AMD Calculations of Medium-Heavy Hypernuclei with the ΛNN Three-Body Force in the Nijmegen Potential

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Collaborators: Y. Yamamoto, Th.A. Rijken

Grand challenges of hypernuclear physics

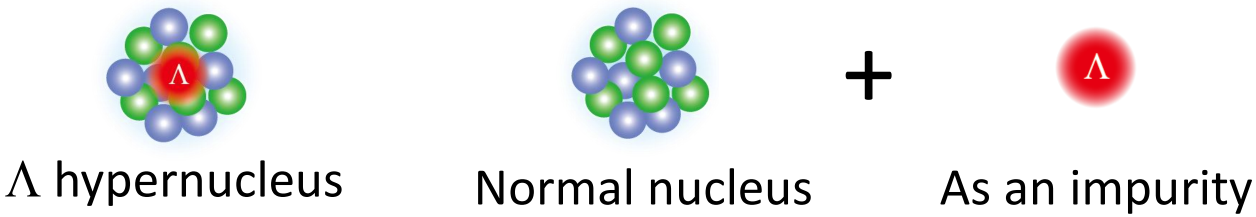
Interaction: To understand baryon-baryon interaction

- **2 body interaction between baryons (nucleon, hyperon)**
  - hyperon-nucleon (YN)
  - hyperon-hyperon (YY)
  - Three-body force

Structure: To understand many-body system of nucleons and hyperon

- **Addition of hyperon(s) shows us new features of nuclear structure**

  Ex.) Structure change by hyperon(s)
  - No Pauli exclusion between N and Y
  - YN interaction is different from NN

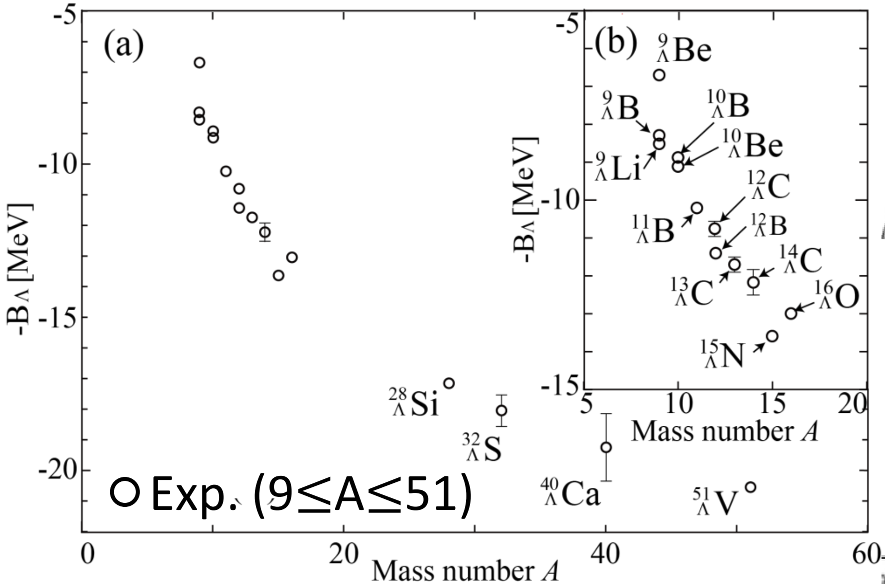


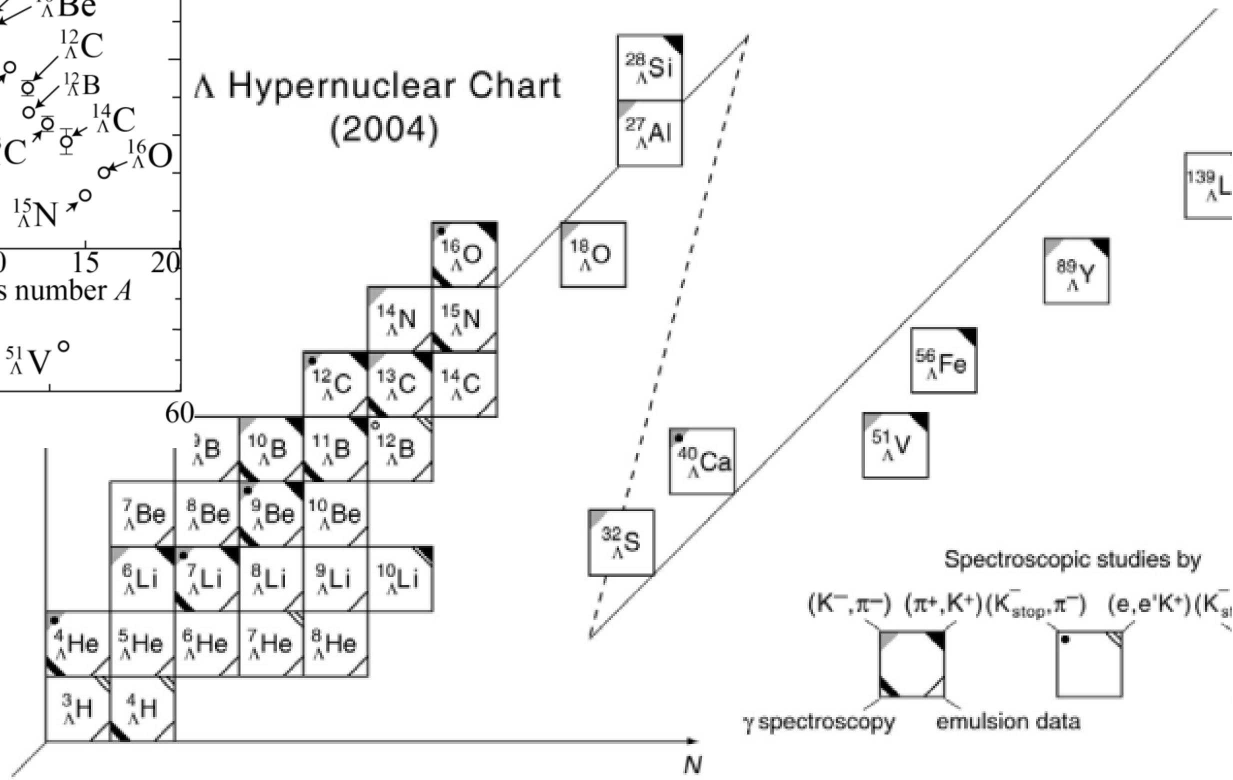
Today's talk: three-body force effects based on structure calculations

Studies of Λ hypernuclei

Λ hypernuclei observed so far

- **Concentrated in light Λ hypernuclei with A ≲ 10**





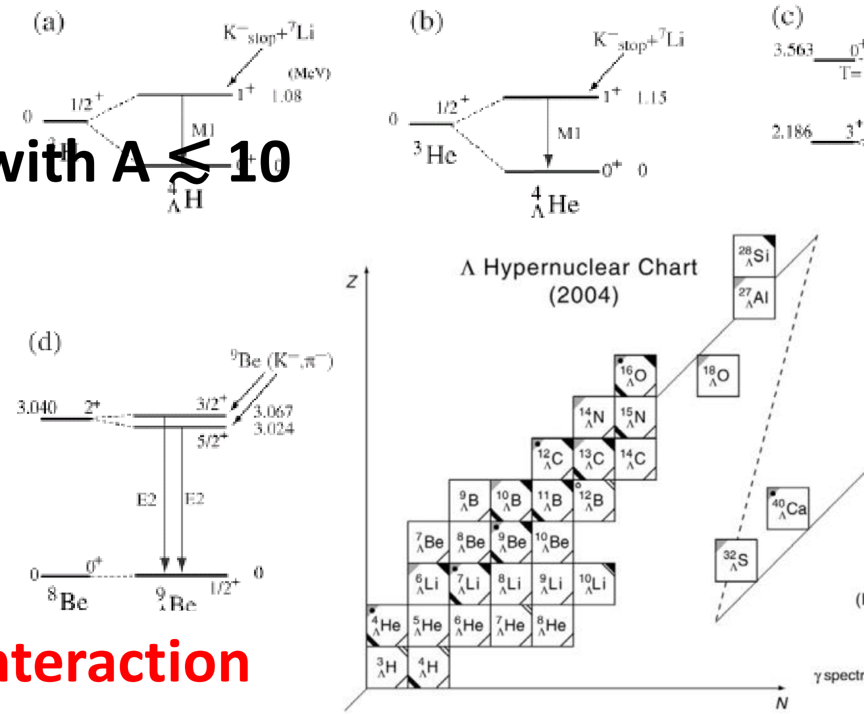
O. Hashimoto and H. Tamura, PPNP **57**(2006),564.

Studies of Λ hypernuclei: *What achieved?*

Λ hypernuclei observed so far

- **Concentrated in light Λ hypernuclei with A ≲ 10**
  - **Accurate solution of few-body problems**
  - **G-matrix calculation for ΛN interactions**
  - **Increases of experimental information**

E. Hiyama, NPA **805** (2008), 190c.
Y. Yamamoto, *et al.*, PTP Suppl. **117** (1994),361.
O. Hashimoto and H. Tamura, PPNP **57** (2006), 564.



→ **Knowledge of ΛN two-body interaction**

Developments of effective interactions

In this study,

G-matrix interaction derived from Nijmegen potential (YNG)

- Nijmegen potential: a meson exchange model
- G-matrix calculation takes into account medium effects

YNG interaction has density (Fermi momentum k_F) dependence coming from ΛN-ΣN coupling effects

"Hyperon puzzle" in neutron star physic

Hyperon puzzle

Massive ($2M_\odot$) neutron stars

2010	PSR J1614-2230	$(1.97 \pm 0.04) M_\odot$
2013	PSR J0348-0432	$(2.01 \pm 0.04) M_\odot$

?

Softening of EOS by hyperon mixing

How do we resolve?

Baryon-baryon three-body force

If strong repulsions exist not only *NNN* channel but *YNN*, *YYN*, and *YYY*, EOS of neutron star matter becomes stiff

S. Nishizaki, *et al.*, Prog. Theor. Phys. **105**, 607 (2001); 108, 703 (2002).

Our aim: to reveal effects of ΛNN 3-body force in Λ hypernuclear data

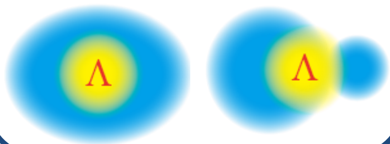
Density dependent three-body force based on YNG ΛN interaction

Toward heavier Λ hypernuclei

◆ **Future experiments: heavier hypernuclei will be produced!**

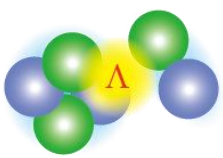
Various structures in the ground states

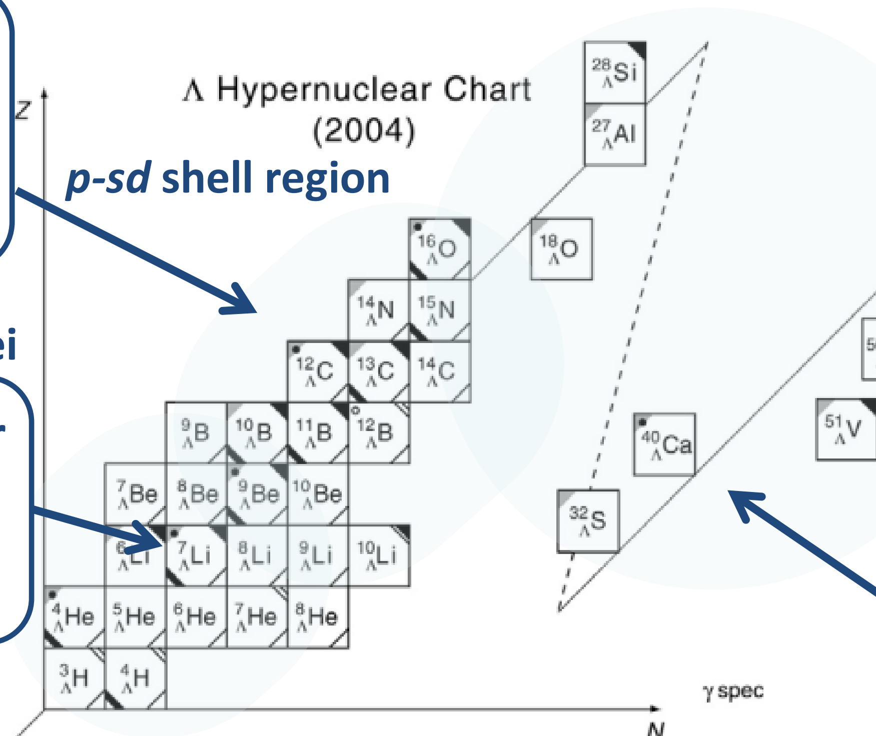
Coexistence of shell and cluster



p-shell Λ hypernuclei

Developed cluster

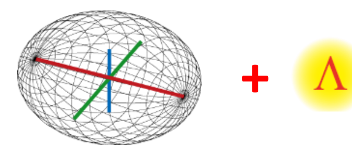




From O. Hashimoto and H. Tamura, PPNP **57**(2006),564.

