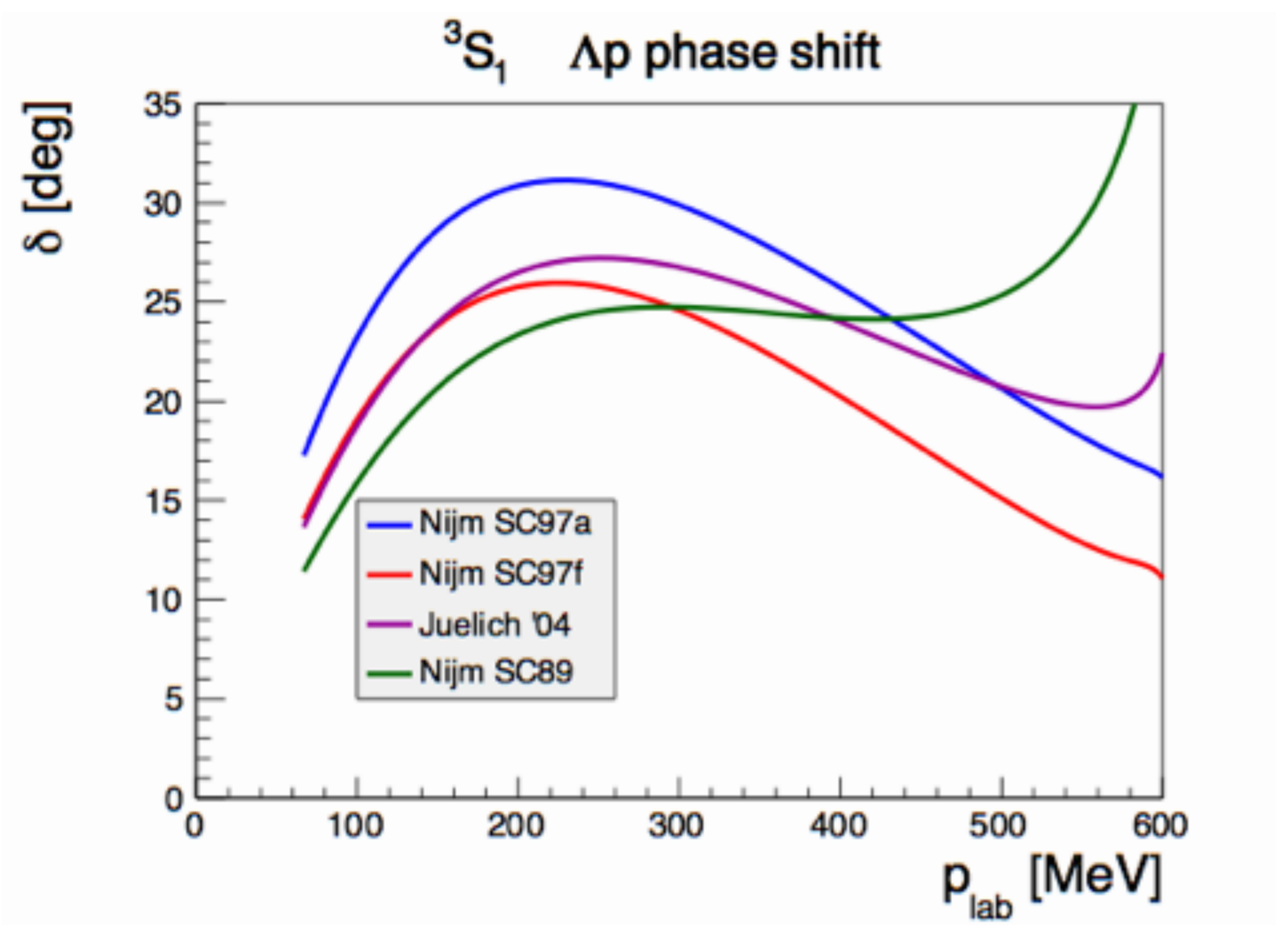
Hypernuclear interactions

35 YN data, no YN bound state, large uncertainties

→ no partial wave analysis possible

YN interaction models (Jülich 89/04, Nijmegen 89/97a-f, ESC, …)

describe all data **more than perfectly**, but are not phase equivalent



	1	3
SC97a	-0.7	-2.15
SC97b	-0.9	-2.11
SC97c	-1.2	-2.06
SC97d	-1.7	-1.93
SC97e	-2.1	-1.83
SC97f	-2.5	-1.73
SC89	-2.6	-1.38
Jülich ´04	-2.6	-1.73

How to further constrain the YN interactions?

Hypernuclei

- *ΛN interactions are generally weaker than the NN interaction*
  - *naively:* ***core nucleus + hyperons***
  - *„separation energies“ are almost independent from NN(+3N) interaction*
- *no Pauli blocking of Λ in nuclei*
  - *good to study nuclear structure*
  - *even light hypernuclei exist in* ***several spin states***
- *size of YNN interactions?*
- ***non-trivial constraints*** *on the YN interaction even from lightest ones*

$$^{3}_{\Lambda}\mathrm{H}\left(\frac{1}{2}^{+}\right)$$

$$^{4}_{\Lambda}\mathrm{H}\left(0^{+}\right) - ^{4}_{\Lambda}\mathrm{He}\left(0^{+}\right)$$

$$^{4}_{\Lambda}\mathrm{H}\left(1^{+}\right) - ^{4}_{\Lambda}\mathrm{He}\left(1^{+}\right)$$

(from Panda@FAIR web page)

Numerical technique

non-rel. Schrödinger equation $\Psi = G_0 V \Psi$ $G_0 = \frac{1}{E - H_0}$

→ decomposition in five **Yakubovsky components**

$$\Psi = (1+P)(\psi_{1A} + \psi_{1B} + \psi_{2A} + \psi_{2B}) + (1 - P_{12})(1+P)\psi_{1C}$$

solution of the **Yakubovsky equations**

$$\psi_{1A} = G_0 t_{12} P(\psi_{1A} + \psi_{1B} + \psi_{2A}) + (1 + G_0 t_{12}) G_0 V_{123}^{(3)} \Psi$$

$$\psi_{1B} = G_0 t_{12}((1 - P_{12})(1 - P_{23})\psi_{1B} + P\psi_{2B})$$

$$\psi_{1C} = G_0 t_{14}(\psi_{1A} + \psi_{1B} + \psi_{2A} - P_{12}\psi_{1C} + P_{12}P_{23}\psi_{1C} + P_{13}P_{23}\psi_{2B})$$

$$\psi_{2A} = G_0 t_{12}((P_{12} - 1)P_{13}\psi_{1C} + \psi_{2B})$$

$$\psi_{2B} = G_0 t_{34}(\psi_{1A} + \psi_{1B} + \psi_{2A})$$

$(P = P_{12}P_{23} + P_{13}P_{23})$

→ improved convergence in terms of partial waves

we carefully checked convergence with respect to partial waves, stability with respect to mesh points, ...

