AMD Calculations of Medium-Heavy Hypernuclei with the \( \Delta NN \) Three-Body Force in the Nijmegen Potential

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Collaborators: Y. Yamamoto, Th.A. Rijken
Grand challenges of hypernuclear physics

**Interaction:** To understand baryon-baryon interaction

- 2 body interaction between baryons (nucleon, hyperon)
  - hyperon-nucleon (YN)
  - hyperon-hyperon (YY)
  - Three-body force

**Structure:** To understand many-body system of nucleons and hyperon

- Addition of hyperon(s) shows us new features of nuclear structure
  
  Ex.) Structure change by hyperon(s)
  - No Pauli exclusion between N and Y
  - YN interaction is different from NN

Today’s talk: three-body force effects based on structure calculations
Studies of $\Lambda$ hypernuclei

$\Lambda$ hypernuclei observed so far

- Concentrated in light $\Lambda$ hypernuclei with $A \lesssim 10$

Studies of \(\Lambda\) hypernuclei: What achieved?

\(\Lambda\) hypernuclei observed so far

- Concentrated in light \(\Lambda\) hypernuclei with \(A \lesssim 10\)
  - Accurate solution of few-body problems
  - \(G\)-matrix calculation for \(\Lambda N\) interactions
  - Increases of experimental information

E. Hiyama, NPA 805 (2008), 190c.

Knowledge of \(\Lambda N\) two-body interaction

Developments of effective interactions

In this study,

\(G\)-matrix interaction derived from Nijmegen potential (YNG)

- Nijmegen potential: a meson exchange model
- \(G\)-matrix calculation takes into account medium effects

YNG interaction has density (Fermi momentum \(k_F\)) dependence coming from \(\Lambda N-\Sigma N\) coupling effects
“Hyperon puzzle” in neutron star physics

Hyperon puzzle

Massive (2M☉) neutron stars

2010 PSR J1614-2230 (1.97 ± 0.04)M☉
2013 PSR J0348-0432 (2.01 ± 0.04)M☉

Softening of EOS by hyperon mixing

How do we resolve?

Baryon-baryon three-body force
If strong repulsions exist not only NNN channel but YNN, YYN, and YYY, EOS of neutron star matter becomes stiff


Our aim: to reveal effects of ΛNN 3-body force in Λ hypernuclear data

Density dependent three-body force based on YNG ΛN interaction
Toward heavier $\Lambda$ hypernuclei

- Future experiments: heavier hypernuclei will be produced!

Various structures in the ground states

Coexistence of shell and cluster

$p$-shell $\Lambda$ hypernuclei

Developed cluster

$p$-$sd$ shell region

$\Lambda$ Hypernuclear Chart (2004)

From O. Hashimoto and H. Tamura, PPNP 57(2006), 564.

Various deformations

Triaxial deformation

Deformations coexist

Structure of core nuclei could affect $\Lambda$ binding energy $B_\Lambda$

“clustering/deformations”, “density dependence of interactions”
$^{41}_{\Lambda}Ca$: How does structure affect $B_\Lambda$ values?

Example: $^{41}_{\Lambda}Ca$

$B_\Lambda$ is smaller in deformed states

M. Isaka, et al., PRC 89, 024310 (2014)

$^{40}Ca$

$^{41}Ca$

$B_\Lambda = 19.15$ MeV

$B_\Lambda = 19.45$ MeV

$B_\Lambda = 18.01$ MeV

$1/2^+_2$ ND

$1/2^+_2$ SD

$1/2^+_3$ SD

$0^+_2$ ND

$0^+_3$ SD

$\beta = 0.40$

$\beta = 0.60$

$\beta = 0.55$

$\beta = 0.40$

$\beta = 0.55$

$k_F = 1.26$ [fm$^{-3}$] calculated from g.s. density by Averaged Density Approximation (ADA)

$$\langle \rho \rangle = \int dr^3 \rho_N(r) \rho_\Lambda(r) \quad k_F = \left( \frac{3\pi^2 \langle \rho \rangle}{2} \right)^{1/3}$$
$^{41}_{\Lambda}$Ca: How does structure affect $B_{\Lambda}$ values?