Various deformations

Triaxial deformation



Deformations coexist

Structure of core nuclei could affect Λ binding energy B_Λ

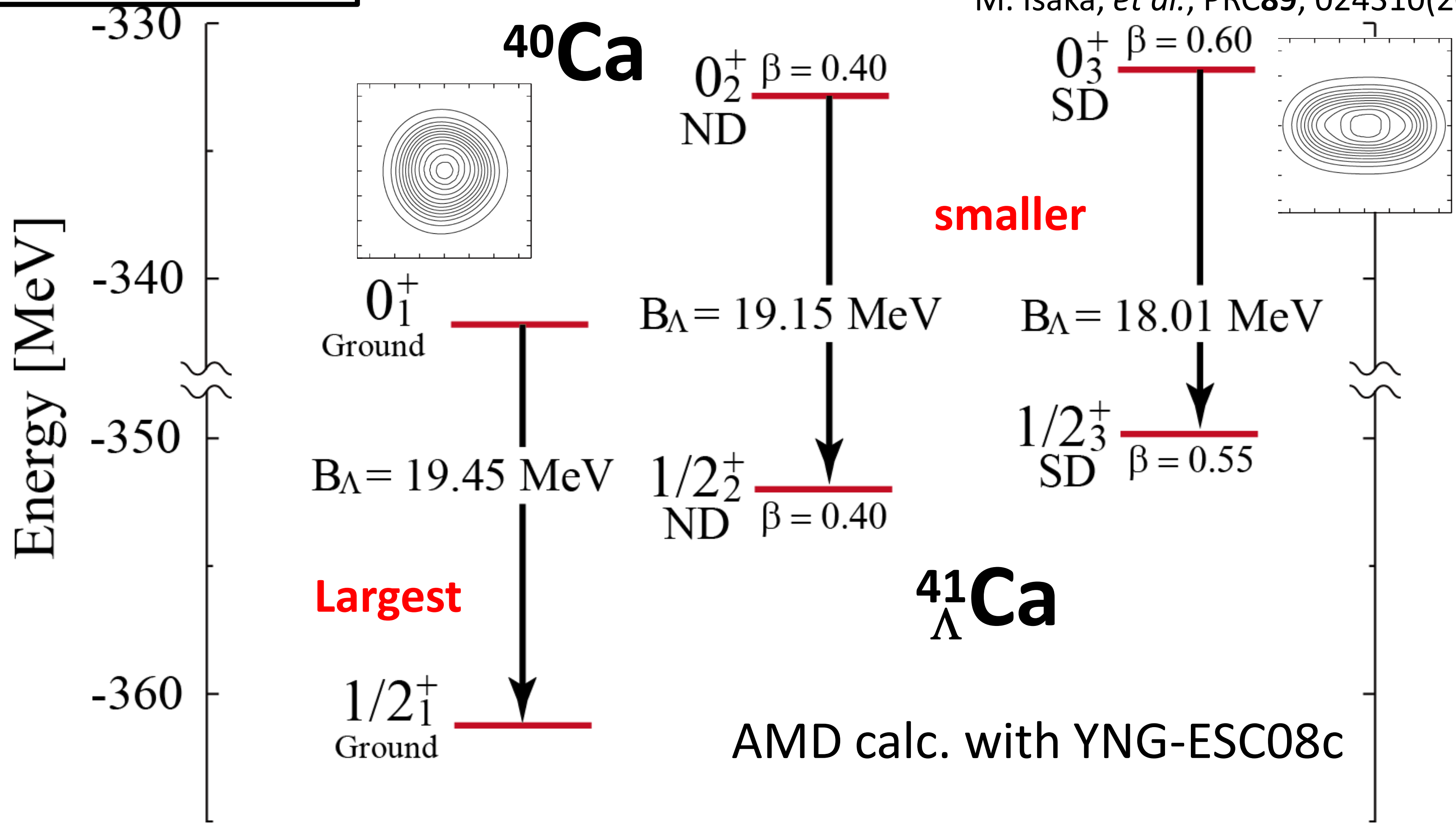
“clustering/deformations”, “density dependence of interactions”

$^{41}_{\Lambda}$Ca: How does structure affect B_Λ values?

Example: $^{41}_{\Lambda}$Ca

B_Λ is smaller in deformed states

M. Isaka, *et al.*, PRC**89**, 024310(2014)



$k_F = 1.26$ [fm^{-3}] calculated from g.s. density by Averaged Density Approximation (ADA)

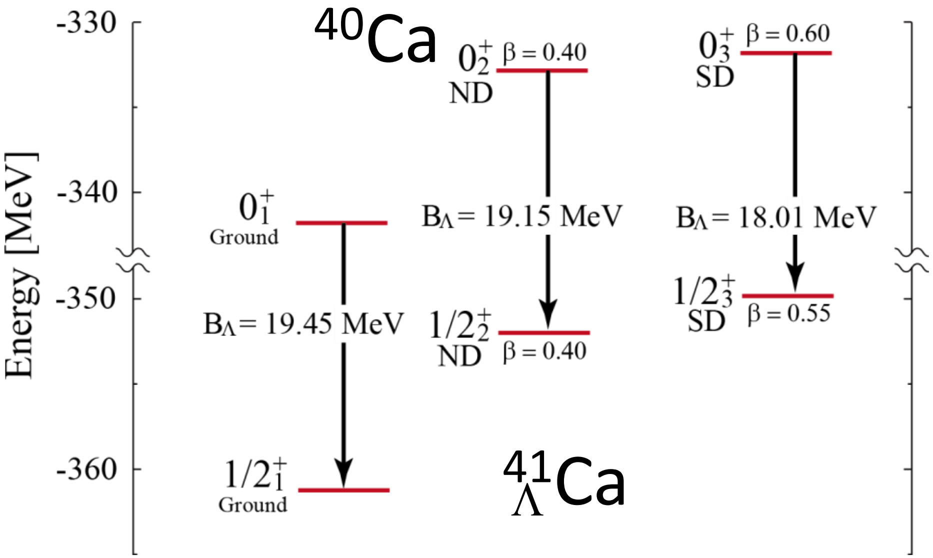
$$\langle\rho\rangle = \int dr^3 \rho_N(\mathbf{r})\rho_\Lambda(\mathbf{r}) \qquad k_F = \left(\frac{3\pi^2\langle\rho\rangle}{2}\right)^{1/3}$$

M. Isaka, *et al.*, PRC**89**, 024310(2014)

$^{41}{}_{\Lambda}$Ca: How does structure affect B_{Λ} values?

Why? **--- Overlap between Λ and core nucleus is essential!**

Overlap between Λ and N becomes smaller as nuclear deformation increases



Overlap between the Λ and nucleons

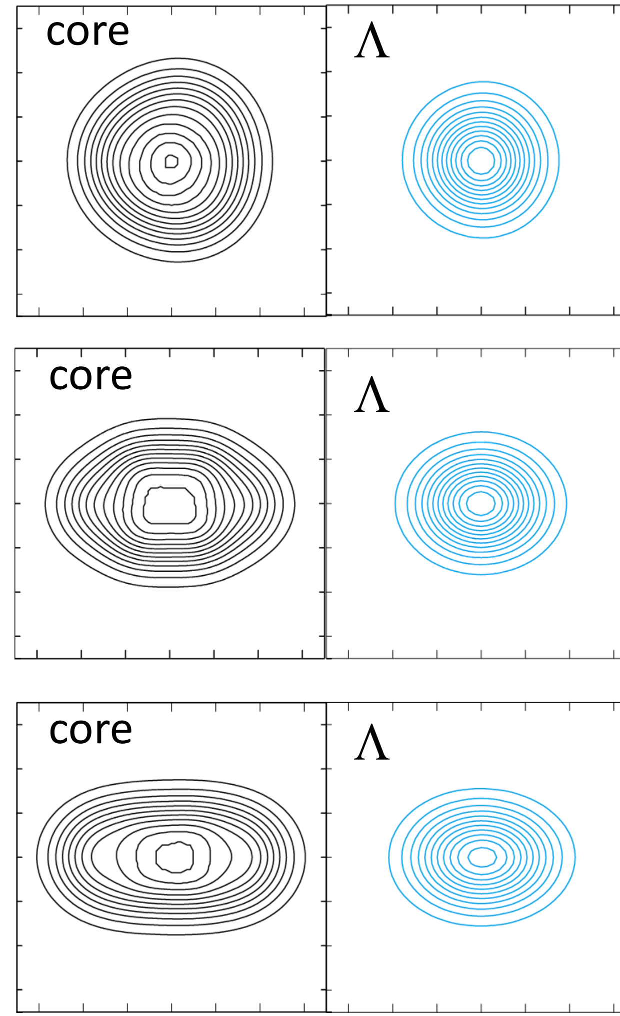
$$I = \int d^3r \rho_N(\mathbf{r}) \rho_{\Lambda}(\mathbf{r})$$

GS
β = 0.01
I = 0.1364

ND
β = 0.35
I = 0.1356

SD
β = 0.55
I = 0.1336

I [fm^{-3}]



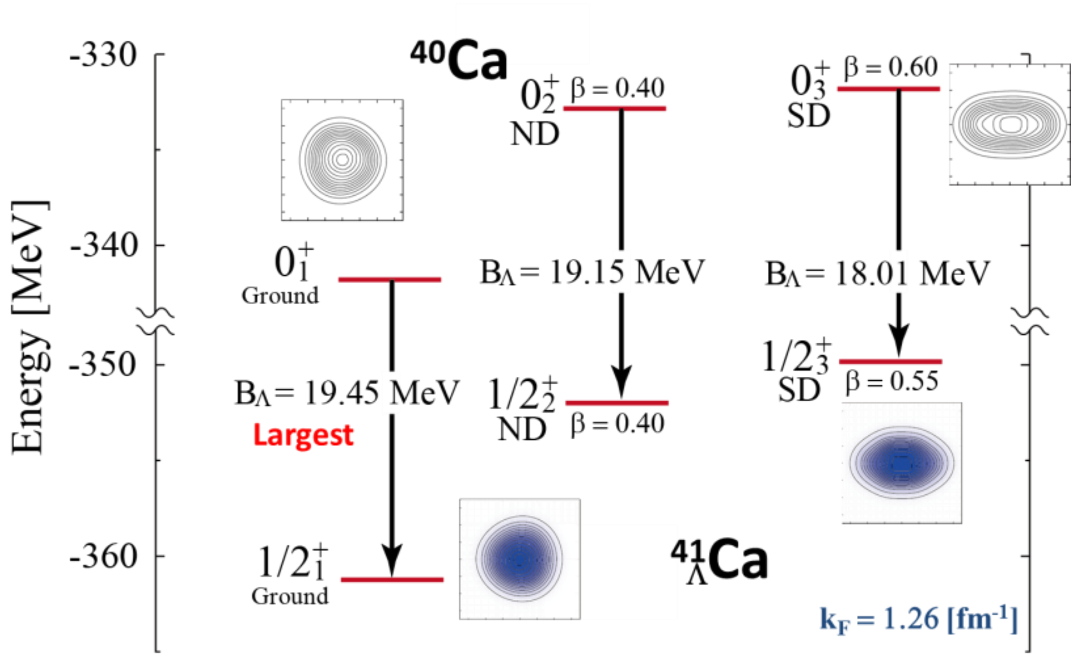
Decrease of overlap makes $V_{\Lambda N}$ shallower

Relation between B_Λ and nuclear structure

- **B_Λ values are related to nuclear structure in two ways.**
  - **Overlap between Λ and nucleons**