(see Nogga et. al., PRL 88,172501 (2002))

Known results I: independendence of NN force

- Λ separation energies

$$E_\Lambda = E(core) - E(hypernucleus)$$

are not strongly dependent on the NN interaction

$^4_\Lambda\mathrm{He}$	0^+		1^+		Δ
	E_B	E_Λ	E_B	E_Λ	
Bonn B	-8.92	1.66	-8.04	0.80	0.84
Nijm 93	-8.55	1.54	-7.69	0.72	0.79
Nijm 93 + TM	-9.32	1.56	-8.35	0.70	0.82

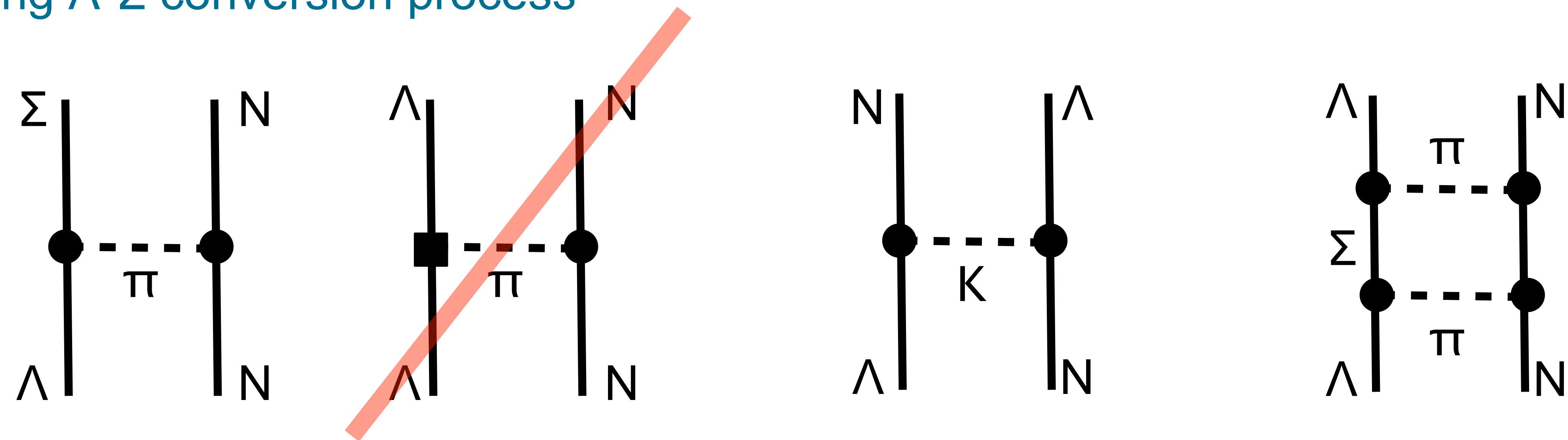
for YN interaction: **SC97e**

(AN, Kamada, Glöckle, 2002)

→ **YN interaction can be discussed independently of an NN and 3N force model**

Known results II:
Λ-Σ conversion is important

strong Λ-Σ conversion process



suppressed by
isospin symmetry

$m_K \approx 500$ MeV

$2m_\pi + m_\Sigma - m_\Lambda \approx 360$ MeV

- strong conversion process
- mass difference comparable to typical momenta
- no π-exchange ΛN-ΛN interaction

→

- *ΛN is **weaker** than NN interaction*
- ***Σs need to be explicitly included in any realistic calculation***

test: use $t_{\Lambda N}$ in Yakubovsky equations (here for a chiral interaction)

	w/ Σ	w/o Σ
E	1.47	1.01
E	0.71	0.49

→ ***effective ΛN interactions are not useful to study YN forces***

Known results III: model dependence

	${}^3_\Lambda\mathrm{H}$ in MeV	${}^4_\Lambda\mathrm{He}$ 0^+ in MeV	1^+	${}^1a_{\Lambda p}$ in fm	${}^3a_{\Lambda p}$	P_Σ
SC97d	-	1.3	0.8	-1.7	-1.9	1.5 %
SC97e	0.02	1.5	0.7	-2.1	-1.8	1.6 %
SC97f	0.08	1.7	0.5	-2.5	-1.7	1.8 %
SC89	0.15	2.1	0.02	-2.6	-1.4	4.1 %
Jülich 04	0.13	1.9	2.3	-2.6	-1.7	0.9 %
Expt	0.13	2.4	1.2	?	?	-

mostly from (AN, Kamada, Glöckle, 2002)

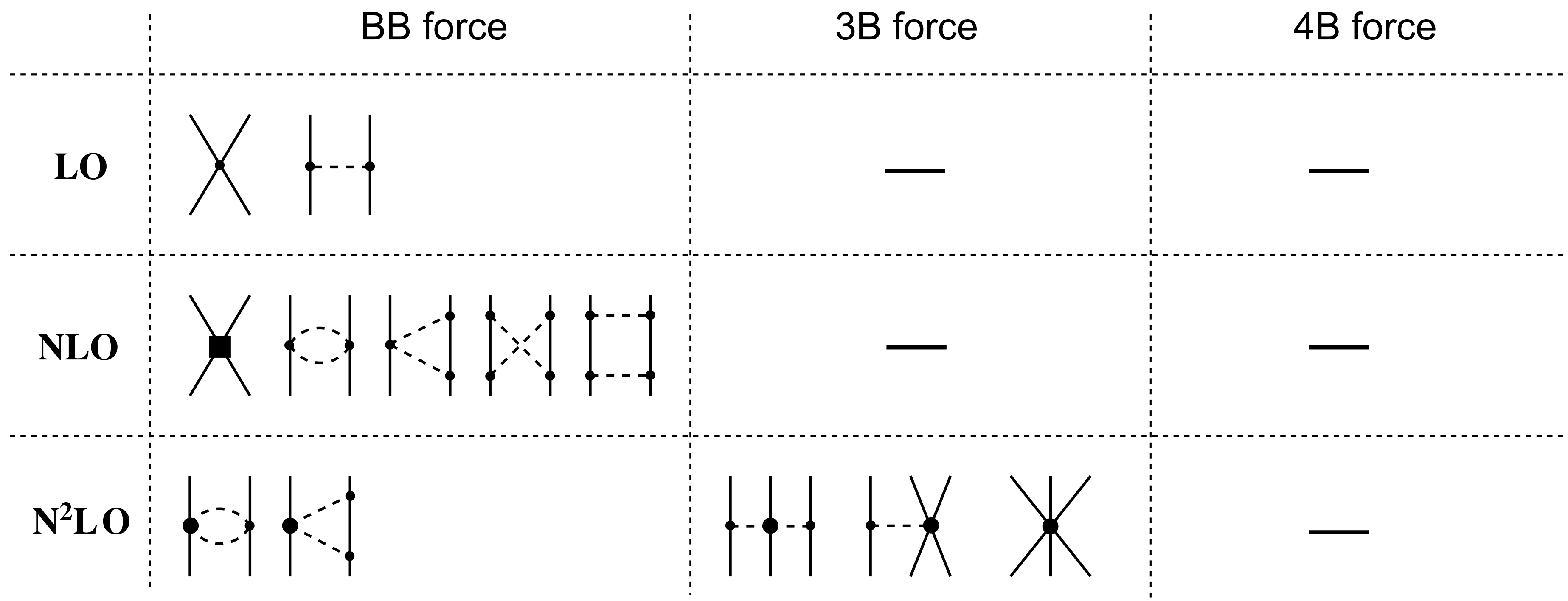
- **none** of these interaction models predicts the hypernuclei correctly
- no strict relation of the scattering lengths to any separation energy

With this in mind:

- **qualitative** study of predications based on LO and NLO interactions w/o SU(3) breaking
- first **attempt** to estimate N2LO/3BF contribution by variation of λ
- qualitative study of **CSB** of ${}^4_\Lambda\mathrm{H} - {}^4_\Lambda\mathrm{He}$

Chiral NN & YN interactions

reminder:



5 NN/YN short range parameters

26 NN/YN short range parameters

(from Epelbaum, 2008)

additional constraints required (only 35 data, but 26 parameters at NLO)