**Why? --- Overlap between $\Lambda$ and core nucleus is essential!**

Overlap between $\Lambda$ and N becomes smaller as nuclear deformation increases

- $^{40}$Ca
  - $0^+_1$ (Ground), $\beta = 0.40$
  - $0^+_2$, $\beta = 0.60$
  - $B_\Lambda = 19.15$ MeV
  - $B_\Lambda = 18.01$ MeV

- $^{41}_{\Lambda}$Ca
  - $1/2^+_1$ (Ground), $\beta = 0.55$

**GS**
- $\beta = 0.01$
- $I = 0.1364$

**ND**
- $\beta = 0.35$
- $I = 0.1356$

**SD**
- $\beta = 0.55$
- $I = 0.1336$

Overlap between the $\Lambda$ and nucleons

$$I = \int d^3r \rho_N(r) \rho_\Lambda(r)$$

Decrease of overlap makes $V_{\Lambda N}$ shallower

---

M. Isaka, et al., PRC 89, 024310(2014)
Relation between $B_\Lambda$ and nuclear structure

- $B_\Lambda$ values are related to nuclear structure in two ways.
  - Overlap between $\Lambda$ and nucleons
    Increasing deformation reduces the overlap between $\Lambda$ and nucleons
    $\rightarrow B_\Lambda$ becomes smaller in deformed states

However, situation is different in dilute (cluster) states ...
10\(^{\Lambda}\)Be: How does structure affect B\(_{\Lambda}\) values?

Example: 10\(^{\Lambda}\)Be

Large difference of overlap makes k\(_{F}\) different

\(\rho_{m}\)  
\(\rho_{p}\)

\(9^{\text{Be}}(1/2^+)\)  
\(\alpha + \alpha + n\)

\(\rho_{m}\)  
\(\rho_{p}\)

\(9^{\text{Be}}(3/2^-)\)  
\(\alpha + \alpha\)

\[k_F = 1.01 \text{ fm}^{-1}\]

Smaller k\(_{F}\) enlarges B\(_{\Lambda}\) in dilute cluster states
$^{10}_\Lambda$Be: How does structure affect $B_\Lambda$ values?

Example: $^{10}_\Lambda$Be

Large difference of overlap makes $k_F$ different
Different $k_F$ makes $B_\Lambda$ different

\[ k_F = 1.01 \text{ fm}^{-1} \]

\[ I = 0.071 \text{ fm}^{-3} \]

\[ I = 0.059 \text{ fm}^{-3} \]

Averaged Density Approximation (ADA)

\[ \langle \rho \rangle = \int d^3r \rho_N(r)\rho_\Lambda(r) \quad k_F = \left( \frac{3\pi^2 \langle \rho \rangle}{2} \right)^{1/3} \]
Relation between $B_\Lambda$ and nuclear structure

- $B_\Lambda$ values are related to nuclear structure in two ways.
  - Overlap between $\Lambda$ and nucleons
    Increasing deformation reduces the overlap between $\Lambda$ and nucleons
    $$\rightarrow B_\Lambda \text{ becomes smaller in deformed states}$$
  - Density dependence of $\Lambda N$ effective interaction.
    Large change of the overlap affects $B_\Lambda$
    through the density dependence
    $$\rightarrow B_\Lambda \text{ becomes larger in light hypernuclei}$$

These effects can appear in systematics of $B_\Lambda$
$B_\Lambda$ as a function of mass number $A$

Observed data of $\Lambda$ binding energy $B_\Lambda$ ($9 \leq A \leq 51$)

Do core nuclei affect the mass dependence of $B_\Lambda$?

“clustering/deformations”, “density dependence of interactions”

Purpose of this study

◆ Purpose

To reveal the many-body force effects on $B_\Lambda$ on the basis of the baryon-baryon interaction model ESC

◆ Individual problems

1) $B_\Lambda$ and Density dependence of the interaction
   Is it possible to describe mass dependence of observed $B_\Lambda$?
   What is essential to reproduce it?

2) Three-body force effects
   Do $\Lambda NN$ three-body effects appear in $B_\Lambda$? How large?
Theoretical framework: HyperAMD

We extended the AMD to hypernuclei

HyperAMD (Antisymmetrized Molecular Dynamics for hypernuclei)

✿ Hamiltonian

\[ \hat{H} = \hat{T}_N + \hat{V}_{NN} + \hat{T}_\Lambda + \hat{V}_{\Lambda N} \]

\[ \text{NN : Gogny D1S} \]
\[ \text{\Lambda N : YNG interactions (ESC08c, ESC08c + \Lambda NN)} \]

