

AFDMC Calculations on Medium-Heavy Hypernuclei

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In collaboration with:

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- ✓ Francesco Pederiva, Trento
- ✓ Francesco Catalano, Uppsala



JLab, March 15, 2016

- ✓ AFDMC wave function: single particle representation

$$\psi_T(R, S) = \prod_{\lambda i} f_c^{\Lambda N}(r_{\lambda i}) \psi_T^N(R_N, S_N) \psi_T^\Lambda(R_\Lambda, S_\Lambda)$$

$$\left\{ \begin{array}{l} \psi_T^\kappa(R_\kappa, S_\kappa) = \prod_{i < j} f_c^{\kappa \kappa}(r_{ij}) \Phi_\kappa(R_\kappa, S_\kappa) \quad \kappa = N, \Lambda \\ \Phi_\kappa(R_\kappa, S_\kappa) = \mathcal{A} \left[\prod_{i=1}^{\mathcal{N}_\kappa} \varphi_\epsilon^\kappa(\mathbf{r}_i, s_i) \right] = \det \left\{ \varphi_\epsilon^\kappa(\mathbf{r}_i, s_i) \right\} \end{array} \right. \begin{array}{l} \swarrow \text{s.p. orbitals} \\ \searrow \text{plane waves} \end{array}$$

$$s_i = \begin{pmatrix} a_i \\ b_i \\ c_i \\ d_i \end{pmatrix}_i = a_i |p \uparrow\rangle_i + b_i |p \downarrow\rangle_i + c_i |n \uparrow\rangle_i + d_i |n \downarrow\rangle_i$$

$$s_\lambda = \begin{pmatrix} u_\lambda \\ v_\lambda \end{pmatrix}_\lambda = u_\lambda |\Lambda \uparrow\rangle_\lambda + v_\lambda |\Lambda \downarrow\rangle_\lambda$$

✓ AFDMC propagation

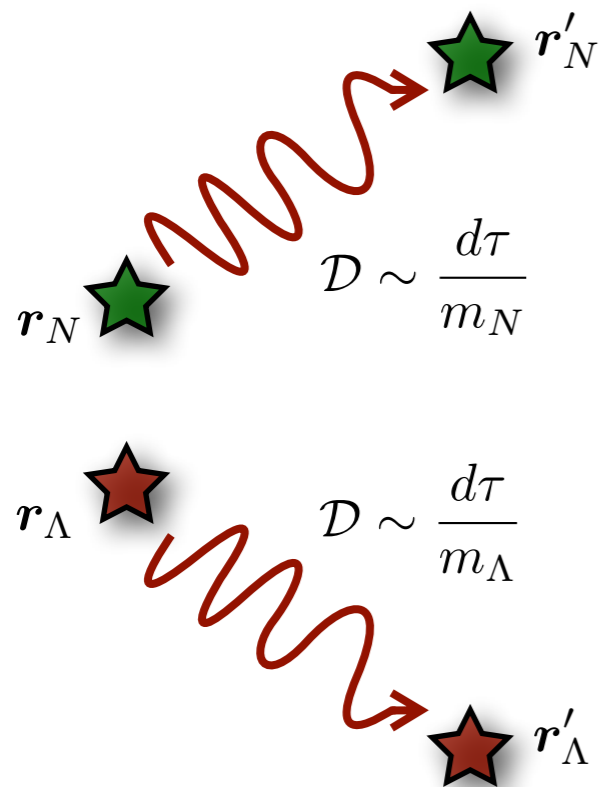
$$\langle SR | \psi(\tau + d\tau) \rangle = \int dR' dS' \langle SR | e^{-(H-E_0)d\tau} | R' S' \rangle \langle S' R' | \psi_T(\tau) \rangle$$

final
walkers

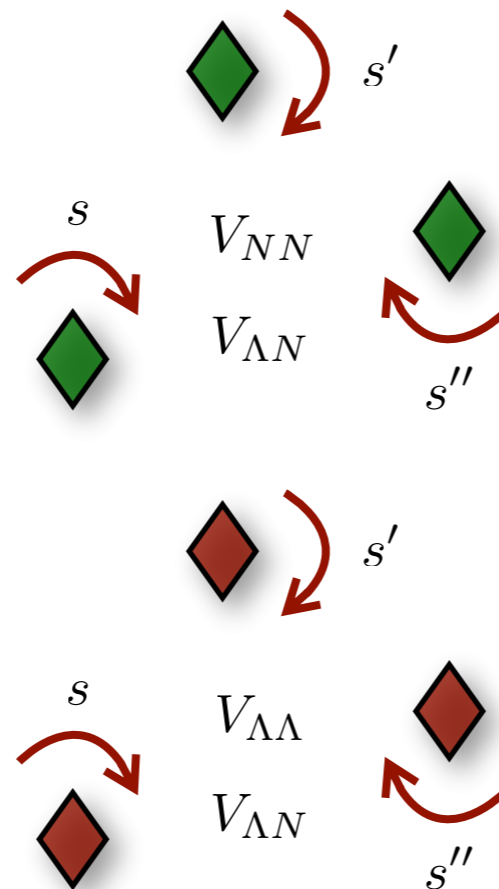
propagator

initial
walkers

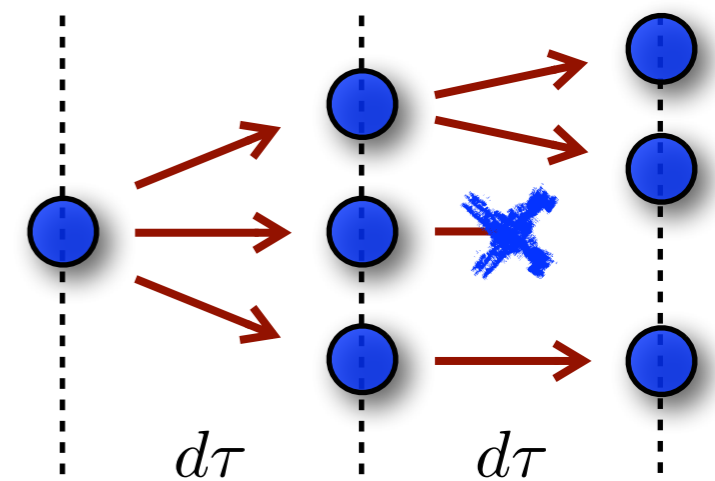
diffusion (DMC): $d\tau$



rotation (AF): $\sqrt{d\tau}$



branching: $d\tau$



✓ AFDMC algorithm

- ▶ imaginary time projection \longrightarrow exact ground state
- ▶ single particle wf + HS transformation \longrightarrow large number of particles
- ▶ stochastic method \longrightarrow error estimate: $\sigma \sim 1/\sqrt{\mathcal{N}}$

✓ AFDMC Hamiltonians

- ▶ nucleon-nucleon phenomenological interaction: Argonne & Urbana

$$H = \sum_i \frac{p_i^2}{2m_N} + \sum_{i < j} v_{ij} + \sum_{i < j < k} v_{ijk}$$

2B: NN scattering + deuteron

3B: nuclei + nuclear matter

✓ AFDMC algorithm

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✓ AFDMC Hamiltonians

- ▶ nucleon-nucleon phenomenological interaction: Argonne & Urbana
- ▶ hyperon-nucleon phenomenological interaction: Argonne like

$$H = \sum_i \frac{p_i^2}{2m_N} + \sum_{i < j} v_{ij} + \sum_{i < j < k} v_{ijk}$$

2B: Λp scattering + $A = 4$ CSB*

$$+ \sum_{\lambda} \frac{p_{\lambda}^2}{2m_{\Lambda}} + \sum_{\lambda, i} v_{\lambda i} + \sum_{\lambda, i < j} v_{\lambda ij}$$

3B:

no unique fit

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✓ AFDMC Hamiltonians

- ▶ nucleon-nucleon phenomenological interaction: Argonne & Urbana
- ▶ hyperon-nucleon phenomenological interaction: Argonne like

$$H = \sum_i \frac{p_i^2}{2m_N} + \sum_{i < j} v_{ij} + \sum_{i < j < k} v_{ijk}$$

$$+ \sum_{\lambda} \frac{p_{\lambda}^2}{2m_{\Lambda}} + \sum_{\lambda, i} v_{\lambda i} + \sum_{\lambda, i < j} v_{\lambda ij}$$



use QMC to fit hyp. exp. data

$$B_{\Lambda} = E({}^{A-1}Z) - E({}_{\Lambda}^AZ)$$

3B:

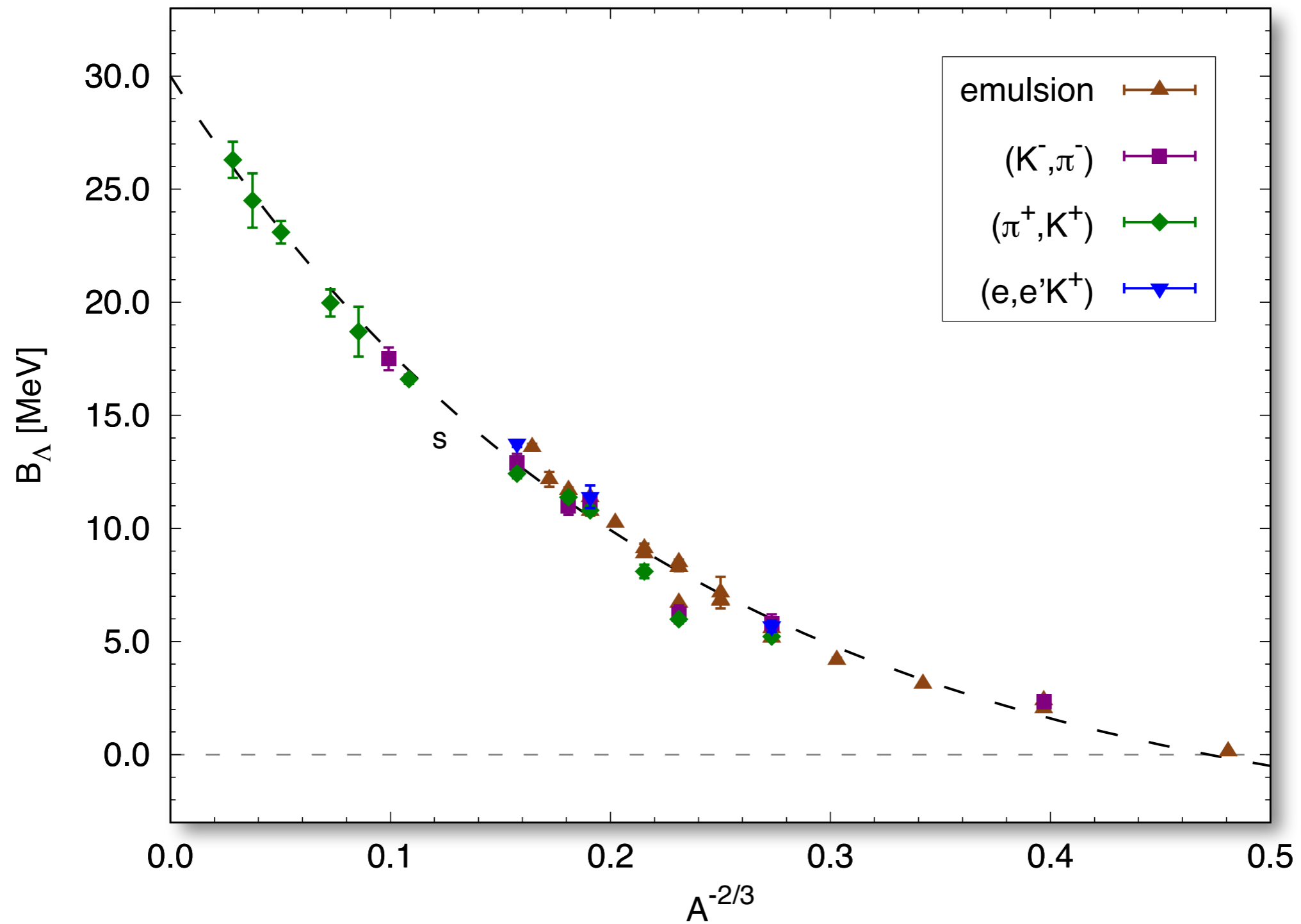
no unique fit

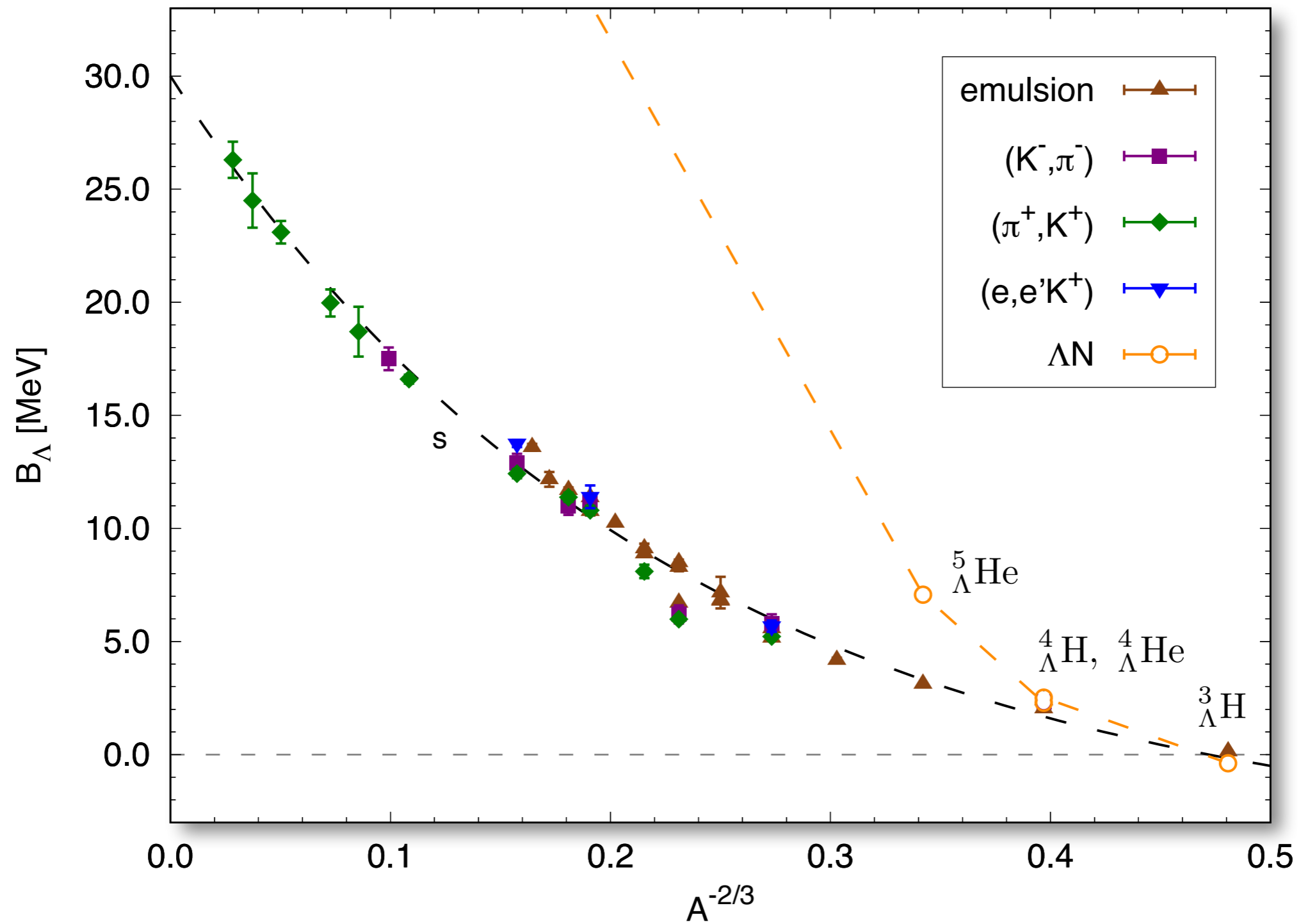
nucleon-nucleon interaction

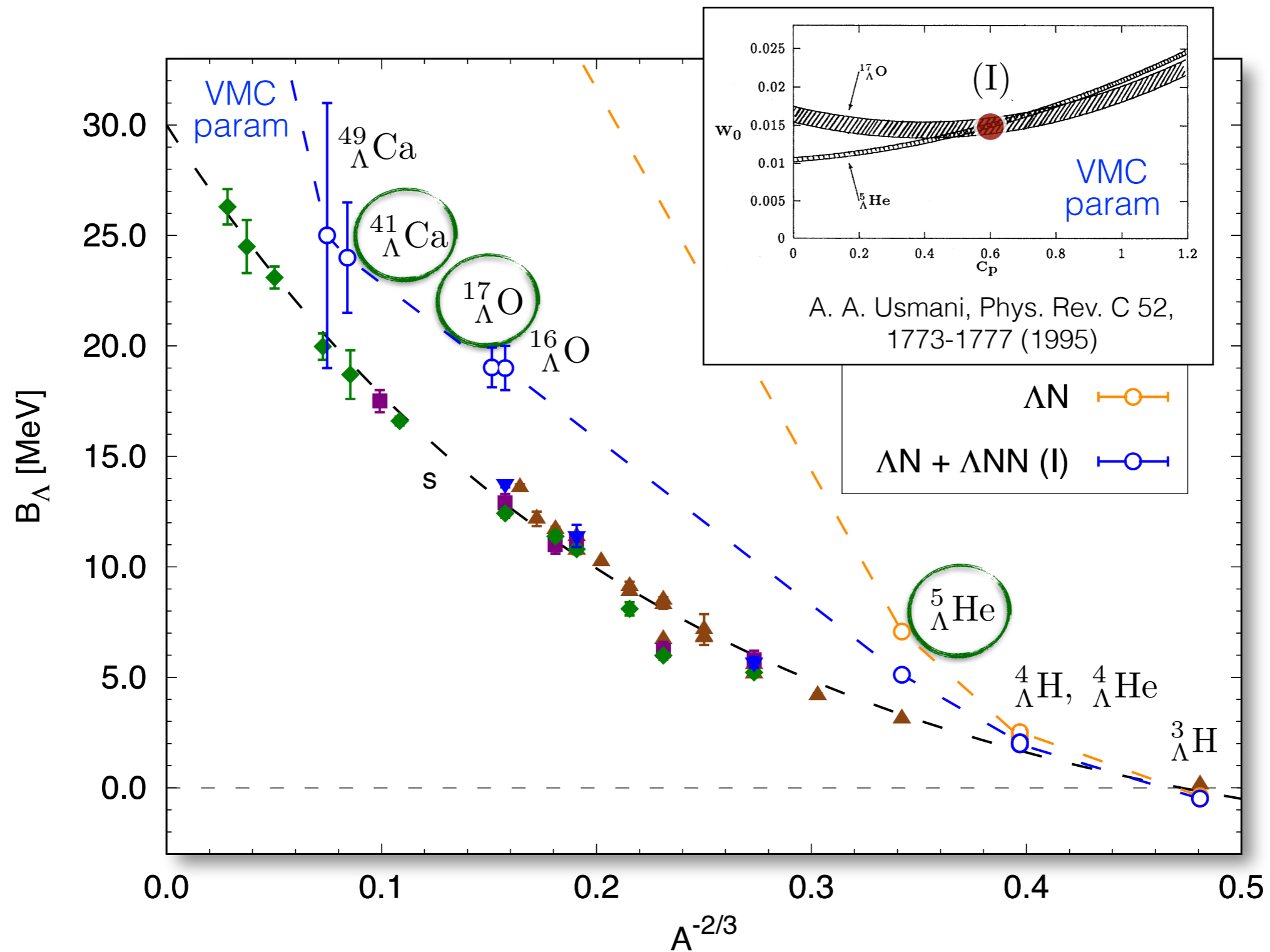
nucleus	AV4'	AV6'	AV7'	AV4'+UIX _c	exp
⁴ He (0 ⁺)	-32.83(5)	-27.09(3)	-25.7(2)	-26.63(2)	-28.295
¹⁵ O (½ ⁻)	—	—	—	-99.43(2)	-111.955
¹⁶ O (0 ⁺)	-180.1(4)	-115.6(3)	-90.6(4)	-119.9(2)	-127.619
³⁹ K (¾ ⁺)	—	—	—	-360.8(2)	-333.724
⁴⁰ Ca (0 ⁺)	-597(3)	-322(2)	-209(1)	-383.3(3)	-342.051
⁴⁴ Ca (0 ⁺)	—	—	—	-397.8(5)	-380.960
⁴⁷ K (½ ⁺)	—	—	preliminary	-386.3(2)	-400.199
⁴⁸ Ca (0 ⁺)	-645(3)	—	—	-413.2(3)	-416.001