Increasing deformation reduces the overlap between Λ and nucleons

→ B_Λ becomes smaller in deformed states



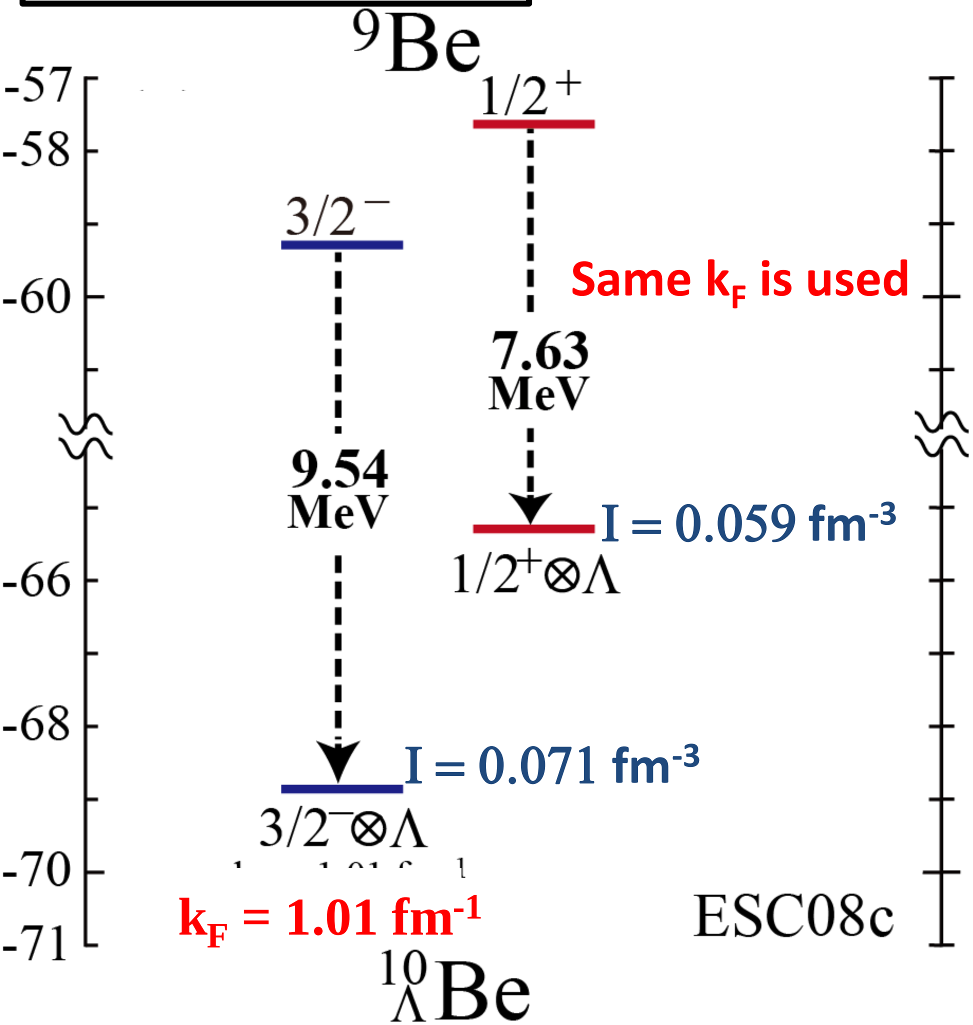
However, situation is different in dilute (cluster) states …

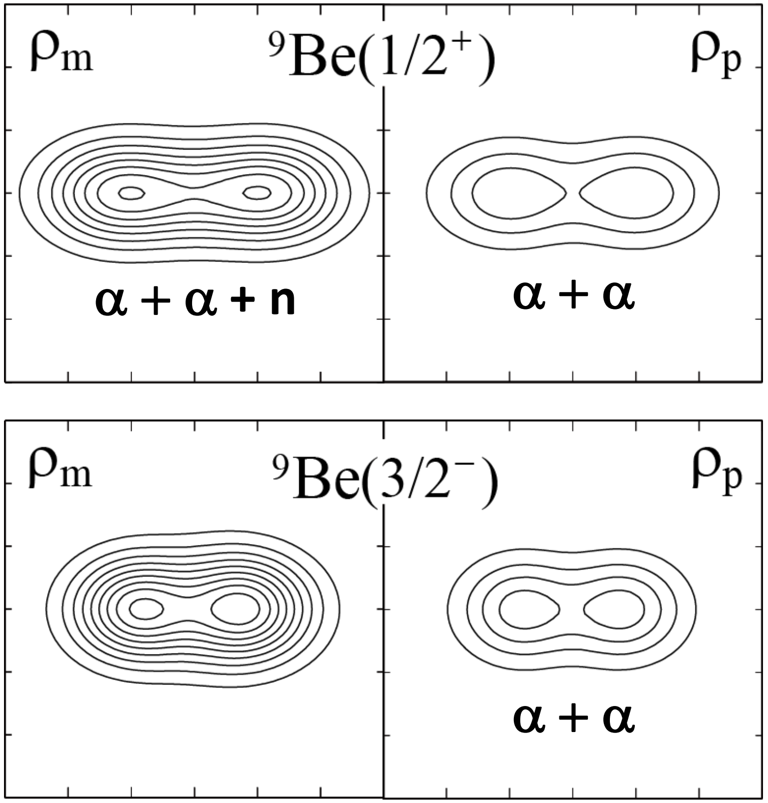
M. Isaka, M. Kimura, PRC**92**, 044326(2015)

$^{10}{}_{\Lambda}$Be: How does structure affect B_Λ values?

Example: $^{10}{}_{\Lambda}$Be

Large difference of overlap makes k_F different





Smaller k_F enlarges B_Λ in dilute cluster states

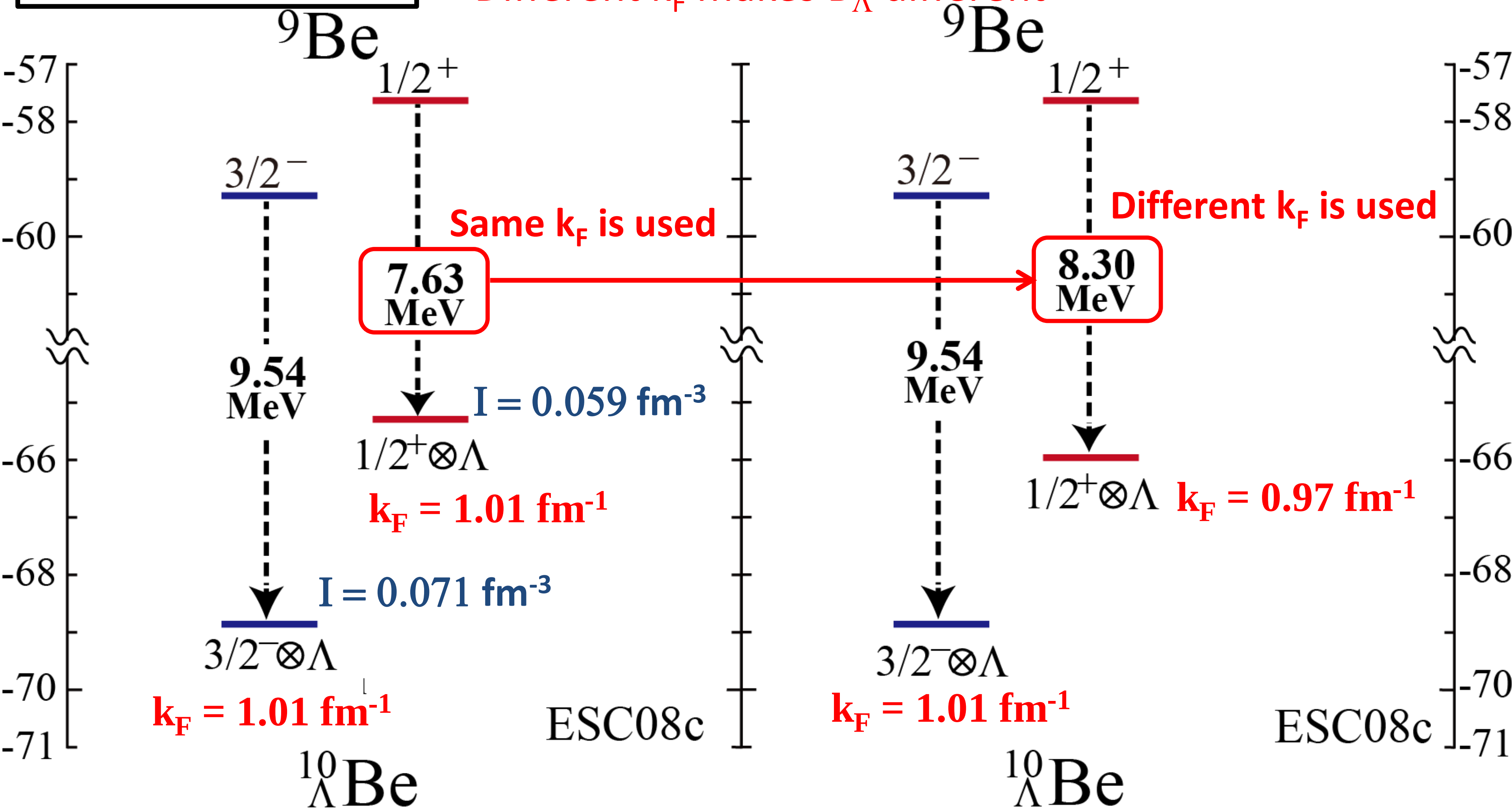
M. Isaka, M. Kimura, PRC**92**, 044326(2015)

${}^{10}_{\Lambda}$Be: How does structure affect B_Λ values?

Example: ${}^{10}_{\Lambda}$Be

Large difference of overlap makes k_F different

Different k_F makes B_Λ different



Averaged Density Approximation (ADA) $\langle\rho\rangle = \int dr^3 \rho_N(\mathbf{r})\rho_\Lambda(\mathbf{r}) \quad k_F = \left(\frac{3\pi^2\langle\rho\rangle}{2}\right)^{1/3}$

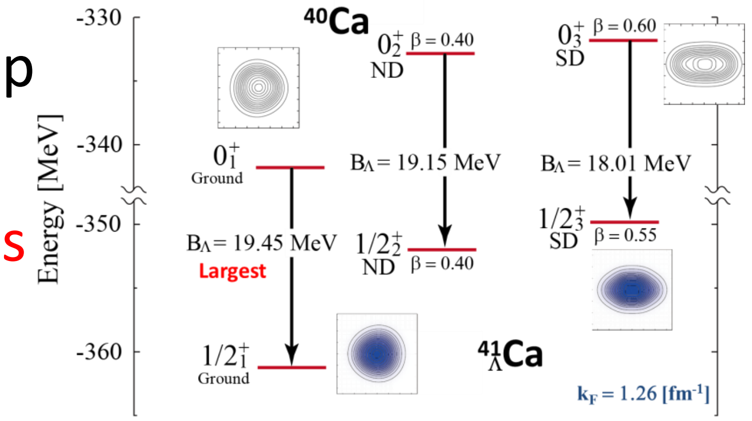
Relation between B_Λ and nuclear structure

◆ **B_Λ values are related to nuclear structure in two ways.**

- **Overlap between Λ and nucleons**

  Increasing deformation reduces the overlap between Λ and nucleons

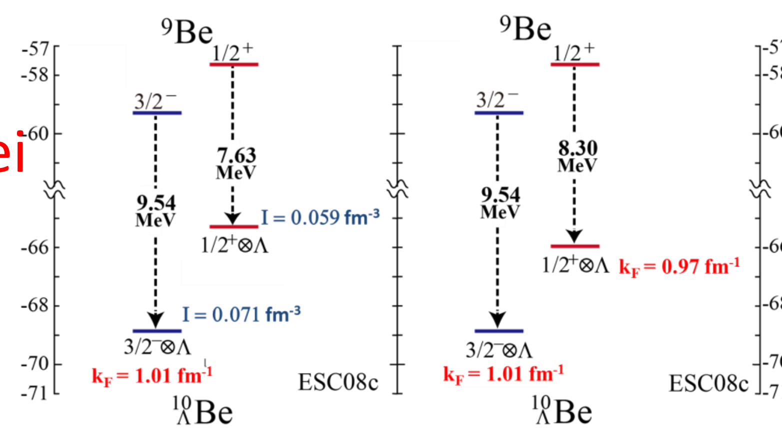
  ⟶ B_Λ becomes smaller in deformed states



