Hyperon Studies at JPAC

Who we are and what we do

General approach: Role of reaction theory

Hyperon Studies

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http://www.indiana.edu/~jpac/index.html
There may be hadrons that look like …

dibaryon

pentaquark

 glueball

diquark + di-antiquark
dimeson molecule

$q\bar{q}g$ hybrid

…but before we know this it is necessary to identify resonances
...we need to know how to interpret “peaks”

\[ \Lambda_b \rightarrow K^- pJ/\psi \]

a resonance in \( pJ/\psi \) ?

... or a \( \bar{K} p \) reflection ?
S-matrix principles: Crossing, Analyticity, Unitarity

Crossing

\[ A(s, t) = \sum_l A_l(s) P_l(z_s) \]

Analyticity

\[ A_l(s) = \lim_{\epsilon \to 0} A_l(s + i\epsilon) \]

bumps/peaks on the real axis (experiment) come from singularities in unphysical sheets

These singularities come from QCD
Amplitude Analysis @ JPAC

Events, X-sections, MC

QCD Predictions

Amplitude analysis: based on S-matrix principles:
- analyticity
- unitarity
- crossing

Global effort
JLab/IU/GWU Physics Analysis Center

$A(s, t) = \sum_{l=0}^{\infty} f_l(s) P_l(z_s) = \sum_{l=0}^{\infty} g_l(t) P_l(z_t)$
QCD on the Lattice: simulated scattering experiment

\[ Z(E_i = \text{"data"}) = T(E_i) \]

\( E_i \) = discrete energy spectrum of states in the lattice

(known kinematical function) (infinite volume amplitude)

in general "solution" of the Lusher condition requires an analytical model for \( T \)

D. Wilson et. al.
**JPAC : Example of Analysis Projects**

Light meson decays and light quark resonance

\[ \omega/\phi \to 3\pi, \pi\gamma \ (\text{dispersive}) \]
\[ \omega \to 3\pi \ (\text{Veneziano, B4}) \]
\[ \eta \to 3\pi, \eta'/f1 \to \eta\pi\pi, \ (\text{Khuri-Treiman, B4}) \]
\[ J/\Psi \to \gamma\pi0\pi0 \]

Photo-production: (production models, FESR and duality)

\[ \gamma p \to \pi0p \]
\[ \gamma p \to pK+K- \ (\text{and } Kp) \]
\[ \gamma p \to \pi+\pi-\rho, \pi0\eta\rho, \omega\rho \]

Exotica and XYZ’s: Launched in the Fall of 2013

\[ \pi-\rho \to \pi-\eta\rho \ & \pi-\rho \to \pi-\eta'\rho \ (\text{FESR}) \]
\[ B^0 \to \Psi' \pi^- K^+ u, \Psi(4260) \to J/\Psi \pi+\pi-, \Lambda_b \to K^- pJ/\Psi \]
\[ J/\Psi \to 3\pi, KK\pi \ (\text{Veneziano, B4}) \]

>20 analysis/papers published
Adam Szczepaniak (IU/JLab)
Mike Pennington (JLab)
Tim Londergan (IU)
Geoffrey Fox (IU)
Emilie Passemard (IU/JLab)
Cesar Fernandez-Ramirez (Jlab → Mexico)
Vincent Mathieu (IU)
Micheal Doering (GWU)
Ron Workman (GWU)

BESIII collaboration
Medina Ablikim (Beijing)
Ryan Mitchell, (IU)

LHCb collaboration
T.Skwarnicki (Syracuse)
J.Rademacker, (Bristol)

http://www.indiana.edu/~jpac/

GlueX collaboration
Matthew Shepherd (IU)
Justin Stevens (JLab)

CLAS collaboration
Diane Schott (GWU/JLab)
Viktor Mokeev (JLab)

HASPECT
Marco Battaglieri (Genova)
Derek Glazier (Glasgow)
Raffaella De Vita (Genoa)

COMPASS collaboration
Mikhail Mikhasenko (Bonn)
Fabian Krinner (TUM)
Boris Grube (TUM)

BaBar collaboration
Antimo Palano (Bari)


Vladyslav Pauk (Mainz → JLab)
Alessandro Pilloni (Rome → JLab)
Astrid Blin (Valencia)
Andrew Jackura (IU)
Lingyun Dai (IU/JLab → Valencia)
Meng Shi (JLab → Beijing)
Igor Danilkin (JLab → Mainz)
Peng Guo (IU/JLab → CSU)

…
We present the model published in [Mat15a].

