

# Hyperonic Three-Body Forces & Neutron Stars

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This study is part of the Ph.D.  
Thesis of **Domenico Logoteta**  
(Univ. Coimbra, 2013)



in collaboration also with



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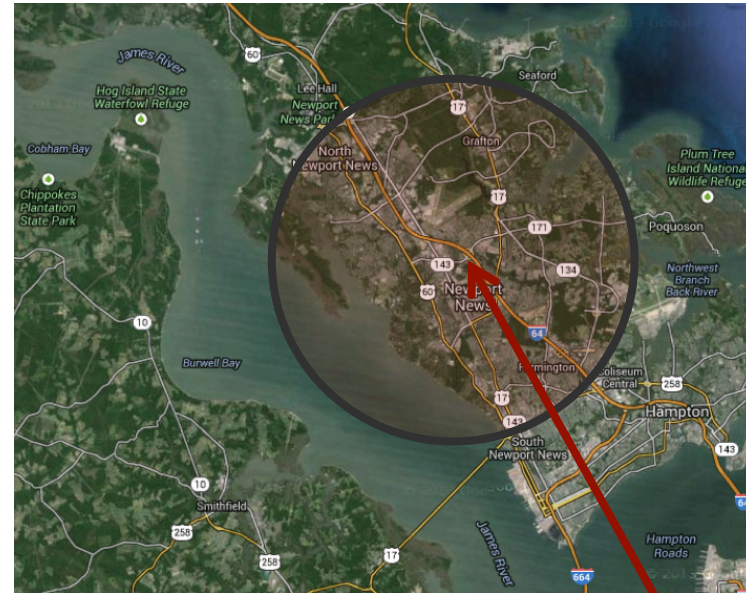
# Some known facts about Neutron Stars

- Formed in: type II, Ib or Ic SN
- Mass:  $M \sim 1 - 2 M_{\odot}$
- Radius:  $R \sim 10 - 12 \text{ km}$
- Density:  $\rho \sim 10^{14} - 10^{15} \text{ g/cm}^3$

$$\rho_{\text{universe}} \sim 10^{-30} \text{ g/cm}^3$$

$$\rho_{\text{sun}} \sim 1.4 \text{ g/cm}^3$$

$$\rho_{\text{earth}} \sim 5.5 \text{ g/cm}^3$$



**You are here !!**

- Baryonic number:  $N_b \sim 10^{57}$  (“giant nuclei”)
- Magnetic field:  $B \sim 10^{8...15} \text{ G}$  ( $10^{4...11} \text{ T}$ )

$0.3 - 0.5 \text{ G}$



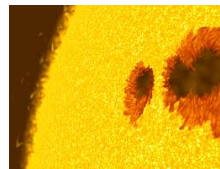
Earth

$10^3 - 10^4 \text{ G}$



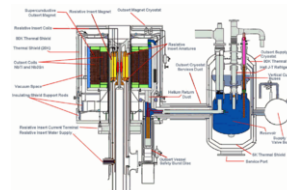
Magnet

$10^5 \text{ G}$



Sunspots

$4.5 \times 10^5 \text{ G}$



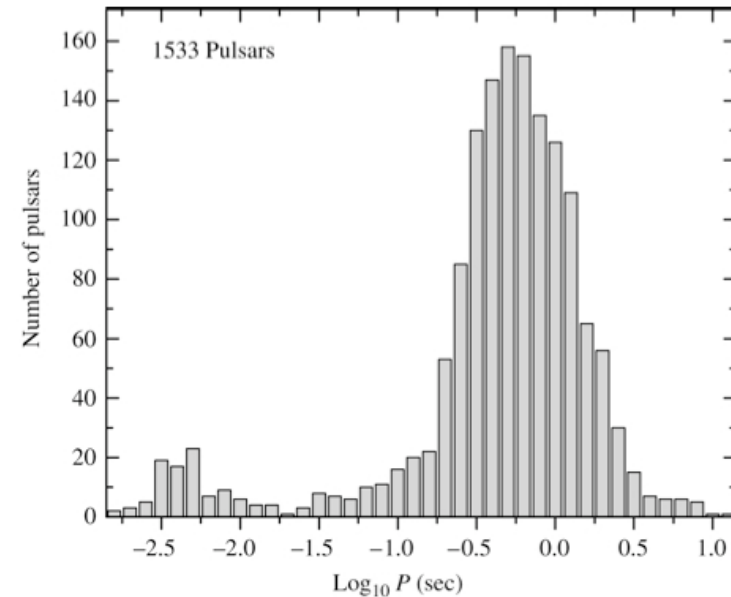
Largest continuous field in lab. (FSU, USA)

$2.8 \times 10^7 \text{ G}$



Largest magnetic pulse in lab. (Russia)

- Electric field:  $E \sim 10^{18}$  V/cm
- Temperature:  $T \sim 10^{6...11}$  K
- Rotational period distribution  
     → two types of pulsars:
  - pulsars with  $P \sim$  s
  - pulsars with  $P \sim$  ms



Shortest rotational period PSR in Terzan 5:  $P_{J1748-2446ad} = 1.39$  ms

- Accretion rates:  $10^{-10}$  to  $10^{-8}$   $M_{\odot}$ /year

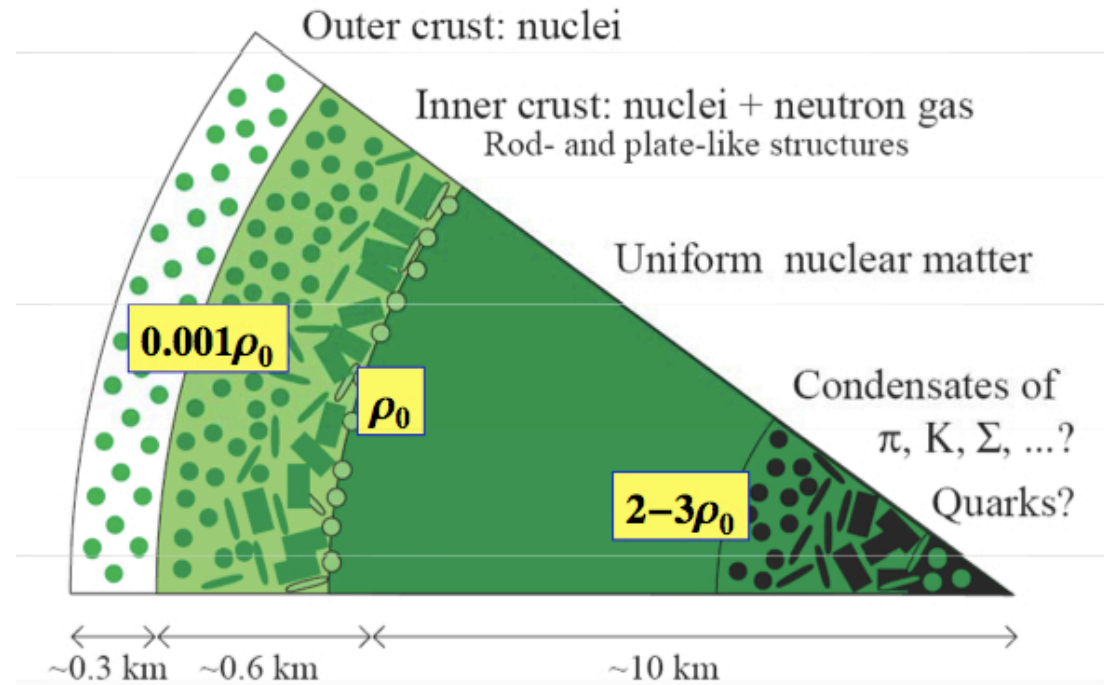
# Anatomy of a Neutron Star

Equilibrium composition  
determined by

- ✓ Charge neutrality

$$\sum_i q_i \rho_i = 0$$

- ✓ Equilibrium with respect to weak interacting processes



$$\begin{array}{l}
 b_1 \rightarrow b_2 + l + \bar{\nu}_l \\
 b_2 + l \rightarrow b_1 + \nu_l
 \end{array}
 \longrightarrow
 \mu_i = b_i \mu_n - q_i (\mu_e - \mu_{\nu_e}), \quad \mu_i = \frac{\partial \varepsilon}{\partial \rho_i}$$

# Hyperons in Neutron Stars

Hyperons in NS considered by many authors since the pioneering work of Ambartsumyan & Saakyan (1960)



## Phenomenological approaches

- ✧ **Relativistic Mean Field Models:** Glendenning 1985; Knorren et al. 1995; Shaffner-Bielich & Mishustin 1996, Bonano & Sedrakian 2012, ...
- ✧ **Non-relativistic potential model:** Balberg & Gal 1997
- ✧ **Quark-meson coupling model:** Pal et al. 1999, ...
- ✧ **Chiral Effective Lagrangians:** Hanauske et al., 2000
- ✧ **Density dependent hadron field models:** Hofmann, Keil & Lenske 2001



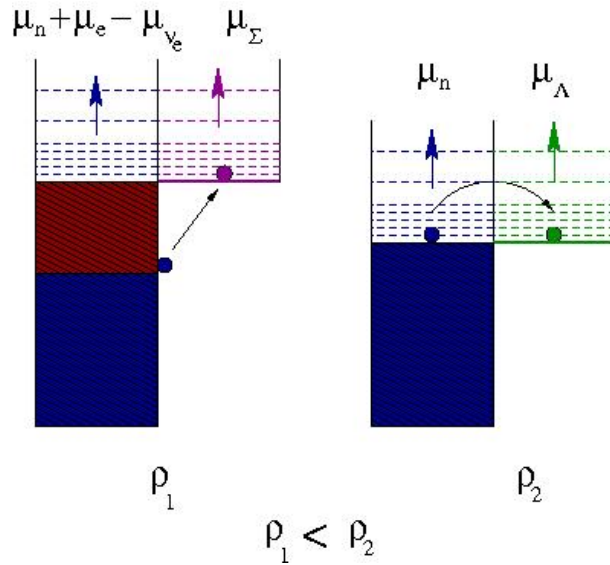
## Microscopic approaches

- ✧ **Brueckner-Hartree-Fock theory:** Baldo et al. 2000; I. V. et al. 2000, Schulze et al. 2006, I.V. et al. 2011, Burgio et al. 2011, Schulze & Rijken 2011
- ✧ **DBHF:** Sammarruca (2009)
- ✧  **$V_{\text{low } k}$ :** Djapo, Schaefer & Wambach, 2010



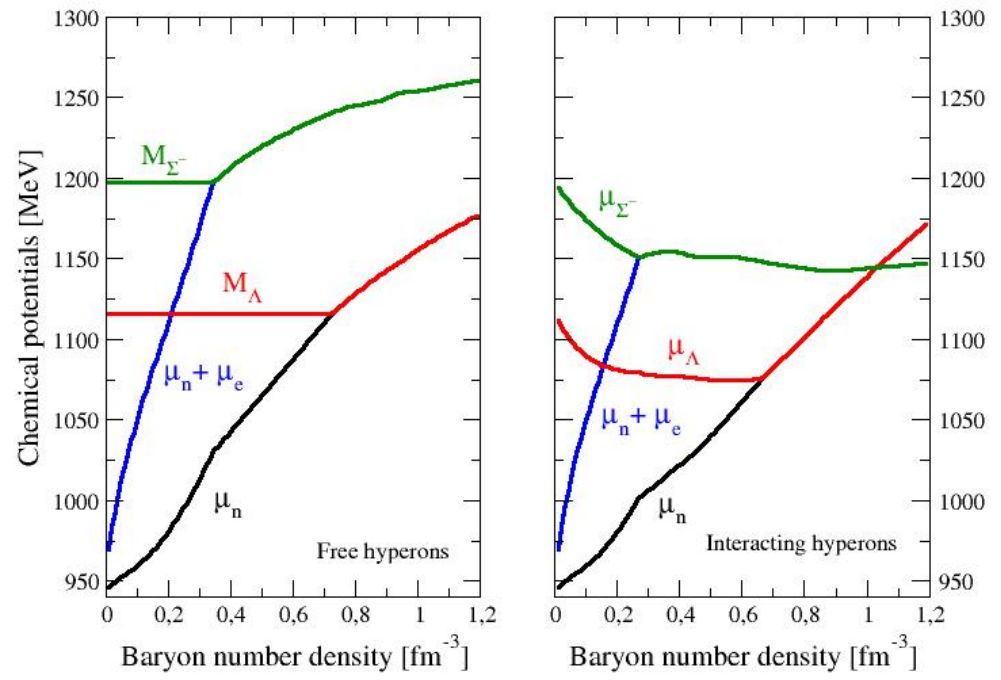
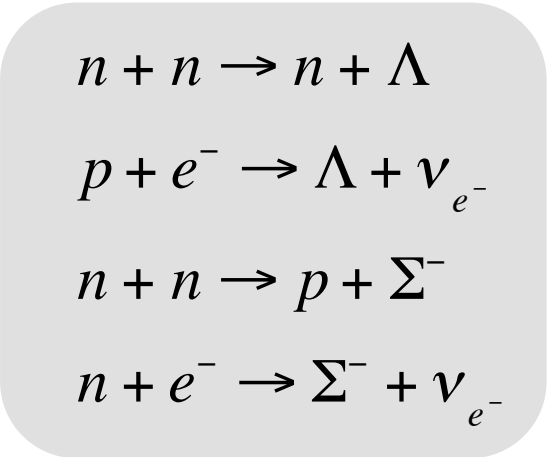
Sorry if I missed somebody

Hyperons are expected to appear in the core of neutron stars at  $\rho \sim (2-3)\rho_0$  when  $\mu_N$  is large enough to make the conversion of N into Y energetically favorable.