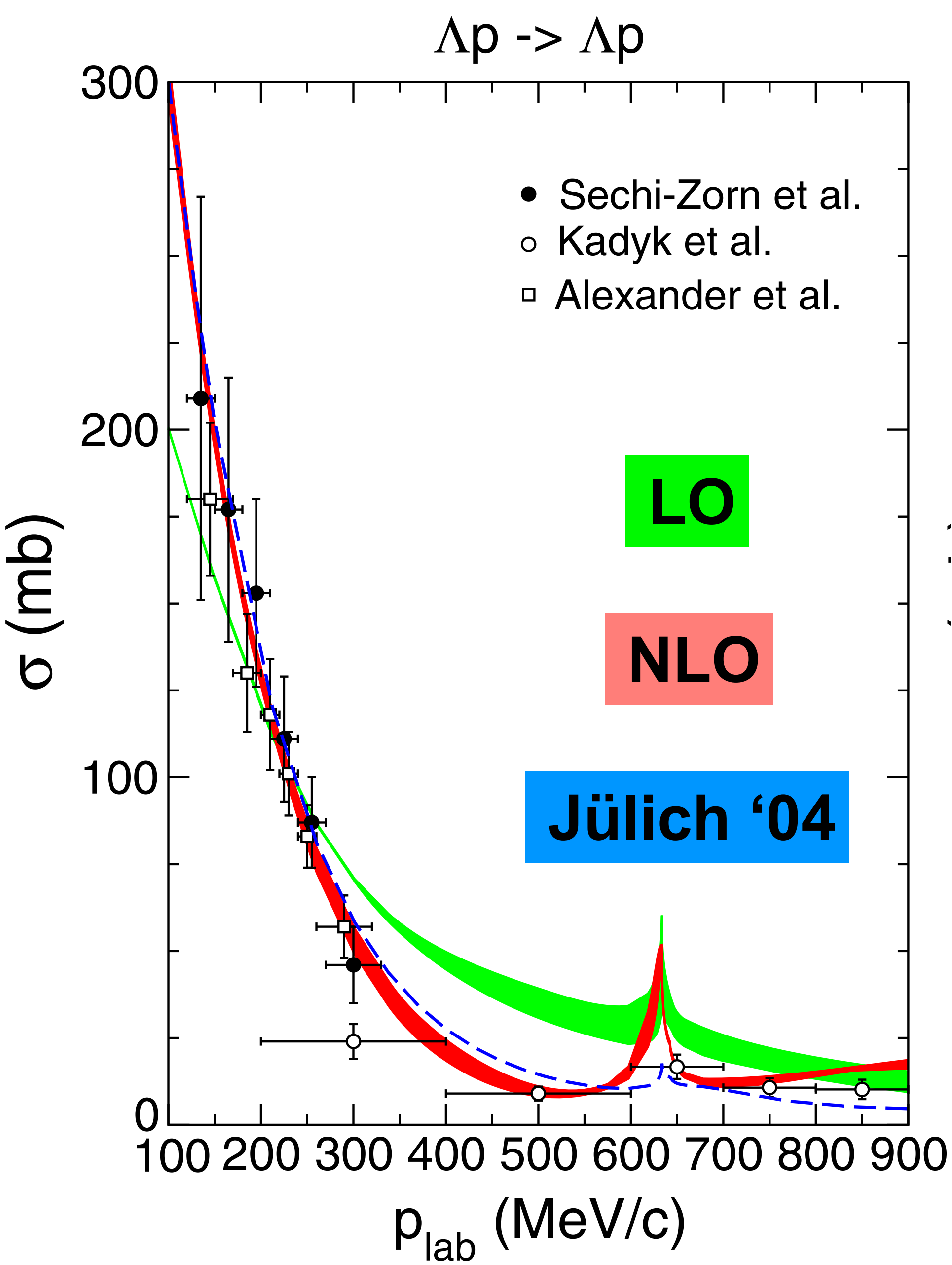
- SU(3) broken by physical m_π,m_K,m_η
- no SU(3) breaking in contact terms (although expected) → 23 contact terms
- no SU(3) breaking in F_π,F_K,F_η"
- „minimize" P-waves and $^1P_1$-$^3P_1$ mixing → only 13 parameters determined by data
- realizations for λ = 450 ... 700 MeV

→ one possible realization at NLO

more constraints required

(J. Haidenbauer et al., 2013
& previous talk)

Chiral interactions at LO, NLO

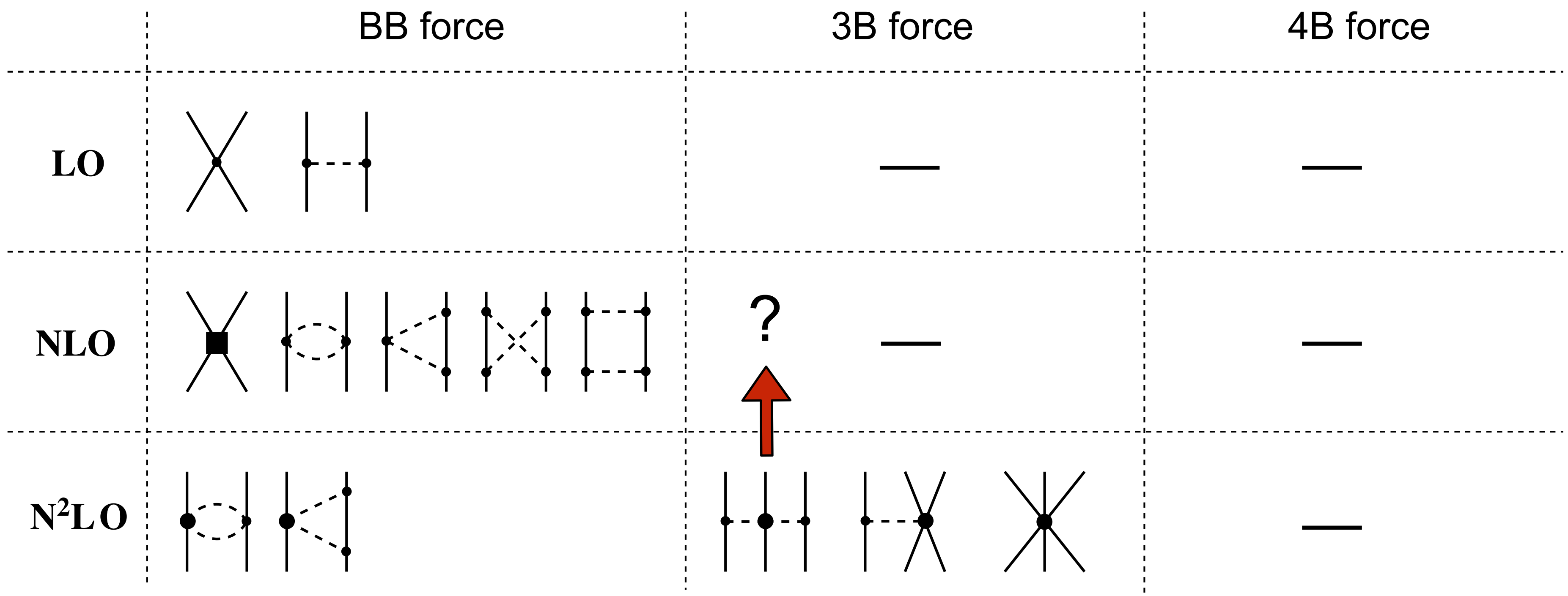


	1	3
LO	-1.9	-1.2
NLO	-2.9	-1.5 ... -1.7
Jülich ´04	-2.6	-1.7

(Polinder et al., NPA 779, 244 (2006),
Haidenbauer et al. , NPA 915, 24 (2013)
see Johann Haidenbauer's talk)

- hypertriton binding energy provides constraint on spin dependence of the YN interaction
- better description of the energy dependence in NLO
- significantly increased scattering lengths in NLO compared to LO

How important are 3B forces?



