Measuring the nΛ scattering length via the nnΛ resonance

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Brief Outline

- What do we know about nΛ scattering?
- Experimental data in hand.
- The nnΛ bound state?
- Where do we stand?
- What is possible (theoretically)?
- nnΛ resonance properties

- We have measured almost nothing about nΛ scattering.
- We have inferred something from pΛ scattering plus the binding energies of few-body hypernuclei. Yet we know, in the true sense of the words, essentially nothing in terms of actual two-body scattering.
- Starting from the sparse pΛ data we have inferred a Charge Symmetry Breaking (CSB) or difference between pΛ and nΛ scattering from the A=4 mirror hypernuclei. Yet, we know **not** whether the observed A=4 CSB comes from the fundamental two-body NΛ interaction or from a possible NNΛ three-body force effect.
- The HypHI collaboration reported seeing a ${}^{3}_{\Lambda}$n bound state. With our knowledge of the nn interaction, a ${}^{3}_{\Lambda}$n bound state would provide a strong constraint on the nΛ interaction. Furthermore, JLab would be the ideal facility to obtain such data using the ^{3}H(e,e$'$K^{+})${}^{3}_{\Lambda}$n reaction.

Alas, there is no ${}^{3}_{\Lambda}$n bound state. What is possible (theoretically)?

Question: How can we model the nΛ interaction, when we have only limited data regarding pΛ scattering?

Few-body hypernuclei:

- Λ hypernuclei provide weak constraints
  - ${}^{3}_{\Lambda}$H is weakly bound [$B_\Lambda({}^{3}_{\Lambda}\mathrm{H} = 0.13 \pm 0.05$ MeV]; small separation energy implies that it is one of the largest halo nuclei.
  - The A=4 isodoublet seems to exhibit significant Charge Symmetry Breaking, some 2-3 times that in the ^{3}H-^{3}He isodoublet.
  - The uncertainty in the pΛ data implies a potentially wide range of variation in the nΛ interaction.
- Recent experiments have decreased the apparent size of the A=4 CSB.
  - Esser *et al.*, PRL **114**, 232501 (2015) report a value for $\mathrm{B}_\Lambda({}^{4}_{\Lambda}\mathrm{H})$ of 2.12 ± 0.01(stat.) ± 0.09(syst.).
  - Yamamoto *et al.*, PRL **115**, 222501 (2015) have measured the gamma decay in ${}^{4}_{\Lambda}$H and obtained a value of 1.406 ± 0.02 ± 0.02 MeV, indicating that CSB in the 1^+ excited states is quite small.

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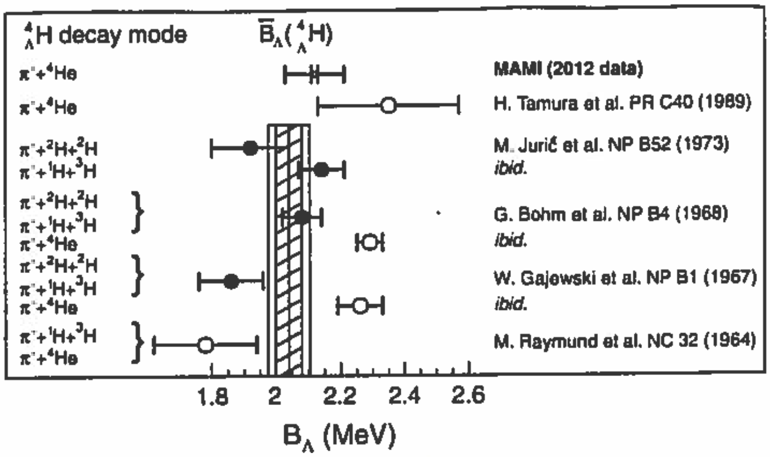
PRL **114,** 232501 (2015)