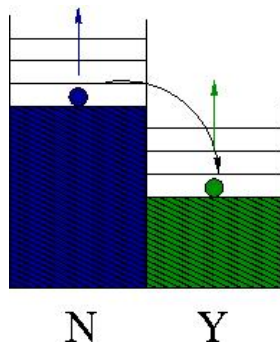
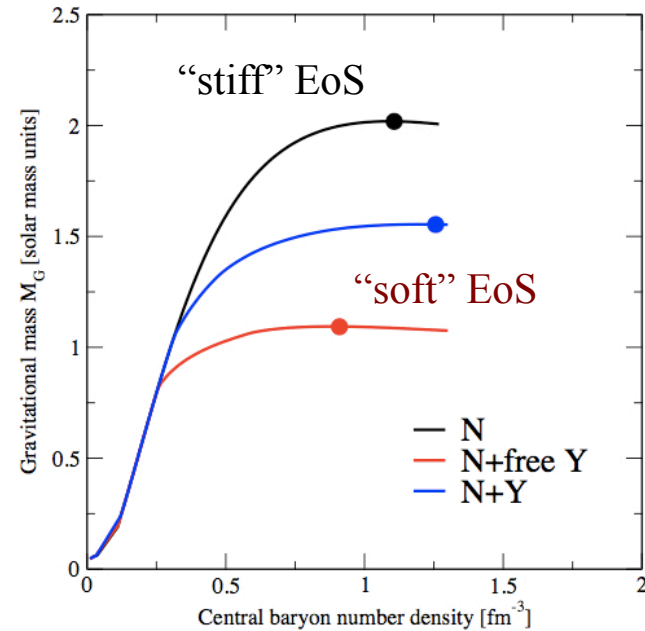
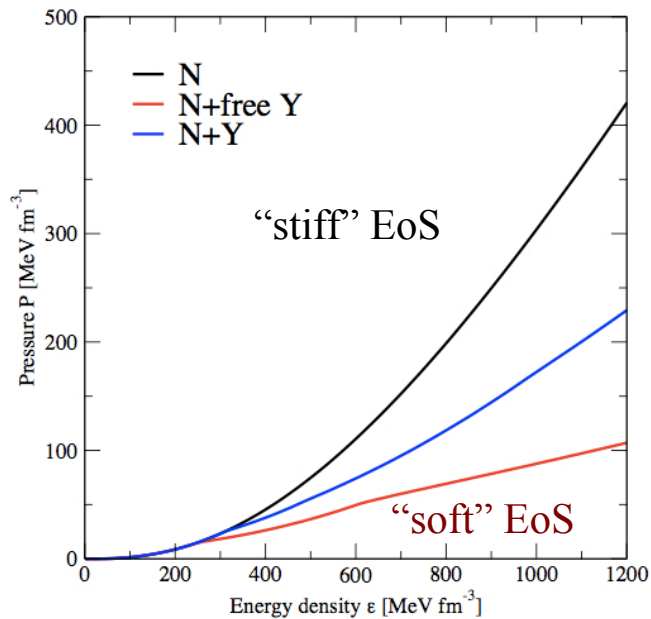


$$\mu_{\Sigma^-} = \mu_n + \mu_{e^-} - \mu_{\nu_{e^-}}$$

$$\mu_{\Lambda} = \mu_n$$



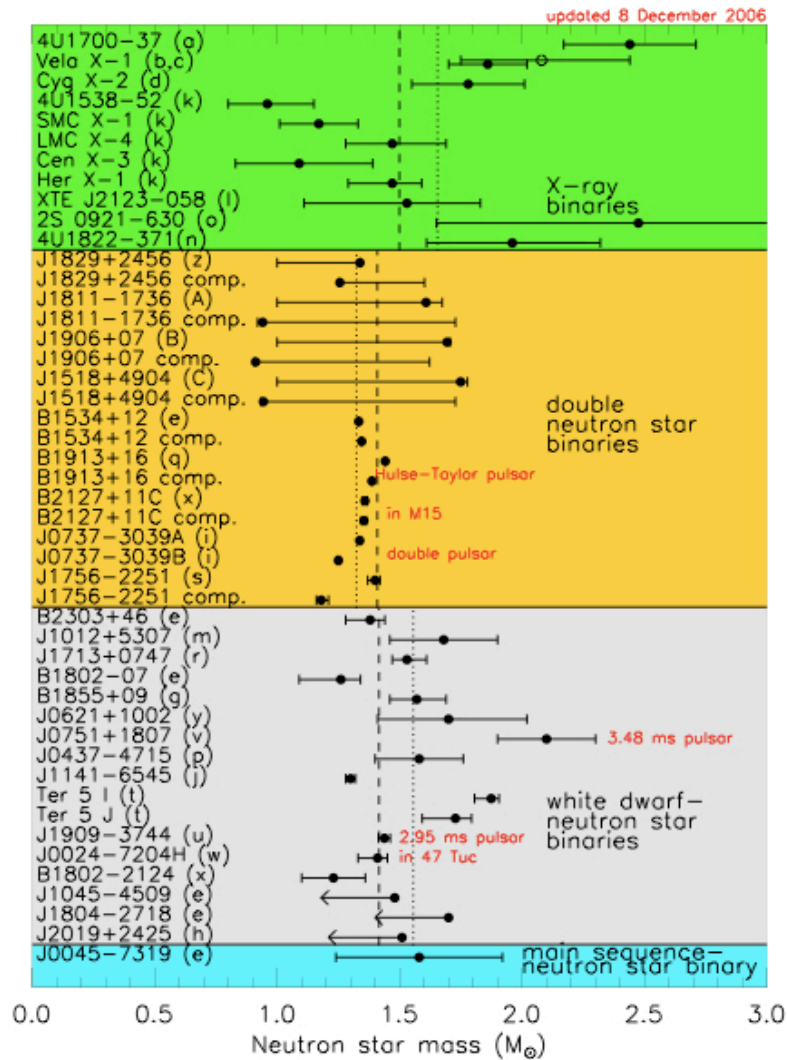
# Effect of Hyperons in the EoS and Mass of Neutron Stars



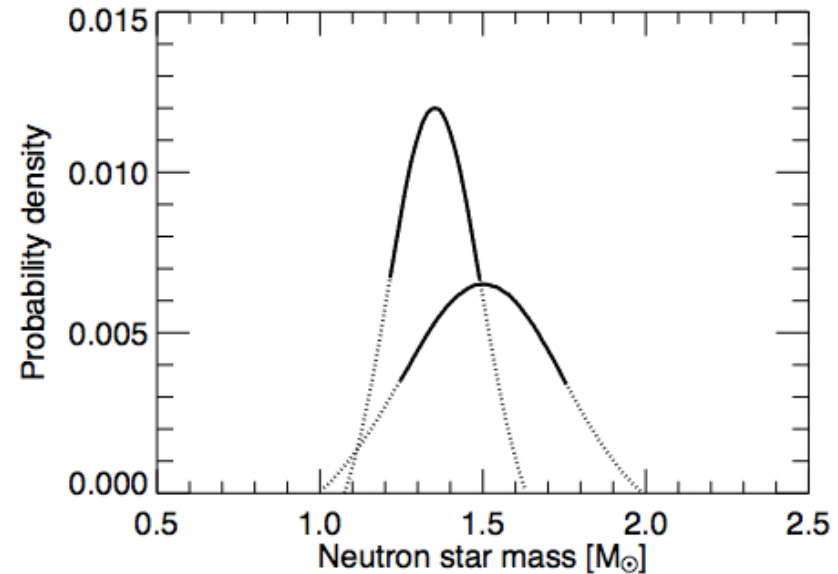
Relieve of Fermi pressure due to the appearance of hyperons →  
EoS softer → reduction of the mass



# Measured Neutron Star Masses (up to ~ 2006-2008)



(Lattimer & Prakash 2007)

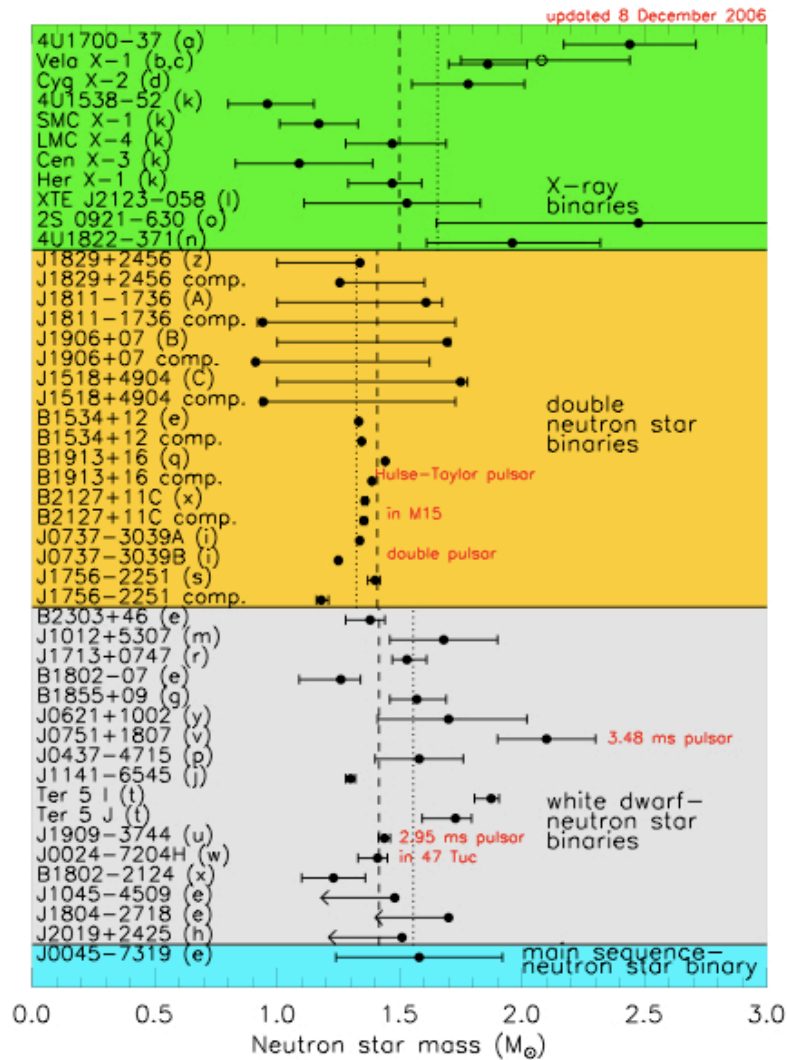


up to ~ 2006-2008 any valid  
 EoS should predict

$$M_{\max} [EoS] > 1.4 - 1.5 M_{\odot}$$

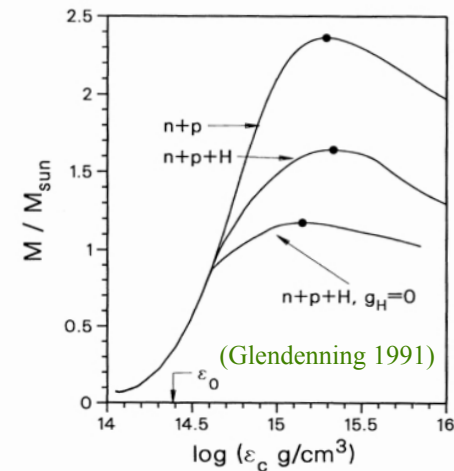
# Hyperons in NS

(up to ~ 2006-2008)

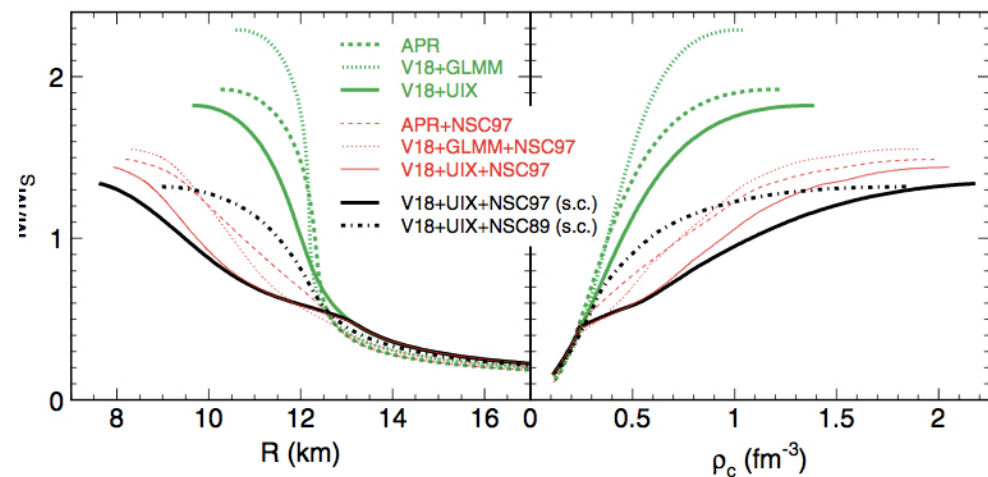


(Lattimer & Prakash 2007)