(from Epelbaum, 2008)

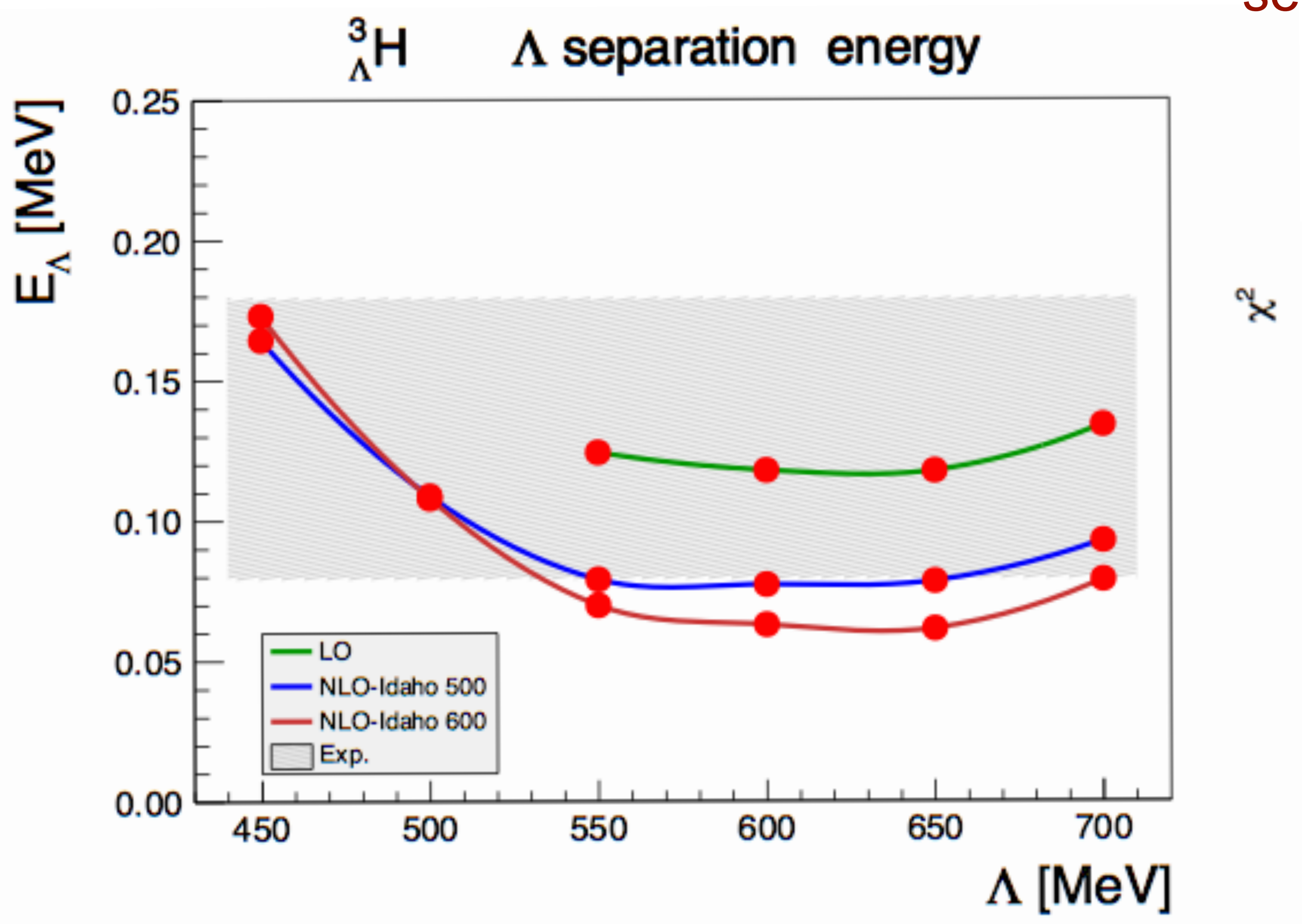
- we explicitly include the Σ ! (otherwise the 3BF should be LO)

- the missing 3BF are either **short-ranged** or induced by **decouplet** baryons (**Σ***, Δ)

Important tool to estimate 3BF in absence of explicit calculations:

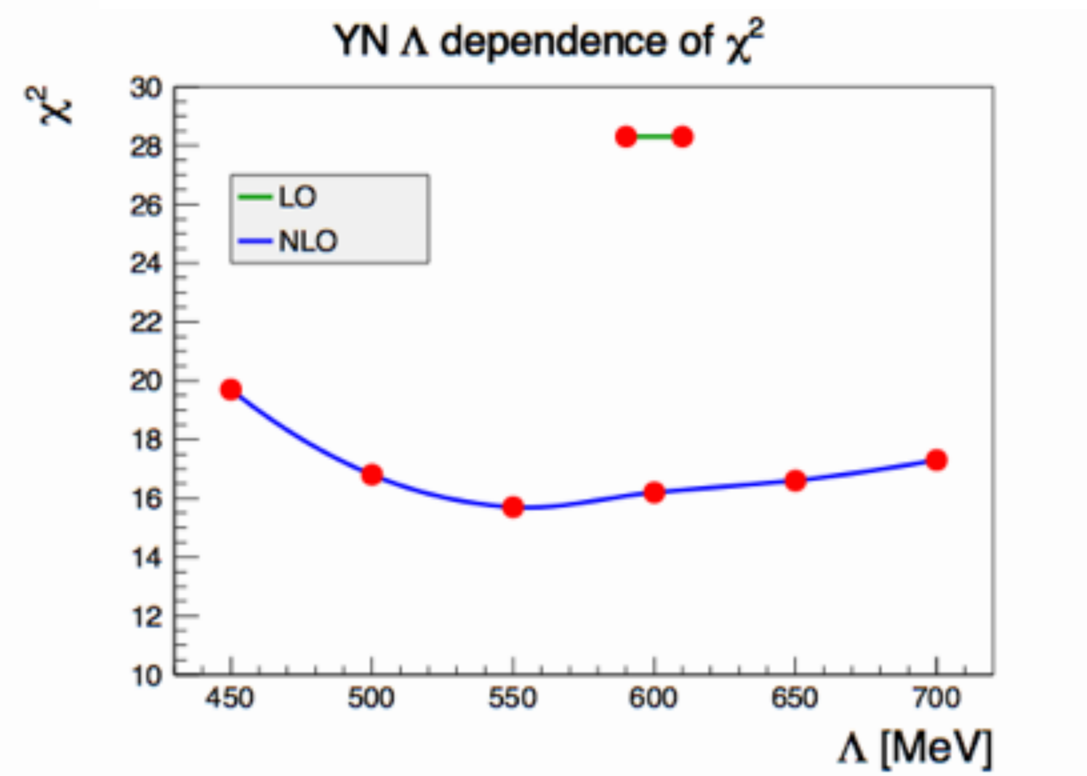
cutoff variations allow one to get lower bounds on their contribution