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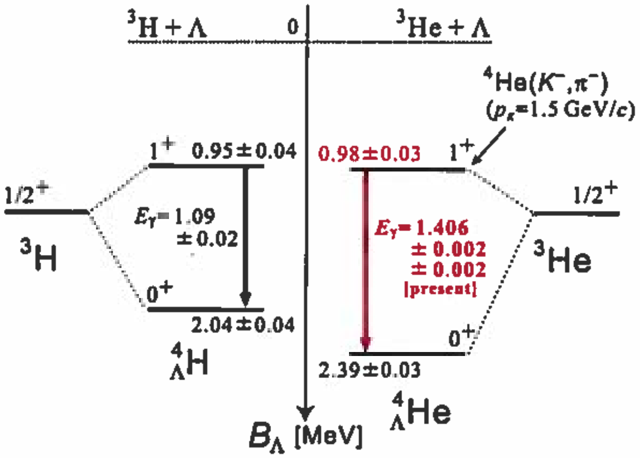

FIG. 1 (color online). Level schemes of the mirror hypernuclei, $^4_\Lambda$H and $^4_\Lambda$He. Λ binding energies (B_Λ) of $^4_\Lambda$H(0^+) and $^4_\Lambda$He(0^+) are taken from past emulsion experiments [2]. $B_\Lambda(^4_\Lambda\mathrm{He}(1^+))$ and $B_\Lambda(^4_\Lambda\mathrm{H}(1^+))$ are obtained using the present data and past γ-ray data [6–8], respectively. Recently, $B_\Lambda(^4_\Lambda\mathrm{H}(0^+)) = 2.12 \pm 0.01(\mathrm{stat}) \pm 0.09(\mathrm{syst})$ MeV was obtained with an independent technique [5].

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- Nevertheless, we do not understand CSB in the mirror Λ hypernuclei. Is it NΛ or NNΛ CSB? Therefore, we cannot accurately model the nΛ interaction.

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A ${}^{3}_{\Lambda}$n hypernucleus?

- The HypHI collaboration reported a bound nnΛ system (${}^{3}_{\Lambda}$n).
  - C. Rappold *et al.*, Phys. Rev. C **88**, 041001(R) (2013).
  - They observed both two-body and three-body decay modes.
  - ${}^{3}_{\Lambda}$n would be the lightest neutron-rich hypernucleus observed.
  - Such a bound state would provide a significant constraint on the nΛ interaction, because the nn interaction is well known.
  - Such a bound state could be observed directly in a ^{3}H(e,e$'$K^{+})${}^{3}_{\Lambda}$n experiment at JLab, although a weakly bound system would imply a small cross section.
  - Alternative reactions at J-PARC would be ${}^{3}\mathrm{H}(\mathrm{K}^{-},\pi^{0}){}^{3}_{\Lambda}\mathrm{n}$ and ${}^{3}\mathrm{He}(\mathrm{K}^{-},\pi^{+}){}^{3}_{\Lambda}\mathrm{n}$. The latter, being a double-charge-exchange reaction, suggests a very small cross section.

A ${}^{3}_{\Lambda}$n hypernucleus?

A ${}^{3}_{\Lambda}$n bound state has been strongly questioned:

- H. Garcilazo and A. Valcarce, Phys. Rev. C **89** 057001 (2014).
- E. Hiyama *et al.*, Phys. Rev. C **89** 061302 (2014).
- A. Gal and H. Garcilazo, Phys. Lett. **B736**, 93 (2014).

Simple physics suggests that one would not expect a bound state.

- The hypertriton is barely bound and its core is a deuteron.
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Could there instead exist an nnΛ three-body resonance?

If so, then one could still utilize the JLab electro-production reaction (or perhaps the J-PARC strangeness-exchange reaction) to constrain the nΛ interaction.

Where do we stand?

The Science

- *No published* nΛ data exist **!** Our numerous NΛ potential models have never been tested against nΛ data.
- Were a bound ${}^{3}_{\Lambda}$n hypernucleus to exist, our knowledge of the nn interaction would permit us to significantly constrain the low-energy properties of the nΛ system. *Existence of such a bound state has been ruled out theoretically* **!**
- However, either a strong sub-threshold resonance or an actual physical resonance in the nnΛ system should exist. The resonance position and the shape of the spectrum in a K^+ electro-production measurement from a tritium target at JLab leading to an nnΛ final state would provide a significant constraint upon the low-energy properties of the heretofore unmeasured nΛ interaction.