The differential cross section for $\gamma p \to \pi^0 p$ is computed with Regge amplitudes in the domain $E_t \leq 4.5 \text{ GeV}$ and $0.01 \leq |t| \leq 3 \text{ (in GeV)}$. The formula can be extrapolated outside these intervals. We use the CGLN invariant amplitudes $A_i$ defined in [Chew57a].

See the section Formalism for the definition of the variables. The fitting procedure is detailed in [Mat15a]. We report here only the main feature of the model.

Formalism

The differential cross section is a function of 2 variables. The first is the beam energy in the laboratory frame $E_b$ (in GeV) or the total energy squared $s$ (in GeV$^2$). The second is the cosine of the scattering angle in the rest frame $\cos \theta$ or the momentum transferred squared $t$ (in GeV$^2$).

The momenta of the particles are $k$ (photon), $q$ (pion), $p_2$ (target) and $p_4$ ( reco). The pion mass is $\mu$ and the proton mass is $M$. The Mandelstam variables, $s = (k + p_2)^2$, $t = (k - q)^2$, $u = (k - p_4)^2$ are related through $s + t + u = 2M^2 + \mu^2$.

The differential cross section is expressed in terms of the parity conserving helicity invariant amplitudes in the $t$ -channel $F_i$.

\[
\frac{d\sigma}{dt} = \frac{389.4}{64\pi} \frac{k_t^2}{M^2 s^2 E_b} \left[ 2s^2 t^2 \left[ t F_1^2 + 4u^2 F_2^2 \right] + (1 - c_0^2) k_t^2 \right]
\]

The differential cross section is expressed in $\mu$b/GeV$^2$. We used (hc)$^2$.

The $t$ -channel is the rest frame of the process $\gamma p \to pp$. In the $t$ -channel, the momenta of the nucleon $p_b$ and the pion $q_k$.

\[
k_t = \frac{1}{2}\sqrt{s - 4M^2}, \quad q_t = -2\sqrt{t}
\]

The invariant amplitudes $F_i$ are related through the CGLN $A_i$ amplitudes:

\[
F_1 = -A_2 + 2M A_4,
F_2 = A_1 + t A_2,
F_3 = 2MA_1 - t A_4,
F_4 = A_3
\]

The $F_i$ amplitudes have good quantum numbers of the $t$ -channel. The naturality $n = P(-1)^2$ and the product $CP$. 

special thanks to Vincent Mathieu
Bridge between light (u,d) and heavy (c,b) quark baryons

Test Quark Model vs QCD (lattice)

Photon couples to quarks is, glueballs, hybrids or use in associated production of K*'s and Hyperons

Hyperon spectrum less understood e.g. Λ(1405) only recently pole positions have started to be reported by the PDG
Some quark model states have not been seen yet

$$2^- \ (L=2, S=1)$$

<table>
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<tr>
<th>$I^G$</th>
<th>naturality $= \text{P}(-1)^J$</th>
<th>twist $= +1$ if $J=0,2,...$ $= -1$ if $J=1,3,...$</th>
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<td>+1</td>
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<td>$f_1, f_3, ...$</td>
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<tr>
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<td>+1</td>
<td>$h_0, h_2, ... \ (0^+, 2^-, ...)$</td>
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<td>-1</td>
<td>$\omega/\phi_1, \omega/\phi_3, ...$</td>
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<tr>
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<td>-1</td>
<td>+1</td>
<td>$\omega/\phi_0, \omega/\phi_2, ... \ (0^+, 2^-, ... : \text{not seen})$</td>
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<td>-1</td>
<td>$h_1, h_3, ...$</td>
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<td>+1</td>
<td>$b_0, b_2, ... \ (0^+, 2^-)$</td>
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<td>-1</td>
<td>$\rho_1, \rho_3, ...$</td>
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<tr>
<td>$1^+$</td>
<td>-1</td>
<td>+1</td>
<td>$\rho_0, \rho_2, ... \ (0^-, 2^- : \text{not seen})$</td>
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<tr>
<td>$1^+$</td>
<td>-1</td>
<td>-1</td>
<td>$b_1, b_3, ...$</td>
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<tr>
<td>$1^-$</td>
<td>+1</td>
<td>+1</td>
<td>$a_0, a_2, ...$</td>
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<tr>
<td>$1^-$</td>
<td>+1</td>
<td>-1</td>
<td>$\pi_1, \pi_3, ... \ (1^+, 3^+)$</td>
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<td>-1</td>
<td>$a_1, a_3, ...$</td>
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</table>
Analyticity is a powerful constraint

cf. Bonn/Gatchina, EBAC, Julich, Giessen,GWU, Mainz, Zagreb,)

cf. Regge phenomenology

\[ s < (2 \text{GeV})^2 \]

s-unitarity

t-channel pole (cut) develops

\[ s > (2 \text{GeV})^2 \]
Can use cross-channel reggeons to study direct channel resonances.
PWA for KN