- **Density dependence of ΛN effective interaction.**

  Large change of the overlap affects B_Λ through the density dependence

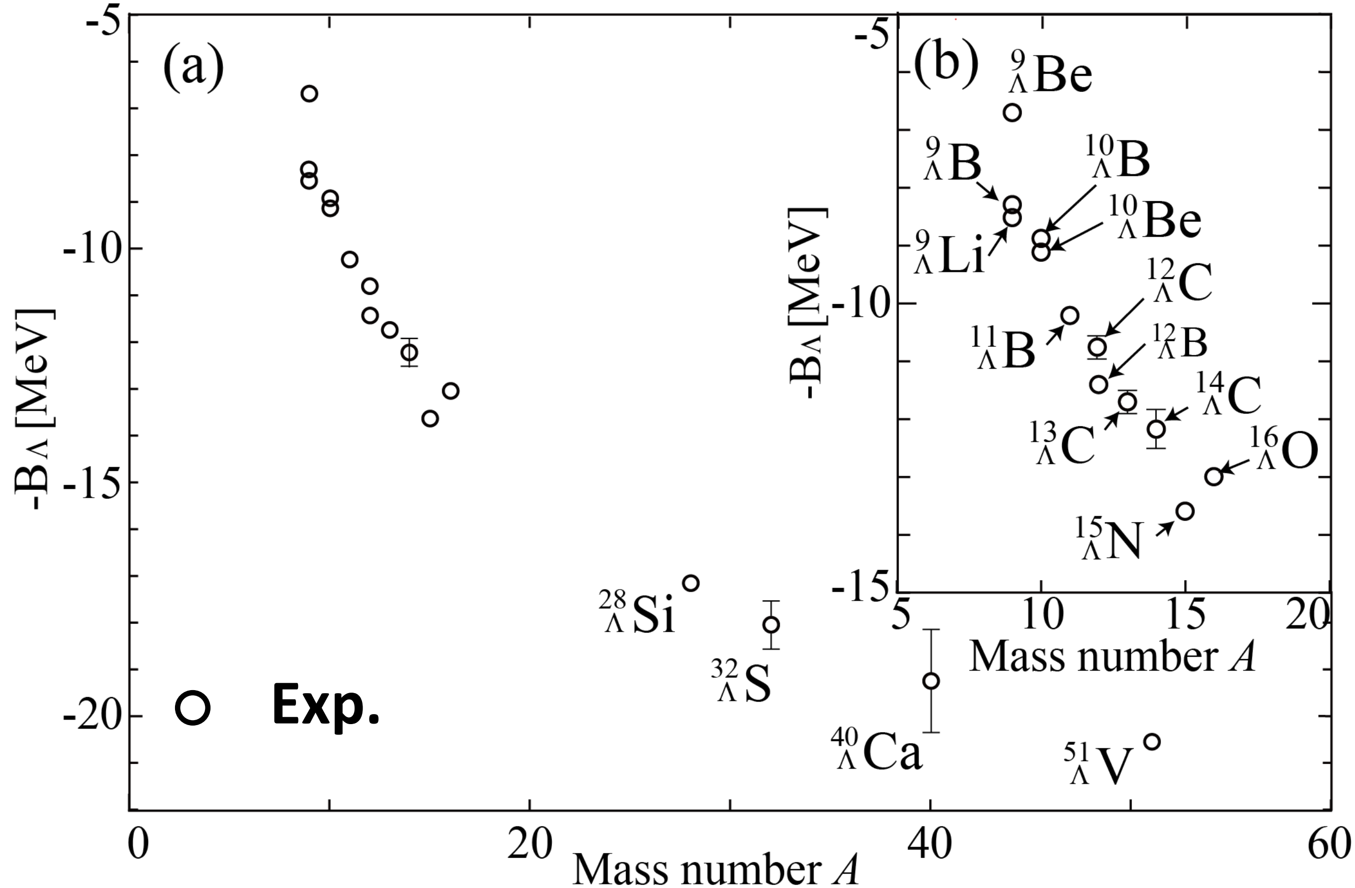
  ⟶ B_Λ becomes larger in light hypernuclei



These effects can appear in systematics of B_Λ

B_Λ as a function of mass number *A*

Observed data of Λ binding energy B_Λ ($9 \leq A \leq 51$)



Do core nuclei affect the mass dependence of B_Λ?

"clustering/deformations", "density dependence of interactions"

Bertini *et al.*, NPA**83**,306(1979), Davis, Juric , *et al.*, NPB**52**(1973), Davis, NPA**547**,369(1992);NPA**754**,3c(2005), Ajimura *et. al.*, NPA**639**(1998)93c, Pile *et al.*, PRL**66**,2585(1991), Hotchi *et al.*, PRC**64**, 044302(2001), Hashimoto and Tamura, PPNP**57**,564(2006), Tang, *et. al.*, PRC**90**,034320(2014).

Purpose of this study

◆ Purpose

- **To reveal the many-body force effects on B_Λ on the basis of the baryon-baryon interaction model ESC**

◆ Individual problems

1) B_Λ and Density dependence of the interaction

Is it possible to describe mass dependence of observed B_Λ?
What is essential to reproduce it?

2) Three-body force effects

Do ΛNN three-body effects appear in B_Λ? How large?

Theoretical framework: HyperAMD

M.Isaka, *et al.*, PRC**83**(2011) 044323
M. Isaka, *et al.*, PRC**83**(2011) 054304

We extended the AMD to hypernuclei

HyperAMD (Antisymmetrized Molecular Dynamics for hypernuclei)

◆Hamiltonian

$$\hat{H} = \hat{T}_N + \hat{V}_{NN} + \hat{T}_\Lambda + \hat{V}_{\Lambda N}$$

NN : Gogny D1S
ΛN : YNG interactions (ESC08c, ESC08c + ΛNN)

◆Wave function

- **Nucleon part : Slater determinant**
  Spatial part of single particle w.f. is described as Gaussian packet

$$\varphi_N(\vec{r}) = \frac{1}{\sqrt{A!}} \det\left[\varphi_i(\vec{r}_j)\right]$$

$$\varphi_i(r) \propto \exp\left[-\sum_{\sigma=x,y,z} \nu_\sigma (r - Z_i)_\sigma^2\right] \chi_i \eta_i \qquad \chi_i = \alpha_i \chi_\uparrow + \beta_i \chi_\downarrow$$

- **Single particle w.f. of Λ hyperon:**
  Superposition of Gaussian packets

$$\varphi_\Lambda(r) = \sum_m c_m \varphi_m(r)$$

$$\varphi_m(r) \propto \exp\left[-\sum_{\sigma=x,y,z} \mu\nu_\sigma (r - z_m)_\sigma^2\right] \chi_m \qquad \chi_m = a_m \chi_\uparrow + b_m \chi_\downarrow$$

- **Total w.f. :**

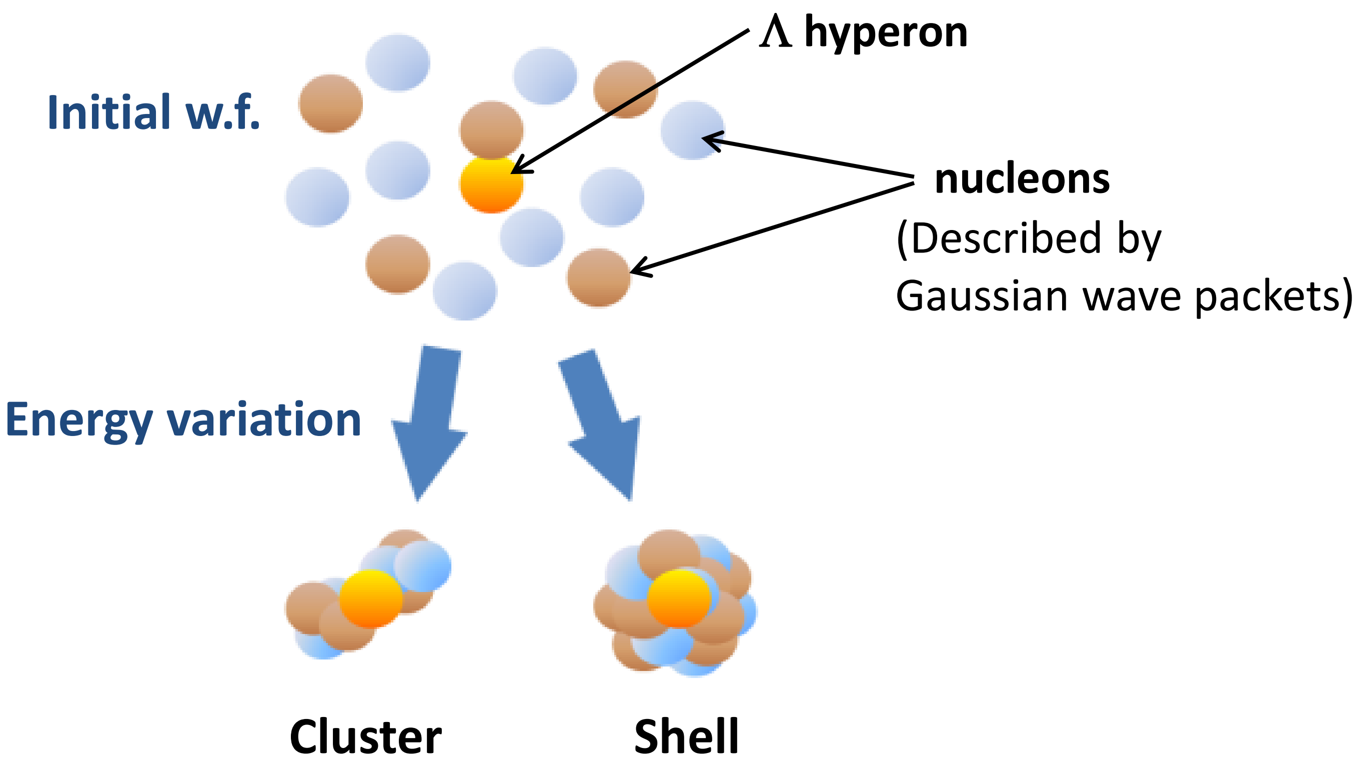
$$\psi(\vec{r}) = \sum_m c_m \varphi_m(r_\Lambda) \otimes \frac{1}{\sqrt{A!}} \det\left[\varphi_i(\vec{r}_j)\right]$$

Theoretical framework: HyperAMD

◆ Procedure of the calculation

Variational Calculation

- Imaginary time development method $\frac{dX_i}{dt} = \frac{\kappa}{\hbar} \frac{\partial H^{\pm}}{\partial X_i^*}$ $\quad \kappa < 0$
- Variational parameters: $X_i = Z_i, z_i, \alpha_i, \beta_i, a_i, b_i, \nu_i, c_i$

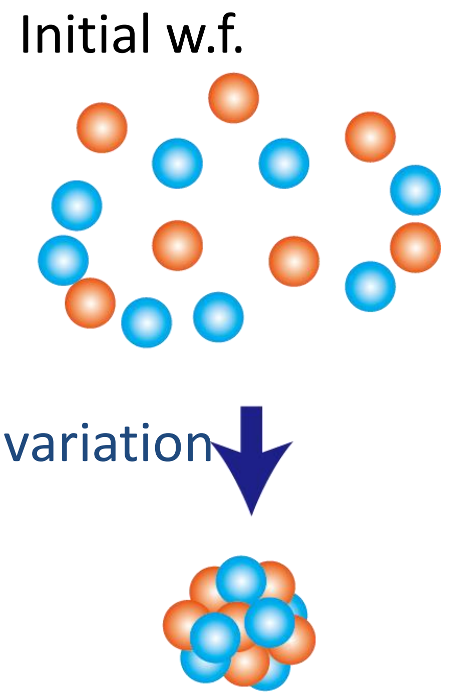


Actual calculation of HyperAMD

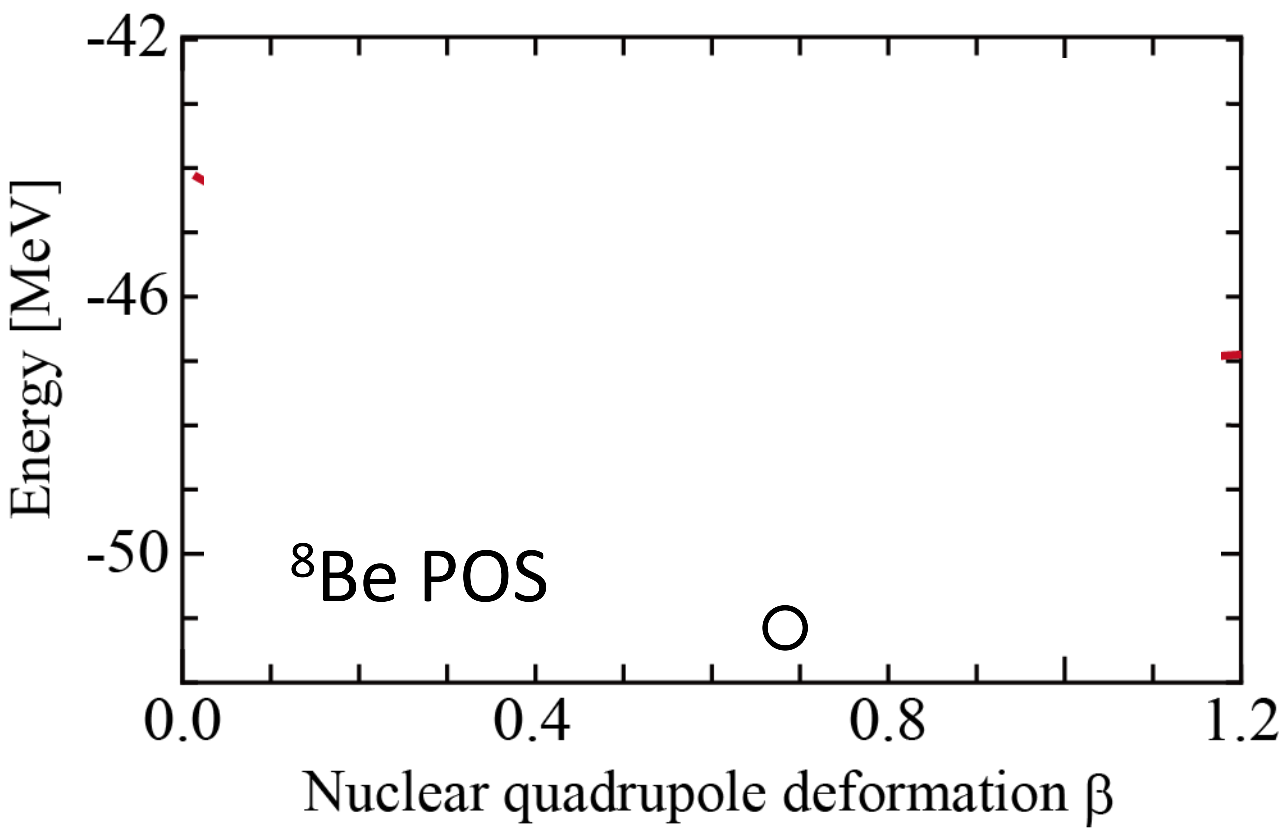
M.Isaka, *et al.*, PRC**83**(2011) 044323
M. Isaka, *et al.*, PRC**83**(2011) 054304