✿ Wave function

- **Nucleon part**: Slater determinant
  
  Spatial part of single particle w.f. is described as Gaussian packet
  
  \[ \varphi_N(\vec{r}) = \frac{1}{\sqrt{A!}} \det[\varphi_i(\vec{r}_j)] \]
  
  \[ \varphi_i(r) \propto \exp \left[ -\sum_{\sigma=x,y,z} v_{i,\sigma}(r-Z_i)^2 \right] \chi_i n_i \quad \chi_i = \alpha_i \chi_\uparrow + \beta_i \chi_\downarrow \]

- **Single particle w.f. of \Lambda hyperon**: Superposition of Gaussian packets
  
  \[ \varphi_\Lambda (r) = \sum_{m} c_m \varphi_m (r) \]
  
  \[ \varphi_m (r) \propto \exp \left[ -\sum_{\sigma=x,y,z} \mu_{i,\sigma}(r-z_m)^2 \right] \chi_m \quad \chi_m = a_m \chi_\uparrow + b_m \chi_\downarrow \]

- **Total w.f.**

  \[ \psi(\vec{r}) = \sum_m c_m \varphi_m (r_\Lambda) \otimes \frac{1}{\sqrt{A!}} \det[\varphi_i(\vec{r}_j)] \]
Theoretical framework: HyperAMD

Procedure of the calculation

Variational Calculation
- Imaginary time development method
- Variational parameters: $X_i = Z_i, z_i, \alpha_i, \beta_i, a_i, b_i, \nu_i, c_i$

\[
\frac{dX_i}{dt} = \frac{\kappa}{\hbar} \frac{\partial H^\pm}{\partial X_i} \quad \kappa < 0
\]
Actual calculation of HyperAMD

Energy variation with constraint on nuclear quadrupole deformation

Ex.) $^8$Be

Initial w.f.

variation

\begin{align*}
\text{Energy [MeV]} \quad & \text{Nuclear quadrupole deformation } \beta \\
-50 \quad & 0.0 \\
-46 \quad & 0.4 \\
-42 \quad & 0.8 \\
\end{align*}

$^8$Be POS

w/o constraint on $\beta$
Actual calculation of HyperAMD

Energy variation with constraint on nuclear quadrupole deformation

Ex. $^8$Be

Initial w.f.

Variation

with constraint on $\beta$

$^8$Be POS

Energy [MeV]

Nuclear quadrupole deformation $\beta$
Actual calculation of HyperAMD

Energy variation with constraint on nuclear quadrupole deformation

Ex.) $^8$Be

Initial w.f.

variation

Energy [MeV]

$^8$Be POS

Nuclear quadrupole deformation $\beta$
Actual calculation of HyperAMD

- For hypernuclei

**Diagram:**

- **Binding energy (MeV)**

  - **$^8\text{Be core}$**
  - **$^8\text{Be} \otimes \Lambda$**
  - **$^9\Lambda\text{Be}$**

- **Quadrupole deformation parameter $\beta$**
Theoretical framework: HyperAMD

Procedure of the calculation

Variational Calculation
- Imaginary time development method
- Variational parameters: \( X_i = Z_i, z_i, \alpha_i, \beta_i, a_i, b_i, \nu_i, c_i \)

\[
\frac{dX_i}{dt} = \kappa \frac{\partial H^\pm}{\partial X_i} \quad \kappa < 0
\]

Angular Momentum Projection

\[
\begin{align*}
\left| \Phi^s_K; JM \right> &= \int d\Omega D^J_K(\Omega) R(\Omega) \left| \Phi^{s+} \right>
\end{align*}
\]

Generator Coordinate Method (GCM)
- Superposition of the w.f. with different configuration
- Diagonalization of \( H^{\pm}_{sK,s'K'} \) and \( N^{\pm}_{sK,s'K'} \)

\[
H^{\pm}_{sK,s'K'} = \left< \Phi^s_K; J^\pm M \right| \hat{H} \left| \Phi^{s'}_{K'}; J^\pm M \right>
\]

\[
N^{\pm}_{sK,s'K'} = \left< \Phi^s_K; J^\pm M \right| \Phi^{s'}_{K'}; J^\pm M \right>
\]

\[
\left| \Psi^{J^\pm M} \right> = \sum_{sK} g_{sK} \left| \Phi^s_K; J^\pm M \right>
\]
ΛNN three-body force used