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The Impact

- Understanding Λ hypernuclei (ground states and single-particle spectra) measured at facilities around the world (JLab, RHIC, Mainz, J-PARC, DaΦne) requires knowledge of the nΛ interaction. Scattering data exist only for pΛ.
- Data on an nnΛ (sub-threshold or physical) resonance would enhance our knowledge of the critical low-energy properties of the nΛ interaction.
- Such nΛ interaction knowledge would elucidate the important Charge Symmetry Breaking relative to the measured pΛ interaction, provide a realistic basis for understanding the long existing Λ hypernuclei data, enhance our calculations of neutron-rich hypernuclei, and constrain our modeling of neutron stars.

Our nnΛ Three-Body Model

- pairwise s-wave interactions of rank one separable form

$$V(k,k') = g(k)Cg(k') \qquad g(k) = 1/(k^2+\beta^2)$$

- nn potential strength and range fitted to effective range parameters:

a_{nn} = -18.9 ± 0.4 fm and r_{nn} = 2.75 ± 0.11 fm

- $n\Lambda$ strength and range fitted to the Nijmegen model D values:

a_s = -2.03 ± 0.32 fm and r_s = 3.66 ± 0.32 fm
a_t = -1.84 ± 0.10 fm and r_t = 3.32 ± 0.11 fm

M. M. Nagels, T. A. Rijken, & J. J. deSwart, PRD **15**, 2547 (1977)

- Separable potentials allow us to simply analytically continue onto the second sheet of the energy plane.
- We search for the resonance poles by examining the eigenvalue specturm of the kernel of the Faddeev equations for the $nn\Lambda$ system
- We previously used such a technique to explore $\Lambda - d$ scattering:

I. R. Afnan and B. F. Gibson, PRC **47**, 1000 (1993).

Searching for Resonances in the nnΛ System

We must analytically continue the Faddeev equations onto the second energy sheet.

- For two identical Fermions interacting via Yamaguchi pairwise potentials, the homogeneous integral equation is of the form

$$\lambda_n(E)\,\phi_{n,k_\alpha}(q,E) = \sum_{k_\beta} \int_0^\infty dq'\; K^{JT}_{k_\alpha,k_\beta}(q,q';E)\;\phi_{n;k_\beta}(q',E)\;, \quad (1)$$

  where the kernel of the integral equation is given by

$$K^{JT}_{k_\alpha,k_\beta}(q,q';E) = Z^{JT}_{k_\alpha,k_\beta}(q,q';E)\;\tau_{k_\beta}[E-\epsilon_\beta(q')]\;q'^2\;. \quad (2)$$

- We analytically continue onto the second energy sheet by utilizing the transformation

$$q \to q\,e^{-i\theta} \qquad q' \to q'\,e^{-i\theta} \qquad \text{with} \qquad \theta > 0\;. \quad (3)$$

- One limitation on the rotation angle θ is imposed by singularities of the kernel; the Born amplitude $Z^{JT}_{k_\alpha,k_\beta}$ requires that $\theta < \frac{\pi}{2}$, which gives us the region $\Im(E) < 0$ on the second Riemann sheet. The other source of singularity is the quasi-particle propagator $\tau_{k_\beta}[E-\epsilon_\beta(q')]$, but because there are no two-body bound states, this does not limit the rotation.

Results of the Eigenvalue Search

Let us consider a specific example: we utilize the nn and the ${}^1S_0$ and ${}^3S_1$ nΛ potentials defined previously.

- nn potential strength and range fitted to effective range parameters:

  $$a_{nn} = -18.9 \pm 0.4 \text{ fm and } r_{nn} = 2.75 \pm 0.11 \text{ fm}$$

- nΛ strength and range fitted to the Nijmegen model D values:

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  $$a_t = -1.84 \pm 0.10 \text{ fm and } r_t = 3.32 \pm 0.11 \text{ fm}$$

- We searched in the complex energy plane for the largest eigenvalue of the kernel of **1** and found a pole at:

  $$E = -0.154 - 0.753\, i \text{ MeV} \quad \text{with eigenvalue} \quad \lambda(E) = 1.0000 - 0.0001\, i\,.$$

- Because $\Re(E) < 0$, this pole corresponds to a sub-threshold resonance, one that lies below the breakup threshold in a region inaccessible by experiment.