Energy variation with constraint on nuclear quadrupole deformation

Ex.) ^{8}Be



w/o constraint on β



Actual calculation of HyperAMD

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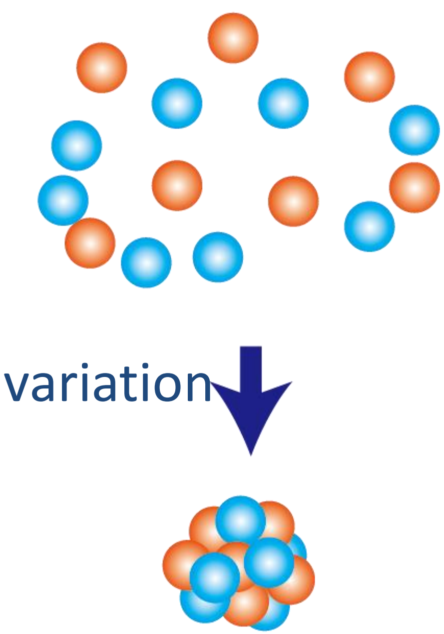
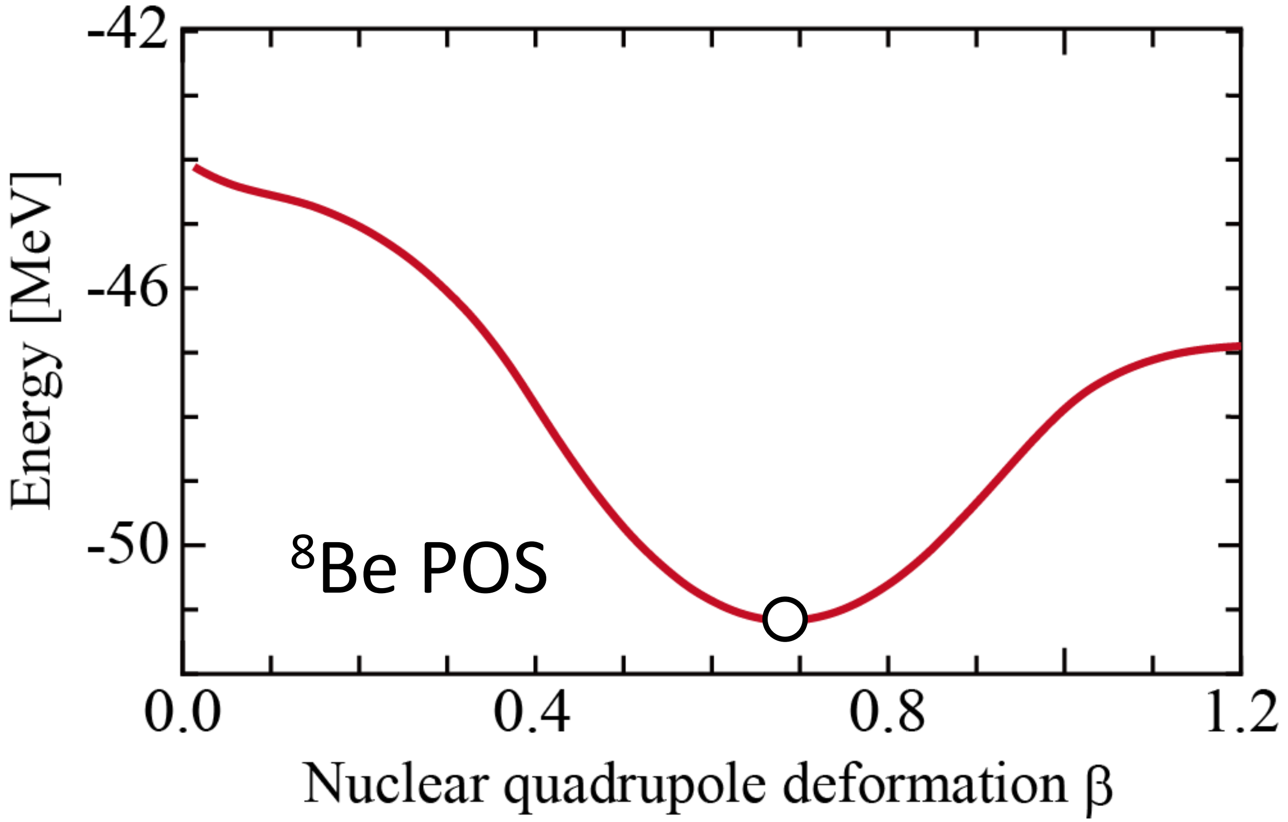
Energy variation with constraint on nuclear quadrupole deformation

Ex.) ^{8}Be

Initial w.f.



with constraint on β



Actual calculation of HyperAMD

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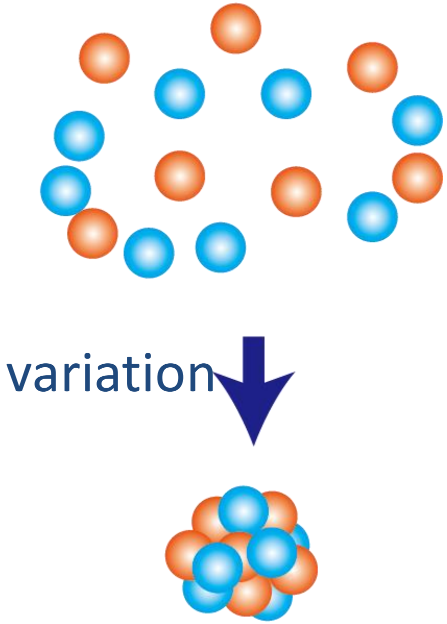

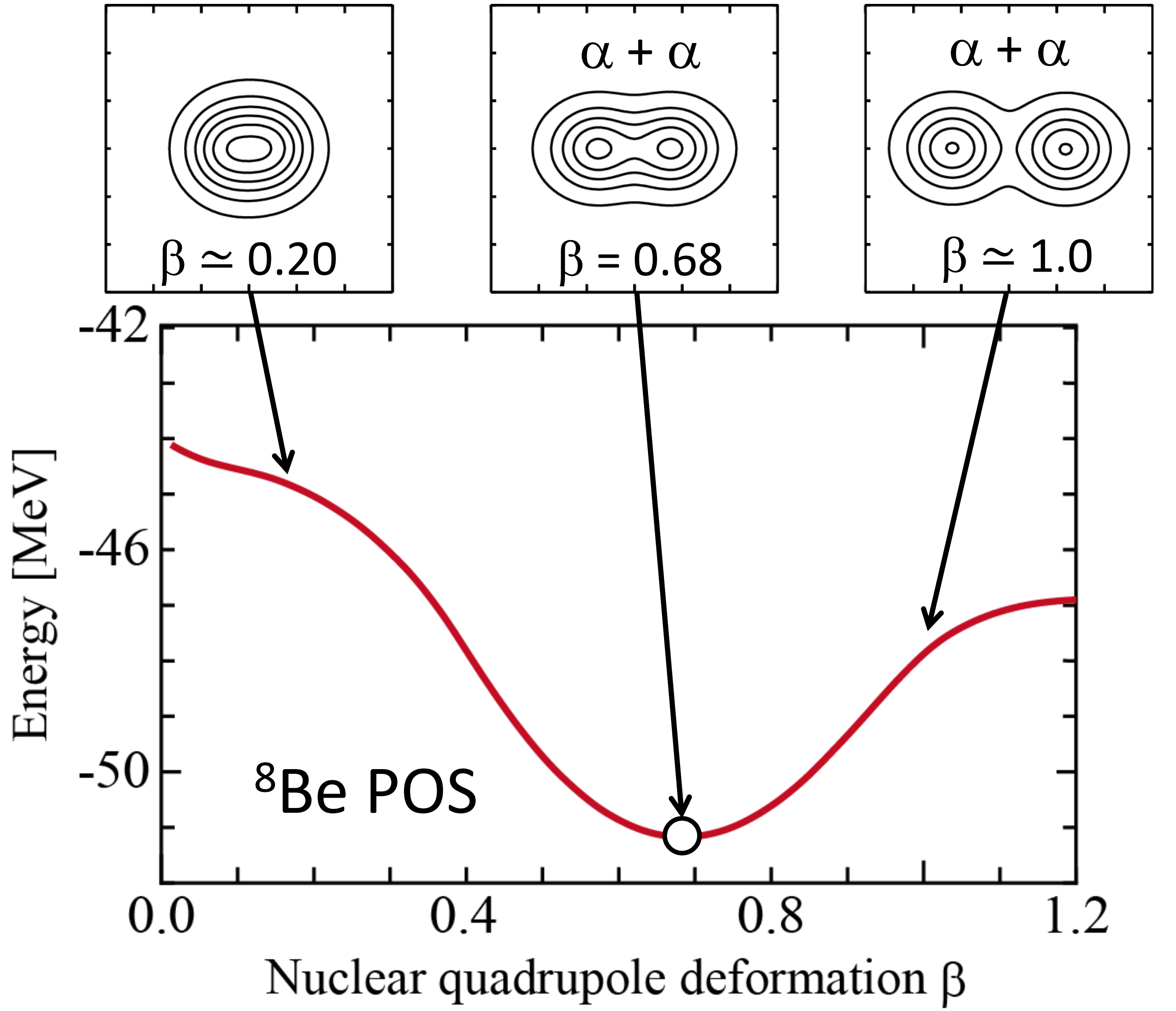

Actual calculation of HyperAMD

M.Isaka, *et al.*, PRC**83**(2011) 044323
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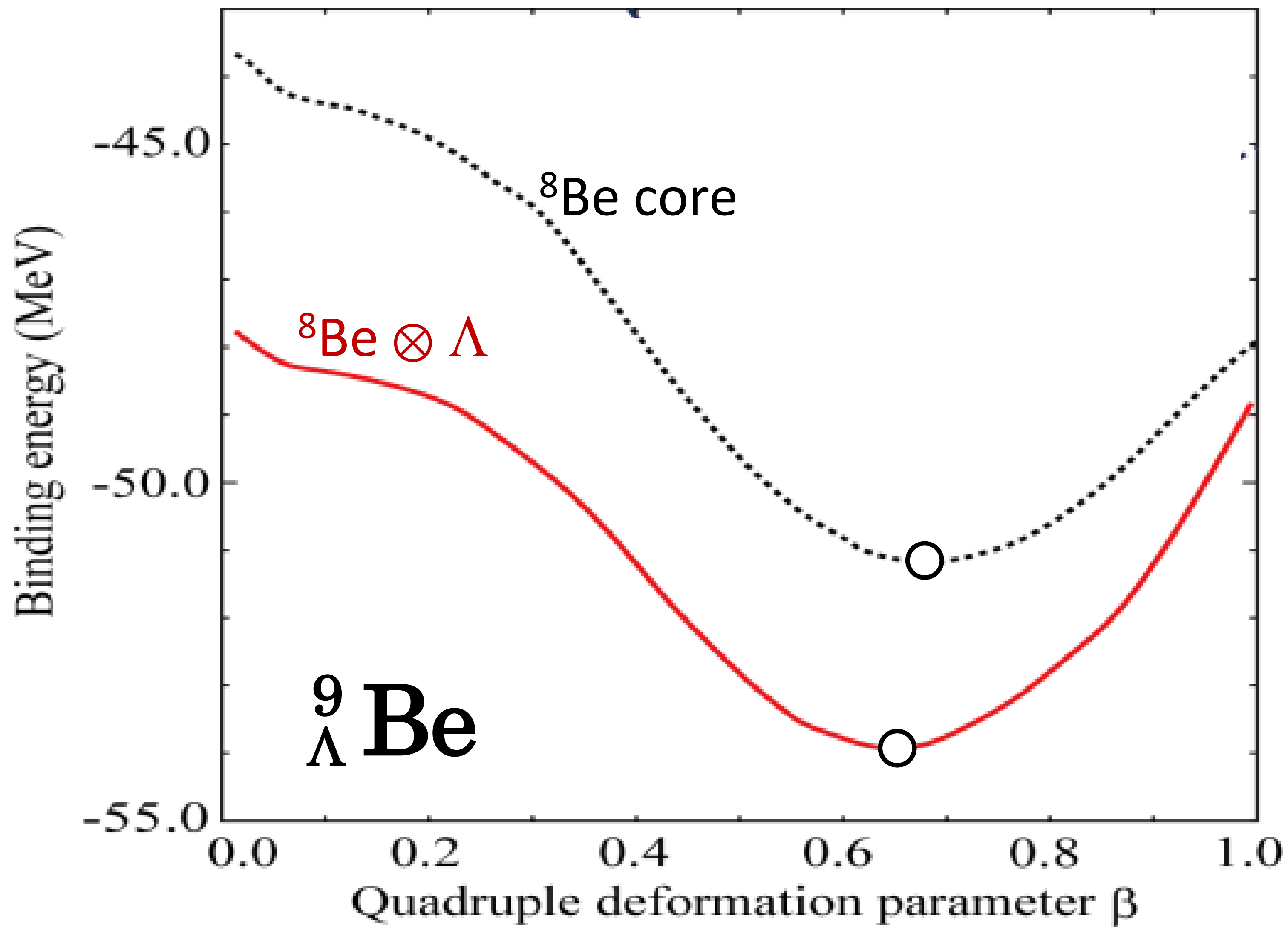
◆ For hypernuclei