◆ G-matrix interaction

ESC08c + MPP + TBA

Additional (ΛNN) 3 body force

MPP: repulsion which works at high dens.
TBA: phenomenological 3-body attraction

ESC08c

\[ V_{\Lambda N}(r; k_F) = \sum_{i=1}^{3} \left( a_i + b_i k_F + c_i k_F^2 \right) \exp \left( -r^2 / \beta_i^2 \right) \]

MPP + TBA

\[ \Delta V_{\Lambda N}(k_F; r) = (a + bk_F + ck_F^2) \exp \left( -r^2 / 0.9^2 \right) \]

ESC08c: effective ΛN force including ΛN-ΣN coupling effects
MPP: giving 2M⊙ neutron star mass
TBA: to reproduce observed spectra of \(^{89}_\Lambda\)Y by spherical SHF calculation

Yamamoto, Furumoto, Yasutake and Rijken, PRC88,022801(2013); PRC90,045805(2014).

◆ \(k_F\) determined by density

● Averaged density approximation(ADA):

\[ \langle \rho \rangle = \int d^3r \rho_N(r) \rho_{\Lambda}(r) \]

\[ k_F = \left( \frac{3\pi^2 \langle \rho \rangle}{2} \right)^{1/3} \]
HYPERON MIXING AND UNIV. MANY-BODY REPULSION IN NEUTRON STARS

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MPP + TBA

MPP: “universal” repulsion for 3 baryons
For NNN sector: MPP + TBA is determined by $^{16}\text{O} + ^{16}\text{O}$ elastic scattering data at E/A = 70 MeV
For hyperon sector: MPP is the same as NNN, TBA is determined by $^{89}\Lambda\text{Y}$ data

TABLE III. Energy spectra (in MeV) of $^{89}\Lambda\text{Y}$ calculated with MPa and ESC in comparison with experimental values. Averaged values of $k_F$ (in fm$^{-1}$) are in parentheses.

<table>
<thead>
<tr>
<th></th>
<th>s</th>
<th>p</th>
<th>d</th>
<th>f</th>
</tr>
</thead>
<tbody>
<tr>
<td>MPa</td>
<td>-23.8</td>
<td>-17.4</td>
<td>-10.6</td>
<td>-3.8</td>
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<tr>
<td></td>
<td>(1.27)</td>
<td>(1.23)</td>
<td>(1.16)</td>
<td>(1.08)</td>
</tr>
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<td>ESC</td>
<td>-23.7</td>
<td>-16.8</td>
<td>-9.8</td>
<td>-3.0</td>
</tr>
<tr>
<td></td>
<td>(1.28)</td>
<td>(1.23)</td>
<td>(1.17)</td>
<td>(1.09)</td>
</tr>
<tr>
<td>Expt.</td>
<td>-23.7</td>
<td>-17.6</td>
<td>-10.9</td>
<td>-3.7</td>
</tr>
</tbody>
</table>

$^{16}\text{O} + ^{16}\text{O}$ scattering data

MPP gives stiff EOS enough to give 2$M_{\odot}$
Results and Discussions

1) $B_\Lambda$ and Density dependence of the interaction
   
   Is it possible to describe mass dependence of observed $B_\Lambda$? What is essential to reproduce it?
   
   Core structure, in particular core deformation

2) Three-body force effects

   Do $\Lambda$NN three-body effects appear in $B_\Lambda$? How large?

   Comparison of the results: “ESC08c only” and “ESC08c + $\Lambda$NN force”
\( B_\Lambda \) as a function of mass number \( A \)

**ESC08c + MPP + TBA**

repulsive attraction

\( k_F \) is determined from ground-state density

\[
\langle \rho \rangle = \int d^3r \rho_N(r)\rho_\Lambda(r) \quad k_F = \left( \frac{3\pi^2 \langle \rho \rangle}{2} \right)^{1/3}
\]