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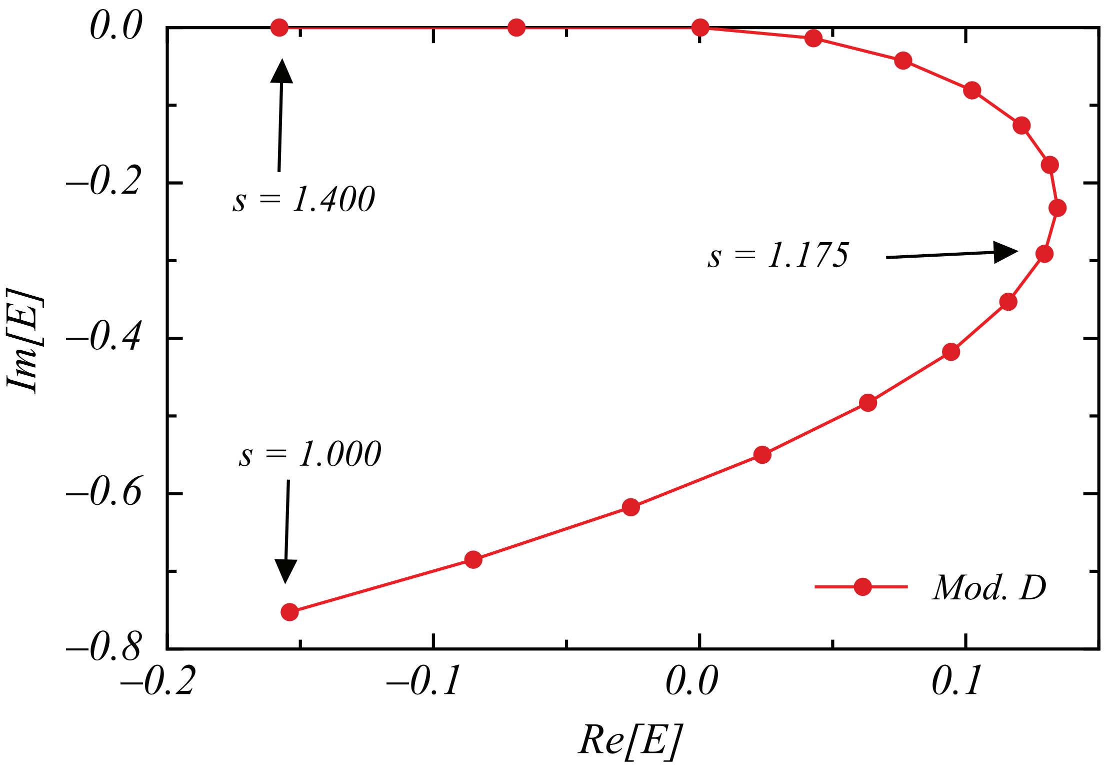
- Because $\Re(E) < 0$, this pole corresponds to a sub threshold resonance, one that lies below the breakup threshold in a region inaccessible to experiment.

Because the pole lies just below the breakup threshold, we may ask how easy it might be to convert the pole into an observable resonance or even a bound state.

- We scale the strength of the ${}^1\mathrm{S}_0$ and ${}^3\mathrm{S}_1$ nΛ potentials by the factor s.
- We follow the path of the pole as it turns into a "resonance" and then into a bound state.
- We observe that a change in strength of ∼7% produces a resonance above the three-body breakup threshold.
- A change of 35% is required to produce a ${}^3_\Lambda$n bound state.

Trajectory of the nnΛ "Resonance" Pole

In the figure one follows the trajectory of the "resonance" pole as the strength s of the nΛ interaction is increased from a value of 1.0 in increments (Δs) of 0.025. One starts from a sub-threshold resonance at values of $s = 1.000$ up to $s = 1.050$. For $s = 1.075$ up to $s = 1.350$ we obtain a resonance; in particular, we obtain a resonance with $E = 0.129 - 0.291i$ MeV at $s = 1.175$. As s is further increased, we obtain a bound state with energy $E = -0.069$ MeV at $s = 1.375$ and $E = -0.158$ at $s = 1.400$. Thus, for this particular model one can see that an nΛ potential whose parameters lie within the uncertainty of the observed low energy pΛ scattering parameters could produce a resonance in the nnΛ system.