Phenomenological:  
 $M_{\max}$  compatible with 1.4-1.5  $M_{\odot}$



Microscopic :  $M_{\max} < 1.4-1.5 M_{\odot}$



(Schulze, Polls, Ramos & IV 2006)

Recent measurements of high masses → life of hyperons more difficult

## Eccentric Binary Millisecond Pulsars

Paulo C. C. Freire

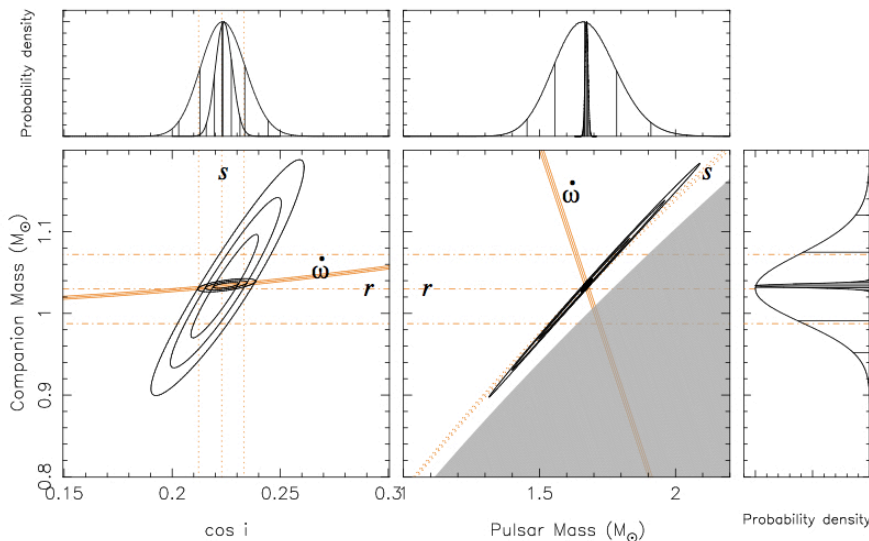
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**Abstract.** In this paper we review recent discovery of millisecond pulsars (MSPs) in eccentric binary systems. Timing these MSPs we were able to estimate (and in one case precisely measure) their masses. These results suggest that, as a class, MSPs have a much wider range of masses ( $1.3$  to  $> 2M_{\odot}$ ) than the normal and mildly recycled pulsars found in double neutron star (DNS) systems ( $1.25 < M_p < 1.44M_{\odot}$ ). This is very likely to be due to the prolonged accretion episode that is thought to be required to form a MSP. The likely existence of massive MSPs makes them a powerful probe for understanding the behavior of matter at densities larger than that of the atomic nucleus; in particular, the precise measurement of the mass of PSR J1903+0327 ( $1.67 \pm 0.01M_{\odot}$ ) excludes several "soft" equations of state for dense matter.

**Keywords:** Neutron Stars, Pulsars, Binary Pulsars, General Relativity, Nuclear Equation of State

**PACS:** 97.60.Gb; 97.60.Jd; 97.80.Fk; 95.30.Sf; 26.60; 91.60.Fe

The precise measurement of the mass of PSR J1903+0328 ( $1.67 \pm 0.01 M_{\text{sun}}$ ) excludes several "soft" EoS for dense matter



- ✓ binary system (P=95.17 d)
- ✓ high eccentricity ( $\epsilon=0.437$ )
- ✓ companion mass:  $\sim 1M_{\odot}$
- ✓ pulsar mass:  $M = 1.67 \pm 0.11M_{\odot}$

# Two-solar mass neutron star measured

## LETTER

Nature 464, 1081 (2010)

doi:10.1038/nature09466

### A two-solar-mass neutron star measured using Shapiro delay

P. B. Demorest<sup>1</sup>, T. Pennucci<sup>2</sup>, S. M. Ransom<sup>1</sup>, M. S. E. Roberts<sup>3</sup> & J. W. T. Hessels<sup>4,5</sup>

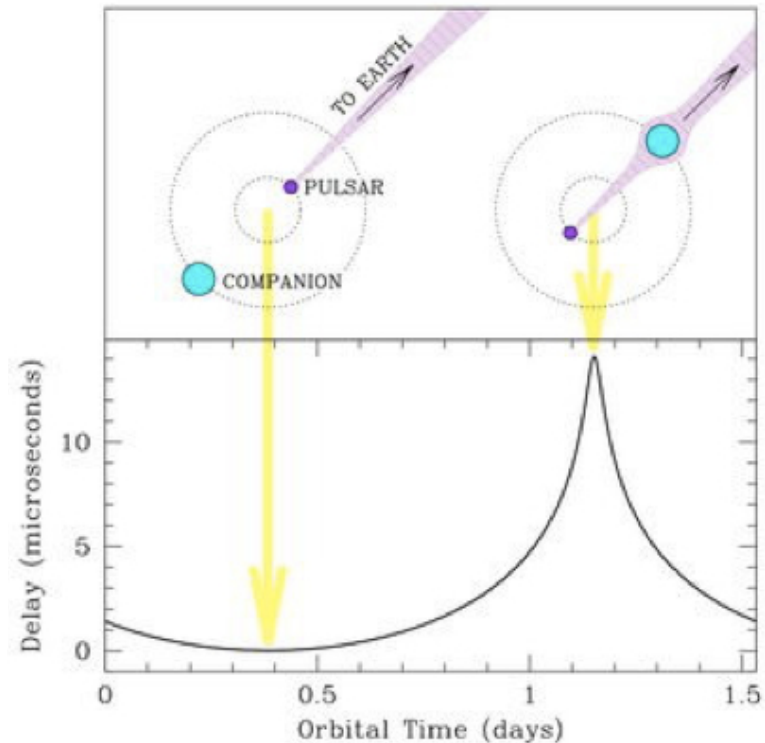
Neutron stars are composed of the densest form of matter known to exist in our Universe, the composition and properties of which are still theoretically uncertain. Measurements of the masses or radii of these objects can strongly constrain the neutron star matter equation of state and rule out theoretical models of their composition<sup>1,2</sup>. The observed range of neutron star masses, however, has hitherto been too narrow to rule out many predictions of 'exotic' non-nucleonic components<sup>3-6</sup>. The Shapiro delay is a general-relativistic increase in light travel time through the curved space-time near a massive body<sup>7</sup>. For highly inclined (nearly edge-on) binary millisecond radio pulsar systems, this effect allows us to infer the masses of both the neutron star and its binary companion to high precision<sup>8,9</sup>. Here we present radio timing observations of the binary millisecond pulsar J1614-2230<sup>10,11</sup> that show a strong Shapiro delay signature. We calculate the pulsar mass to be  $(1.97 \pm 0.04)M_{\odot}$ , which rules out almost all currently proposed<sup>2-5</sup> hyperon or boson condensate equations of state ( $M_{\odot}$ , solar mass). Quark matter can support a star this massive only if the quarks are strongly interacting and are therefore not 'free' quarks<sup>12</sup>.

long-term data set, parameter covariance and dispersion measure variation can be found in Supplementary Information.

As shown in Fig. 1, the Shapiro delay was detected in our data with extremely high significance, and must be included to model the arrival times of the radio pulses correctly. However, estimating parameter values and uncertainties can be difficult owing to the high covariance between many orbital timing model terms<sup>14</sup>. Furthermore, the  $\chi^2$  surfaces for the Shapiro-derived companion mass ( $M_2$ ) and inclination angle ( $i$ ) are often significantly curved or otherwise non-Gaussian<sup>15</sup>. To obtain robust error estimates, we used a Markov chain Monte Carlo (MCMC) approach to explore the post-fit  $\chi^2$  space and derive posterior probability distributions for all timing model parameters (Fig. 2). Our final results for the model

Table 1 | Physical parameters for PSR J1614-2230

Parameter	Value
Ecliptic longitude ( $\lambda$ )	245.78827556(5) <sup>o</sup>
Ecliptic latitude ( $\beta$ )	-1.256744(2) <sup>o</sup>
Proper motion in $\lambda$	9.79(7) mas yr <sup>-1</sup>
Proper motion in $\beta$	-30(3) mas yr <sup>-1</sup>
Parallax	0.5(6) mas



Binary millisecond pulsar PSR J1614+2230  
Shapiro delay signature

$$\Delta t = -\frac{2GM}{c^3} \log(1 - \vec{R} \cdot \vec{R}')$$

The mass  $1.97 \pm 0.04 M_{\text{sun}}$  of the pulsar PSR J1614+2230 rules out almost all currently proposed hyperon or boson condensate EoS. Quark matter can support such a massive star only if quarks are strongly interacting (not "free quarks")

# On April 26<sup>th</sup> 2013 the discovery of the most massive (up to now) pulsar (PSR J0348+0432) was made public

Science 26 April 2013:  
Vol. 340 no. 6131  
DOI: 10.1126/science.1233232

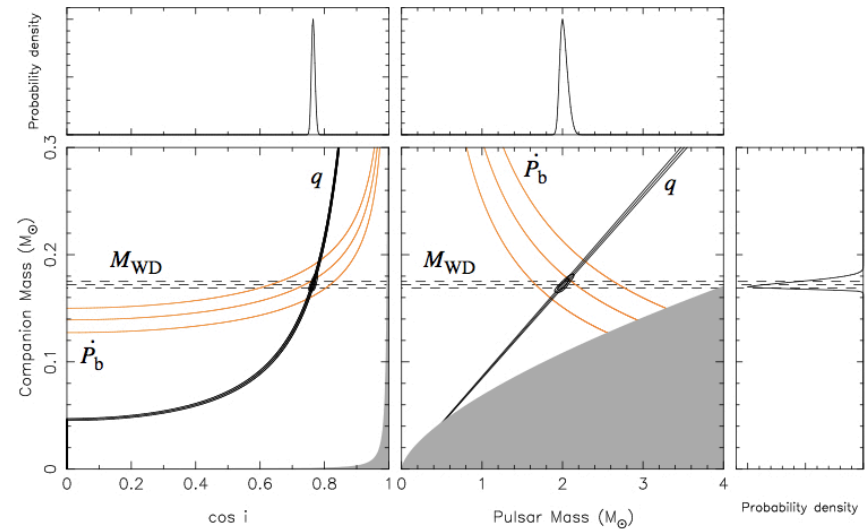
## A Massive Pulsar in a Compact Relativistic Binary

John Antoniadis<sup>1,2</sup>, Paulo C. C. Freire<sup>3</sup>, Norbert Wex<sup>4</sup>, Thomas M. Tauris<sup>2,3</sup>, Ryan S. Lynch<sup>3</sup>, Marten H. van Kerkwijk<sup>4</sup>, Michael Kramer<sup>1,5</sup>, Cees Bassa<sup>6</sup>, Vik S. Dhillon<sup>6</sup>, Thomas Driebe<sup>7</sup>, Jason W. T. Hessels<sup>8,9</sup>, Victoria M. Kaspi<sup>3</sup>, Vladislav I. Kondratiev<sup>8,10</sup>, Norbert Langer<sup>2</sup>, Thomas R. Marsh<sup>11</sup>, Maura A. McLaughlin<sup>12</sup>, Timothy T. Pennucci<sup>13</sup>, Scott M. Ransom<sup>14</sup>, Ingrid H. Stairs<sup>15</sup>, Joeri van Leeuwen<sup>8,9</sup>, Joris P. W. Verbiest<sup>1</sup>, David G. Whelan<sup>13</sup>