<table>
<thead>
<tr>
<th>( \beta )</th>
<th>( \gamma )</th>
<th>( \langle \rho \rangle )</th>
<th>( k_F )</th>
<th>( -B_\Lambda^{\text{calc}} )</th>
<th>( -B_\Lambda^{\text{exp}} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>(^9)Li</td>
<td>0.50</td>
<td>2°</td>
<td>0.072</td>
<td>1.02</td>
<td>-8.1</td>
</tr>
<tr>
<td>(^9)Be</td>
<td>0.87</td>
<td>1°</td>
<td>0.060</td>
<td>0.96</td>
<td>-8.1</td>
</tr>
<tr>
<td>(^9)B</td>
<td>0.45</td>
<td>2°</td>
<td>0.072</td>
<td>1.02</td>
<td>-8.2</td>
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<tr>
<td>(^{10})Be</td>
<td>0.57</td>
<td>1°</td>
<td>0.077</td>
<td>1.04</td>
<td>-9.0</td>
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<td>(^{10})B</td>
<td>0.58</td>
<td>1°</td>
<td>0.075</td>
<td>1.04</td>
<td>-9.1</td>
</tr>
<tr>
<td>(^{11})B</td>
<td>0.50</td>
<td>29°</td>
<td>0.108</td>
<td>1.06</td>
<td>-10.0</td>
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<tr>
<td>(^{12})B</td>
<td>0.39</td>
<td>48°</td>
<td>0.083</td>
<td>1.07</td>
<td>-11.3</td>
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<td>(^{12})C</td>
<td>0.41</td>
<td>34°</td>
<td>0.086</td>
<td>1.08</td>
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<td>(^{13})C</td>
<td>0.45</td>
<td>60°</td>
<td>0.090</td>
<td>1.10</td>
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<td>(^{14})C</td>
<td>0.45</td>
<td>31°</td>
<td>0.093</td>
<td>1.11</td>
<td>-12.5</td>
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<tr>
<td>(^{15})C</td>
<td>0.28</td>
<td>60°</td>
<td>0.098</td>
<td>1.13</td>
<td>-12.9</td>
</tr>
<tr>
<td>(^{16})O</td>
<td>0.02</td>
<td>-</td>
<td>0.105</td>
<td>1.16</td>
<td>-13.0</td>
</tr>
<tr>
<td>(^{17})O</td>
<td>0.30</td>
<td>3°</td>
<td>0.110</td>
<td>1.18</td>
<td>-14.3</td>
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<tr>
<td>(^{27})Mg</td>
<td>0.36</td>
<td>36°</td>
<td>0.126</td>
<td>1.23</td>
<td>-16.2</td>
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<td>(^{28})Si</td>
<td>0.32</td>
<td>53°</td>
<td>0.125</td>
<td>1.23</td>
<td>-16.6</td>
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<td>(^{32})S</td>
<td>0.28</td>
<td>0°</td>
<td>0.130</td>
<td>1.24</td>
<td>-17.6</td>
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<td>(^{40})K</td>
<td>0.01</td>
<td>-</td>
<td>0.136</td>
<td>1.26</td>
<td>-19.4</td>
</tr>
<tr>
<td>(^{40})Ca</td>
<td>0.03</td>
<td>-</td>
<td>0.136</td>
<td>1.26</td>
<td>-19.3</td>
</tr>
<tr>
<td>(^{41})Ca</td>
<td>0.13</td>
<td>12°</td>
<td>0.136</td>
<td>1.26</td>
<td>-19.5</td>
</tr>
<tr>
<td>(^{48})K</td>
<td>0.01</td>
<td>-</td>
<td>0.141</td>
<td>1.28</td>
<td>-20.2</td>
</tr>
<tr>
<td>(^{48})Ca</td>
<td>0.13</td>
<td>12°</td>
<td>0.136</td>
<td>1.26</td>
<td>-19.5</td>
</tr>
<tr>
<td>(^{51})V</td>
<td>0.18</td>
<td>2°</td>
<td>0.151</td>
<td>1.31</td>
<td>-20.3</td>
</tr>
<tr>
<td>(^{59})Fe</td>
<td>0.26</td>
<td>23°</td>
<td>0.142</td>
<td>1.28</td>
<td>-21.7</td>
</tr>
</tbody>
</table>

HyperAMD calculation nicely reproduces \( B_\Lambda \) in wide mass regions
What is essential to reproduce $B_\Lambda$?

"Description of the core structure"

"Full calc." vs. "Spherical calc."

![Graph showing comparison between full and spherical calculations](image)
What is essential to reproduce $B_\Lambda$

Ex. $^9_\Lambda$Be  

“Full calc.” vs. “Spherical calc.”

“Full calc.”: all of w.f. on energy curve in GCM calc.