Hypertriton separation energies



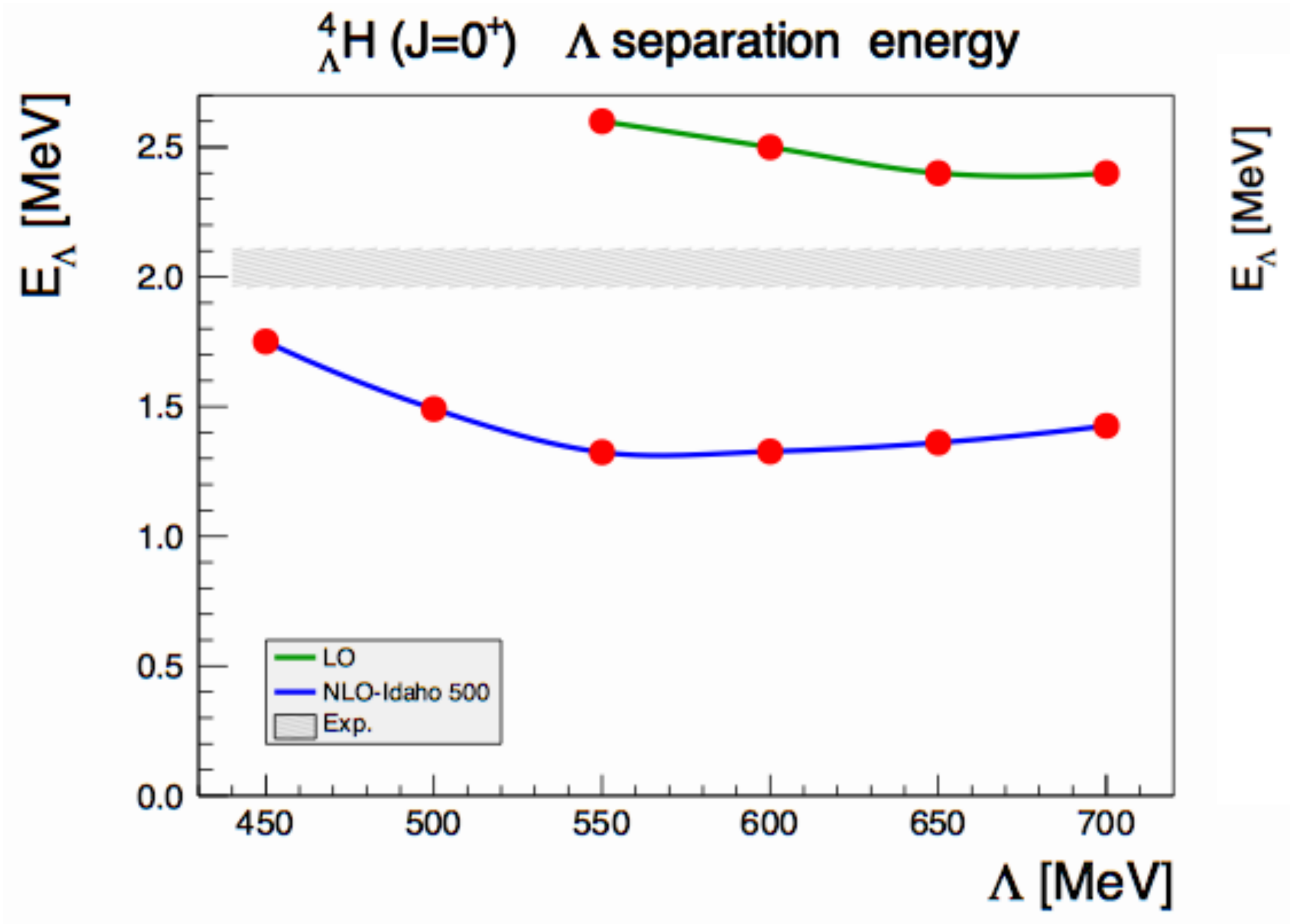
separation energies:

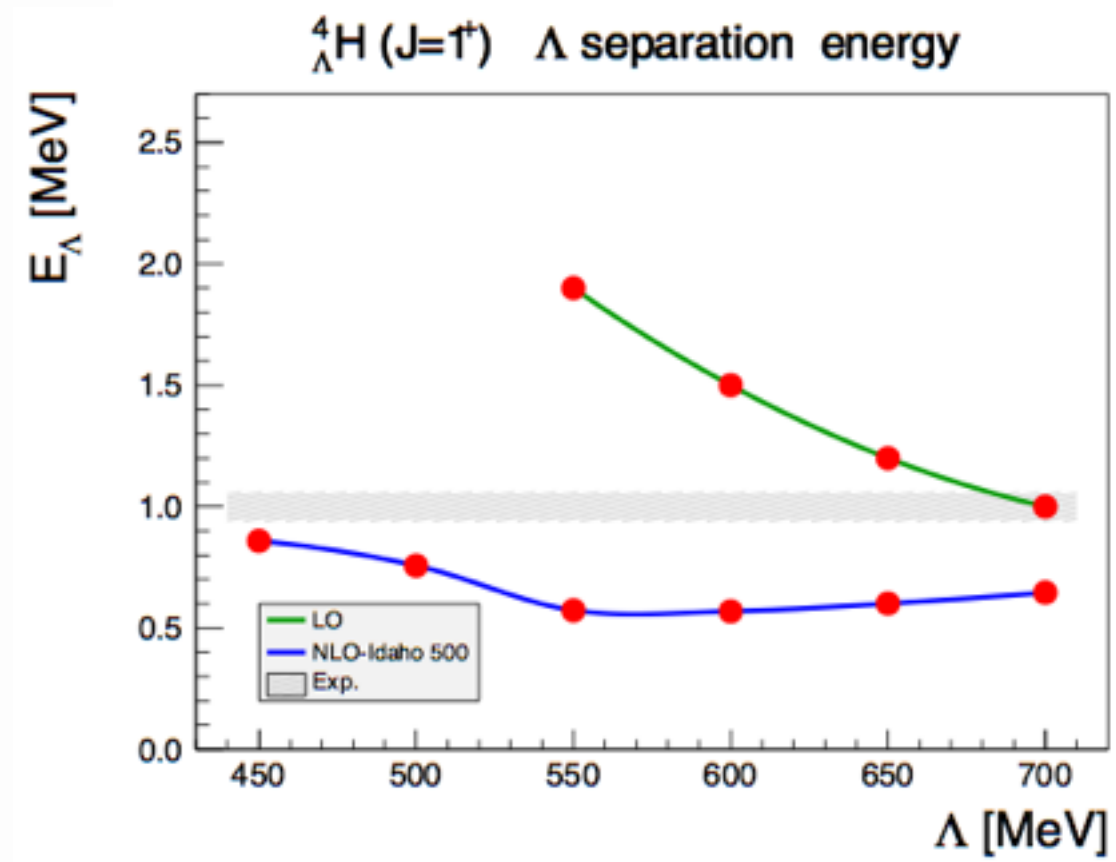
$$E_\Lambda = E(core) - E(hypernucleus)$$



- singlet scattering length for one cutoff chosen so that hypertriton binding energy is OK
- cutoff variation
  - is **lower bound** for magnitude of higher order contributions
  - correlation with χ^2 of YN interaction ?
- long range 3BFs need to be explicitly estimated