Model the amplitude
Fit to data
Analytically continue to complex values of energy to search for poles

Partial-wave analysis (Lmax= 5), Coupled channels, Unitarity
Analyticity: Right threshold behavior (angular momentum barrier), Resonances and backgrounds are incorporated “by-hand” through K matrices

In the range 2.19<s<4.70 GeV² (8000 data points, 7500 data points, 5000 data points) We fit the KSU analysis single-energy partial waves [Zhang et al., PRC 88, 035204 (2015)]
Caveat: we lose correlations among partial waves

Cesar Fernandez Ramirez et al., arXiv:1510.07065 [hep-ph]
\[ S_\ell = I + 2i \left[ C_\ell(s) \right]^{1/2} T_\ell(s) \left[ C_\ell(s) \right]^{1/2} \]

\[ T_\ell(s) = \left[ K^{-1}(s) - i\rho_\ell(s) \right]^{-1} \]

\[ [i\rho_\ell(s)]_{kk} = \frac{s - s_k}{\pi} \int_{s_k}^{\infty} \frac{[C_\ell(s)]_{kk}}{s' - s} ds' \]

\[ k = \pi\Sigma, KN, \pi\Lambda, \pi\Sigma(1385), \pi\Lambda(1520), \eta\Sigma, \eta\Lambda, K^*N, \pi\Delta(1232), \pi\pi\Sigma, \pi\pi\Lambda \]

**Resonance**

\[ [K_a(s)]_{kj} = \frac{x_k^a}{M_a^2 - s} x_j^a \]

**Background**

\[ [K_b(s)]_{kj} = \frac{x_k^b}{M_b^2 + s} x_j^b \]

Generates pole in the 2nd Riemann sheet

Generates pole in the real axis for \( s < 0 \) in the 1st Riemann sheet
Phase Space/Analiicticity

\[ [C_\ell(s)]_{kk} = \frac{q_k(s)}{q_0} \left[ \frac{q_k^2(s)r^2}{1 + q_k^2(s)r^2} \right] ^\ell \]

- Right threshold behavior
- Angular momentum barrier
- Right high-energy behavior
- \( r = 1 \) fm (interaction radius)

\[ [q_k(s)]^2 = \frac{m_1m_2}{s_k} [s - s_k] \]

\[ [i\rho_\ell(s)]_{kk} = \frac{s - s_k}{\pi} \int_{s_k}^{\infty} \frac{[C_\ell(s')]_{kk}}{s' - s} \frac{ds'}{s' - s_k} = -a_0 a^{\ell} \frac{\Gamma(\ell)}{\pi \Gamma(\ell)} \left[ \frac{\Gamma(\ell)(s - s_k)\sqrt{s_k - s}}{1 + a(s - s_k)} \right] \]

- Valid for \( \ell \) real and bigger than -1/2
Partial Waves
Partial Waves
Pole Positions
Resonances as Regge Poles

near the resonance pole

\[ T_l \sim \frac{1}{\alpha'(m_l^2 - s)} = \frac{1}{l - (l - \alpha'_m l^2 + \alpha's)} \]

if \( l = \alpha_0 + \alpha'_m l^2 \) than \( T_l \sim \frac{1}{l - \alpha(s)} \) with

\[ \alpha(s) = \alpha_0 + \alpha's \]

In general \( T = T(l, s) \) and a pole corresponds to a trajectory in the l,s space

A pole in s at a fixed integer l is connected to another pole at a different integer l
(3 *) Σ(1940) nobody gets it, but there is a gap in Ragge trajectory.
On the nature of $\Lambda(1405)$

- Puzzle since the 60’s
- Quantum numbers those of a uds state
- Constituent quark models fail to reproduce the mass
  - 1550 MeV [Capstick, Isgur, PRD 34, 2809 (1986)]
  - 1524 MeV [Löring, Metsch, Petry, EPJA 10, 447 (2001)]
- Amplitude analysis of KN scattering and $\pi\Sigma K^+$ data finds two poles [Mai, Meiβner, EPJA 51, 30 (2015)]
  - 1429-12i MeV
  - 1325-90i MeV
- Lattice says: KN molecule [Hall et al., PRL 114, 132002 (2015)]
- Lattice says: three-quark state [Engel et al., PRD 87, 034502 (2013); PRD 87, 074504 (2013)]
- Quark-diquark models obtain one $\Lambda(1405)$ with the right energy
  - 1430 MeV [Santopinto, Ferretti, PRC 92, 025202 (2015)]
  - 1406 MeV [Faustov, Galkin, PRD 92, 054005 (2015)]
\( \Lambda(1405) \)
A new non-ordinary baryon resonance. In the following states is a non-ordinary state, making the trajectories nearly degenerate. However, the different can be obtained by fitting the Regge trajectory of ordinary, different. It is motivated by the relation between the imaginary part of the Regge trajectory and the width constants. It can be expected that the procedure is repeated, each time obtaining a new resonance. The results are summarized in Table 1.

