Reducing ambiguity of antikaon-nucleon amplitude using modern experimental data

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What is $\Lambda(1405)$ made of?

- **Quark model**
  - genuine $qqq$ state
  - or even more exotic: hybrids, active glue, ...
  
  *Capstick, Isgur (1986)*

- **Dynamically generated from coupled-channel effects**
  - K-matrix
  - unitarized coupled-channel amplitude from ChPT
    - two pole solution
    - many (confirming) works followed
    - accepted by PDG in 2015!
  
  *Dalitz, Tuan (1960!)*
  
  *Kaiser, Siegel, Weise (1995)*
  
  *Oller, Meißner (2001)*

- **Lattice QCD**
  - $32^2 \times 64$ full-QCD ensembles
  - Magnetic form factor of s-quark vanishes
    - $\Lambda(1405)$ is dominated by a molecular $\bar{K}N$ state
  
  *Hall et al. (2014)*
Experimental situation

- **Total cross sections on** $K^- p \rightarrow K^- p, \bar{K}^0 n, ...$
  - various bubble chamber experiments
    - LNL Berkeley (1960s), Rutherford Laboratory (1981s), ...
  - huge error bars
  - large deviations btw. experiments
    - weak constraints on $\bar{K}N$ amplitude

- **$\pi \Sigma$ mass distribution**
  - 2m bubble chamber @ CERN
  - multistep production
  - low energy resolution
    - not very restictive

![Bubble chamber @ Lawrence Radiation Laboratory](image)

![Hemingway (1985)](image)
Experimental situation

- **Threshold amplitudes**
  - $\bar{K}H$ strong energy shift and width in the SIDDHARTA exp.  
    $\Rightarrow a_{\bar{K}-p}$ from the Deser-type formula  
    $\Rightarrow A_{\bar{K}d}$ from the Deser-type formula  
    $\Rightarrow a_1, a_0$ from Faddeev equations/ (Static Approximation + Recoil Corrections)  
    DAΦNE (????), J-PARC (????)

- **$pp$ collisions**
  - high quality data  
  - theoretical analysis very intricate

- **$\pi\Sigma$ mass distribution**
  - electro- and photoproduction: $\gamma p \rightarrow (K^+)\Lambda(1405) \rightarrow \pi\Sigma$  
  - $J^P = \frac{1}{2}^-$ “confirmed” experimentally  
  - high statistics and good angular resolution  
    $\Rightarrow$ new contraints on $\bar{K}N$ amplitude (?)  
I. Meson-baryon scattering
General framework

- ChPT is an appropriate tool to study low-energy hadronic interactions.
  
  **Here it has to fail! Because:**
  
  1. Kaon mass is large → convergence
  2. Relevant thresholds are widely separated → convergence
  3. Resonance just below $\bar{K}N$ threshold → non-perturbative effect

- Non-perturbative methods:
  
  → Dispersion relations, $N/D$, Roy-Steiner equations
  → K-Matrix, JÜLICH-BONN model, ...
  → IAM, *Chiral Unitary Models*, ...

- **Chiral Unitary Models** - driving term

  \[
  V(q_2, q_1; p) = A_{WT}(q_1 + q_2) + \text{Born}(s) + \text{Born}(u) \\
  + A_{14}(q_1 \cdot q_2) + A_{57}[q_1, q_2] + A_M + A_{811}\left(q_2(q_1 \cdot p) + q_1(q_2 \cdot p)\right)
  \]

  ⇒ \(A_{\cdot}\) depend on low energy constants ⇒ free parameters
Resummation

- Bethe-Salpeter equation

\[ T(\not{q}_2, \not{q}_1; p) = V(\not{q}_2, \not{q}_1; p) + i \int \frac{d^d l}{(2\pi)^d} \frac{V(\not{q}_2, l; p) T(l, \not{q}_1; p)}{((\not{q} - l) - m + i\epsilon)(l^2 - M^2 + i\epsilon)} \]

→ Intermediate particles are off-shell
  ⇒ exactly corresponding to a series of Feynman loop diagrams

⇒ BSE can be solved analytically, if \( V \sim \) local terms
  ⇒ drop the Born graphs

→ Loop integrals → Passarino-Veltman reduction → dim. reg.

→ Bubble chain in \( s \) direction → topologies are missing
  ⇒ scale dependence does not cancel out
  ⇒ additional model parameters
Fits and results

- Off-shell effects are moderate
  ⇒ for an efficient scan of parameter space (20 dim.!) use on-shell approximation → performance × 30
  ⇒ later gradually turn on the off-shell effects

- Fit strategy
  → Data: threshold amplitudes, cross sections - 155 data points
  → Randomly chosen sets of starting values (# ≈ 10000)
  → Solutions having poles on I. RS sorted out

- Results: 8 best fits obtained
  → similar \( \chi^2_{d.o.f.} \)

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<tr>
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<td>( \chi^2_{d.o.f.} )</td>
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<td>1.14</td>
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<td>1.06</td>
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MM, Meißner (2013)
Fits and results