S. Gandolfi, A. Lovato, J. Carlson, K. E. Schmidt, Phys. Rev. C 90, 061306(R) (2014)

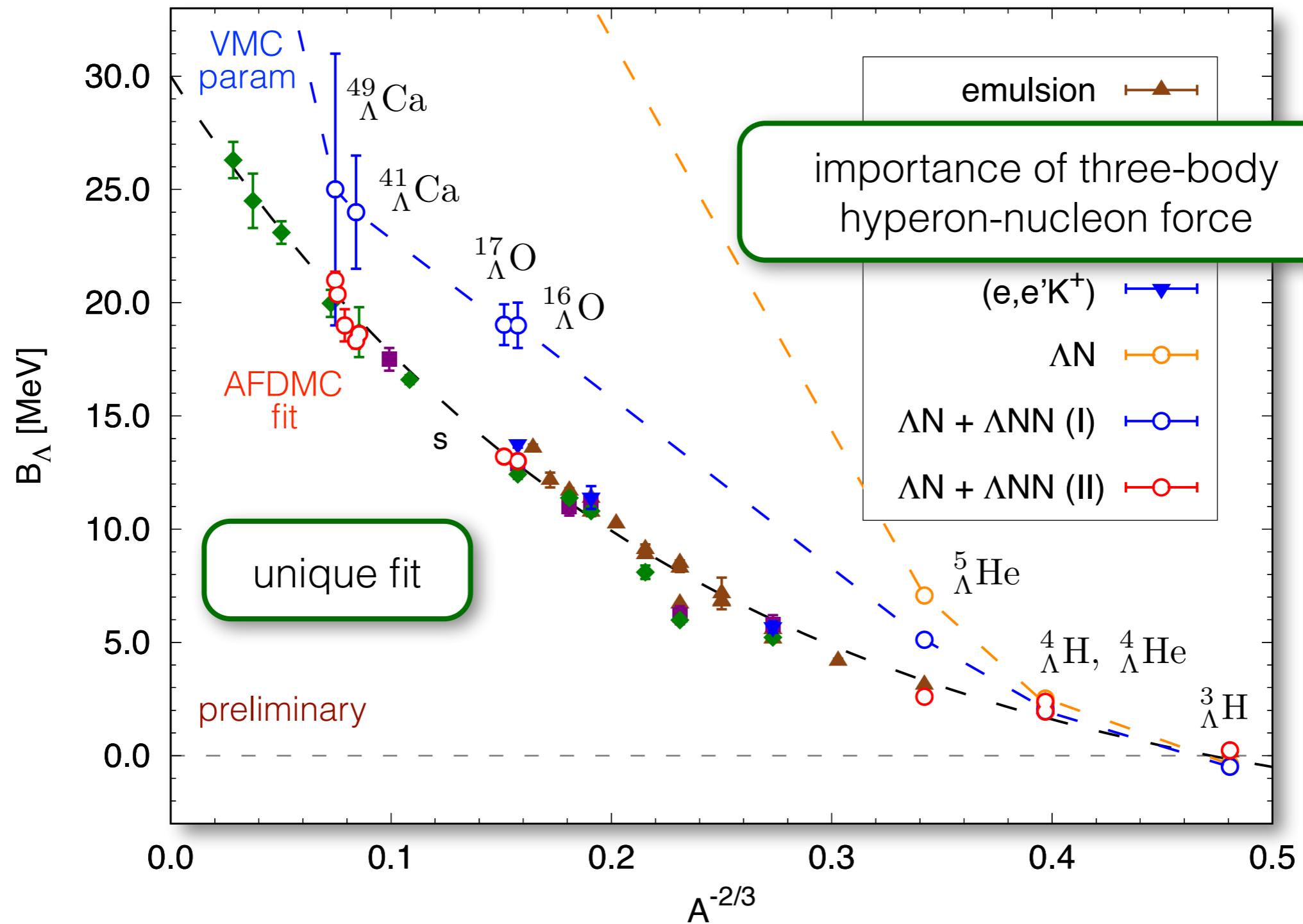
F. Pederiva, F. Catalano, D. L., A. Lovato, S. Gandolfi, arXiv:1506.04042 (2015)





