On the importance of Kpi scattering for Phenomenology

Emilie Passemar
Indiana University/Jefferson Lab.

Physics with Neutral Kaon Beam at JLab Workshop
Thomas Jefferson National Accelerator Facility
Newport News, VA, February 2, 2016
Outline

1. Introduction and Motivation
2. Test of ChPT
3. Hadron spectroscopy
4. Test of the SM and new physics
5. Conclusion and outlook
1. Introduction and Motivation
1.1 Why $K\pi$ scattering is important?

- Hadron spectroscopy: determine resonances and their nature
  - P-wave: $K^*(892), K^*(1410), K^*(1680), \ldots$
  - S-wave: “$\kappa(\sim 800)$”, \ldots
  - Exotics,\ldots

- $\pi\pi$ and $K\pi$ building blocks for hadronic physics:
  - Test of Chiral Dynamics
  - Extraction of fundamental parameters of the Standard Model
  - Look for physics beyond the Standard Model: High precision at low energy as a key to new physics?

Very important when *Final State Interactions* at play!
1.2 Ex: $K\pi$ scattering: P-wave

$\tau \rightarrow K\pi\nu_\tau$

See also lattice QCD
Dudek et al. Wilson et al.'14

$K^*\pi$ threshold

Tau data

threshold parameters

$LASS$

Estabrooks et al.

Boito, Escribano & Jamin'10

Emilie Passemard
2. Using $K\pi$ scattering to test ChPT
$K\pi$ scattering: P-wave

ChPT

K*\pi threshold

$\tau \rightarrow K\pi\nu_\tau$

Tau data

$K\pi$ scattering: P-wave

Emilie Passemar

Boito, Escribano & Jamin’10

threshold parameters

LASS

Estabrooks et al.
2.1 Chiral Symmetry

- Limit \( m_k \to 0 \)

\[
\mathcal{L}_{QCD} \to \mathcal{L}_{QCD}^0 = -\frac{1}{4} G_{\mu\nu} G^{\mu\nu} + \bar{q}_L i\gamma^\mu D_\mu q_L + \bar{q}_R i\gamma^\mu D_\mu q_R, \quad q = \begin{pmatrix} u \\ d \\ s \end{pmatrix}
\]

with \( q_{L/R} \equiv \frac{1}{2}(1 \mp \gamma_5)q \)

Symmetry: \( G \equiv SU(3)_L \otimes SU(3)_R \to SU(3)_V \)

- Chiral Perturbation Theory: dynamics of the Goldstone bosons (kaons, pions, eta)

- Goldstone bosons interact weakly at low energy and \( m_u, m_d \ll m_s < \Lambda_{QCD} \)

Expansion organized in external momenta and quark masses

Weinberg’s power counting rule

\[
\mathcal{L}_{\text{eff}} = \sum_{d \geq 2} \mathcal{L}_d, \quad \mathcal{L}_d = \mathcal{O}(p^d), \quad p \equiv \{q, m_q\}
\]

\( p \ll \Lambda_H = 4\pi F_\pi \sim 1 \text{ GeV} \)
2.2 Chiral expansion

- \( \mathcal{L}_{ChPT} = \mathcal{L}_2 + \mathcal{L}_4 + \mathcal{L}_6 + \ldots \)

  \begin{align*}
  \text{LO} & : \mathcal{O}(p^2) \\
  \text{NLO} & : \mathcal{O}(p^4) \\
  \text{NNLO} & : \mathcal{O}(p^6)
  \end{align*}

- The structure of the lagrangian is fixed by chiral symmetry but not the coupling constants \( \rightarrow \) LECs appearing at each order

- The method has been rigorously established and can be formulated as a set of calculational rules:

  \begin{align*}
  \text{LO} : & \quad \text{tree level diagrams with } \mathcal{L}_2 \\
  \mathcal{L}_2 & : F_0, B_0 \\
  \text{NLO} : & \quad \text{1-loop diagrams with } \mathcal{L}_2 \\
  \mathcal{L}_4 & = \sum_{i=1}^{10} L_i O_4^i \\
  \text{NNLO} : & \quad \text{2-loop diagrams with } \mathcal{L}_2 \\
  \mathcal{L}_6 & = \sum_{i=1}^{90} C_i O_6^i \\
  \text{NNLO} : & \quad \text{1-loop diagrams with one vertex from } \mathcal{L}_4
  \end{align*}