- Results: 8 best fits obtained

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→ similar threshold ratios

→ error bars are twofold
  1. parameter: variation of best fit parameters, such that $\Delta \chi^2_{d.o.f.} < 1.15$
  2. systematic: spread of solutions
Results

→ similar cross sections
Results - complex plane

- Analytic continuation to the complex energy plane

→ two poles in all solutions on II. RS
→ stable position of the narrow pole
→ position of the second pole is rather unstable
II. CLAS data on $\gamma p \rightarrow K^{+}\pi\Sigma$
Framework

Data

- $\Lambda(1405)$ lineshape from double meson photoproduction JLAB
  - 9 energy bins
  - 60 values of $M_{\pi\Sigma}$ - 5 MeV resolution
  - three channels: $\pi^+\Sigma^-$, $\pi^-\Sigma^+$, $\pi^0\Sigma^0$

Photoproduction amplitude

I. Gauge invariant approaches

1. Turtle approximation
   - attach photon everywhere to off-shell hadronic amplitude
     Gross, Riska (1987), Kvinikhidze, Blankleider (1999) and Borasoy et al. (2005)
   - single meson case is done for the NLO-kernel
   - double meson case is tedious ... work in progress

2. Gauged vertices
   - photon attached to meson production amplitude at the tree level
   - unitary meson-baryon amplitude as a FSI
   - done for LO driving term: Nakamura, Jido (2014)
     ⇒ no good fit to CLAS data
     ⇒ good fit with additional vector meson d.o.f. - 15 per energy bin!
II Test model

- most simple ansatz to test the hadronic solution:

\[ M_j(W, M_{\pi \Sigma}) = C_i(W) \cdot G_i(M_{\pi \Sigma}) \cdot T_{i \rightarrow j}^{on}(M_{\pi \Sigma}) \]

- flexible enough for the CLAS data
  \( \Rightarrow \) less free parameters (15 \( \mapsto \) 10)
- no gauge invariance, parameters are not physical
  \( \Rightarrow \) global fit is meaningless
  \( \Rightarrow \) *no access* to microscopic features of the spectrum
  \( \Rightarrow \) **conservative** test of the hadronic solutions

*Oset, Roca (2013)*
Results

- Test of hadronic solutions

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<td>$\chi^2_{d.o.f.}$ (hadr.)</td>
<td>1.35</td>
<td>1.14</td>
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<td>1.06</td>
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</tr>
<tr>
<td>$\chi^2_{p.p.}$ (CLAS)</td>
<td>3.18</td>
<td>1.94</td>
<td>2.56</td>
<td>1.77</td>
<td>1.90</td>
<td>6.11</td>
<td>2.93</td>
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- Hadronic fits #2, #4 and #5 lead to *good* fits

- Hadronic fits #1, #3, #6, #7 and #8 do not!!!
⇒ after comparison with Hemingway data \((K^- p \rightarrow \Sigma^+ \pi^- \pi^+ \pi^-)\) two solutions remain: #2 and #4

⇒ both solutions have similar pole positions

... also similar to the estimation by Oset and Roca (2013)

⇒ universal feature demanded by CLAS data!
III. New scattering data???
Pseudo scattering data

- What is the desired accuracy on $\sigma_{KN\rightarrow \cdots}$ measurement?
- Generate pseudodata: benchmark - fit #4
  - Assume uniformly distributed data for $p_{lab} = 100\ldots300$ MeV
    ... with energy bins of the size of $\Delta E = 5, 10, 20$ MeV
  - Assume error bars of $\Delta \sigma = 2.5, 5, 10$ mb for charged
    ... and $\Delta \sigma = 5, 10, 20$ mb for neutral channels
Pseudo scattering data

- **Compare** $\chi^2_{\text{d.o.f.}} / \chi^2_{\text{d.o.f.}}$ (#4)
  - threshold ratios, SIDDHARTA
  - pseudo and real scattering data
  ⇒ $\Delta \sigma < 5(10)$ mb and $\Delta E < 10$ MeV desired

→ threshold ratios, SIDDHARTA
→ pseudo scattering data
  ⇒ much larger values of $\Delta \sigma$ and $\Delta E$ are sufficient
Summary

- The NLO chiral unitary $\bar{K}N$ amplitude used to analyze hadronic data

- 8 solutions are found in the on-shell approximation
  - the position of the narrow pole is quite certain
  - broad pole has large systematic uncertainty

- Photoproduction amplitude constructed from the hadronic part
  - simple, but very flexible ansatz ... conservative test
  - 5 solutions disagree with the CLAS data, 2 remain after all tests

- New data can actually reduce the ambiguity of the $\bar{K}N$ amplitude
  - desired accuracy is not a part of science-fiction
THANK YOU
● Qualitative comparison with Hemingway data \((K^- p \rightarrow \Sigma^+ \pi^- \pi^+ \pi^-)\)

→ Fit #2 and #4 are fine

→ Fit #5 is completely off