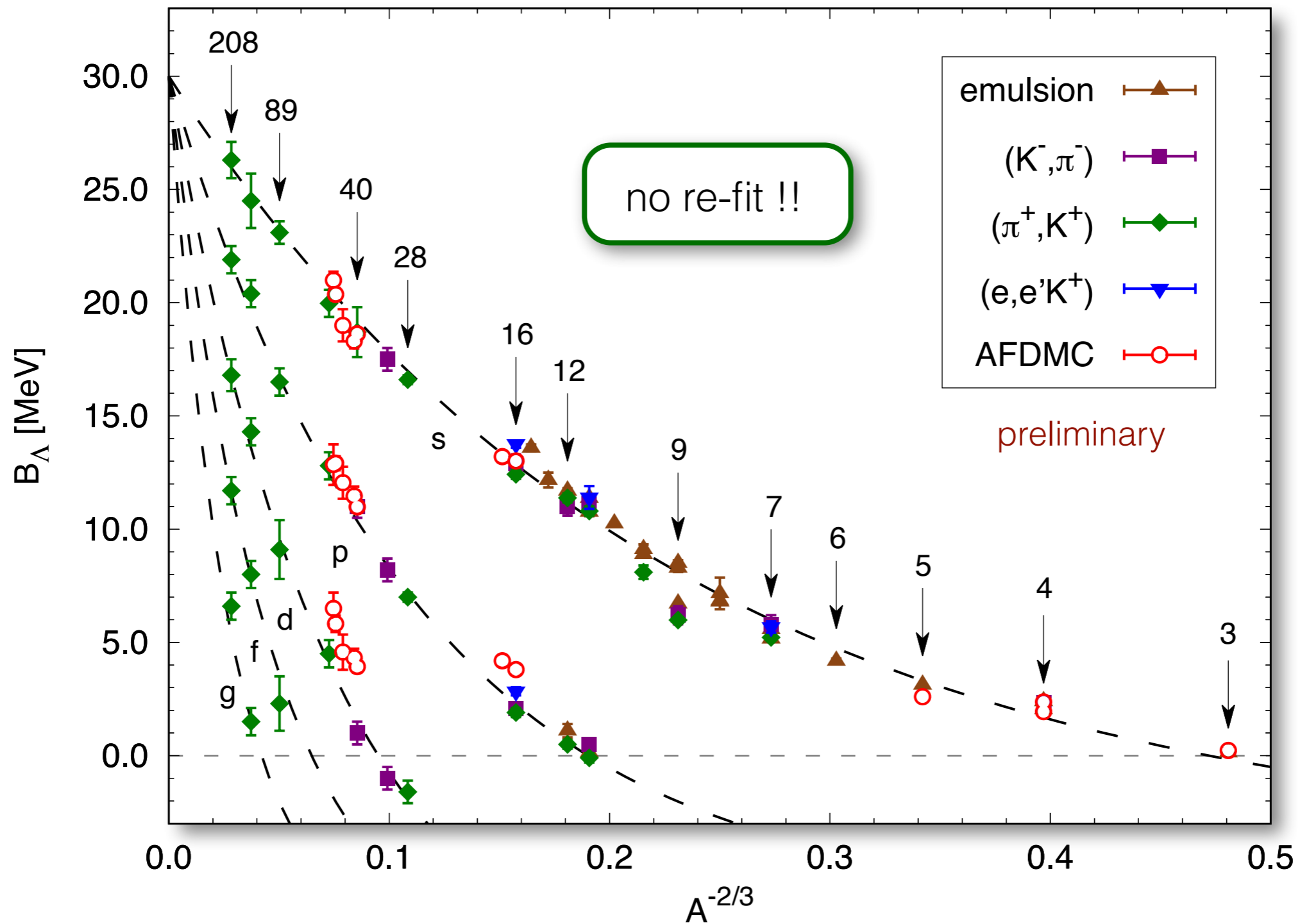


D. L., F. Pederiva, S. Gandolfi, Phys. Rev. C 89, 014314 (2014)



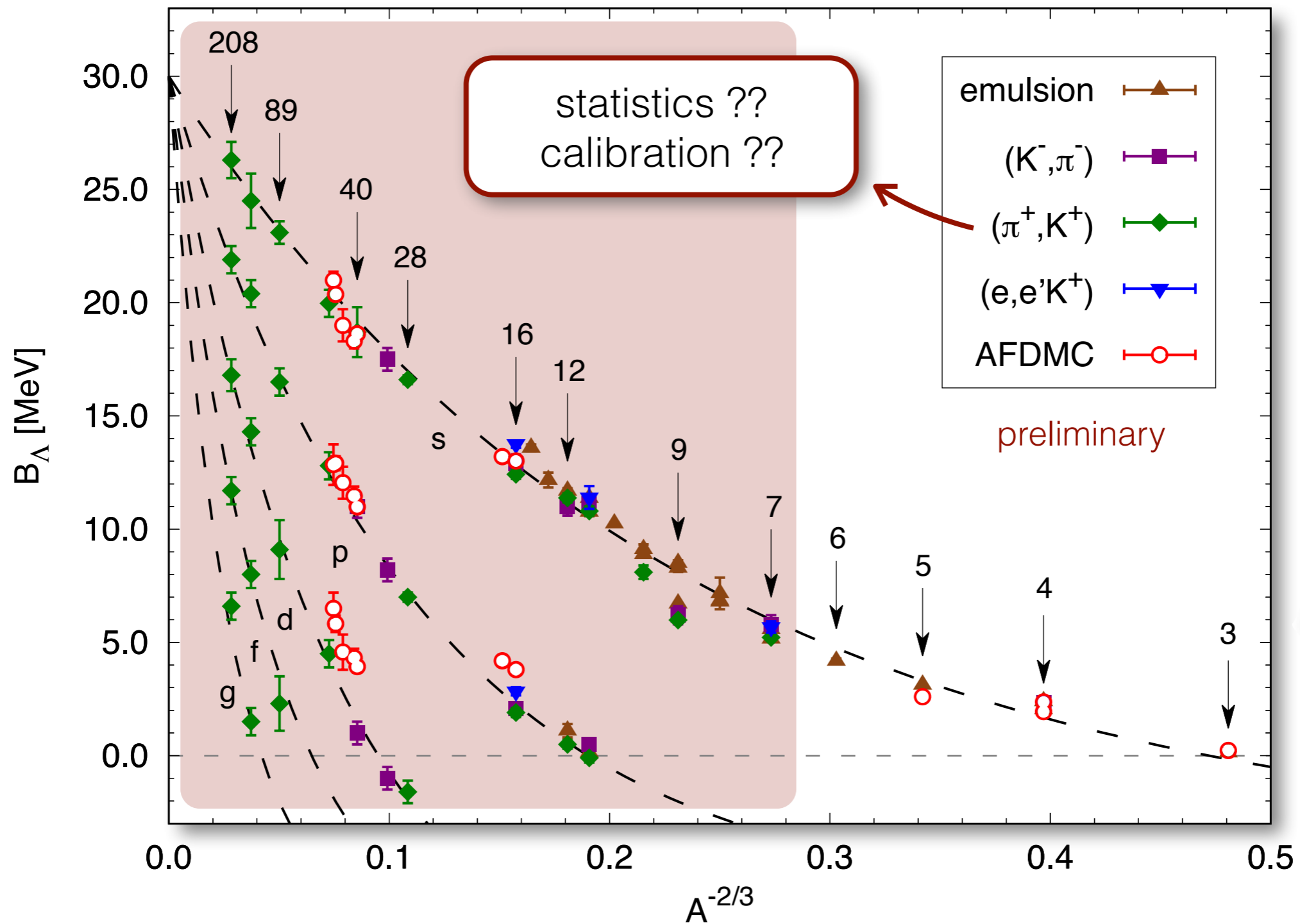
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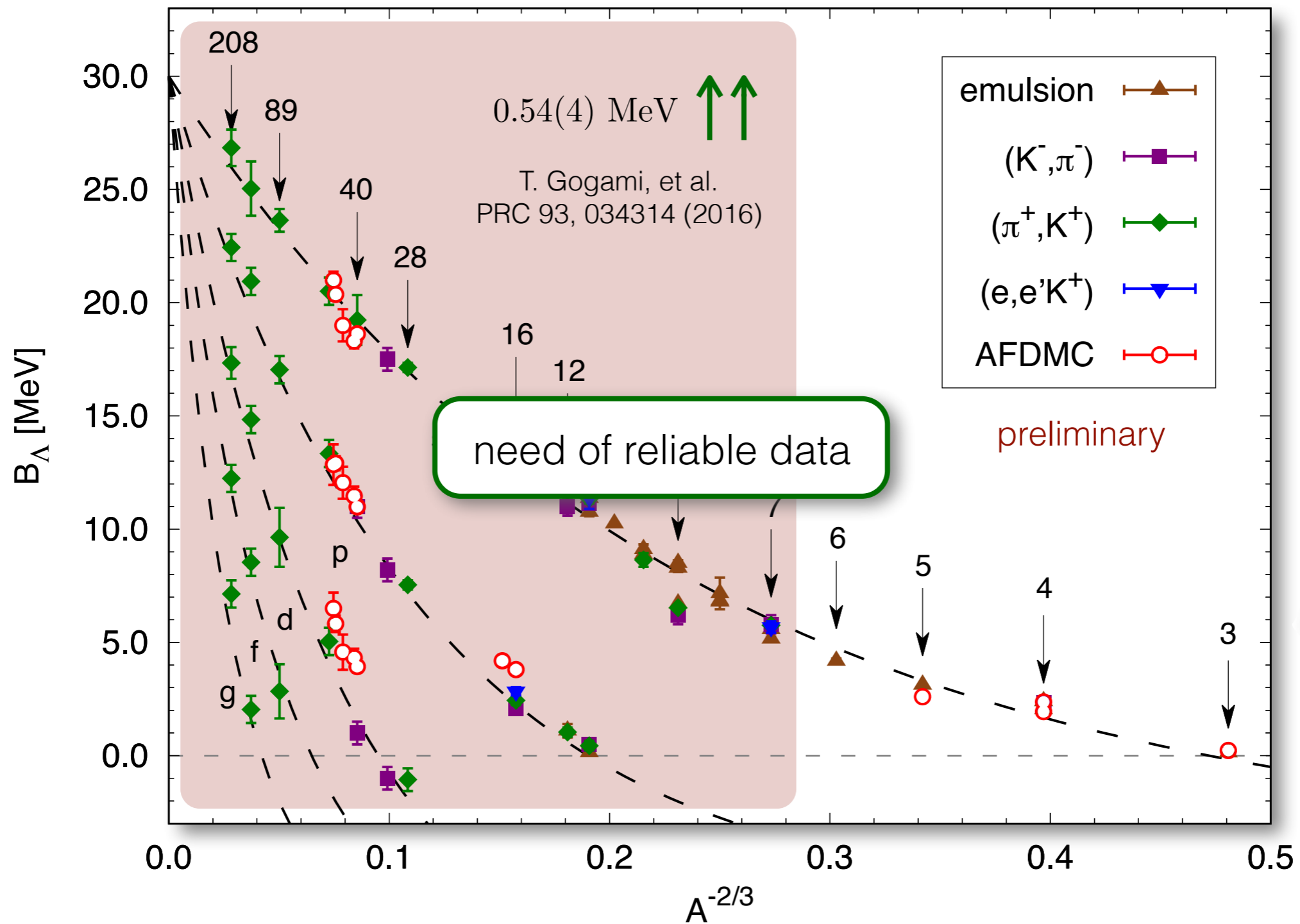
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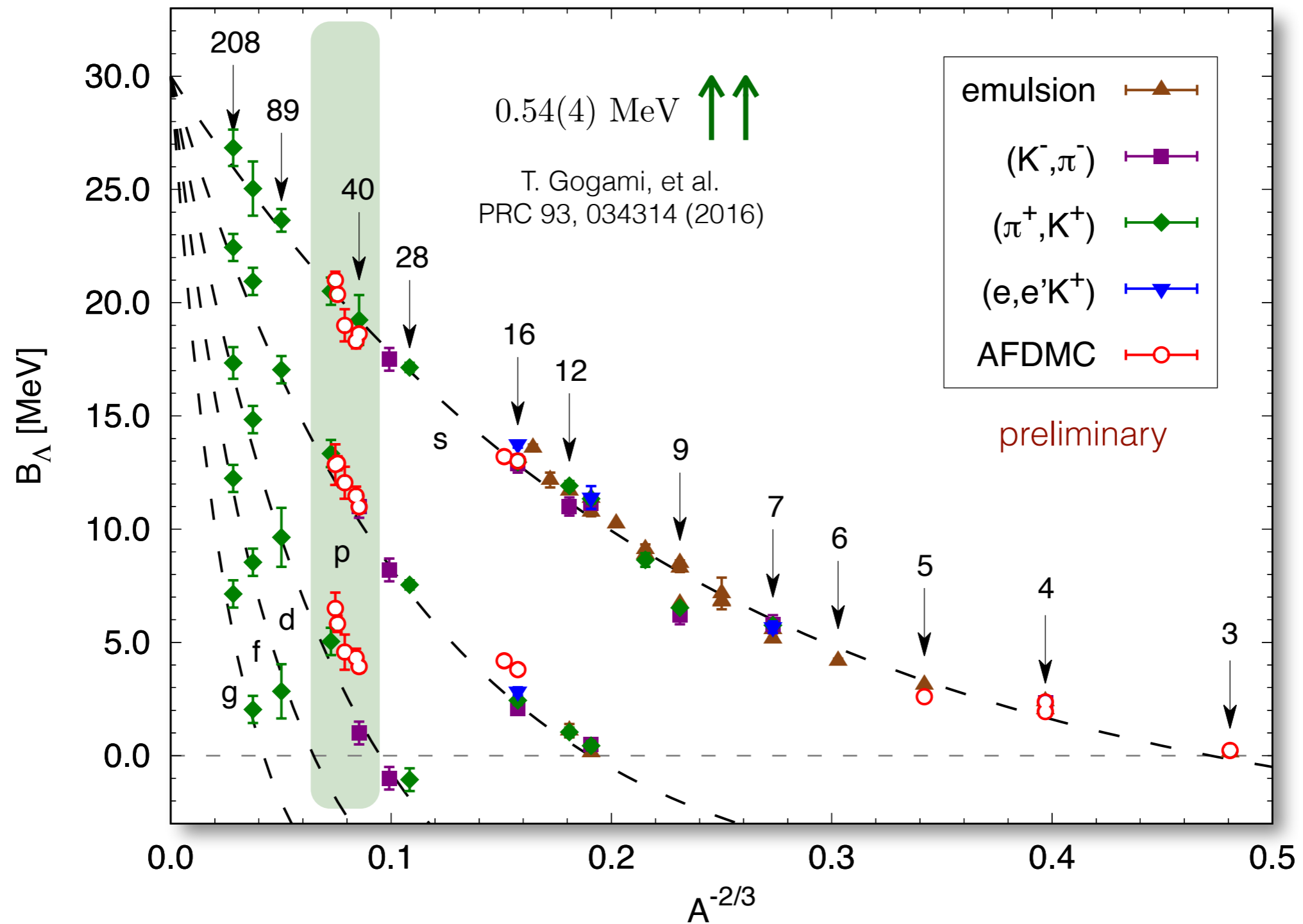