- Renormalizable and unitary order by order in the expansion
2.3 ChPT in the meson sector: precision calculations

- Today’s standard in the meson sector: 2-loop calculations

- Main obstacle to reaching high precision: determination of the LECs: $O(p^6)$ LECs proliferation makes the program to pin down/estimate all of them prohibitive

- In a specific process, only a limited number of LECs appear

- The LECs calculable if QCD solvable, instead
  - Determined from experimental measurement
  - Estimated with models: Resonances, large $N_C$
  - Computed on the lattice
2.4 Test of SU(3) ChPT

- Interesting framework to test ChPT is offered by the kaons: $K_{l3}$, $K_{l4}$, $K \rightarrow 3\pi$, etc

- A very interesting quantity is the scattering length: first term in the expansion:

$$\frac{2}{\sqrt{s}} \text{Re} t_l^I(s) = \frac{1}{2q} \sin 2\delta^I_l(q) = q^{2l} \left[ a^I_l + b^I_l q^2 + c^I_l q^4 + O(q^6) \right]$$

- For $\pi\pi$: SU(2) ChPT very successful!
\( \pi \pi \) scattering lengths

\[ a_0^2 \]

- Universal band
- Tree (66), one loop (83), two loops (96)
- Prediction (ChPT + dispersion theory, 2001)
- DIRAC (2005)
- NA48 \( K \rightarrow 3 \pi \) (2005)
- NA48 isospin corrected
- E865 isospin corrected
2.4 Test of SU(3) ChPT

• Interesting framework to test ChPT is offered by the kaons: $K_{l3}$, $K_{l4}$, $K \rightarrow 3\pi$, etc

• A very interesting quantity is the scattering length: first term in the expansion:

$$\frac{2}{\sqrt{s}} \text{Re} t_l^I(s) = \frac{1}{2q} \sin 2\delta_l^I(q) = q^{2l} [a_l^I + b_l^I q^2 + c_l^I q^4 + \mathcal{O}(q^6)]$$

• For $\pi\pi$: SU(2) ChPT very successful!

• What about SU(3) ChPT?
  In principle slower convergence if convergence at all!
**Kπ scattering lengths: S-wave**

Buettiker, Descotes-Genon, Moussallam’04

Janowski’14

Emilie Passemard
Roy-Steiner equations for $K\pi$

- Unitarity effects can be calculated exactly using dispersive methods

- Unitarity, analyticity and crossing symmetry $\equiv$ Roy-Steiner equations

- **Input:** Data on $K\pi \rightarrow K\pi$ and $\pi\pi \rightarrow KK$ for $E \geq 1$ GeV
  
  two subtraction constants, e.g. $a_0^0$ and $a_2^0$

- **Output:** the full $K\pi$ scattering amplitude below 1 GeV
  
  In poor agreement with the experimental data

- Numerical solutions of the Roy-(Steiner) equations:
  
  - $\pi\pi$: Pennington-Protopopescu, Basdevant-Froggatt-Petersen (70s)
    Bern group: Ananthanarayan et al.’00, Caprini et al.’11
    Orsay group: Descotes-Genon, Fuchs, Girlanda and Stern’01
    Madrid-Cracow group: Garcia-Martin, et al.’11
  
  - $K\pi$: Buettiker, Descotes-Genon, Moussallam’04
  
  - $KN$: Ruiz de Elvira et al’15
$K\pi$ scattering lengths: P-wave

$ChPT$

$K^*\pi$ threshold

Tau data

$\tau \to K\pi\nu_\tau$

Boito, Escribano & Jamin’10
**Kπ scattering lengths: P-wave**

\[
\frac{2}{\sqrt{s}} \text{Re} \, t^I_l (s) = \frac{1}{2q} \sin 2 \delta^I_l (q) = q^{2l} \left[ a^I_l + b^I_l q^2 + c^I_l q^4 + \mathcal{O}(q^6) \right]
\]