“Spherical calc.”
What is essential to reproduce $B_\Lambda$

- $B_\Lambda$ in “Spherical calc.” are shallower than those in “Full calc.” with $A < 16$

  Originated in density dependence of interaction

<table>
<thead>
<tr>
<th>$k_F$ values used</th>
<th>$k_F$ [fm$^{-1}$]</th>
<th>Full</th>
<th>Spherical</th>
</tr>
</thead>
<tbody>
<tr>
<td>$^{11}$B</td>
<td>1.05</td>
<td>1.12</td>
<td></td>
</tr>
<tr>
<td>$^{12}$B</td>
<td>1.06</td>
<td>1.15</td>
<td></td>
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<td>$^{13}$C</td>
<td>1.09</td>
<td>1.14</td>
<td></td>
</tr>
</tbody>
</table>
What is essential to reproduce $B_\Lambda$

"Description of the core structure"

Ex.: $^{11}\text{B}$

More sophisticated treatment: GCM calc. on ($\beta$, $\gamma$) plane

T. Suhara and Y. Kanada-En'yo, PTP123,303(2010)

$^{11}\text{B NEG}$

$^{11}\text{B} \text{ (Spherical)}$

$B_\Lambda = 9.5 \text{ MeV}$

($k_F = 1.16 \text{ fm}^{-1}$)

$^{12}\Lambda \text{ B (Exp)}$

$B_\Lambda = 11.4 \pm 0.02 \text{ MeV}$

$^{12}\Lambda \text{ B (Spherical)}$

$B_\Lambda = 11.3 \text{ MeV}$

($k_F = 1.07 \text{ fm}^{-1}$)

$B(2^+) e^2 \text{ fm}^4$

$7/2^-$

$1.9 \pm 0.4$

$6$

$6$

$3/2_2^-$

$5/2^-$

$14 \pm 3$

$1/2^-$

$3/2_1^-$
Results and Discussions

1) $B_\Lambda$ and Density dependence of the interaction
   Is it possible to describe mass dependence of observed $B_\Lambda$? What is essential to reproduce it?
   Core structure, in particular core deformation

2) Three-body force effects
   Do $\Lambda$NN three-body effects appear in $B_\Lambda$? How large?

Comparison of the results: “ESC08c only” and “ESC08c + $\Lambda$NN force”
Comparison with the results with ESC08c only

◆ Effects of many-body force

**ESC:** ESC08c only

**MPa:** ESC08c + **MPP + TBA**

ΛNN three-body effects

### Over-binding with ESC08c only

<table>
<thead>
<tr>
<th>[MeV]</th>
<th>ESC</th>
<th>MPa</th>
<th>Exp.</th>
</tr>
</thead>
<tbody>
<tr>
<td>$^{13}_{\Lambda}$C</td>
<td>$-11.5$</td>
<td>$-11.7$</td>
<td>$-11.69 \pm 0.19$</td>
</tr>
<tr>
<td>$^{16}_{\Lambda}$O</td>
<td>$-13.3$</td>
<td>$-13.0$</td>
<td>$-12.96 \pm 0.05$</td>
</tr>
<tr>
<td>$^{28}_{\Lambda}$Si</td>
<td>$-17.7$</td>
<td>$-16.6$</td>
<td>$-17.1 \pm 0.2$</td>
</tr>
<tr>
<td>$^{40}_{\Lambda}$K</td>
<td>$-21.5$</td>
<td>$-19.4$</td>
<td>–</td>
</tr>
<tr>
<td>$^{48}_{\Lambda}$K</td>
<td>$-22.6$</td>
<td>$-20.2$</td>
<td>–</td>
</tr>
</tbody>
</table>

• Additional dens. dep. of ΛNN 3-body force makes $B_{\Lambda}$ different

• Systematic data of $B_{\Lambda}$ will provide a new insight to many-body force
Current status of observed $B_\Lambda$

Observations are not enough with $A > 16$

Systematic and accurate data of observed $B_\Lambda$ are desired

HyperAMD + GCM was applied with ESC08c + MPP + TBA interaction

Observation of $B_\Lambda$ are successfully reproduced in wide mass regions

Structure of the core nuclei
- Spherical shape: deviate from observed $B_\Lambda$
- Description of core deformation is essential

→ Sophisticated treatment of hypernuclei is indispensable

Many-body (MPP + TBA) force effects
- $\Lambda NN(MPP + TBA)$ force brings additional density dependence

→ Systematic observations of $B_\Lambda$ is necessary to confirm/give constraints

Future plan
- To reveal reasons for deviation of $B_\Lambda$ with $A < 9$ (e.g. $^9\Lambda$Be)
- Further study on model dependence of three-body force effects