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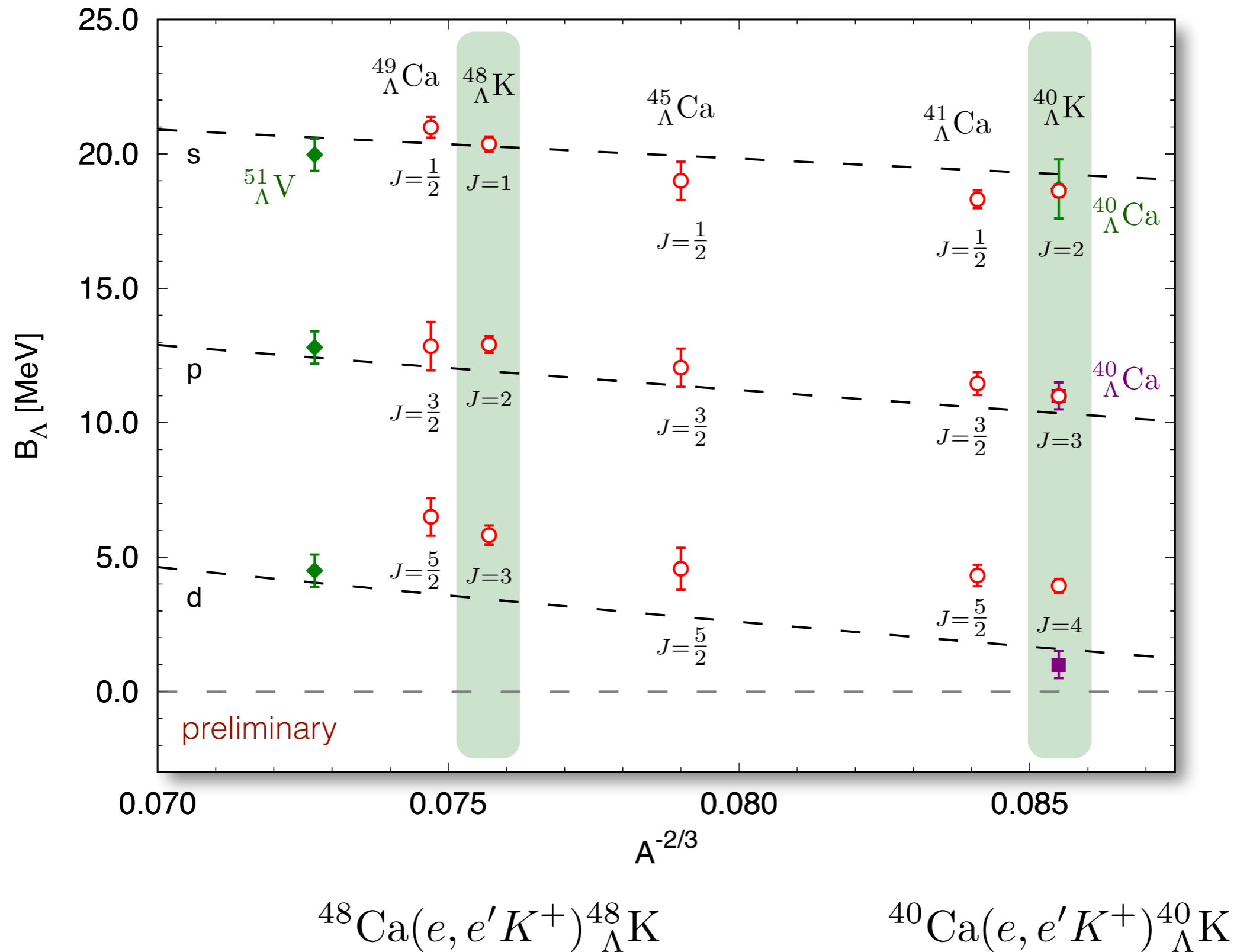
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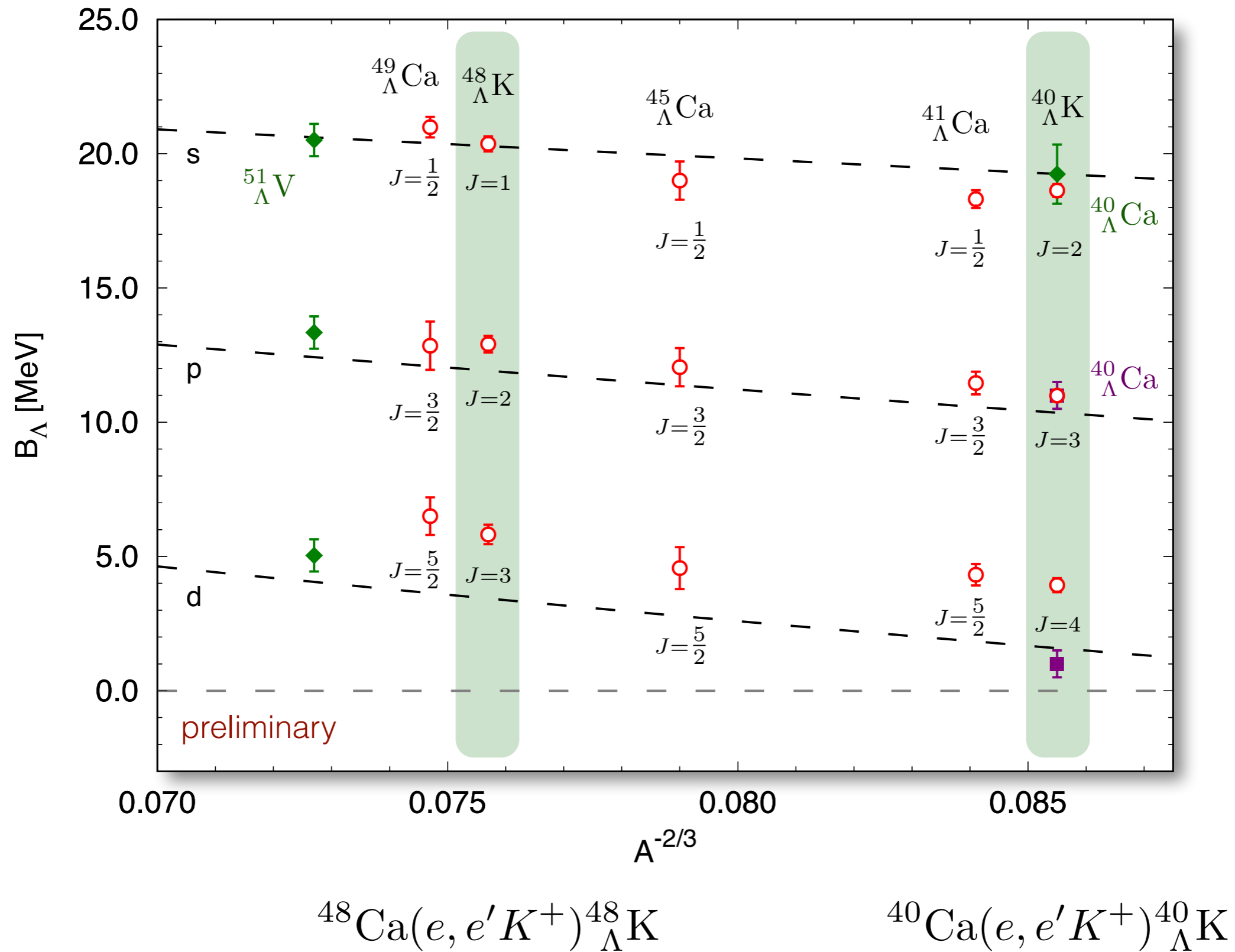
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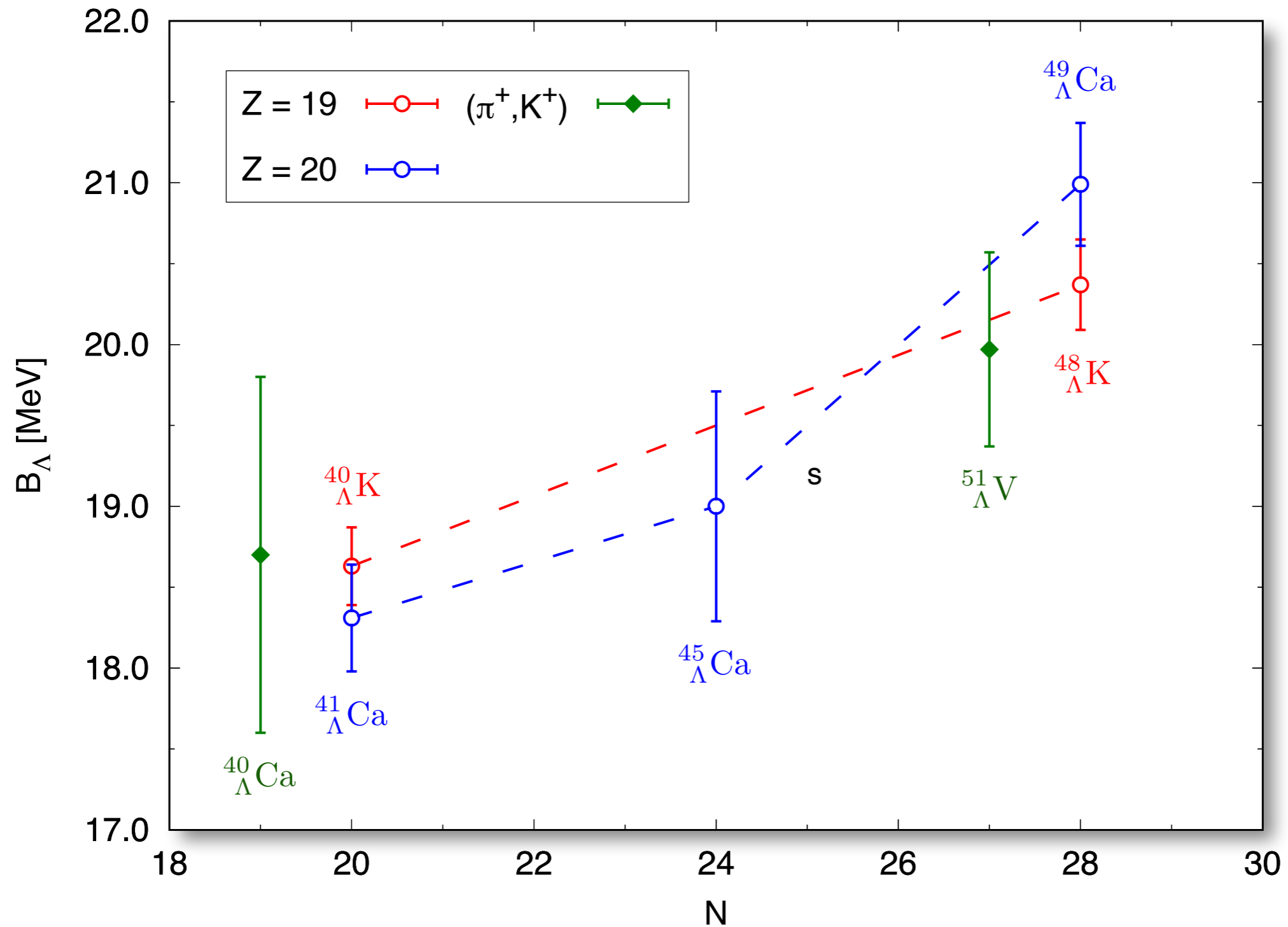