<table>
<thead>
<tr>
<th></th>
<th>Tau data</th>
<th>ChPT $\mathcal{O}(p^4)$</th>
<th>RChPT $\mathcal{O}(p^4)$</th>
<th>ChPT $\mathcal{O}(p^6)$</th>
<th>Roy-Steiner</th>
</tr>
</thead>
<tbody>
<tr>
<td>$m_\pi^3 a_1^{1/2} \times 10$</td>
<td>0.166(4)</td>
<td>0.16(3)</td>
<td>0.18(3)</td>
<td>0.18</td>
<td>0.19(1)</td>
</tr>
<tr>
<td>$m_\pi^5 b_1^{1/2} \times 10^2$</td>
<td>0.258(9)</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>0.18(2)</td>
</tr>
<tr>
<td>$m_\pi^7 c_1^{1/2} \times 10^3$</td>
<td>0.90(3)</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>0.71(11)</td>
</tr>
</tbody>
</table>

Recent analysis combining $K_{l3}$, tau and D data: $0.249 \pm 0.011$ — Bernard’14

- Bernard, Kaiser, Meissner’91
- Bernard, Kaiser, Meissner’91
- Bijnens, Dhonte, Talavera’04
- Buettiker, Descotes-Genon, Moussallam’04

- Poor agreement — need more data

[Page 17]
3. Hadron spectroscopy
3.1 Determining of pole and width

- Once one gets Kpi scattering amplitude
  analytical continuation into the complex plane

*Poles on the second sheet correspond to zeros on the first sheet!*
$K\pi$ scattering lengths: P-wave

Fit to $\tau \rightarrow K\pi\nu_{\tau}$ with restrictions from $K*\pi$ threshold

Parameters

Tau data

$\tau \rightarrow K\pi\nu_{\tau}$

threshold

Boito, Escribano & Jamin’10

Emilie Passemard
### 3.2 $K^*(892)$ mass and width

#### CHARGED ONLY, HADROPRODUCED

<table>
<thead>
<tr>
<th>VALUE (MeV)</th>
<th>EVTS</th>
<th>DOCUMENT ID</th>
<th>TECN</th>
<th>CHG</th>
<th>COMMENT</th>
</tr>
</thead>
<tbody>
<tr>
<td>891.66±0.26 OUR AVERAGE</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>889.6 ± 0.5</td>
<td>5840</td>
<td>BAUBILLIER 84B</td>
<td>HBC</td>
<td>–</td>
<td>8.25 $K^- p \to \bar{K}_S^0 \pi^- p$</td>
</tr>
<tr>
<td>888 ± 3</td>
<td></td>
<td>NAPIER 84 SPEC</td>
<td>+</td>
<td></td>
<td>200 $\pi^- p \to 2K_S^0 X$</td>
</tr>
<tr>
<td>891 ± 1</td>
<td></td>
<td>NAPIER 84 SPEC</td>
<td>–</td>
<td></td>
<td>200 $\pi^- p \to 2K_S^0 X$</td>
</tr>
<tr>
<td>891.7 ± 2.1</td>
<td>3700</td>
<td>BARTH 83 HBC</td>
<td>+</td>
<td></td>
<td>70 $K^+ p \to K^0 \pi^+ X$</td>
</tr>
<tr>
<td>891 ± 1</td>
<td>4100</td>
<td>TOAFF 81 HBC</td>
<td>–</td>
<td></td>
<td>6.5 $K^- p \to \bar{K}_S^0 \pi^- p$</td>
</tr>
<tr>
<td>892.8 ± 1.6</td>
<td></td>
<td>AJINENKO 80 HBC</td>
<td>+</td>
<td></td>
<td>32 $K^- p \to K^0 \pi^+ X$</td>
</tr>
<tr>
<td>890.7 ± 0.9</td>
<td>1800</td>
<td>AGUILAR-... 78B HBC</td>
<td>±</td>
<td></td>
<td>0.76 $\bar{p}p \to K^\mp K_0^\pm \pi^\pm X$</td>
</tr>
<tr>
<td>886.6 ± 2.4</td>
<td>1225</td>
<td>BALAND 78 HBC</td>
<td>±</td>
<td></td>
<td>12 $\bar{p}p \to (K\pi)^\pm X$</td>
</tr>
<tr>
<td>891.7 ± 0.6</td>
<td>6706</td>
<td>COOPER 78 HBC</td>
<td>±</td>
<td></td>
<td>0.76 $\bar{p}p \to (K\pi)^\pm X$</td>
</tr>
<tr>
<td>891.9 ± 0.7</td>
<td>9000</td>
<td>1 PALER 75 HBC</td>
<td>–</td>
<td></td>
<td>14.3 $K^- p \to (K\pi)^\mp X$</td>
</tr>
<tr>
<td>892.2 ± 1.5</td>
<td>4404</td>
<td>AGUILAR-... 71B HBC</td>
<td>–</td>
<td></td>
<td>3.94.6 $K^- p \to (K\pi)^\mp X$</td>
</tr>
<tr>
<td>891 ± 2</td>
<td>1000</td>
<td>CRENNEIL 69D DBC</td>
<td>–</td>
<td></td>
<td>3.9 $K^- N \to K^0 \pi^- X$</td>
</tr>
<tr>
<td>890 ± 3.0</td>
<td>720</td>
<td>BARLOW 67 HBC</td>
<td>±</td>
<td></td>
<td>1.2 $\bar{p}p \to (K^0\pi^\pm)^\mp X$</td>
</tr>
<tr>
<td>889 ± 3.0</td>
<td>600</td>
<td>BARLOW 67 HBC</td>
<td>±</td>
<td></td>
<td>1.2 $\bar{p}p \to (K^0\pi^\pm)^\mp X$</td>
</tr>
<tr>
<td>891 ± 2.3</td>
<td>620</td>
<td>DEBAERE 67B HBC</td>
<td>+</td>
<td></td>
<td>3.5 $K^+ p \to K^0 \pi^+ p$</td>
</tr>
<tr>
<td>891.0 ± 1.2</td>
<td>1700</td>
<td>3 WOJCICKI 64 HBC</td>
<td>–</td>
<td></td>
<td>1.7 $K^- p \to \bar{K}_S^0 \pi^- p$</td>
</tr>
</tbody>
</table>