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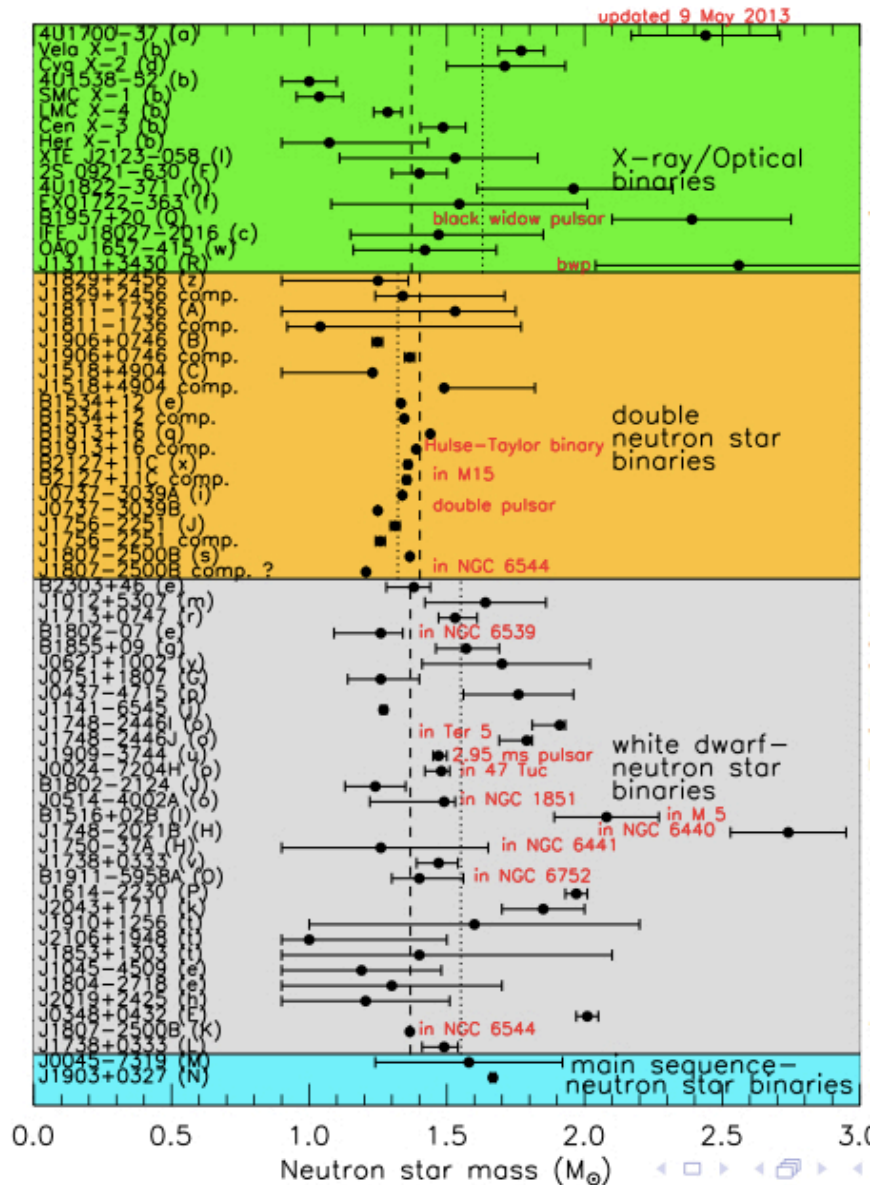
ABSTRACT STRUCTURED ABSTRACT EDITOR'S SUMMARY

Many physically motivated extensions to general relativity (GR) predict substantial deviations in the properties of spacetime surrounding massive neutron stars. We report the measurement of a  $2.01 \pm 0.04$  solar mass ( $M_{\odot}$ ) pulsar in a 2.46-hour orbit with a  $0.172 \pm 0.003 M_{\odot}$  white dwarf. The high pulsar mass and the compact orbit make this system a sensitive laboratory of a previously untested strong-field gravity regime. Thus far, the observed orbital decay agrees with GR, supporting its validity even for the extreme conditions present in the system. The resulting constraints on deviations support the use of GR-based templates for ground-based gravitational wave detectors. Additionally, the system strengthens recent constraints on the properties of dense matter and provides insight to binary stellar astrophysics and pulsar recycling.



- ✓ binary system (P=2.46 h)
- ✓ very low eccentricity
- ✓ companion mass:  $0.172 \pm 0.003 M_{\odot}$
- ✓ pulsar mass:  $M = 2.01 \pm 0.04 M_{\odot}$

# Measured Neutron Star Masses (2014)



Observation of  $\sim 2 M_{\text{sun}}$  neutron stars



Dense matter EoS stiff enough is required such that

$$M_{\text{max}} [EoS] > 2M_{\odot}$$

Can hyperons still be present in the interior of neutron stars in view of this constraint ?

## The Hyperon Puzzle



“Hyperons → “soft (or too soft) EoS” not compatible (mainly in microscopic approaches) with measured (high) masses. However, the presence of hyperons in the NS interior seems to be unavoidable.”



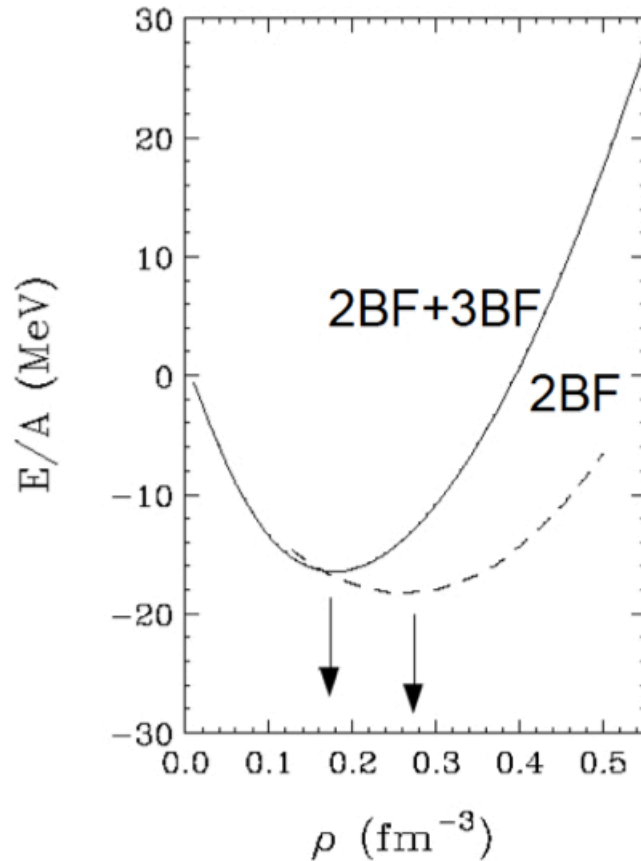
- ✓ can YN & YY interactions still solve it ?
- ✓ or perhaps hyperonic three-body forces ?
- ✓ what about quark matter ?

# Can Hyperonic TBF solve this puzzle ?

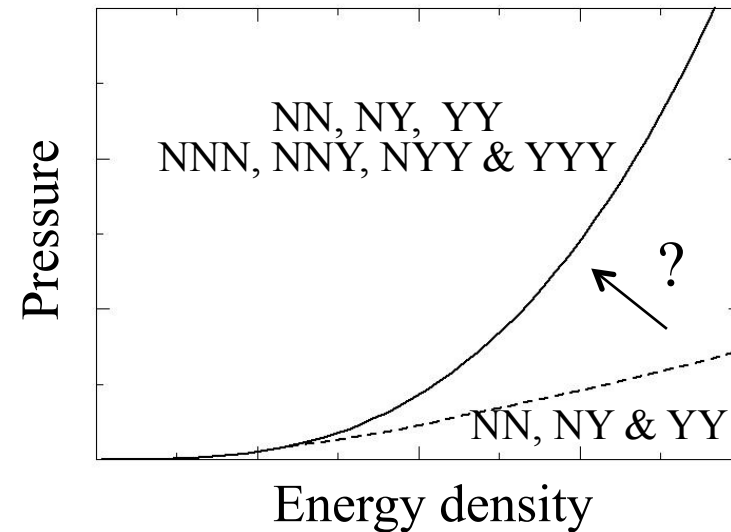
Natural solution based on: **Importance of NNN force in Nuclear Physics**

(Considered by several authors: Chalk, Gal, Usmani, Bodmer, Takatsuka, Loiseau, Nogami, Bahaduri, IV)

## NNN Force



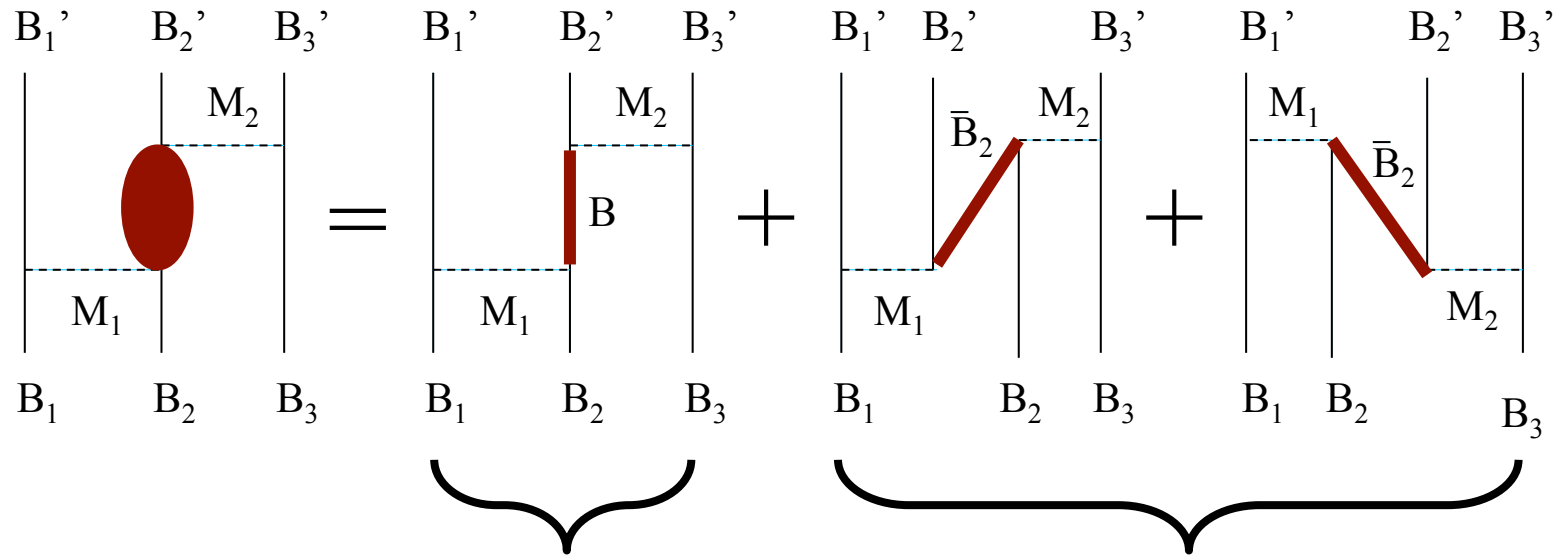
## NNY, NYY & YYY Forces



Can hyperonic TBF provide enough repulsion at high densities to reach  $2M_{\odot}$ ?



## Two-meson exchange Hyperonic TBF



$B_i B_i'$ :  $N, \Lambda, \Sigma$

$B$  - excitation

$Z$  - diagram

$M_i$ :  $\pi, K, \sigma, \omega$

$B$ :  $\Lambda, \Sigma, \Delta, \Sigma^*$

$\bar{B}_2$ :  $\bar{N}, \bar{\Lambda}, \bar{\Sigma}$

Vertices: consistent with YN and YY

Repulsion at high densities due to Z-diagram as in NNN

# Baryon-excitation contribution ( $\pi$ -, $K$ -exchange)

$$V_{NNY}^{M_1 M_2, B} = C_{NNY}^{M_1 M_2, B} \left( \hat{O}_A \{ X_{12}(\vec{r}_{12}), X_{23}(\vec{r}_{23}) \} + \hat{O}_B [ X_{12}(\vec{r}_{12}), X_{23}(\vec{r}_{23}) ] \right)$$

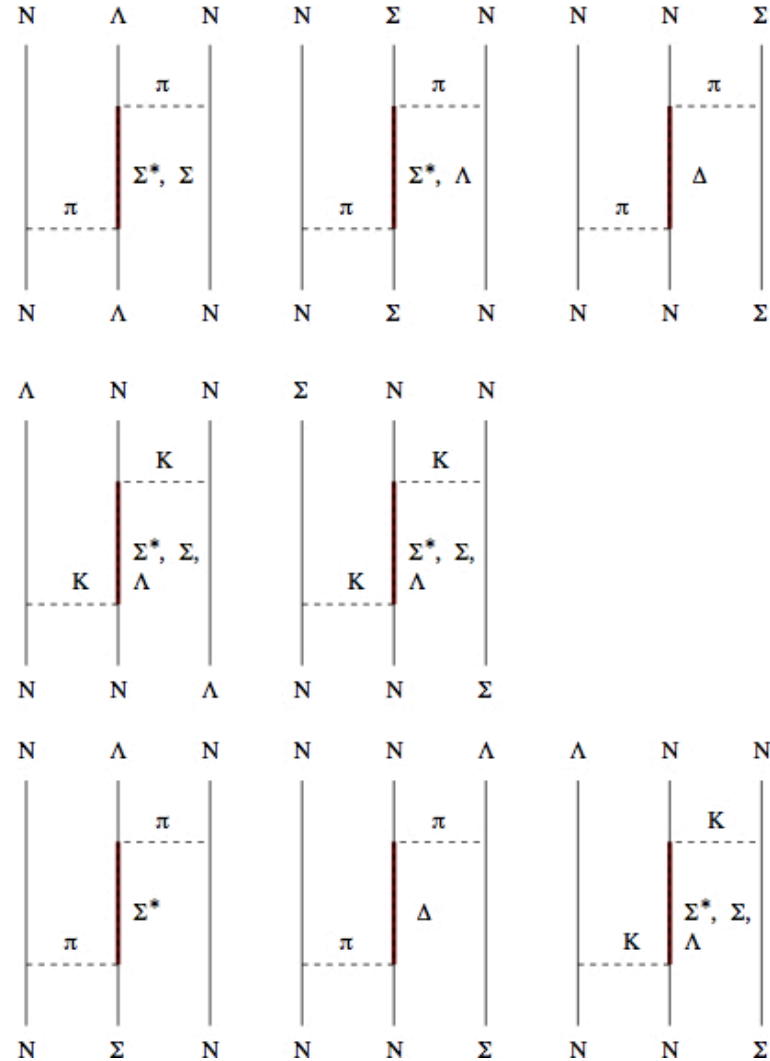
$\hat{O}_A, \hat{O}_B \rightarrow$  isospin structure

$$X_{ij}(\vec{x}) = \vec{\sigma}_i \cdot \vec{\sigma}_j Y_{ij}(x) + \hat{S}_{ij}(\hat{x}) T_{ij}(x)$$

$$Y_{ij}(x) = \frac{\partial^2 Z_{ij}}{\partial x^2} + \frac{2}{x} \frac{\partial Z_{ij}}{\partial x}, \quad T_{ij}(x) = \frac{\partial^2 Z_{ij}}{\partial x^2} - \frac{1}{x} \frac{\partial Z_{ij}}{\partial x}$$

$$Z_{12}(x) = \frac{4\pi}{m_{M_1}} \int \frac{d\vec{k}}{(2\pi)^3} \frac{e^{-i\vec{k}\cdot\vec{x}}}{k^2 + m_{M_1}^2} F_{B_1 B_1 M_1}(k^2) F_{B_2 B M_1}(k^2)$$

$$Z_{23}(x) = \frac{4\pi}{m_{M_2}} \int \frac{d\vec{q}}{(2\pi)^3} \frac{e^{-i\vec{q}\cdot\vec{x}}}{q^2 + m_{M_2}^2} F_{B_3 B_3 M_2}(q^2) F_{B_2 B M_2}(q^2)$$