Theoretical framework: HyperAMD

◆ Procedure of the calculation

Variational Calculation

- Imaginary time development method $$\frac{dX_i}{dt} = \frac{\kappa}{\hbar}\frac{\partial H^{\pm}}{\partial X_i^*} \qquad \kappa < 0$$
- Variational parameters: $X_i = Z_i, z_i, \alpha_i, \beta_i, a_i, b_i, \nu_i, c_i$

Angular Momentum Projection

$$\left|\Phi_K^s; JM\right\rangle = \int d\Omega D_{MK}^{J*}(\Omega) R(\Omega) \left|\Phi^{s+}\right\rangle$$

Generator Coordinate Method(GCM)

- Superposition of the w.f. with different configuration
- Diagonalization of $H_{sK,s'K'}^{J\pm}$ and $N_{sK,s'K'}^{J\pm}$

$$H_{sK,s'K'}^{J\pm} = \left\langle \Phi_K^s; J^{\pm}M \right| \hat{H} \left| \Phi_{K'}^{s'}; J^{\pm}M \right\rangle$$

$$N_{sK,s'K'}^{J\pm} = \left\langle \Phi_K^s; J^{\pm}M \middle| \Phi_{K'}^{s'}; J^{\pm}M \right\rangle$$

$$\left|\Psi^{J\pm M}\right\rangle = \sum_{sK} g_{sK} \left|\Phi_K^s; J^{\pm}M\right\rangle$$

ΛNN three-body force used

- **G-matrix interaction**

Additional (ΛNN) 3 body force

ESC08c + MPP + TBA

MPP: repulsion which works at high dens.

TBA : phenomenological 3-body attraction

ESC08c $$V_{\Lambda N}(r;k_F) = \sum_{i=1}^{3} (a_i + b_i k_F + c_i k_F^2)\exp\left(-r^2/\beta_i^2\right)$$

MPP + TBA $$\Delta V_{\Lambda N}(k_F;r) = (a + bk_F + ck_F^2)\exp\left(-r^2/0.9^2\right)$$

ESC08c: effective ΛN force including ΛN-ΣN coupling effects

MPP: giving $2M_{\odot}$ neutron star mass

TBA: to reproduce observed spectra of $^{89}_{\Lambda}$Y by spherical SHF calculation

Yamamoto, Furumoto, Yasutake and Rijken, PRC**88**,022801(2013); PRC**90**,045805(2014).

- **k_F determined by density**
  - **Averaged density approximation(ADA):**

$$\langle\rho\rangle = \int dr^3 \rho_N(\mathbf{r})\rho_\Lambda(\mathbf{r}) \qquad k_F = \left(\frac{3\pi^2\langle\rho\rangle}{2}\right)^{1/3}$$

ΛNN three-body force used

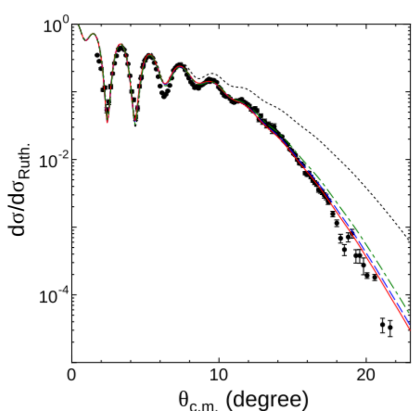
PHYSICAL REVIEW C **90**, 045805 (2014)

Hyperon mixing and universal many-body repulsion in neutron stars

Y. Yamamoto,[1] T. Furumoto,[2] N. Yasutake,[3] and Th. A. Rijken[1,4]
[1]*Nishina Center for Accelerator-Based Science, Institute for Physical and Chemical Research (RIKEN), Wako, Saitama 351-0198, Japan*
[2]*National Institute of Technology, Ichinoseki College, Ichinoseki, Iwate 021-8511, Japan*
[3]*Department of Physics, Chiba Institute of Technology, 2-1-1 Shibazono Narashino, Chiba 275-0023, Japan*
[4]*IMAPP, University of Nijmegen, Nijmegen, The Netherlands*
(Received 9 June 2014; revised manuscript received 1 September 2014; published 30 October 2014)

MPP + TBA

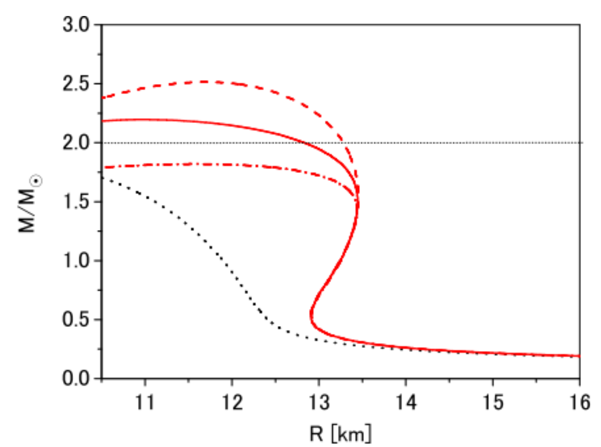
MPP: "universal" repulsion for 3 baryons
For NNN sector: MPP + TBA is determined by ^{16}O + ^{16}O elastic scattering data at E/A = 70 MeV
For hyperon sector: MPP is the same as NNN, TBA is determined by $^{89}_{\Lambda}$Y data



^{16}O + ^{16}O scattering data

TABLE III. Energy spectra (in MeV) of $^{89}_{\Lambda}$Y calculated with MPa and ESC in comparison with experimental values. Averaged values of k_F (in fm^{-1}) are in parentheses.

	s	p	d	f
MPa	−23.8	−17.4	−10.6	−3.8
	(1.27)	(1.23)	(1.16)	(1.08)
ESC	−23.7	−16.8	−9.8	−3.0
	(1.28)	(1.23)	(1.17)	(1.09)
Expt.	−23.7	−17.6	−10.9	−3.7



MPP gives stiff EOS enough to give 2M$_{\odot}$

Results and Discussions

1) B_Λ and Density dependence of the interaction

Is it possible to describe mass dependence of observed B_Λ?
What is essential to reproduce it?

Core structure, in particular core deformation

2) Three-body force effects

Do ΛNN three-body effects appear in B_Λ? How large?

Comparison of the results: “ESC08c only” and “ESC08c + ΛNN force“

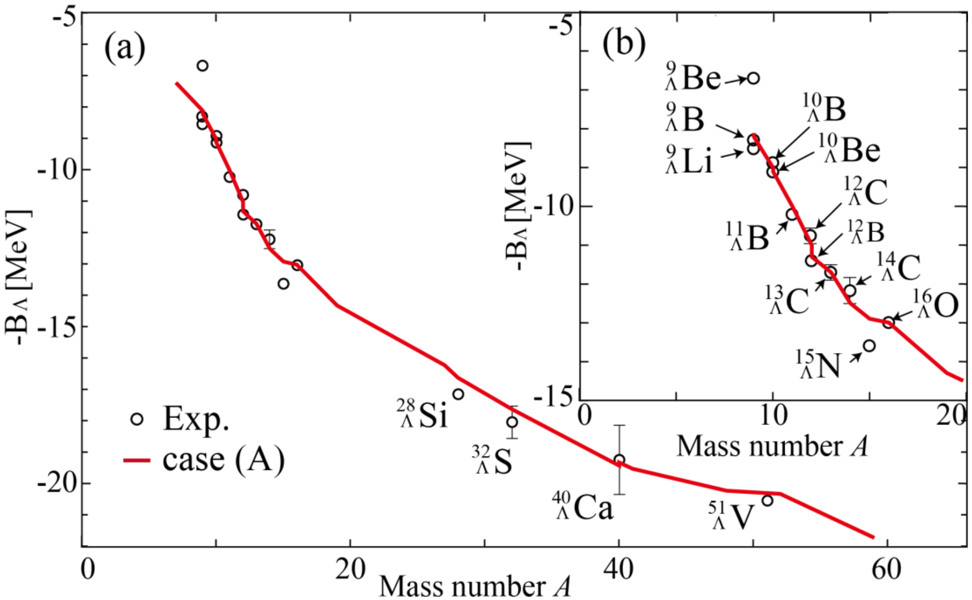
B_Λ as a function of mass number A

ESC08c + MPP + TBA

repulsive **attraction**

k_F is determined from ground-state density

$$\langle\rho\rangle = \int dr^3 \rho_N(\mathbf{r})\rho_\Lambda(\mathbf{r}) \quad k_F = \left(\frac{3\pi^2\langle\rho\rangle}{2}\right)^{1/3}$$