Separation energies for ${}^{4}_{\Lambda}\mathrm{H}$

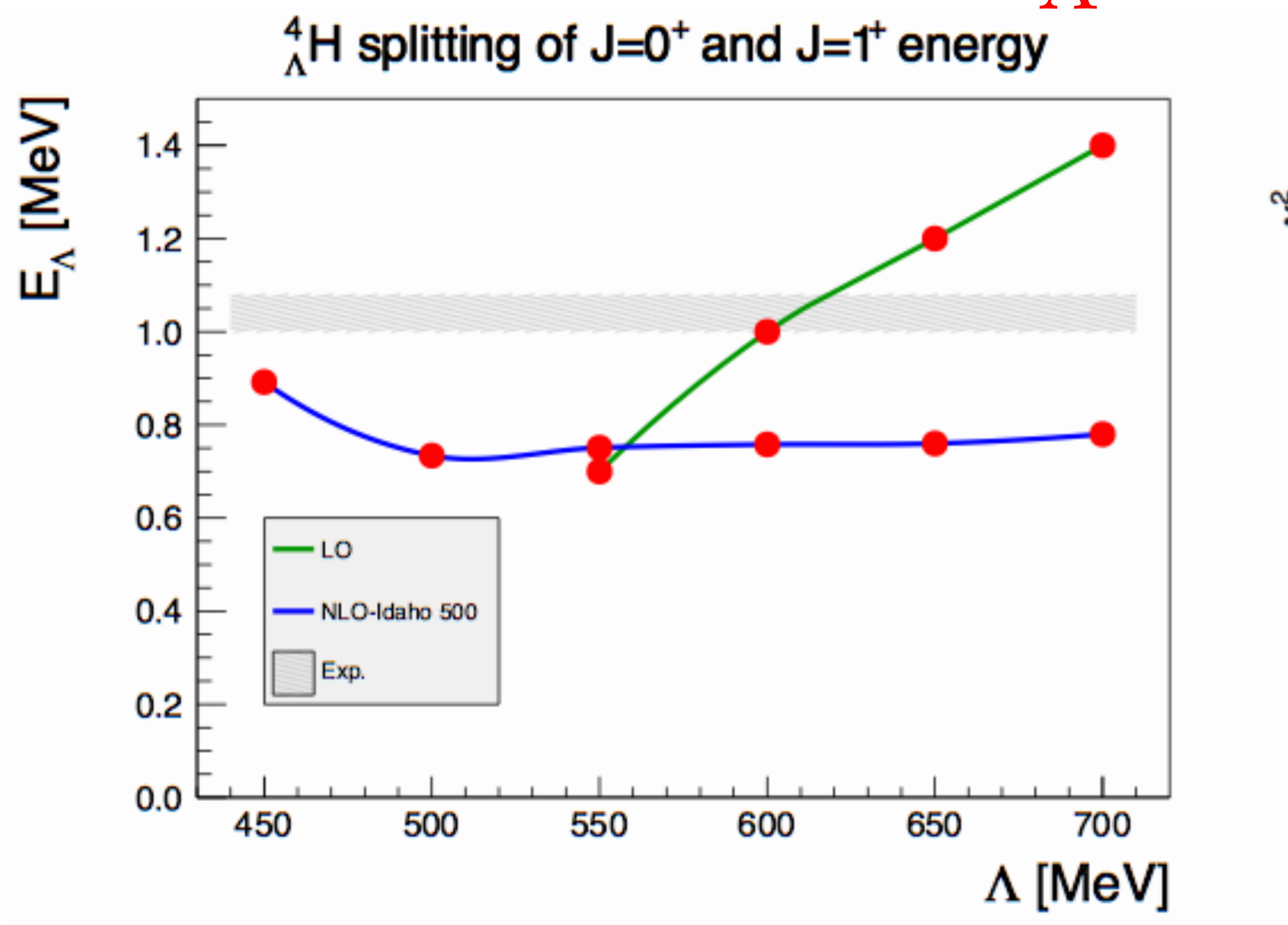


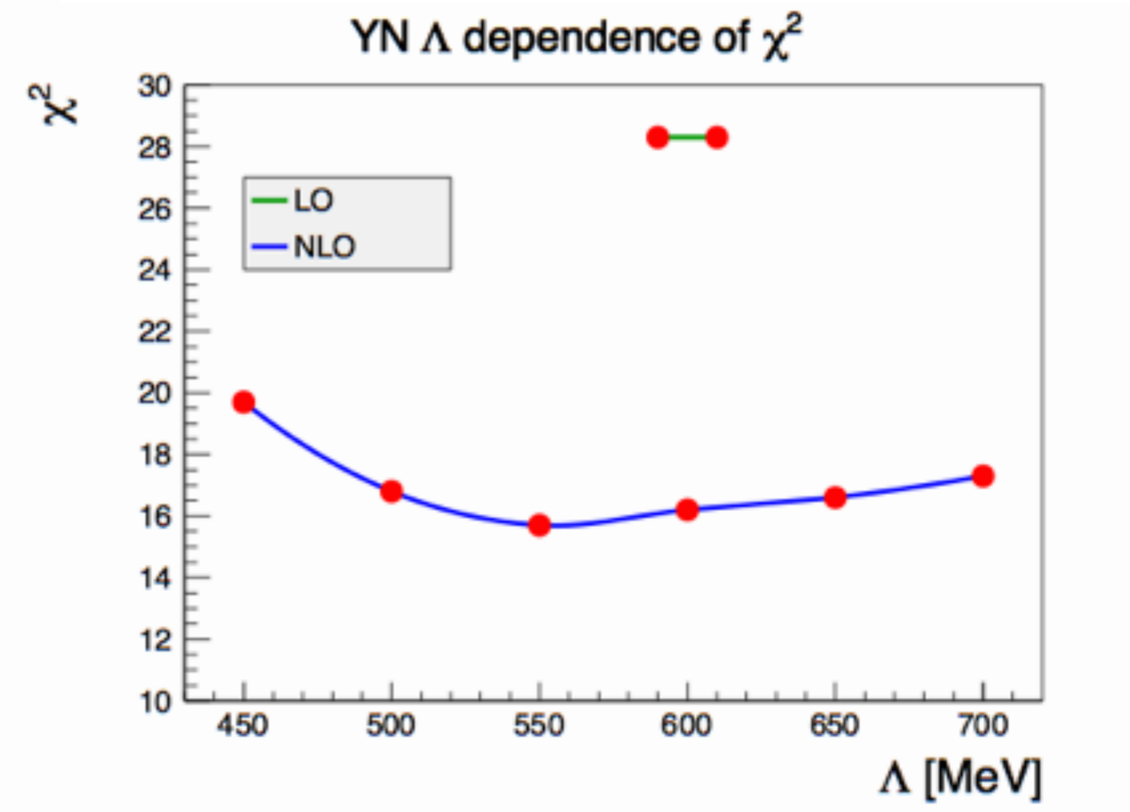


- LO/NLO results: LO uncertainty in 0^+ is underestimated by cutoff variation
- NLO results in line with model results, implies underbinding
- long range 3BFs need to be explicitly estimated

- **but:** for this version of NLO, results are **inconsistent** with experiment
  - note: this NLO does not allow for SU(3) breaking in contact part of YN
  - ad-hoc p-waves