# Isospin structure: operators $\hat{O}_A$ & $\hat{O}_B$

$$V_{NNY}^{M_1 M_2, B} = C_{NNY}^{M_1 M_2, B} \left( \hat{O}_A \{X_{12}(\vec{r}_{12}), X_{23}(\vec{r}_{23})\} + \hat{O}_B [X_{12}(\vec{r}_{12}), X_{23}(\vec{r}_{23})] \right)$$

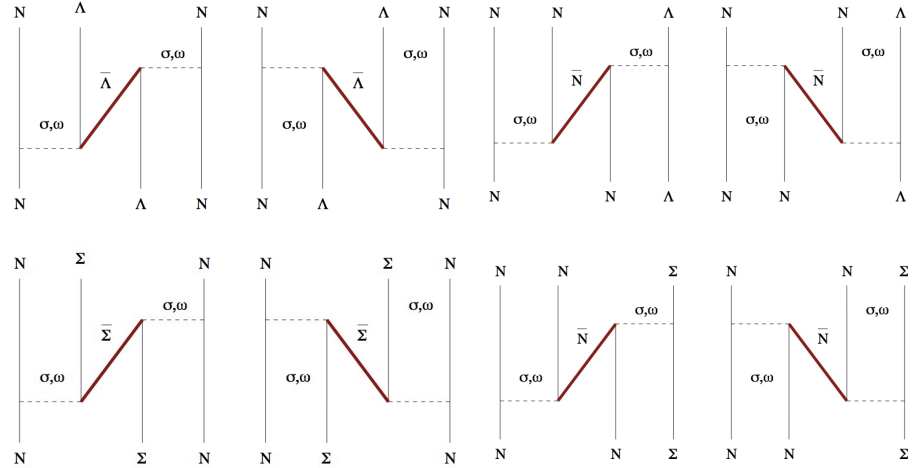
$V_{NNY}^{M_1 M_2, B}$	$\hat{O}_A$	$\hat{O}_B$
$V_{NN\Lambda}^{\pi\pi, \Sigma^*}, V_{NN\Lambda}^{\pi\pi, \Sigma}, V_{NN\Sigma}^{\pi\pi, \Sigma^*}, V_{NN\Sigma}^{\pi\pi, \Lambda}, V_{NN\Sigma}^{KK, \Lambda}, V_{NN\Sigma \leftrightarrow NN\Lambda}^{\pi\pi, \Sigma^*}$	$\vec{\tau}_1 \cdot \vec{\tau}_3$	—
$V_{NN\Sigma}^{\pi\pi, \Delta}$	$\{\vec{\tau}_1 \cdot \vec{\tau}_2, \vec{\tau}_2 \cdot \vec{I}_3\}$	$\frac{1}{4}[\vec{\tau}_1 \cdot \vec{\tau}_2, \vec{\tau}_2 \cdot \vec{I}_3]$
$V_{NN\Sigma \leftrightarrow NN\Lambda}^{\pi\pi, \Delta}$	$\{\vec{\tau}_1 \cdot \vec{\tau}_2, \vec{\tau}_2 \cdot \vec{\rho}_3\}$	$\frac{1}{4}[\vec{\tau}_1 \cdot \vec{\tau}_2, \vec{\tau}_2 \cdot \vec{\rho}_3]$
$V_{NN\Lambda}^{KK, \Sigma^*}$	$\{\vec{1}_1 \cdot \vec{\tau}_2, \vec{\tau}_2 \cdot \vec{1}_3\}$	$-\frac{1}{2}[\vec{1}_1 \cdot \vec{\tau}_2, \vec{\tau}_2 \cdot \vec{1}_3]$
$V_{NN\Lambda}^{KK, \Sigma}$	$\{\vec{1}_1 \cdot \vec{\tau}_2, \vec{\tau}_2 \cdot \vec{1}_3\}$	$[\vec{1}_1 \cdot \vec{\tau}_2, \vec{\tau}_2 \cdot \vec{1}_3]$
$V_{NN\Lambda}^{KK, \Lambda}$	1	—

# Isospin structure: operators $\hat{O}_A$ & $\hat{O}_B$ (cont')

$$V_{NNY}^{M_1 M_2, B} = C_{NNY}^{M_1 M_2, B} \left( \hat{O}_A \{X_{12}(\vec{r}_{12}), X_{23}(\vec{r}_{23})\} + \hat{O}_B [X_{12}(\vec{r}_{12}), X_{23}(\vec{r}_{23})] \right)$$

$V_{NNY}^{M_1 M_2, B}$	$\hat{O}_A$	$\hat{O}_B$
$V_{NN\Sigma}^{KK, \Sigma^*}$	$\{\vec{\tau}_1 \cdot \vec{\tau}_2, \vec{\tau}_2 \cdot \vec{\tau}_3\}$	$-\frac{1}{2}[\vec{\tau}_1 \cdot \vec{\tau}_2, \vec{\tau}_2 \cdot \vec{\tau}_3]$
$V_{NN\Sigma}^{KK, \Sigma}$	$\{\vec{\tau}_1 \cdot \vec{\tau}_2, \vec{\tau}_2 \cdot \vec{\tau}_3\}$	$[\vec{\tau}_1 \cdot \vec{\tau}_2, \vec{\tau}_2 \cdot \vec{\tau}_3]$
$V_{NN\Sigma \leftrightarrow NNA}^{KK, \Sigma^*}$	$\{\vec{\rho}_1 \cdot \vec{\tau}_2, \vec{\tau}_2 \cdot \vec{\tau}_3\}$	$-\frac{1}{2}[\vec{\rho}_1 \cdot \vec{\tau}_2, \vec{\tau}_2 \cdot \vec{\tau}_3]$
$V_{NN\Sigma \leftrightarrow NNA}^{KK, \Sigma}$	$\{\vec{\rho}_1 \cdot \vec{\tau}_2, \vec{\tau}_2 \cdot \vec{\tau}_3\}$	$[\vec{\rho}_1 \cdot \vec{\tau}_2, \vec{\tau}_2 \cdot \vec{\tau}_3]$
$V_{NN\Sigma \leftrightarrow NNA}^{KK, \Lambda}$	$\vec{\rho}_1 \cdot \vec{\tau}_2$	—

# Z-diagram contribution ( $\sigma, \omega$ -exchange)



## ■ $\sigma\sigma$ -exchange contribution

$$\begin{aligned}
 V_{NNY}^{\sigma\sigma, \bar{B}} = & C_{NNY}^{\sigma\sigma, \bar{B}} \left( -4Z_{12}(r_{12})Z_{23}(r_{23})\nabla_{r_2'}^2 - 4Z_{12}'(r_{12})Z_{23}(r_{23})\hat{r}_{12} \cdot \nabla_{r_2'} \right. \\
 & -4Z_{12}(r_{12})Z_{23}'(r_{23})\hat{r}_{23} \cdot \nabla_{r_2'} - (Y_{12}(r_{12})Z_{23}(r_{23}) + Z_{12}(r_{12})Y_{23}(r_{23})) \\
 & -\hat{r}_{12} \cdot \hat{r}_{23}Z_{12}'(r_{12})Z_{23}'(r_{23}) - 2i \left( Z_{12}'(r_{12})Z_{23}(r_{23})\vec{\sigma}_2 \cdot \hat{r}_{12} \times \nabla_{r_2'} \right. \\
 & \left. \left. + Z_{12}(r_{12})Z_{23}'(r_{23})\vec{\sigma}_2 \cdot \hat{r}_{23} \times \nabla_{r_2'} \right) \right) \delta(\vec{r}_1 - \vec{r}_1') \delta(\vec{r}_2 - \vec{r}_2') \delta(\vec{r}_3 - \vec{r}_3')
 \end{aligned}$$

- $\omega\omega$ –exchange contribution

$$\begin{aligned}
V_{NNY}^{\omega\omega, \bar{B}} = & C_{NNY}^{\omega\omega, \bar{B}} \left( \left( (1 + \vec{\sigma}_1 \cdot \vec{\sigma}_2 + \vec{\sigma}_1 \cdot \vec{\sigma}_2 + \vec{\sigma}_2 \cdot \vec{\sigma}_3) \hat{r}_{12} \cdot \hat{r}_{23} - \vec{\sigma}_1 \cdot \hat{r}_{23} \vec{\sigma}_2 \cdot \hat{r}_{12} - \vec{\sigma}_2 \cdot \hat{r}_{23} \vec{\sigma}_3 \cdot \hat{r}_{12} \right. \right. \\
& - \vec{\sigma}_1 \cdot \hat{r}_{23} \vec{\sigma}_3 \cdot \hat{r}_{12} \Big) Z'_{12}(r_{12}) Z'_{23}(r_{23}) - 2i Z'_{12}(r_{12}) Z_{23}(r_{23}) (\vec{\sigma}_2 + \vec{\sigma}_3) \cdot \hat{r}_{12} \times \nabla_{r'_3} \\
& - 2i Z_{12}(r_{12}) Z'_{23}(r_{23}) (\vec{\sigma}_2 + \vec{\sigma}_3) \cdot \hat{r}_{23} \times \nabla_{r'_2} - 4 Z_{12}(r_{12}) Z_{23}(r_{23}) \nabla_{r'_1} \cdot \nabla_{r'_3} \Big) \\
& \delta(\vec{r}_1 - \vec{r}'_1) \delta(\vec{r}_2 - \vec{r}'_2) \delta(\vec{r}_3 - \vec{r}'_3)
\end{aligned}$$

- $\sigma\omega$ –exchange contribution

$$\begin{aligned}
V_{NNY}^{\sigma\omega, \bar{B}} = & C_{NNY}^{\sigma\omega, \bar{B}} \left( \left( (1 + \vec{\sigma}_2 \cdot \vec{\sigma}_3) Z_{12}(r_{12}) Y_{23}(r_{23}) - 2i Z_{12}(r_{12}) Z'_{23}(r_{23}) (\vec{\sigma}_2 + \vec{\sigma}_3) \cdot \hat{r}_{23} \times \nabla_{r'_2} \right. \right. \\
& + 2i Z'_{12}(r_{12}) Z'_{23}(r_{23}) (\vec{\sigma}_2 + \vec{\sigma}_3) \cdot \hat{r}_{12} \times \hat{r}_{23} + 2i Z_{12}(r_{12}) Z'_{23}(r_{23}) \vec{\sigma}_2 \cdot \hat{r}_{23} \times \nabla_{r'_3} \\
& + 2 Z'_{12}(r_{12}) Z_{23}(r_{23}) \hat{r}_{12} \cdot \nabla_{r'_3} + 2 Z_{12}(r_{12}) Z'_{23}(r_{23}) \hat{r}_{23} \cdot \nabla_{r'_3} + 4 Z_{12}(r_{12}) Z_{23}(r_{23}) \nabla_{r'_2} \cdot \nabla_{r'_3} \\
& \left. - \frac{1}{3} \left( \vec{\sigma}_2 \cdot \vec{\sigma}_3 Y_{12}(r_{12}) + \hat{S}_{23}(\hat{r}_{23}) T_{23}(r_{23}) \right) Z_{12}(r_{12}) \right) \\
& + D_{NNY}^{\sigma\omega, \bar{B}} \left( -Y_{12}(r_{12}) + Y_{23}(r_{23}) - 4 Z'_{12}(r_{12}) Z'_{23}(r_{23}) - 3 Z'_{12}(r_{12}) \nabla_{r'_2} \cdot \hat{r}_{12} \right) \\
& + i \vec{\sigma}_2 \cdot \left( 2 \nabla_{r_{23}} \times \nabla_{r_{12}} - 5 \nabla_{r'_2} \times \hat{r}_{23} \right) Z_{12}(r_{12}) Z_{23}(r_{23}) \\
& + \left( \vec{r}_{12} \leftrightarrow \vec{r}_{23}, \vec{r}_1 \leftrightarrow \vec{r}'_1, \vec{r}_2 \leftrightarrow \vec{r}'_2, \vec{\sigma}_1 \leftrightarrow \vec{\sigma}_3 \right) \delta(\vec{r}_1 - \vec{r}'_1) \delta(\vec{r}_2 - \vec{r}'_2) \delta(\vec{r}_3 - \vec{r}'_3)
\end{aligned}$$