D. L., F. Pederiva, S. Gandolfi, Phys. Rev. C 89, 014314 (2014)

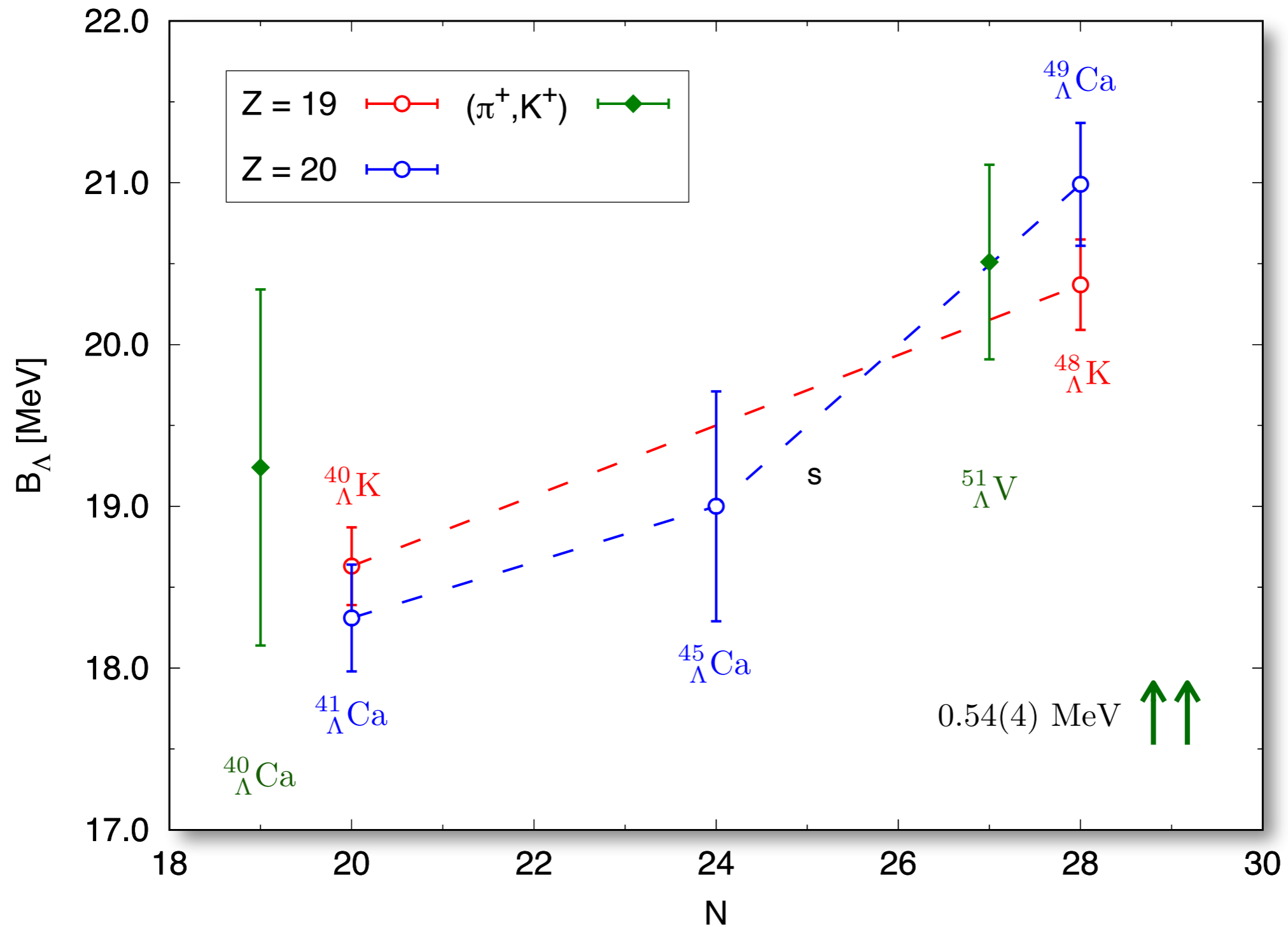
F. Pederiva, F. Catalano, D. L., A. Lovato, S. Gandolfi, arXiv:1506.04042 (2015)