![Diagram](image1.png)

**FIG. 1.** (color online). Chew–Frautschi plot for the leading Λ and Σ Regge trajectories. Dashed lines are displayed to guide the eye.

![Diagram](image2.png)

**FIG. 2.** (color online). Projections of the leading Λ and Σ Regge trajectories onto the (–Im(s_p), J) plane. Dashed lines are displayed to guide the eye.
Compare fits $0^{-}_a$, $0^{-}_b$, $0^{-}_c$

$\Lambda_a(1405) = 1429 - 12i \text{ MeV}$

$\Lambda_b(1405) = 1352 - 90i \text{ MeV}$

$\Lambda_a(1405)$ is closer to the “normal” trajectory
New, analytical model for hyperon spectrum

Need to incorporate Regge constraints
  • in direct channel as a constraint on, eg, K-matrix matrix poles
  • in cross channels, as constrained on p.w. extraction,

Λ(1405): One more piece to the puzzle (more confusion?)
TABLE II. Summary of $\Lambda^*$ pole masses ($M_p = \text{Re} \sqrt{s_p}$) and widths ($\Gamma_p = -2 \text{Im} \sqrt{s_p}$) in MeV. Our poles are depicted in Fig. 5 unless they have a very large imaginary part. In [2] the $\Lambda(1520)$ pole was obtained at ($M_p = 1518.8$, $\Gamma_p = 17.2$). Ref. [5] implements two models labeled as KA and KB (see text). $I$ stands for isospin, $\eta$ for naturality, $J$ for total angular momentum, $P$ for parity, and $\ell$ for orbital angular momentum. For baryons, $\eta = +$, natural parity, if $P = (-1)^{J-1/2}$ and $\eta = -$, unnatural parity, if $P = (-1)^{J-1/2}$ where $P$ stands for parity. Resonances marked with $\dagger$ are unreliable themselves due to systematics and lack of good-quality $\chi^2$/dof. Resonances marked with $\ddagger$ are most likely artifacts of the fits.

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<td></td>
<td>$M_p$ ($\pm$)</td>
<td>$\Gamma_p$ ($\pm$)</td>
<td>$M_p$</td>
<td>$\Gamma_p$</td>
<td>$M_p$</td>
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<td>$0^- \frac{1}{2}^- S$</td>
<td>1435.8 $\pm$ 5.9 $\dagger$</td>
<td>279 $\pm$ 16</td>
<td>1402</td>
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<td>1573 $\ddagger$</td>
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<td>1685 $\pm$ 29 $\dagger$</td>
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<td>1835 $\pm$ 10 $\ddagger$</td>
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<td>1837.2 $\pm$ 3.4 $\dagger$</td>
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<td>1846.36 $\pm$ 0.81</td>
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<td>$0^+ \frac{3}{2}^- D$</td>
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<td>2133 $\pm$ 120 $\ddagger$</td>
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<td>1970</td>
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<td>2199 $\pm$ 52</td>
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<td>$0^+ \frac{5}{2}^+ F$</td>
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<td>$0^- \frac{7}{2}^+ F$</td>
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TABLE III. Summary of $\Sigma^*$ pole masses ($M_p = \text{Re}\sqrt{s_p}$) and widths ($\Gamma_p = -2\text{Im}\sqrt{s_p}$) in MeV. Our poles are depicted in Fig. 5 unless they have a very large imaginary part. Notation is the same as in Table II. Resonances marked with † are unreliable themselves due to systematics and lack of good-quality $\chi^2/dof$.

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<td>1501</td>
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<td>1551</td>
<td>376</td>
<td>$\Sigma(1620)$</td>
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<td>1708</td>
<td>158</td>
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<td>86</td>
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<td>$\Sigma(1750)$</td>
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<td>$1813 \pm 32$†</td>
<td>$227 \pm 43$</td>
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<td>$1990.8 \pm 4.3$†</td>
<td>$173.1 \pm 5.4$</td>
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<td>1940</td>
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