But that's only the beginning of the full story  
there are

**MANY, MANY, MANY** more forces & contributions ....



# BHF approximation of Hyperonic Matter

## 💡 Energy per particle

$$\blacksquare \frac{E}{A}(\rho, \beta) = \frac{1}{A} \sum_B \sum_{k \leq k_{FB}} \left( \frac{\hbar^2 k^2}{2m_B} + \frac{1}{2} \text{Re} [U_B(\vec{k})] \right)$$



Infinite summation of **two-hole line** diagrams

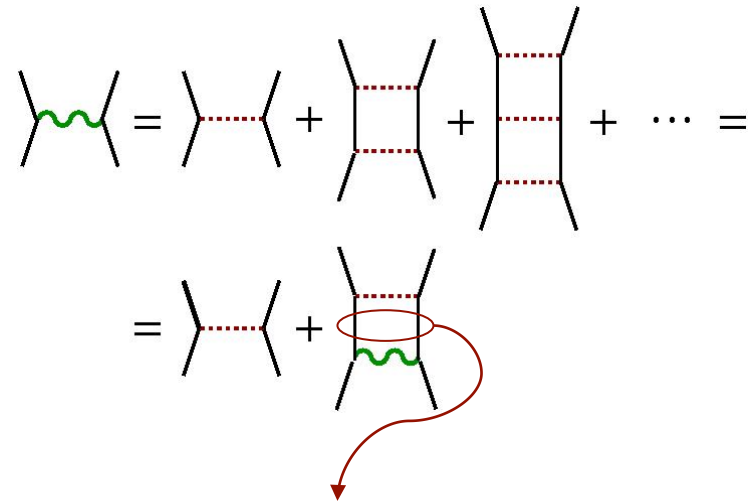
## 💡 Bethe-Goldstone Equation

$$\blacksquare G(\omega) = V + V \frac{Q}{\omega - E - E' + i\eta} G(\omega)$$

$$\blacksquare E_B(k) = \frac{\hbar^2 k^2}{2m_B} + \text{Re} [U_N(k)] + m_B$$

$$\blacksquare U_B(k) = \sum_{B'} \sum_{k' \leq k_{FB'}} \langle \vec{k}\vec{k}' | G(\omega = E_B(k) + E_{B'}(k')) | \vec{k}\vec{k}' \rangle$$

Partial summation of **pp ladder** diagrams



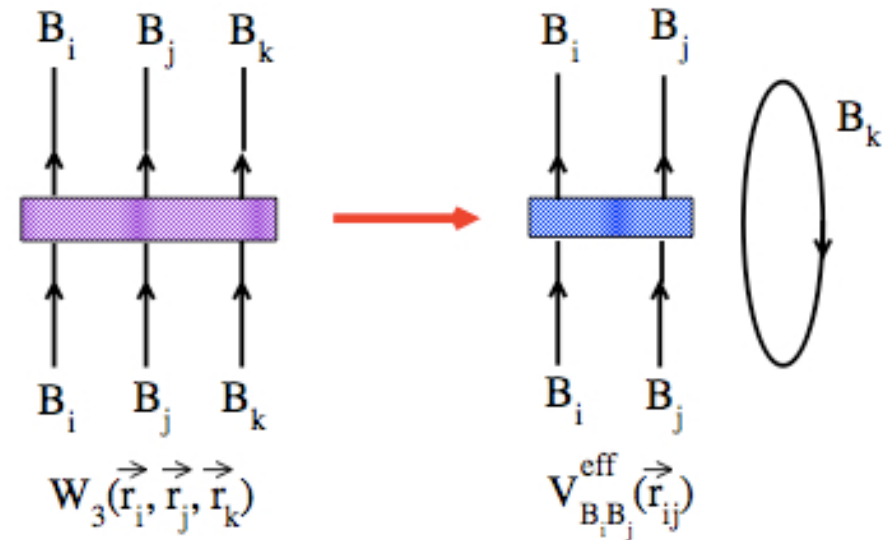
✓ Pauli blocking

✓ Baryon dressing



## Three-Body Forces within the BHF approach

TBF can be introduced in our BHF approach by adding effective density-dependent two body forces to the baryon-baryon interactions  $V$  when solving the Bethe-Goldstone equation

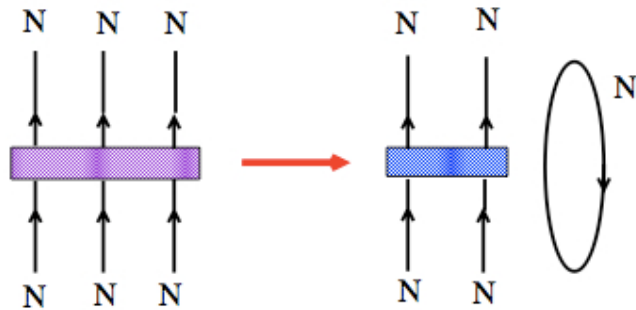


$$V_{B_i B_j}^{eff}(\vec{r}_{ij}) = \int W_3(\vec{r}_i, \vec{r}_j, \vec{r}_k) n(\vec{r}_i, \vec{r}_j, \vec{r}_k) d^3 \vec{r}_k$$

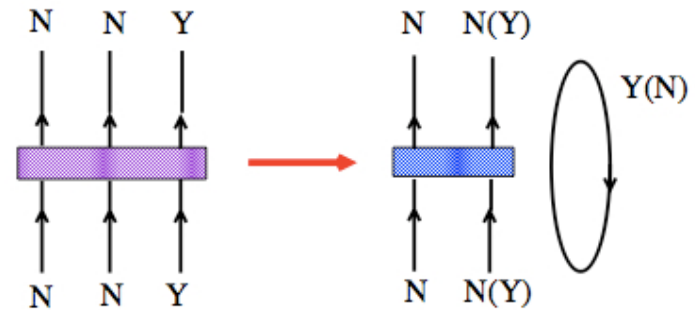
$W_3(\vec{r}_i, \vec{r}_j, \vec{r}_k)$ : genuine TBF     
  $n(\vec{r}_i, \vec{r}_j, \vec{r}_k)$ : three-body correlation function