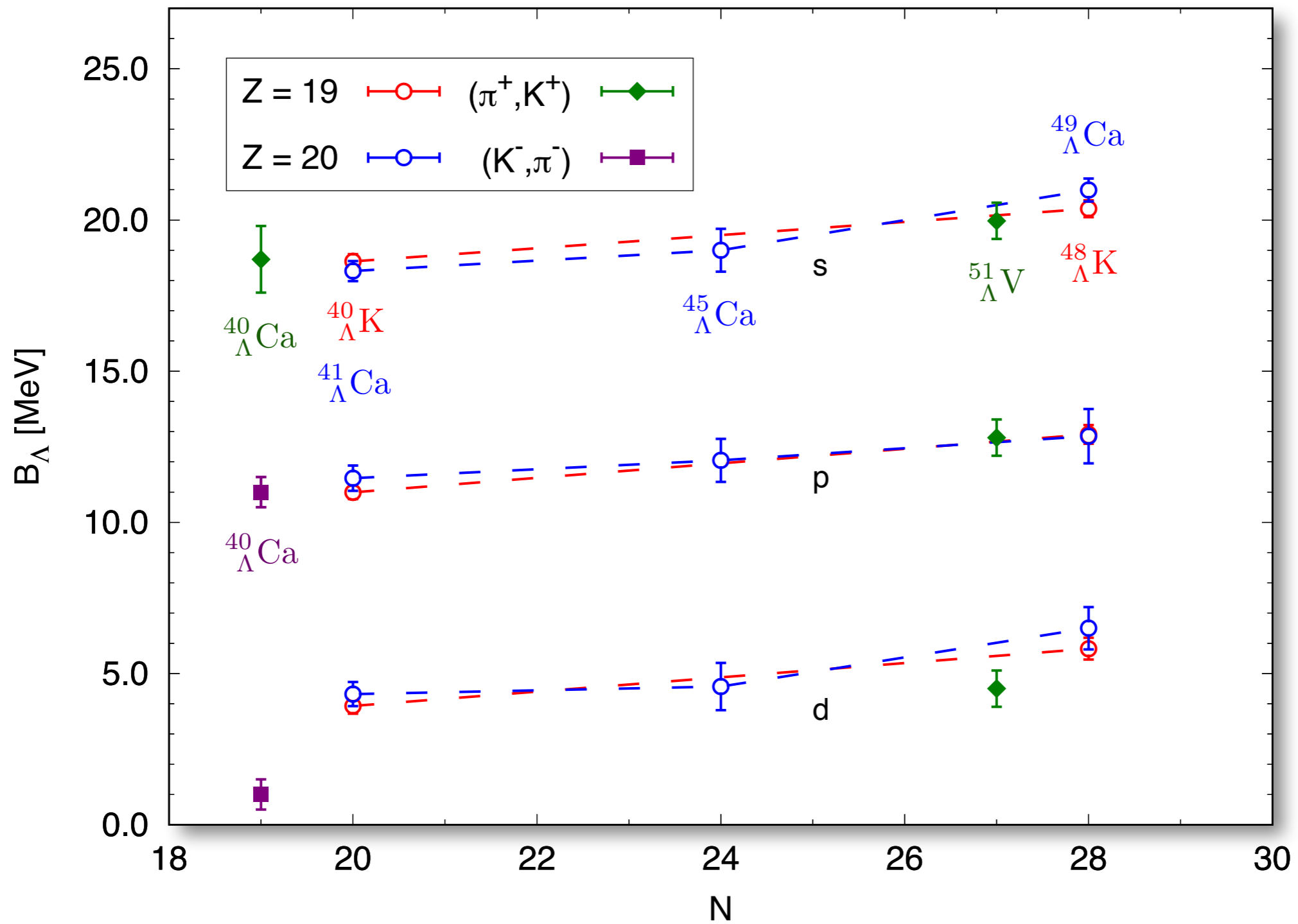




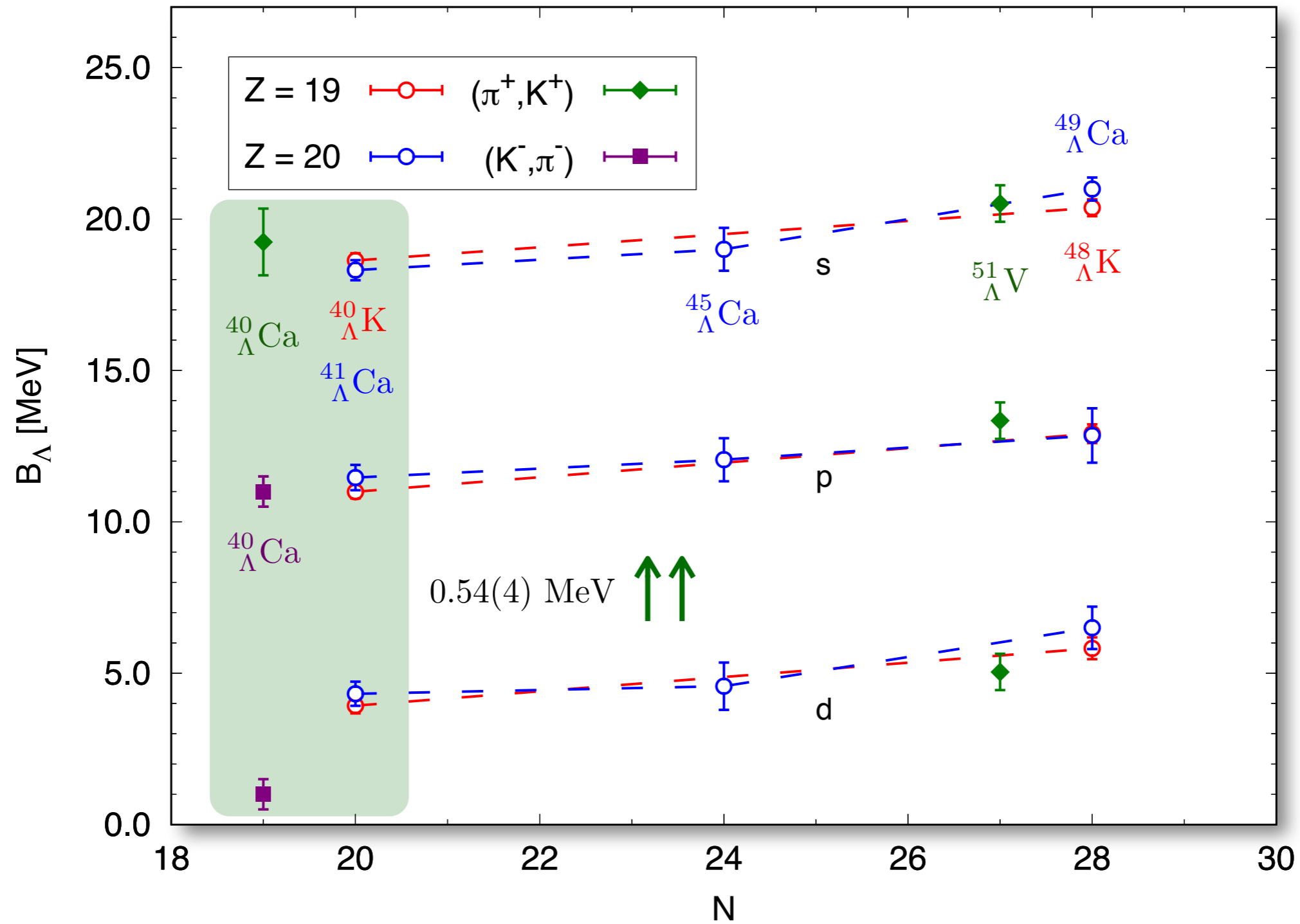
preliminary



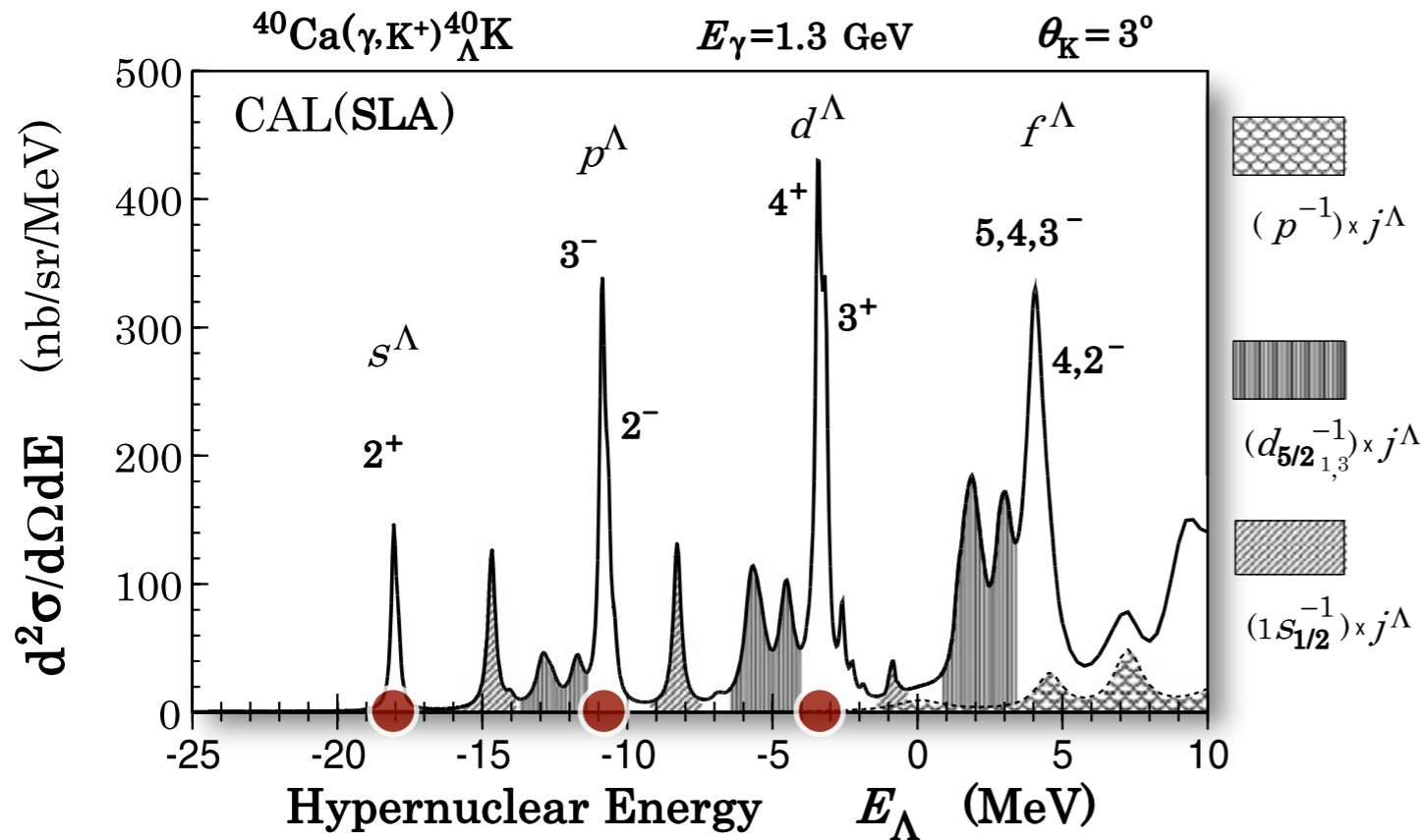
preliminary



preliminary



preliminary



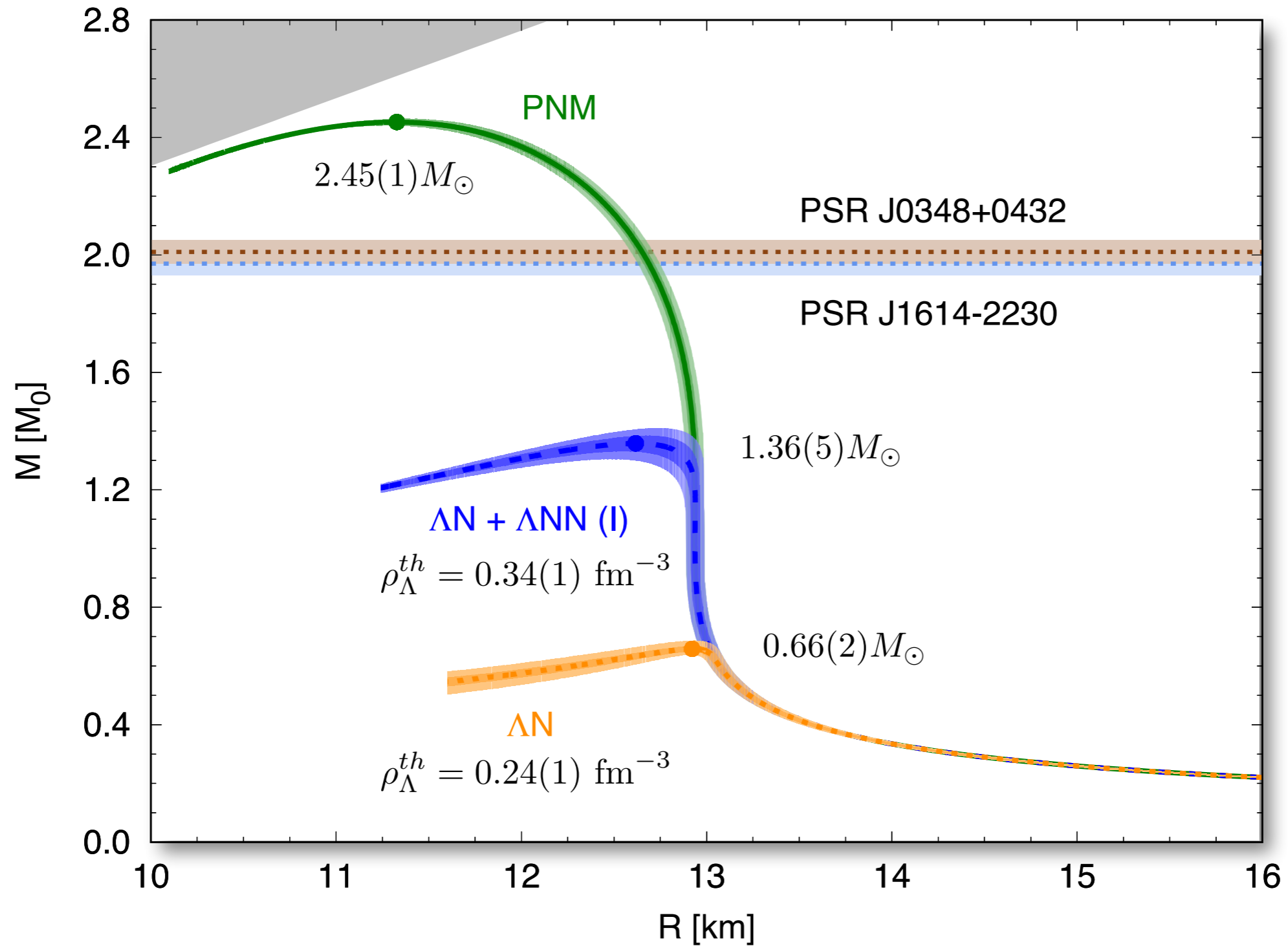
P. Bydžovský, M. Sotona, T. Motoba, K. Itonaga,
K. Ogawa, O. Hashimoto,
Nucl. Phys. A 881 (2012) 199-217

$$B_{\Lambda}^s \simeq 18.0 \text{ MeV}$$

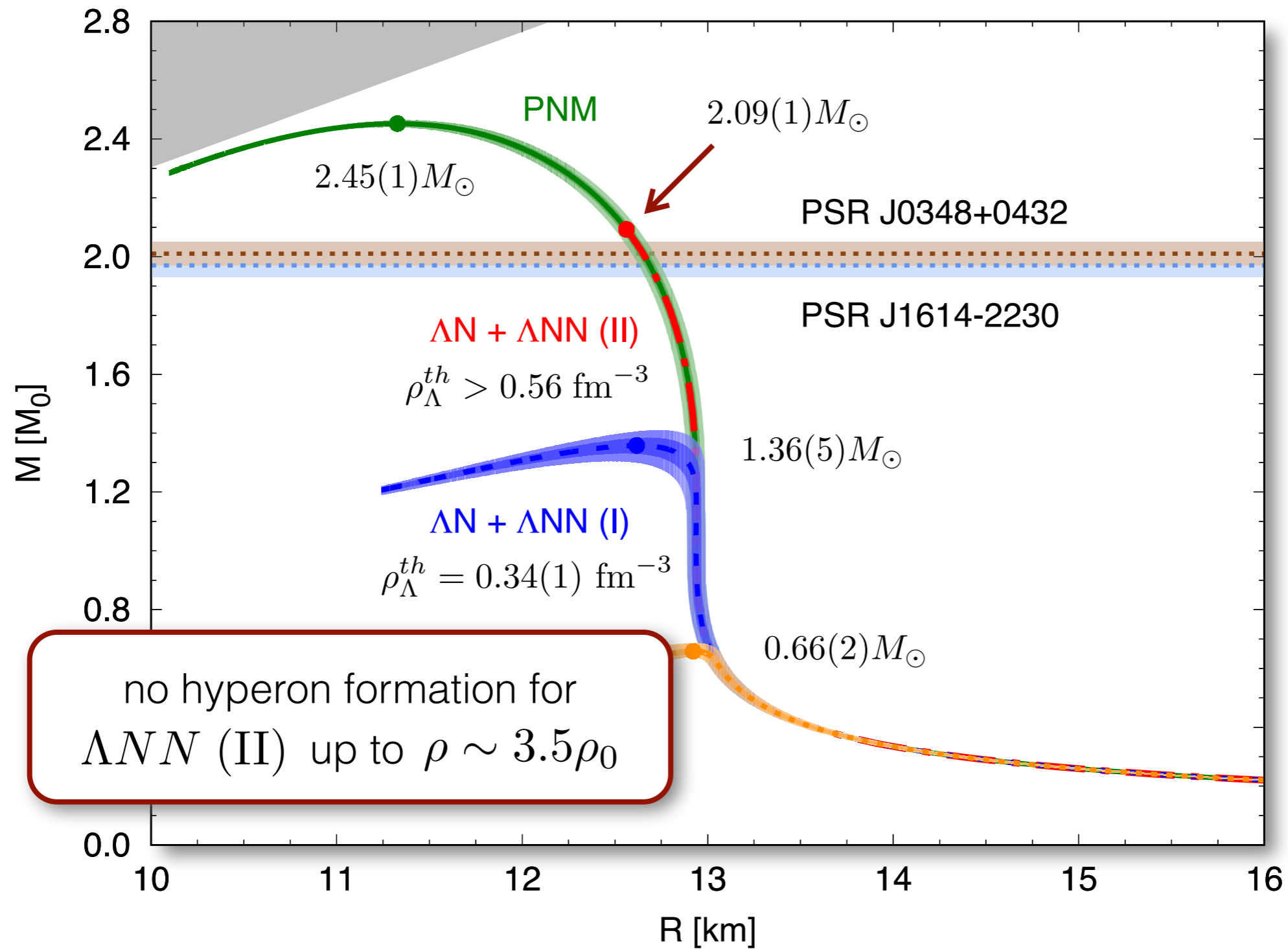
$$B_{\Lambda}^p \simeq 10.7 \text{ MeV}$$

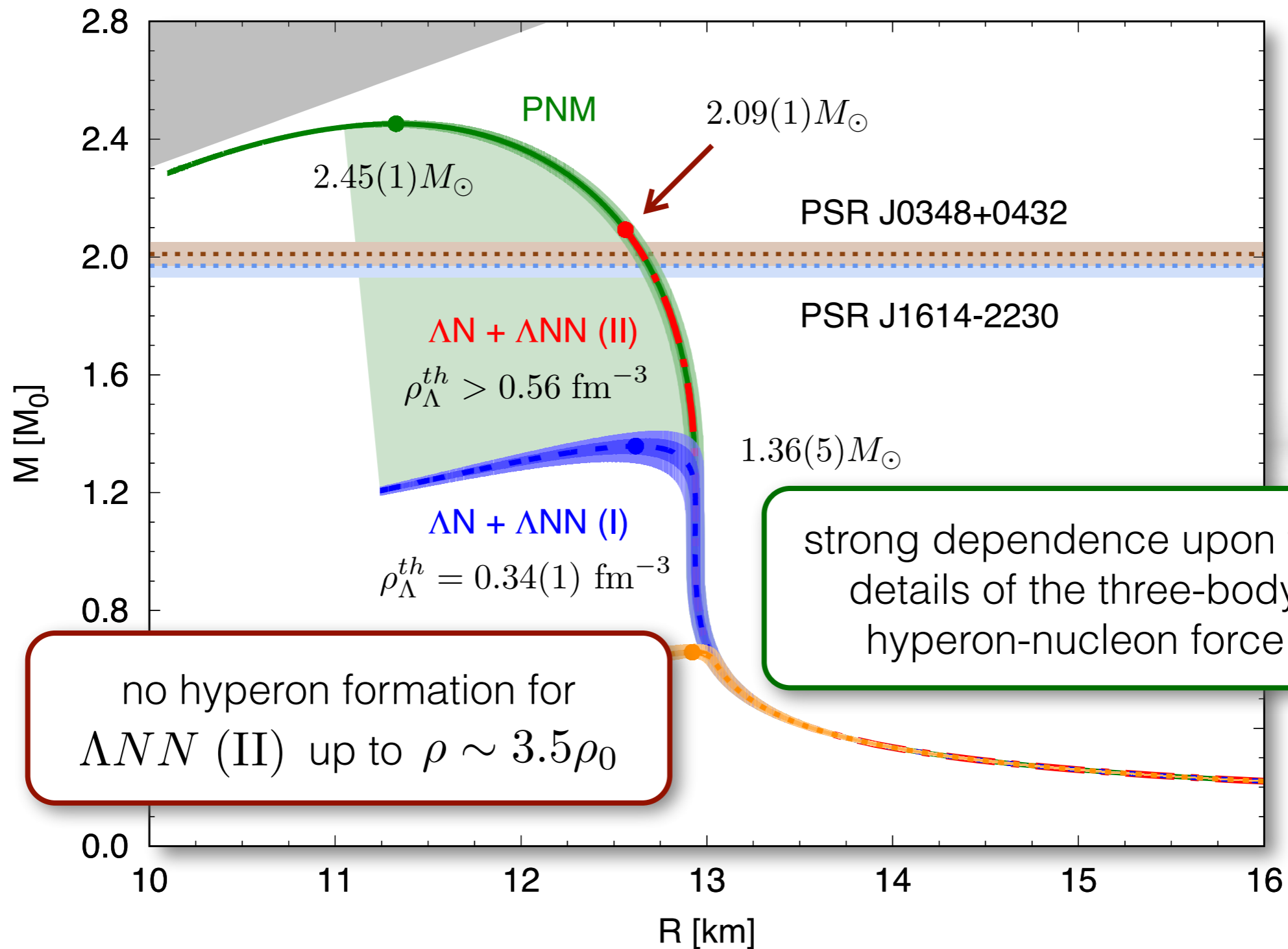
$$B_{\Lambda}^d \simeq 3.3 \text{ MeV}$$

hypernucleus	s-wave	p-wave	d-wave
$^{40}_{\Lambda}\text{K}$ AFDMC	18.63(24)	10.99(22)	3.93(26)
$^{41}_{\Lambda}\text{Ca}$ AFDMC	18.31(33)	11.46(42)	4.32(40)
$^{40}_{\Lambda}\text{Ca} (\pi^+, K^+)$	18.7(1.1)	—	—
$^{40}_{\Lambda}\text{Ca} (K^-, \pi^-)$	—	11.0(5)	1.0(5)