Separation energies of $^4_\Lambda\mathrm{H}$





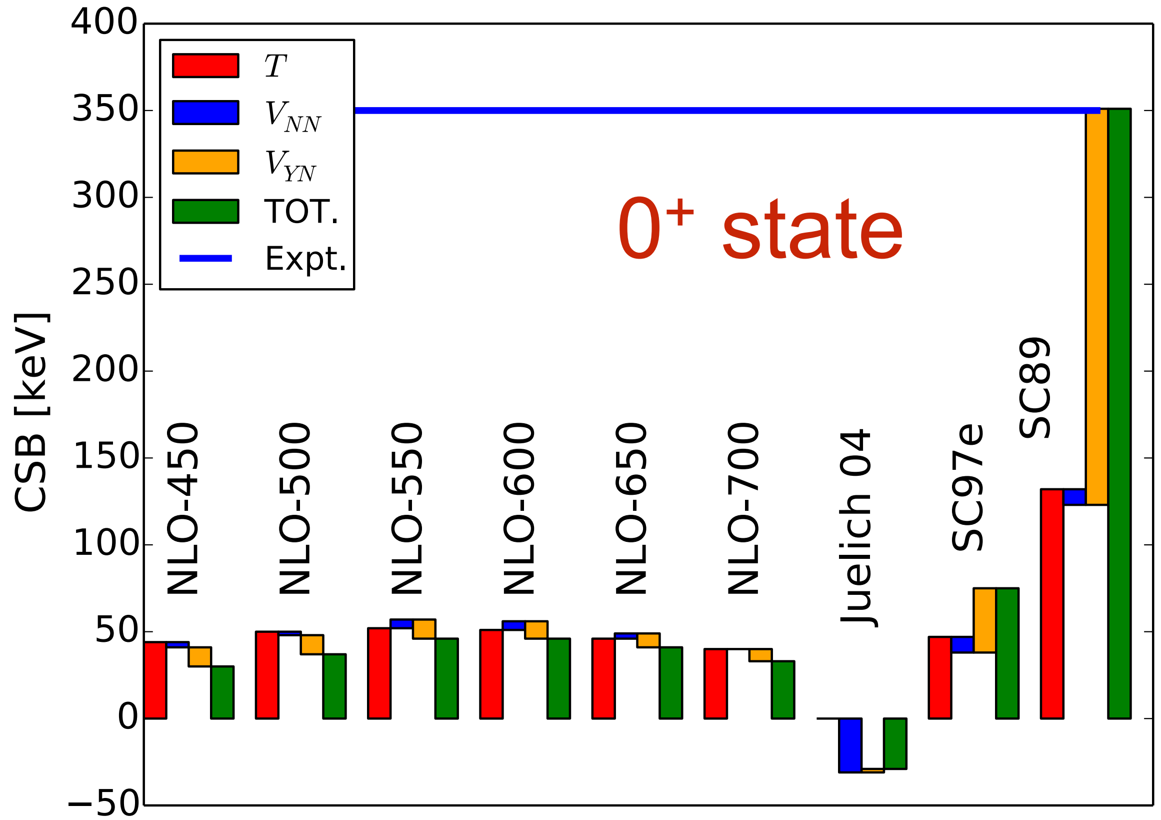
- LO/NLO cutoff dependence does not indicate 3BF contribution

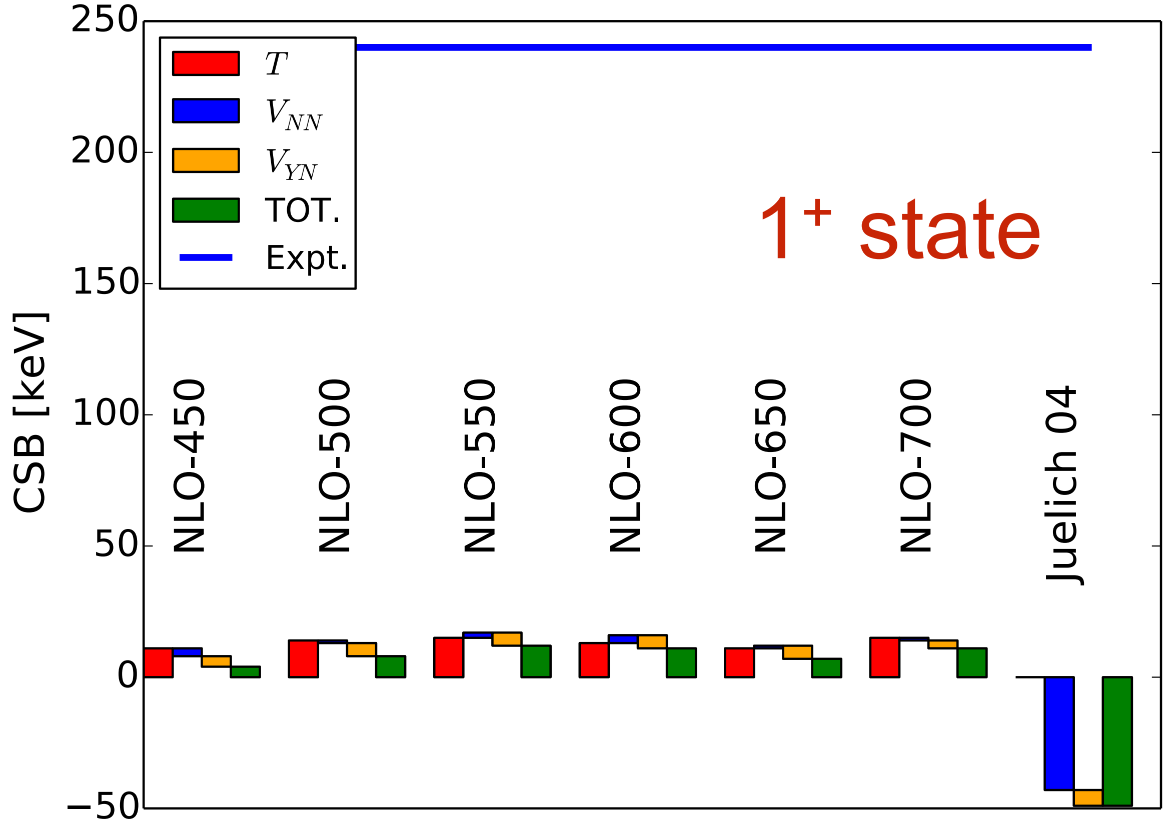
  long range 3BF needs to be studied
- results cutoff dependence for small Λ?

  related to non-optimal description of data?
- LO/NLO: splitting stabilizes
- **but:** NLO results are inconsistent with experiment

CSB at NLO & for model interactions

Contributions to the difference $E_\Lambda\left({}^4_\Lambda\mathrm{H}\right) - E_\Lambda\left({}^4_\Lambda\mathrm{He}\right)$