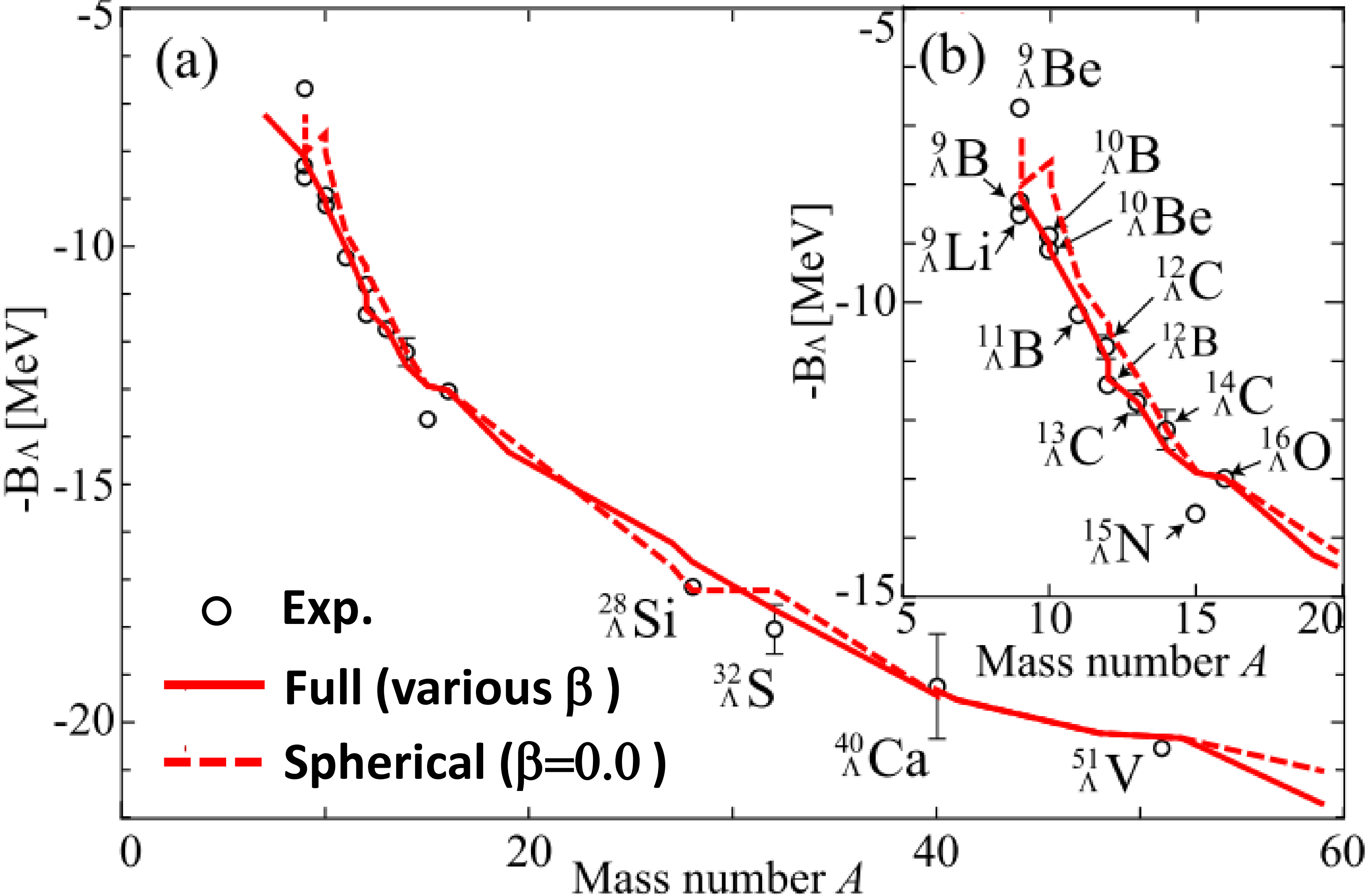
	β	γ	$\langle\rho\rangle$	k_F	$-B_\Lambda^{calc}$	$-B_\Lambda^{exp}$
$^{9}_{\Lambda}$Li	0.50	2°	0.072	1.02	−8.1	−8.50 ± 0.12 [2
$^{9}_{\Lambda}$Be	0.87	1°	0.060	0.96	−8.1	−6.71 ± 0.04 [2
$^{9}_{\Lambda}$B	0.45	2°	0.072	1.02	−8.2	−8.29 ± 0.18 [2
$^{10}_{\Lambda}$Be	0.57	1°	0.077	1.04	−9.0	−9.11 ± 0.22 [2
						−8.55 ± 0.18 [3
$^{10}_{\Lambda}$B	0.58	1°	0.075	1.04	−9.1	−8.89 ± 0.12 [2
$^{11}_{\Lambda}$B	0.50	29°	0.081	1.06	−10.0	−10.24 ± 0.05 [
$^{12}_{\Lambda}$B	0.39	48°	0.083	1.07	−11.3	−11.37 ± 0.06 [
						−11.38 ± 0.02 [
$^{12}_{\Lambda}$C	0.41	34°	0.086	1.08	−11.0	−10.76 ± 0.19 [
$^{13}_{\Lambda}$C	0.45	60°	0.090	1.10	−11.7	−11.69 ± 0.19 [
$^{14}_{\Lambda}$C	0.45	31°	0.093	1.11	−12.5	−12.17 ± 0.33 [
$^{15}_{\Lambda}$N	0.28	60°	0.098	1.13	−12.9	−13.59 ± 0.15 [
$^{16}_{\Lambda}$O	0.02	–	0.105	1.16	−13.0	−12.96 ± 0.05 [2
$^{19}_{\Lambda}$O	0.30	3°	0.110	1.18	−14.3	–
$^{27}_{\Lambda}$Mg	0.36	36°	0.125	1.23	−16.2	–
$^{28}_{\Lambda}$Si	0.32	53°	0.125	1.23	−16.6	−17.1 ± 0.2 [9
$^{32}_{\Lambda}$S	0.28	0°	0.130	1.24	−17.6	−18.0 ± 0.5 [23
$^{40}_{\Lambda}$K	0.01	–	0.136	1.26	−19.4	–
$^{40}_{\Lambda}$Ca	0.03	–	0.136	1.26	−19.3	−19.24 ± 1.1 [2
$^{41}_{\Lambda}$Ca	0.13	12°	0.136	1.26	−19.5	–
$^{48}_{\Lambda}$K	0.01	–	0.141	1.28	−20.2	–
$^{51}_{\Lambda}$V	0.18	2°	0.151	1.31	−20.3	−20.51 ± 0.13 [2
$^{59}_{\Lambda}$Fe	0.26	23°	0.142	1.28	−21.7	–

HyperAMD calculation nicely reproduces B_Λ in wide mass regions

What is essential to reproduce B_Λ?

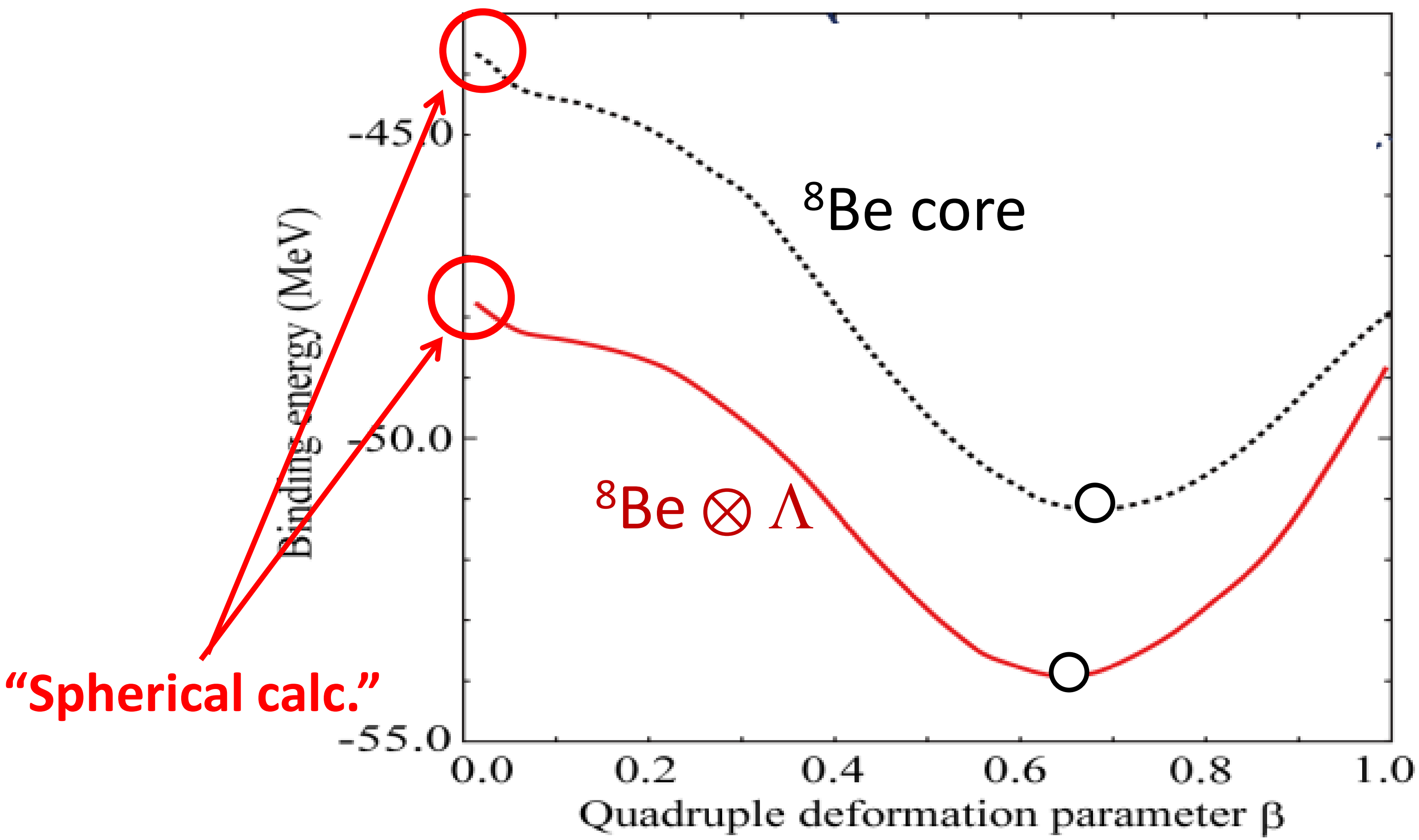
"Description of the core structure"

"Full calc." vs. "Spherical calc."



What is essential to reproduce B_Λ

Ex. ${}^{9}_{\Lambda}$Be **"Full calc." vs. "Spherical calc."**

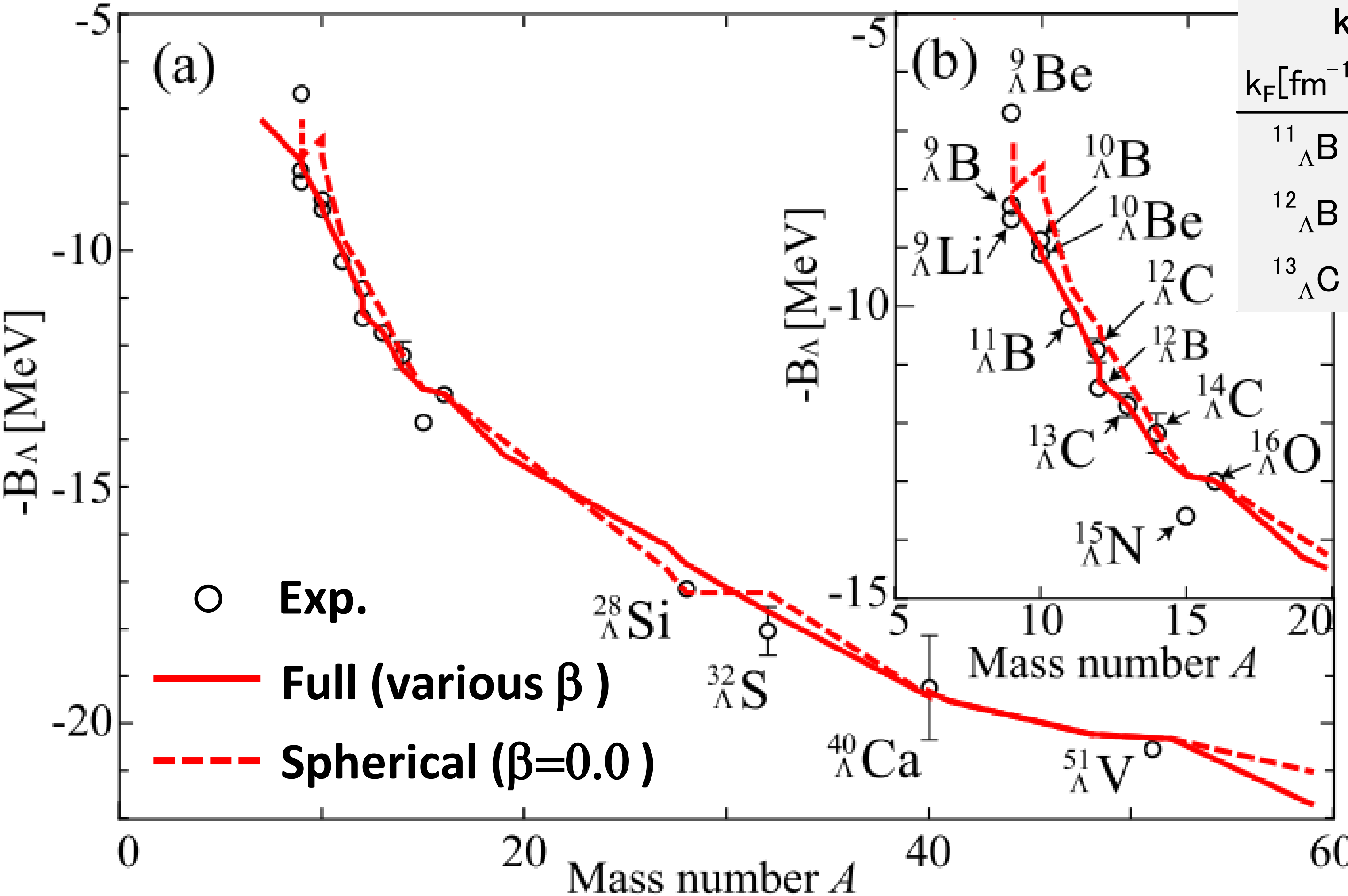


"Full calc.": all of w.f. on energy curve in GCM calc.