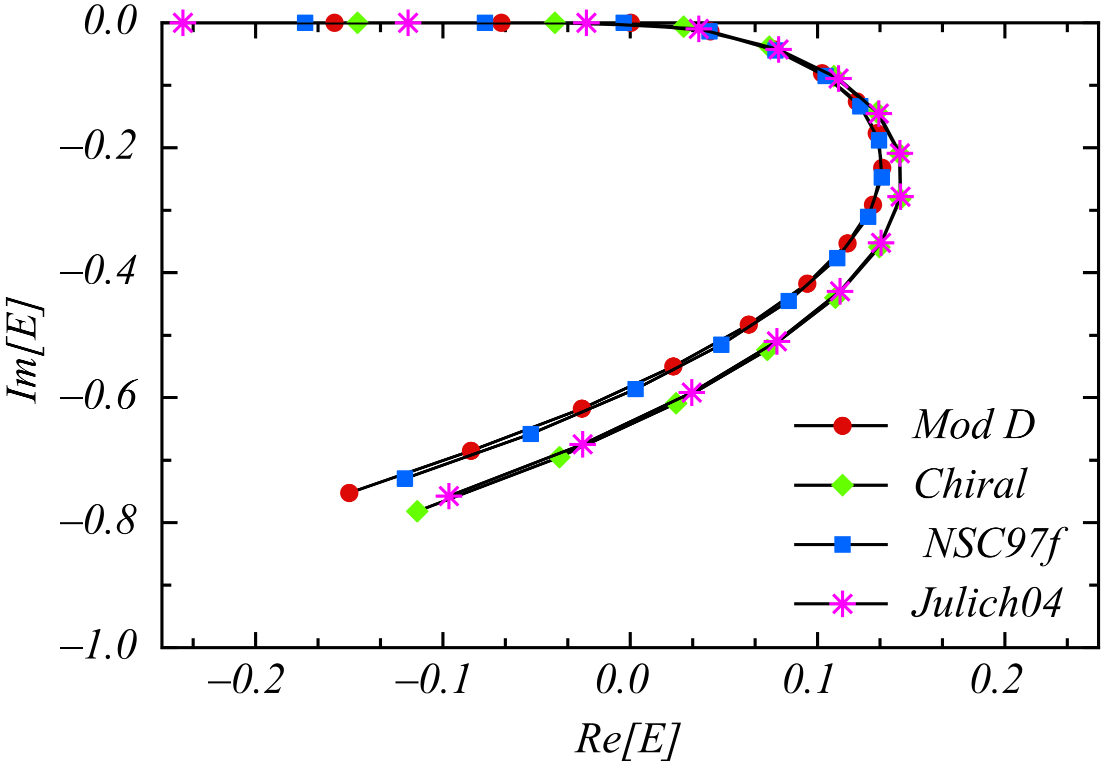


Trajectory of the nnΛ "Resonance" Pole for Four Contemporary Models

To explore the sensitivity to various NΛ potential models

- M. M. Nagels, T. A. Rijken, and J. J. de Swart, "Baryon-baryon scattering in a one-boson-exchange-potential approach, II. Hyperon-nucleon scattering", Phys, Rev. D **15**, 2547 (1977). (*ModD*)
- T. A. Rijken, V. G. J. Stoks, Y. Yamamoto, "Soft-core hyperon-nucleon potentials", Phys. Rev. C **59**, 21 (1999). (*NSC*97*f*)
- J. Haidenbauer and U.-G. Meißner, "Jülich hyperon-nucleon model revisited", Phys. Rev. C **72**, 044005 (2005). (*Jülich*04)
- J. Haidenbauer, *et al.*, "Hyperon-nucleon interaction at next-to-leading order in chiral effective field theory", Nucl. Phys. A **915**, 24 (2013). (*Chiral*)

we repeated the eigenvalue search for each of the potential models. None of the four models produces an observable resonance; each produces a sub-threshold resonance, which lies below the nnΛ threshold. A change in s of as little as 5% produces a resonance above the three-body threshold, but a change of at least 25% is required to produce a bound ${}^{3}_{\Lambda}$n.



What we can do

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