# From the genuine NNN,NNY, NYY and YYY TBF ...

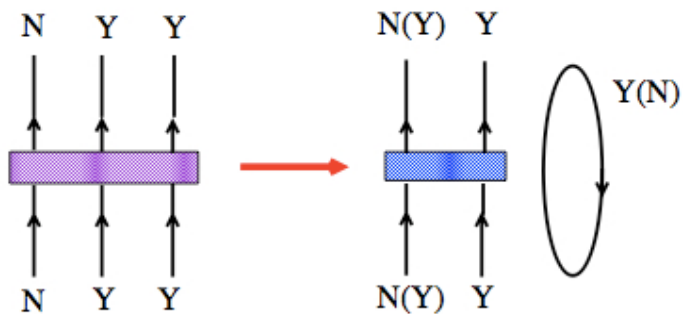
NNN → NN



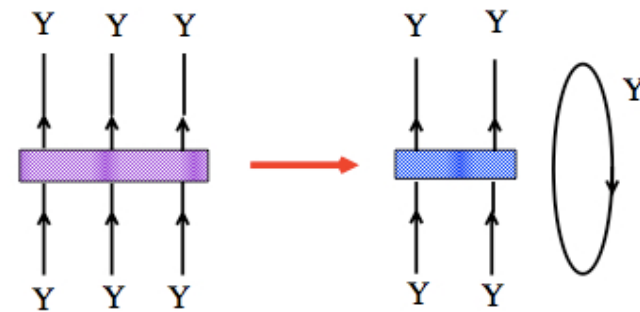
NNY → NN, NY

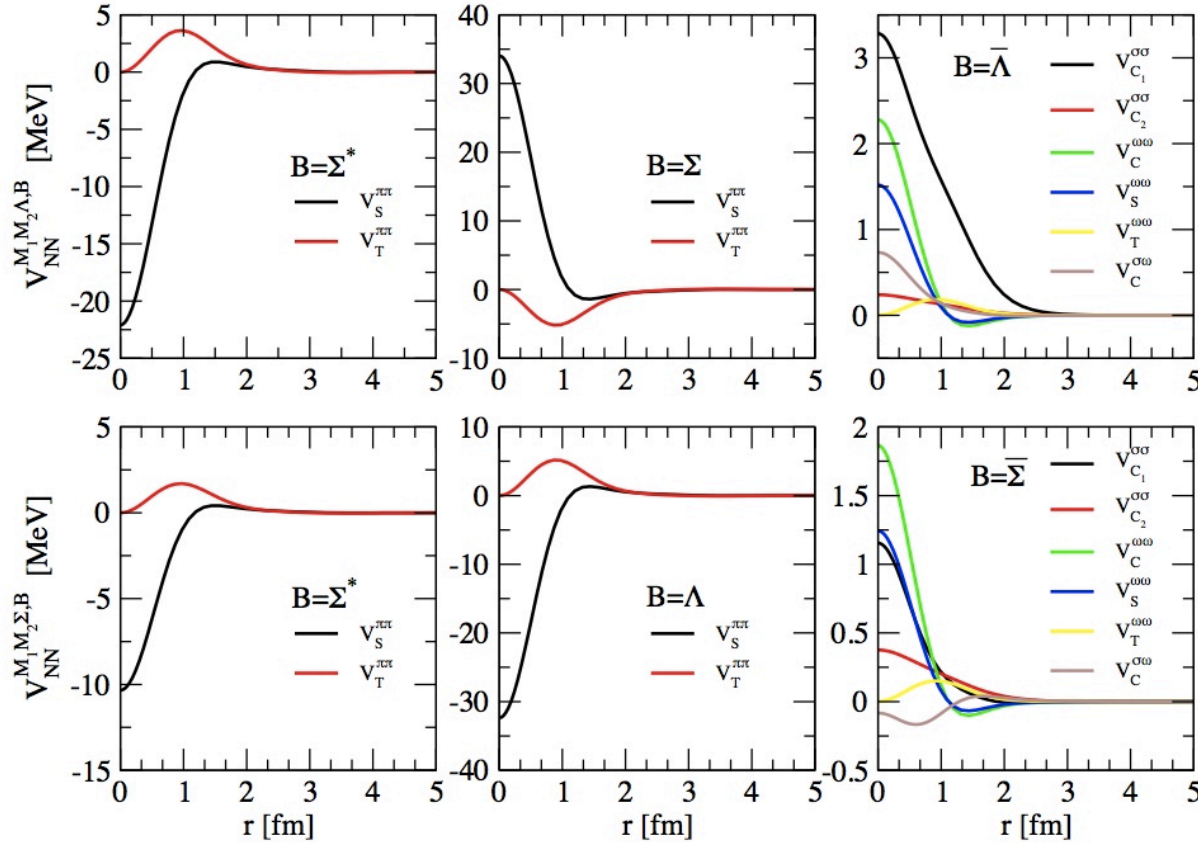


NYY → NY, YY



YYY → YY

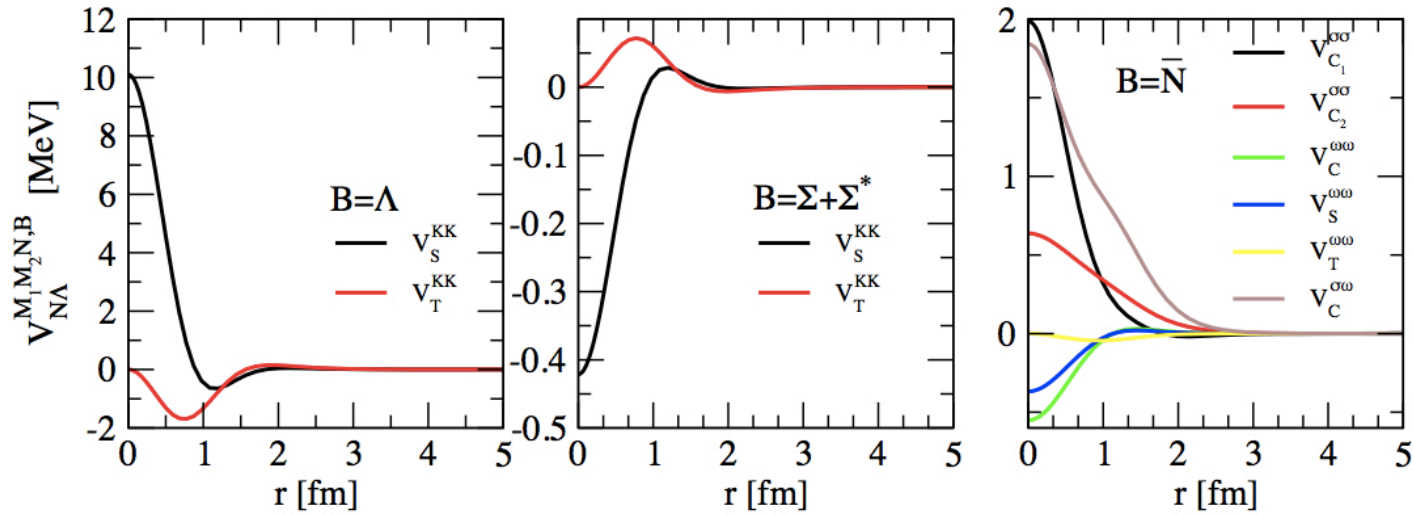




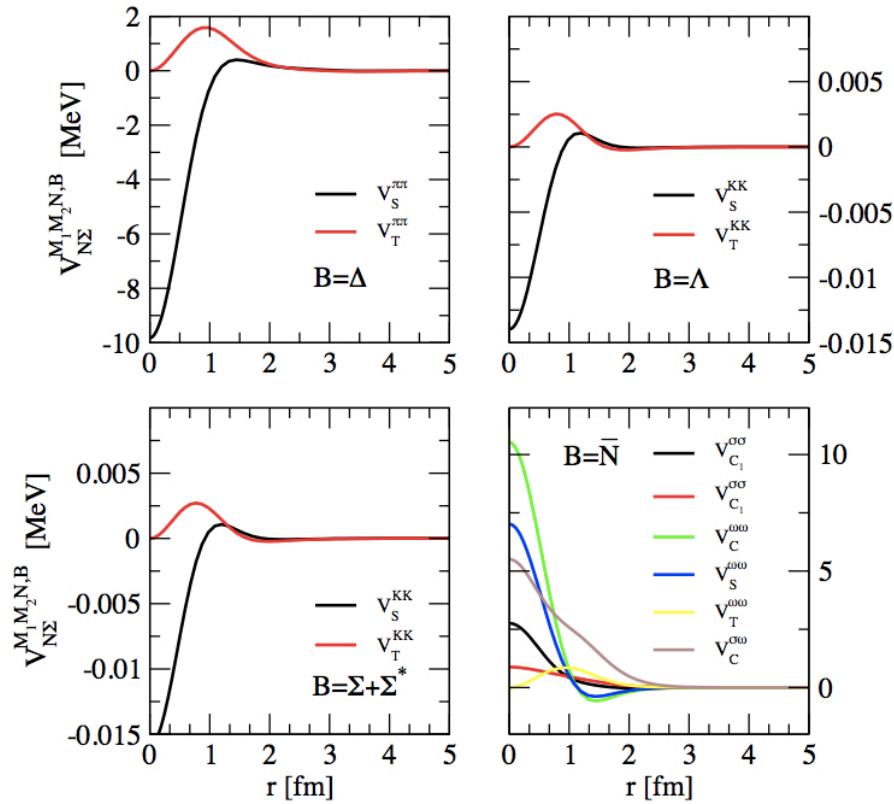
## Effective NN density-dependent 2BF from NNY

- $$V_{NN}^{\pi\pi Y, B}(\vec{r}) = C_{NNY}^{\pi\pi, B} \rho_Y \left[ V_S^{\pi\pi}(\vec{r}) \vec{\sigma}_1 \cdot \vec{\sigma}_2 + V_T^{\pi\pi}(\vec{r}) S_{12}(\hat{r}) \right] \vec{\tau}_1 \cdot \vec{\tau}_2$$
- $$V_{NN}^{\sigma\sigma Y, \bar{B}}(\vec{r}) = C_{NNY}^{\sigma\sigma, \bar{B}} \left[ \rho_N V_{C_1}^{\sigma\sigma}(\vec{r}) + \rho_N^{5/3} V_{C_2}^{\sigma\sigma}(\vec{r}) \right]$$
- $$V_{NN}^{\omega\omega Y, \bar{B}}(\vec{r}) = C_{NNY}^{\omega\omega, \bar{B}} \rho_Y \left[ V_C^{\omega\omega}(\vec{r}) + V_S^{\omega\omega}(\vec{r}) \vec{\sigma}_1 \cdot \vec{\sigma}_2 + V_T^{\omega\omega}(\vec{r}) S_{12}(\hat{r}) \right]$$
- $$V_{NN}^{\sigma\omega Y, \bar{B}}(\vec{r}) = C_{NNY}^{\sigma\omega, \bar{B}} \rho_N V_C^{\sigma\omega}(\vec{r})$$

# Effective $N\Lambda$ density-dependent 2BF from $NN\Lambda$



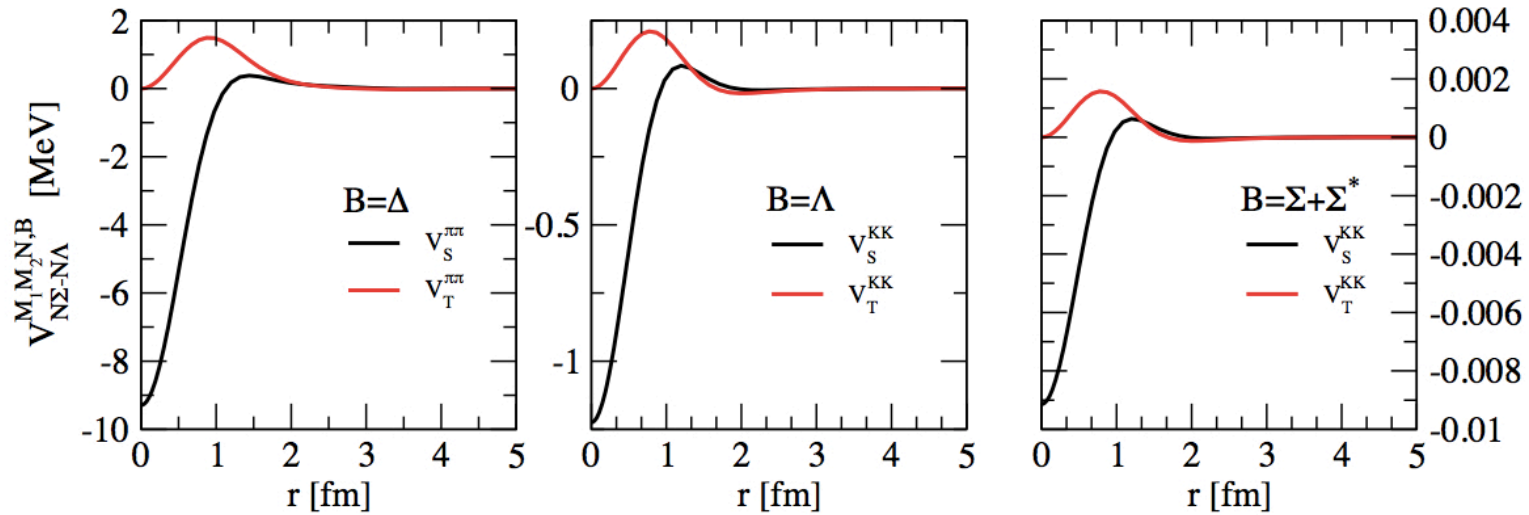
- $V_{N\Lambda}^{KKN, \Lambda}(\vec{r}) = C_{NN\Lambda}^{KK, \Lambda} \rho_N \left[ V_S^{KK}(\vec{r}) \vec{\sigma}_1 \cdot \vec{\sigma}_2 + V_T^{KK}(\vec{r}) S_{12}(\hat{r}) \right]$
- $V_{N\Lambda}^{KKN, \Sigma/\Sigma^*}(\vec{r}) = C_{NN\Lambda}^{KK, \Sigma/\Sigma^*} \rho_N \left[ V_S^{KK}(\vec{r}) \vec{\sigma}_1 \cdot \vec{\sigma}_2 + V_T^{KK}(\vec{r}) S_{12}(\hat{r}) \right] \vec{\tau}_1 \cdot \vec{1}_2$
- $V_{N\Lambda}^{\sigma\sigma N, \bar{N}}(\vec{r}) = C_{NN\Lambda}^{\sigma\sigma, \bar{N}} \left[ \rho_\Lambda V_{C_1}^{\sigma\sigma}(\vec{r}) + \rho_\Lambda^{5/3} V_{C_2}^{\sigma\sigma}(\vec{r}) \right]$
- $V_{N\Lambda}^{\omega\omega N, \bar{N}}(\vec{r}) = C_{NN\Lambda}^{\omega\omega, \bar{N}} \rho_N \left[ V_C^{\omega\omega}(\vec{r}) + V_S^{\omega\omega}(\vec{r}) \vec{\sigma}_1 \cdot \vec{\sigma}_2 + V_T^{\omega\omega}(\vec{r}) S_{12}(\hat{r}) \right]$
- $V_{N\Lambda}^{\sigma\omega N, \bar{N}}(\vec{r}) = C_{NN\Lambda}^{\sigma\omega, \bar{N}} \rho_\Lambda V_C^{\sigma\omega}(\vec{r})$



## Effective $N\Sigma$ density-dependent 2BF from $NN\Sigma$

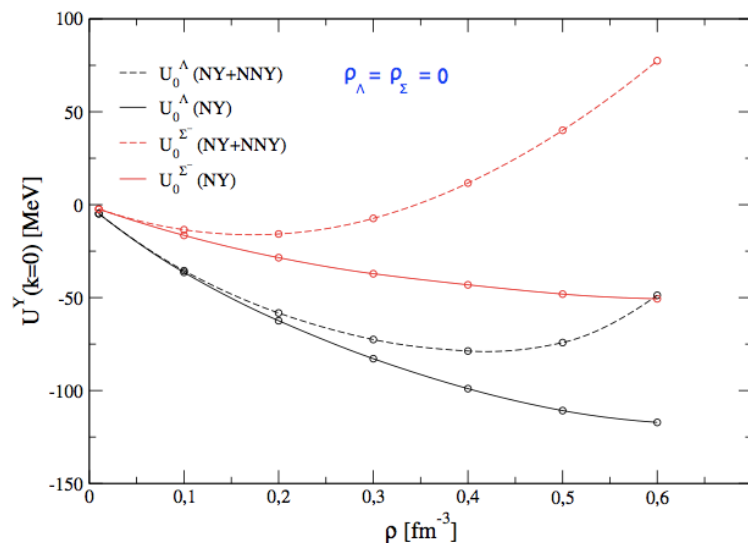
- $V_{N\Sigma}^{\pi\pi N, \Delta}(\vec{r}) = C_{NN\Sigma}^{\pi\pi, \Delta} \rho_N \left[ V_S^{\pi\pi}(\vec{r}) \vec{\sigma}_1 \cdot \vec{\sigma}_2 + V_T^{\pi\pi}(\vec{r}) S_{12}(\hat{r}) \right] \vec{\tau}_1 \cdot \vec{I}_2$
- $V_{N\Sigma}^{KK N, \Lambda/\Sigma}(\vec{r}) = C_{NN\Sigma}^{KK, \Lambda/\Sigma} \rho_N \left[ V_S^{KK}(\vec{r}) \vec{\sigma}_1 \cdot \vec{\sigma}_2 + V_T^{KK}(\vec{r}) S_{12}(\hat{r}) \right] \vec{\tau}_1 \cdot \vec{\tau}_2$
- $V_{N\Sigma}^{KK N, \Sigma^*}(\vec{r}) = C_{NN\Sigma}^{KK, \Sigma^*} \rho_N \left[ V_S^{KK}(\vec{r}) \vec{\sigma}_1 \cdot \vec{\sigma}_2 + V_T^{KK}(\vec{r}) S_{12}(\hat{r}) \right] \vec{\tau}_1 \cdot \vec{1}_2$
- $V_{N\Sigma}^{\sigma\sigma N, \bar{N}}(\vec{r}) = C_{NN\Sigma}^{\sigma\sigma, \bar{N}} \left[ \rho_\Sigma V_{C_1}^{\sigma\sigma}(\vec{r}) + \rho_\Sigma^{5/3} V_{C_2}^{\sigma\sigma}(\vec{r}) \right]$
- $V_{N\Sigma}^{\omega\omega N, \bar{N}}(\vec{r}) = C_{NN\Sigma}^{\omega\omega, \bar{N}} \rho_N \left[ V_C^{\omega\omega}(\vec{r}) + V_S^{\omega\omega}(\vec{r}) \vec{\sigma}_1 \cdot \vec{\sigma}_2 + V_T^{\omega\omega}(\vec{r}) S_{12}(\hat{r}) \right]$
- $V_{N\Sigma}^{\sigma\omega N, \bar{N}}(\vec{r}) = C_{NN\Sigma}^{\sigma\omega, \bar{N}} \rho_\Sigma V_C^{\sigma\omega}(\vec{r})$