What is essential to reproduce B_Λ

- **B_Λ in "Spherical calc." are shallower than those in "Full calc." with *A* < 16**

  Originated in density dependence of interaction



k_F values used

k_F[fm^{-1}]	Full	Spherical
$^{11}{}_\Lambda$B	1.05	1.12
$^{12}{}_\Lambda$B	1.06	1.15
$^{13}{}_\Lambda$C	1.09	1.14

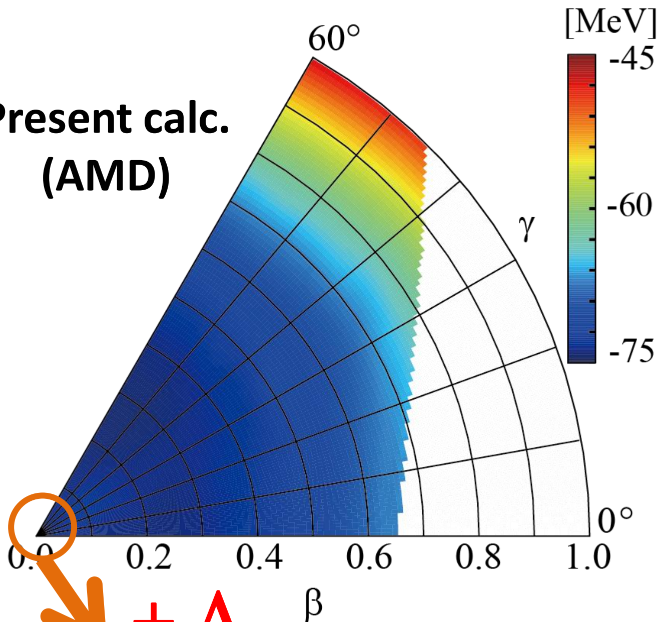
What is essential to reproduce B_Λ

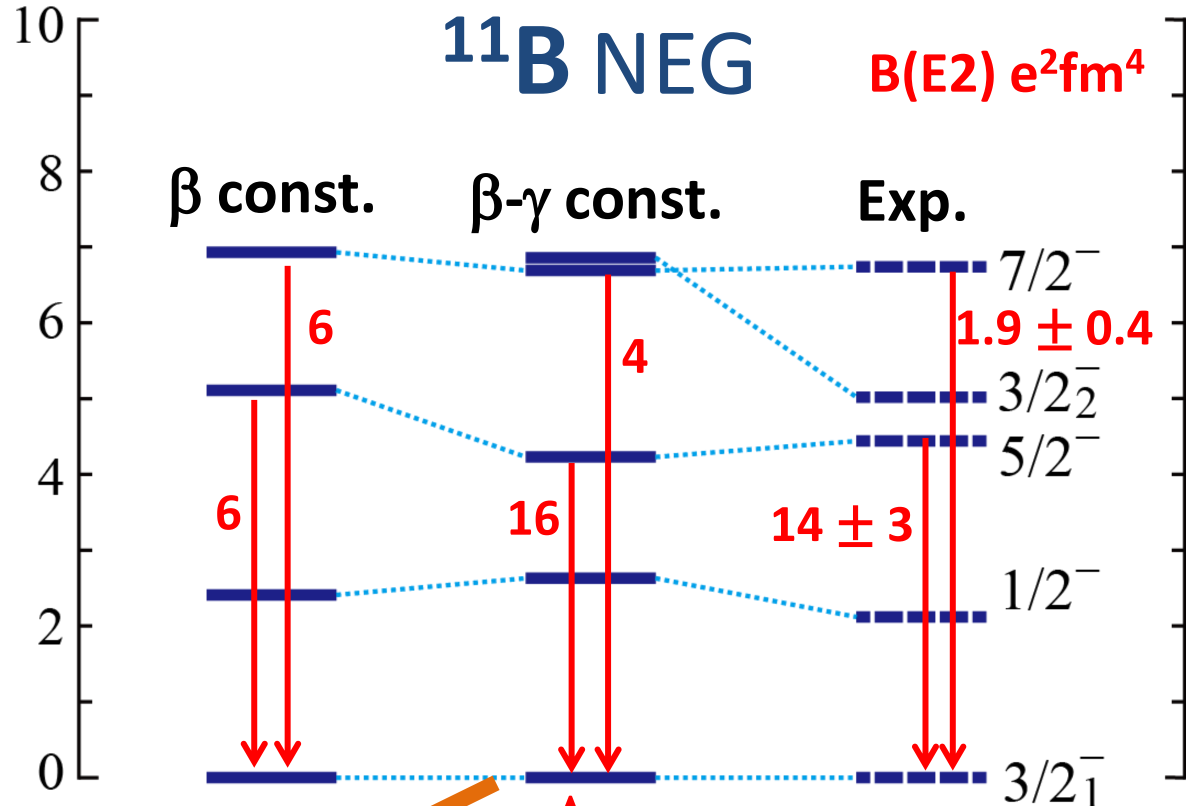
"Description of the core structure"

Ex.: ^{11}B **More sophisticated treatment: GCM calc. on (β, γ) plane**

T. Suhara and Y. Kanada-En'yo, PTP**123**,303(2010)

Present calc. (AMD)





+ Λ

$^{12}_{\Lambda}$B (Spherical)

B_Λ = 9.5 MeV
(k_F = 1.16 fm^{-1})

+ Λ

$^{12}_{\Lambda}$B (β-γ)

B_Λ = 11.3 MeV
(k_F = 1.07 fm^{-1})

$^{12}_{\Lambda}$B (EXP)

B_Λ = 11.4 ±0.02 MeV

Results and Discussions

1) B_Λ and Density dependence of the interaction

Is it possible to describe mass dependence of observed B_Λ?
What is essential to reproduce it?

Core structure, in particular core deformation

2) Three-body force effects

Do ΛNN three-body effects appear in B_Λ? How large?

Comparison of the results: “ESC08c only” and “ESC08c + ΛNN force“

Comparison with the results with ESC08c only

- **Effects of many-body force**

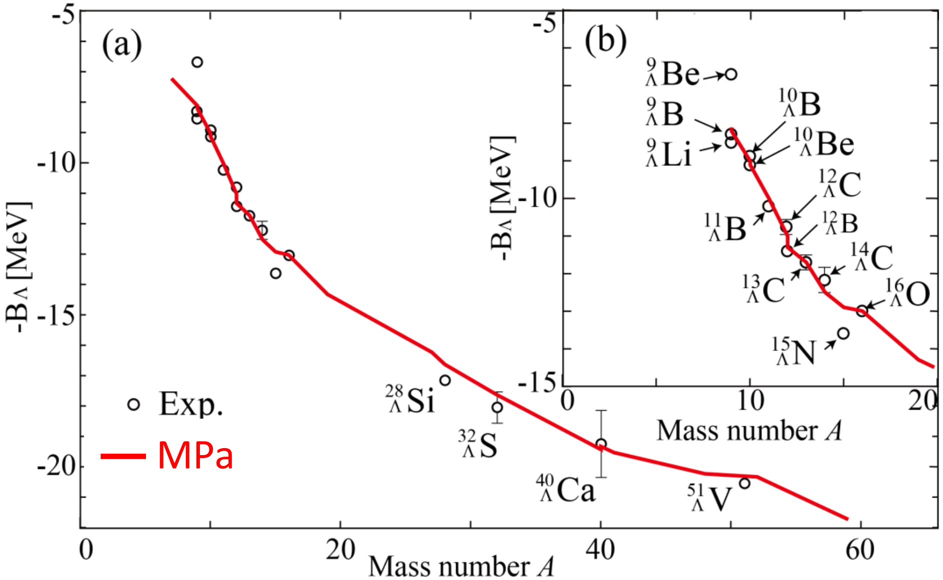
ESC: ESC08c only

MPa: ESC08c + MPP + TBA

ΛNN three-body effects

Over-binding with ESC08c only

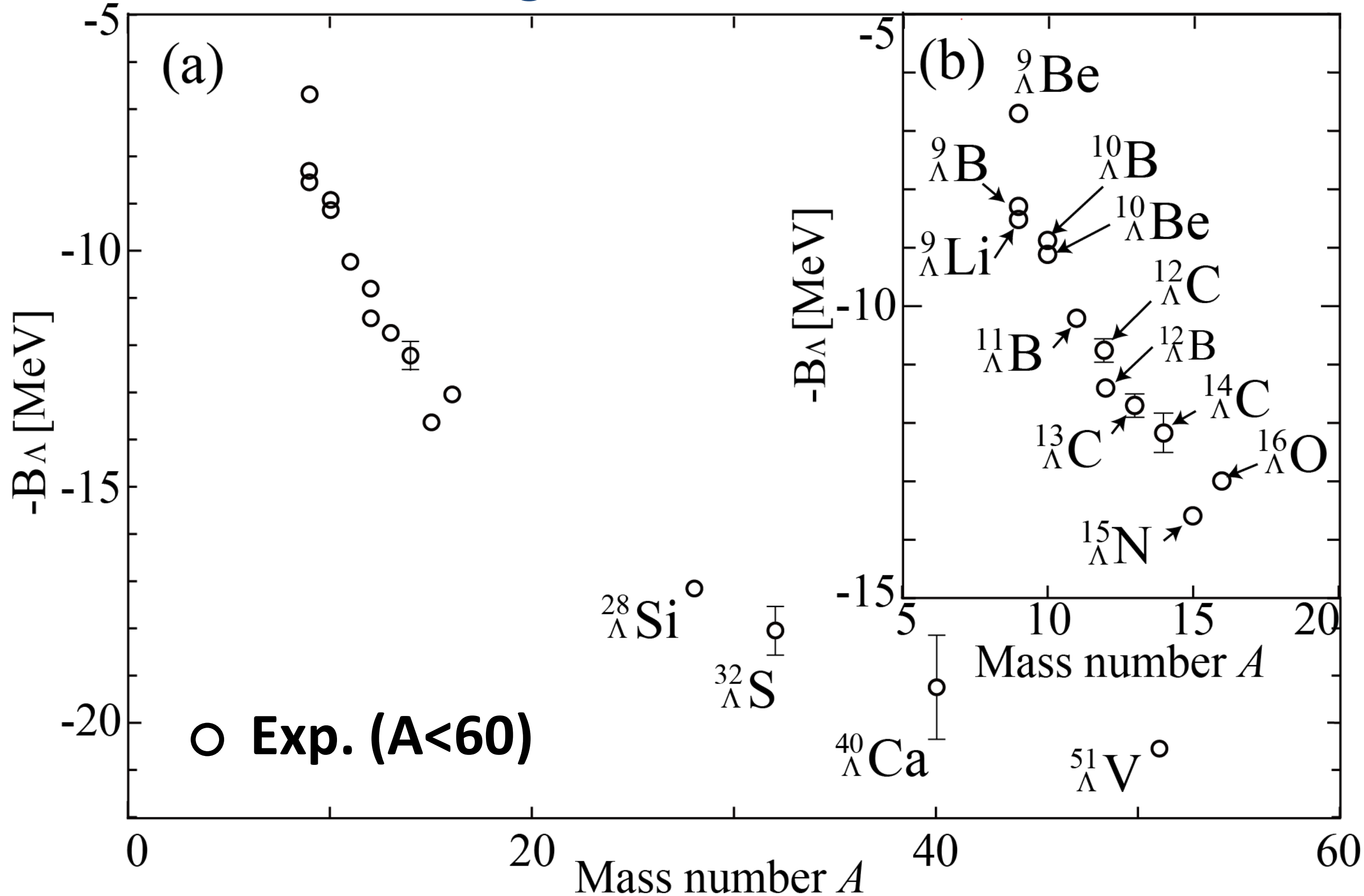
[MeV]	ESC	MPa	Exp.
$^{13}_{\Lambda}$C	-11.5	-11.7	-11.69 ± 0.19
$^{16}_{\Lambda}$O	-13.3	-13.0	-12.96 ± 0.05
$^{28}_{\Lambda}$Si	-17.7	-16.6	-17.1 ± 0.2
$^{40}_{\Lambda}$K	-21.5	-19.4	–
$^{48}_{\Lambda}$K	-22.6	-20.2	–



- **Additional dens. dep. of ΛNN 3-body force makes B_Λ different**
- **Systematic data of B_Λ will provide a new insight to many-body force**

Current status of observed B_Λ

Observations are not enough with A > 16



Systematic and accurate data of observed B_Λ are desired

Bertini *et al*., NPA**83**,306(1979), Davis, Juric , *et al*., NPB**52**(1973), Davis, NPA**547**,369(1992);NPA**754**,3c(2005), Ajimura *et. al*., NPA**639**(1998)93c, Pile *et al*., PRL**66**,2585(1991), Hotchi *et al*., PRC**64**, 044302(2001), Hashimoto and Tamura, PPNP**57**,564(2006), Tang, *et. al*., PRC**90**,034320(2014).

Summary

◆ Summary

- HyperAMD + GCM was applied with ESC08c + MPP + TBA interaction

Observed B_Λ are successfully reproduced in wide mass regions

- **Structure of the core nuclei**
  - Spherical shape: deviate from observed B_Λ
  - Description of core deformation is essential

  ⟶ Sophisticated treatment of hypernuclei is indispensable

- **Many-body (MPP + TBA) force effects**
  - ΛNN(MPP + TBA) force brings additional density dependence

  ⟶ Systematic observations of B_Λ is necessary to confirm/give constraints

◆ Future plan

- To reveal reasons for deviation of B_Λ with A < 9 (e.g. $^{9}{}_{\Lambda}$Be)
- Further study on model dependence of three-body force effects