D. L., A. Lovato, S. Gandolfi, F. Pederiva, Phys. Rev. Lett. 114, 092301 (2015)





3-body interaction



fit on symmetric hypernuclei

ΛNN force: no dependence on
singlet or triplet nucleon isospin state

$$\tau_i \cdot \tau_j = -3 \mathcal{P}^{T=0} - \mathcal{P}^{T=1}$$

isospin projectors



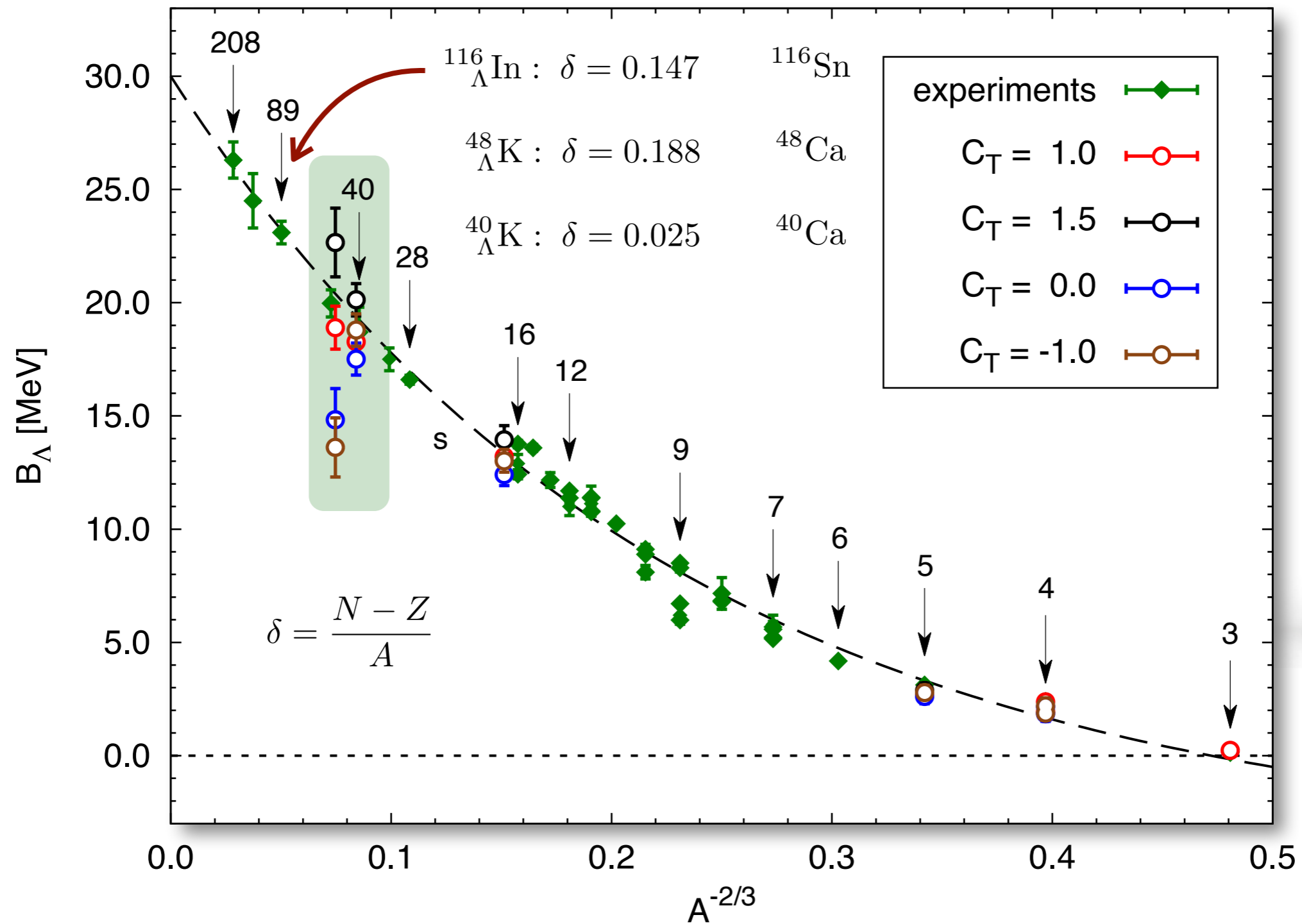
$$-3 \mathcal{P}^{T=0} + C_T \mathcal{P}^{T=1}$$

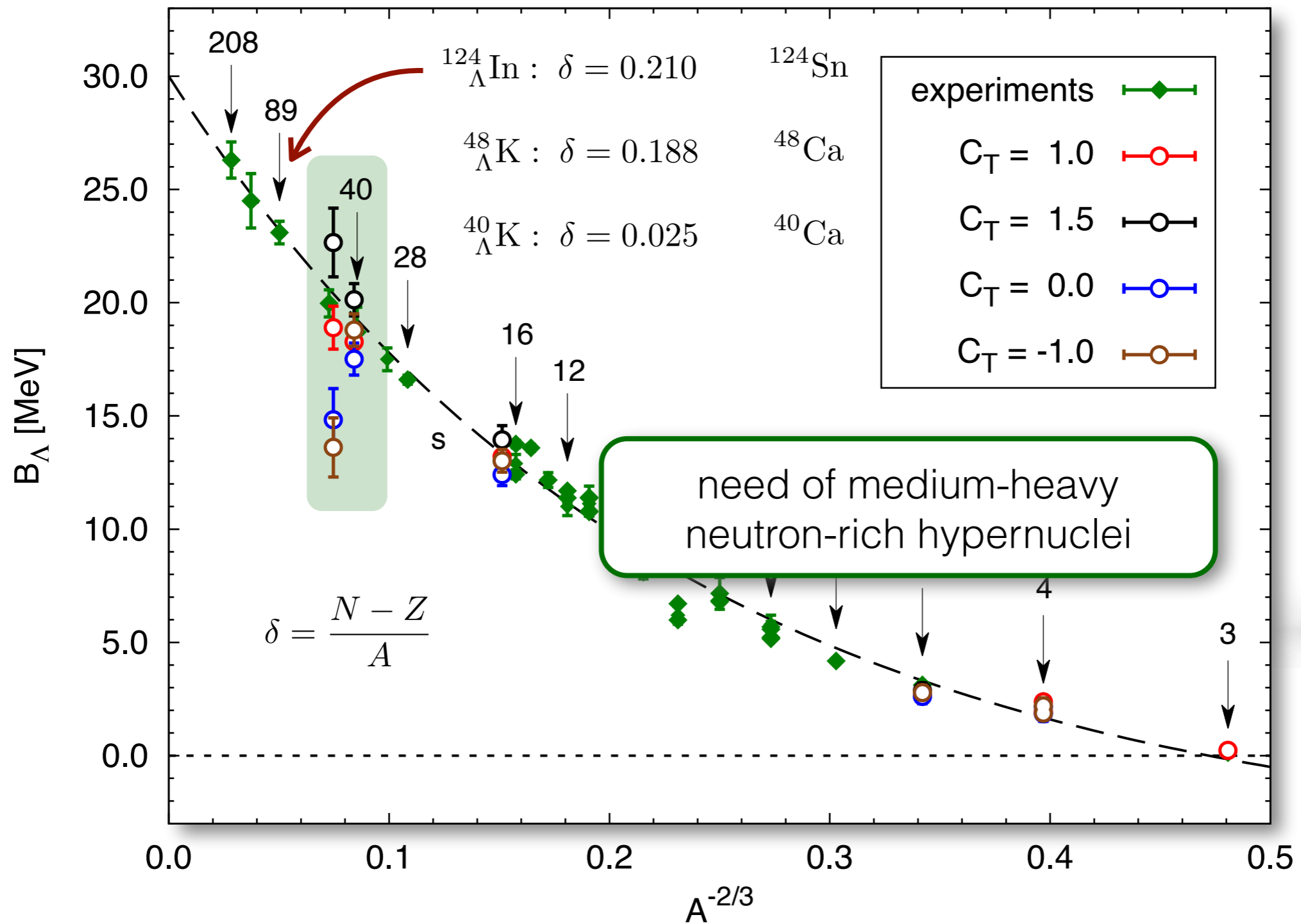


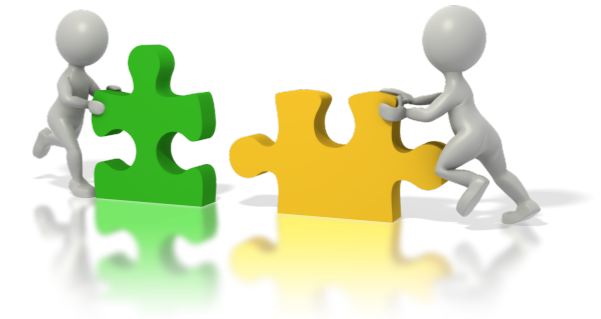
control parameter:
strength and sign of the nucleon
isospin triplet channel

sensitivity study:
light- & medium-heavy hypernuclei









✓ Present status

Substantial improvements in AFDMC calculations for strange systems

- ▶ better performance & better accuracy
- ▶ the medium-mass region ($A=40-50$) can be used to constrain the hyperon-nucleon interaction
- ▶ need of precise experimental input \longrightarrow pin down the isospin dependence of the hyperon-nucleon force

✓ What's next

- ▶ How does the experimental uncertainty of the Λ separation energy affect the prediction of the neutron star structure?
- ▶ Produce a second generation of results extending the progresses reached in AFDMC calculations for nuclei and nuclear matter to the strange sector
- ▶ Use of the hypernuclear EoS in dynamical general relativity calculations



Thank you!!