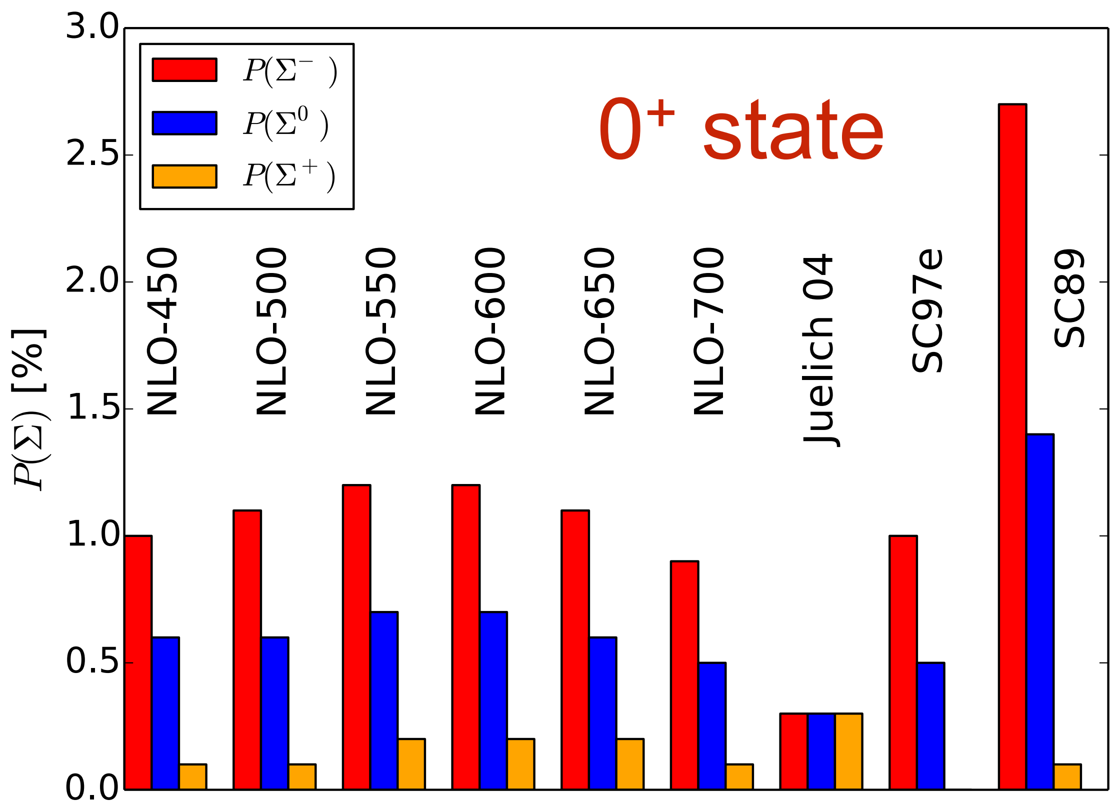


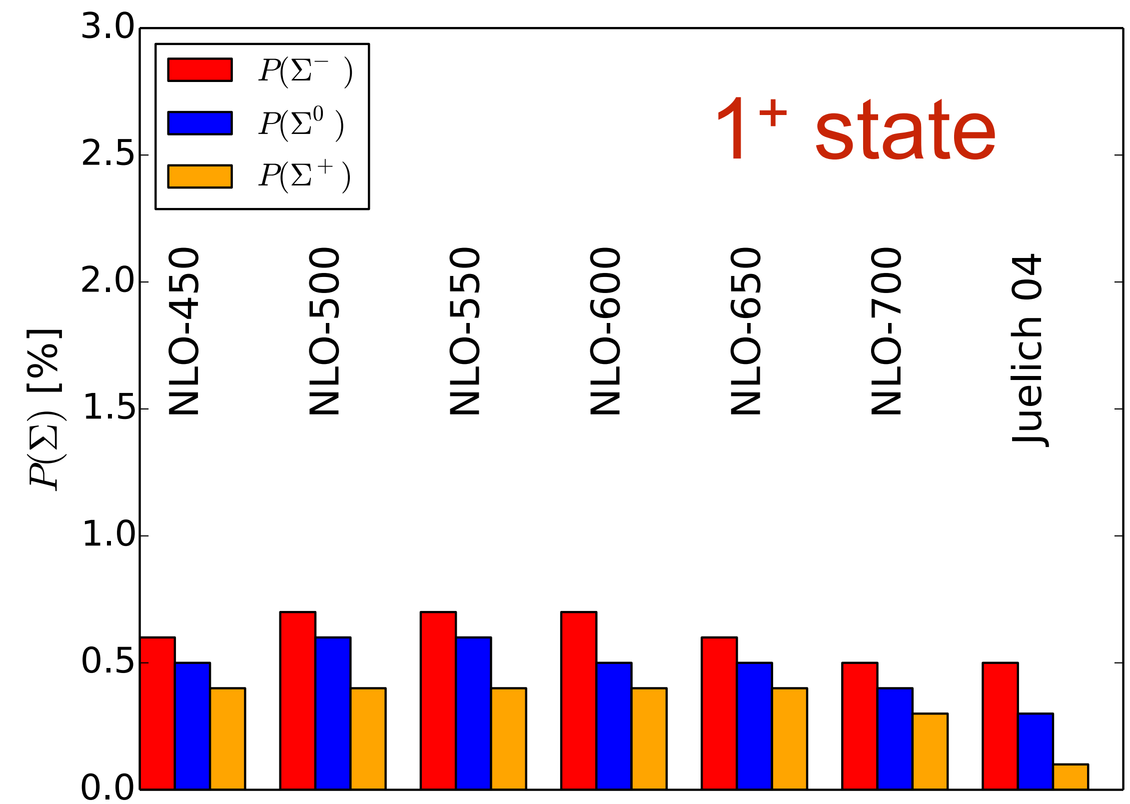


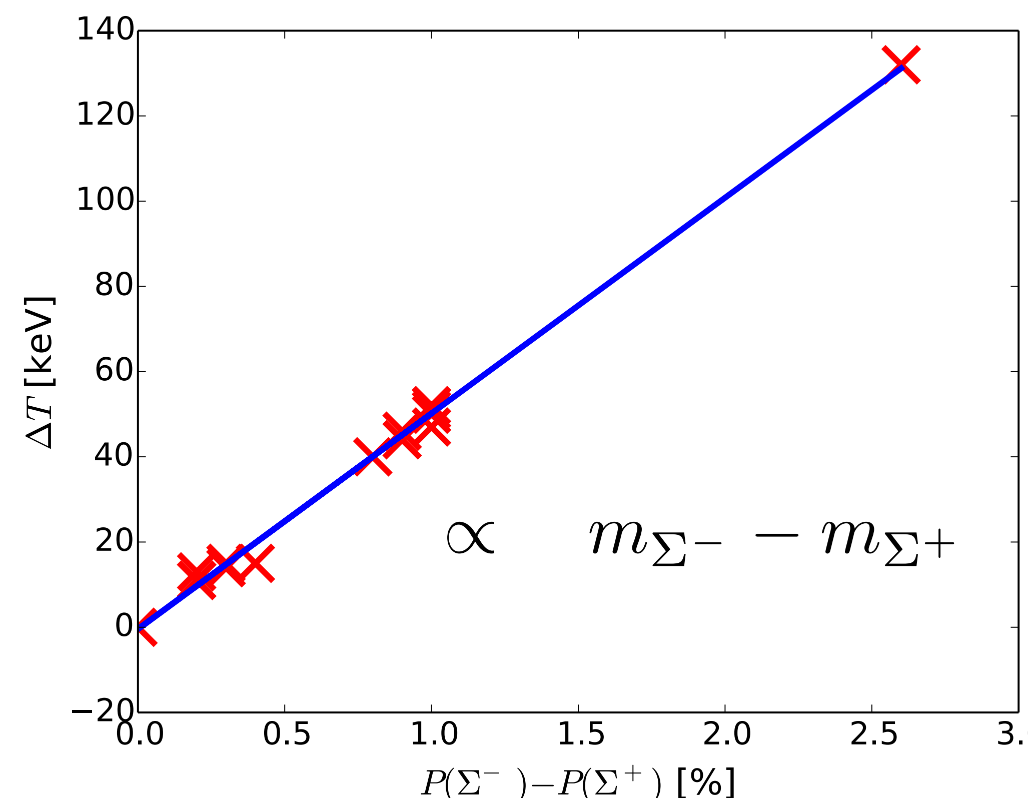
- NN force contribution due to small deviation of Coulomb
- YN force contribution:
  - *SC89 CSB is strong*
  - *NLO CSB is zero, only Coulomb acts (Σ component)*
- **kinetic energy contribution is driven by Σ component**

CSB and Σ probability

Σ probabilities in ${}^{4}_{\Lambda}\mathrm{H}$







- spin/isospin structure of hypernuclei drives Σ components
- **kinetic energy contribution is given linearly by differences of Σ components**

Conclusions & Outlook

- YN interactions are interesting and not well understood
  - *Λ-Σ conversion, explicit chiral symmetry breaking*
  - *well known: YN models fail*
  - *NLO of chiral interactions: still freedom to adjust YN forces*
  - *but: further estimates of **three-baryon interactions** (in progress)*
- hypernuclei are an essential source of information on YN
  - *it is not trivial to describe the simplest systems consistently*
  - *experiments for **very light** hypernuclei are important!*
    *The data needs to be **accurate** (better data for the hypertriton?)*
    *We need to be sure that these data are **reliable**.*
- CSB for four-body hypernuclei is a puzzle
  - *obviously related to Λ-Σ conversion*
    Can we engineer chiral interactions with different conversion strength?
  - *experiments for **very light** hypernuclei are important!*
    Is today's data reliable?
- extension of complete calculations to larger systems **(access more data)**
  (see also Roland Wirth's talk)