# Effective density-dependent transition $N\Sigma - N\Lambda$ from $NN\Sigma - NN\Lambda$

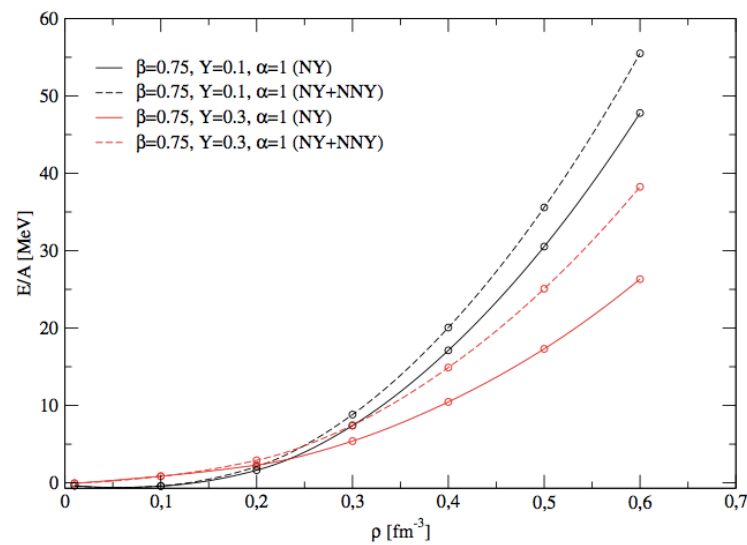
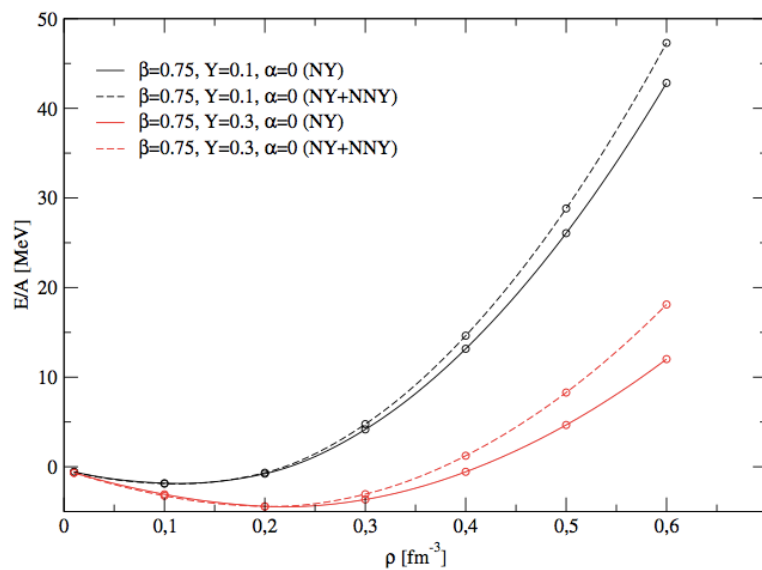


- $$V_{N\Sigma \leftrightarrow N\Lambda}^{\pi\pi N, \Delta}(\vec{r}) = C_{NN\Sigma \leftrightarrow NN\Lambda}^{\pi\pi, \Delta} \rho_N \left[ V_S^{\pi\pi}(\vec{r}) \vec{\sigma}_1 \cdot \vec{\sigma}_2 + V_T^{\pi\pi}(\vec{r}) S_{12}(\hat{r}) \right] \vec{\tau}_1 \cdot \vec{I}_2$$
- $$V_{N\Sigma \leftrightarrow N\Lambda}^{KK N, \Lambda/\Sigma/\Sigma^*}(\vec{r}) = C_{NN\Sigma \leftrightarrow NN\Lambda}^{KK, \Lambda/\Sigma/\Sigma^*} \rho_N \left[ V_S^{KK}(\vec{r}) \vec{\sigma}_1 \cdot \vec{\sigma}_2 + V_T^{KK}(\vec{r}) S_{12}(\hat{r}) \right] \vec{\tau}_1 \cdot \vec{I}_2$$

# Effect of TBF on Mean Field & E/A



- ✓ Only NNY considered (preliminar)
- ✓ Repulsion at high densities due to Z-diagram contribution as in NNN



Work is in progress, many more contributions have to be considered,  
but we can still try to estimate the effect of hyperonic TBF in NS

- 1-. Construct the hyperonic matter EoS within the BHF at 2 body level  
(Av18 NN + NSC89 YN)
- 2-. Add simple phenomenological density-dependent contact terms that  
mimic the effect of TBF.

Density-dependent contact terms: (Balberg & Gal 1997)

Potential of a baryon  $B_y$  in a sea  
of baryons  $B_x$  of density  $\rho_x$

Folding  $V_y(\rho_x)$  with  $\rho_x$ ,  $V_x(\rho_y)$  with  $\rho_y$  and  
combining with weight factors  $\rho_x/\rho$  and  $\rho_y/\rho$

$$V_y(\rho_x) = a_{xy}\rho_x + b_{xy}\rho_x^{\gamma_{xy}}$$

$$\varepsilon_{xy}(\rho_x, \rho_y) = a_{xy}\rho_x\rho_y + b_{xy}\rho_x\rho_y \left( \frac{\rho_x^{\gamma_{xy}} + \rho_y^{\gamma_{xy}}}{\rho_x + \rho_y} \right)$$

attraction

repulsion

larger than 1

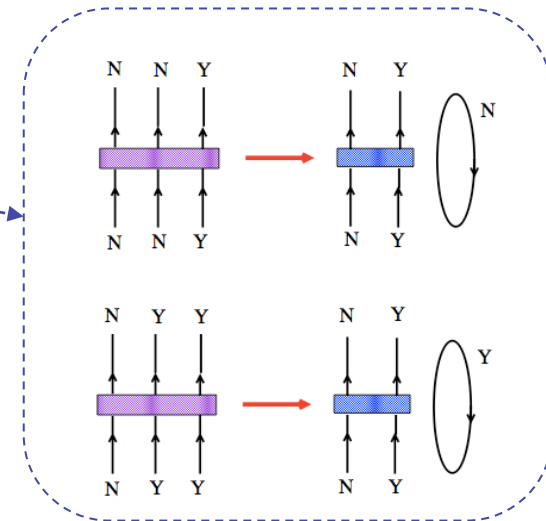
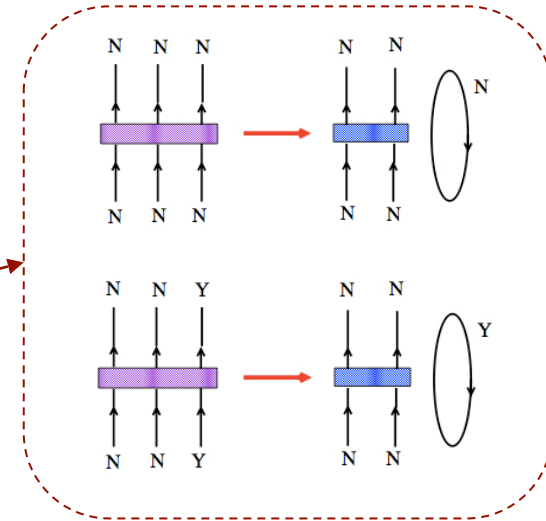


Then, we have ...

$$\varepsilon_{CT} = a_{NN}\rho_N^2 + b_{NN}\rho_N^{\gamma_{NN}}$$

$$+ a_{\Lambda N}\rho_\Lambda\rho_N + b_{\Lambda N}\rho_\Lambda\rho_N\left(\frac{\rho_\Lambda^{\gamma_{\Lambda N}} + \rho_N^{\gamma_{\Lambda N}}}{\rho_\Lambda + \rho_N}\right)$$

$$+ a_{\Sigma N}\rho_\Sigma\rho_N + b_{\Sigma N}\rho_\Sigma\rho_N\left(\frac{\rho_\Sigma^{\gamma_{\Sigma N}} + \rho_N^{\gamma_{\Sigma N}}}{\rho_\Sigma + \rho_N}\right)$$



$$\rho_N = \rho_n + \rho_p, \quad \rho_\Sigma = \rho_{\Sigma^-} + \rho_{\Sigma^0} + \rho_{\Sigma^+}$$

**NYN → NY and YYY → YY**  
not included for consistency

The parameters  $a_{NN}$ ,  $b_{NN}$  and  $\gamma_{NN}$   
 fitted to reproduce  $\rho_0=0.16 \text{ fm}^{-3}$ ,  
 $E/A=-16 \text{ MeV}$  and  $K = 211-285 \text{ MeV}$

$\gamma_{NN}$	$a_{NN}$ [MeV fm <sup>3</sup> ]	$b_{NN}$ [MeV fm <sup>3<math>\gamma_{NN}</math></sup> ]	$K_\infty$ [MeV]
2	-33.44	213.02	211
2.5	-22.08	355.03	236
3	-16.40	665.68	260
3.5	-12.99	1331.36	285

For simplicity, we take  $a_{\Lambda N}=a_{\Sigma N}$ ,  $b_{\Lambda N}=b_{\Sigma N}$  and  $\gamma_{\Lambda N}=\gamma_{\Sigma N}$  with

$$a_{\Lambda N} = xa_{NN}, \quad b_{\Lambda N} = xb_{NN}, \quad x = 0, \frac{1}{3}, \frac{2}{3}, 1$$

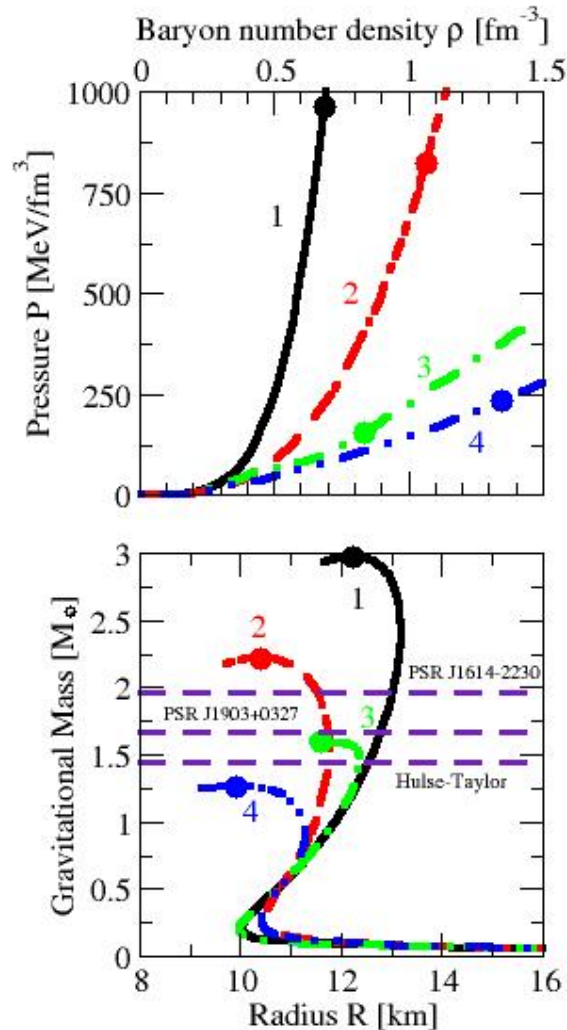
to explore different strength  
of the hyperonic TBF

$\gamma_{\Lambda N}$  is obtained using the value of  
-28 MeV for the binding energy  
of a  $\Lambda$  in nuclear matter

$$\left(\frac{B}{A}\right)_\Lambda = 28 \text{ MeV} = -U_\Lambda(k=0) - a_{YN}\rho_0 - b_{YN}\rho_0^{\gamma_{YN}}$$

$$U_\Lambda(k=0) = -30.8 \text{ MeV}$$

# Effect of hyperonic TBF on $M_{\max}$



$\gamma_{NN}$	$x$	$\gamma_{YN}$	Maximum Mass
2	0	-	1.27 (2.22)
	1/3	1.49	1.33
	2/3	1.69	1.38
2.5	1	1.77	1.41
	0	-	1.29 (2.46)
	1/3	1.84	1.38
3	2/3	2.08	1.44
	1	2.19	1.48
	0	-	1.34 (2.72)
3.5	1/3	2.23	1.45
	2/3	2.49	1.50
	1	2.62	1.54
3.5	0	-	1.38 (2.97)
	1/3	2.63	1.51
	2/3	2.91	1.56
	1	3.05	1.60

Hyperonic TBFs seem not to be the full solution of the “Hyperon Puzzle”, although they probably contribute to its solution

$$1.27 < M_{\max} < 1.6M_{\odot}$$



# Summary & Conclusions

- ❖ Construction of two-meson exchange hyperonic TBF

Repulsion is obtained at high densities (Z-diagram)

D. Logoteta, Ph.D. Thesis (Univ. Coimbra 2013)

- ❖ Simple model to establish numerical lower and upper limits to the effect of hyperonic TBF on the maximum mass of NS.

Assuming the strength of hyperonic TBF  $\leq$  nucleonic TBF:

$$1.27 M_{\odot} < M_{\max} < 1.60 M_{\odot} \quad \text{compatible with } 1.4\text{-}1.5 M_{\odot}$$

but incompatible with observation of very massive NS

PSR J1903+0327  $(1.67 \pm 0.01) M_{\odot}$

PSR J1614-2230  $(1.97 \pm 0.04) M_{\odot}$

PSR J0348+0432  $(2.01 \pm 0.04) M_{\odot}$

## Take away message



Hyperonic Three-Body Forces seem not to be the full solution to the “Hyperon Puzzle”, although they probably can contribute to it

- You for your time & attention
- The organizers for their invitation
- The sponsors for their support