- We do not use the following data for averages, fits, limits, etc.

#### CHARGED ONLY, PRODUCED IN $\tau$ LEPTON DECAYS

<table>
<thead>
<tr>
<th>VALUE (MeV)</th>
<th>EVTS</th>
<th>DOCUMENT ID</th>
<th>TECN</th>
<th>COMMENT</th>
</tr>
</thead>
<tbody>
<tr>
<td>895.47±0.20±0.74</td>
<td>53k</td>
<td>6 EPIFANOV 07 BELL</td>
<td></td>
<td>$\tau^- \to K_S^0 \pi^- \nu_\tau$</td>
</tr>
<tr>
<td>892.0 ± 0.5</td>
<td>7 BOITO 10 RVUE</td>
<td></td>
<td></td>
<td>$\tau^- \to K_S^0 \pi^- \nu_\tau$</td>
</tr>
<tr>
<td>892.0 ± 0.9</td>
<td>8,9 BOITO 09 RVUE</td>
<td></td>
<td></td>
<td>$\tau^- \to K_S^0 \pi^- \nu_\tau$</td>
</tr>
<tr>
<td>895.3 ± 0.2</td>
<td>8,10 JAMIN 08 RVUE</td>
<td></td>
<td></td>
<td>$\tau^- \to K_S^0 \pi^- \nu_\tau$</td>
</tr>
<tr>
<td>896.4 ± 0.9</td>
<td>11970</td>
<td>11 BONVICINI 02 CLEO</td>
<td></td>
<td>$\tau^- \to K^- \pi^- \nu_\tau$</td>
</tr>
<tr>
<td>895 ± 2</td>
<td>12 BARATE 99R ALEP</td>
<td></td>
<td></td>
<td>$\tau^- \to K^- \pi^- \nu_\tau$</td>
</tr>
</tbody>
</table>

#### NEUTRAL ONLY

<table>
<thead>
<tr>
<th>VALUE (MeV)</th>
<th>EVTS</th>
<th>DOCUMENT ID</th>
<th>TECN</th>
<th>COMMENT</th>
</tr>
</thead>
<tbody>
<tr>
<td>895.81±0.19 OUR AVERAGE</td>
<td></td>
<td></td>
<td></td>
<td>Error includes scale factor of 1.4. See the ideogram below.</td>
</tr>
<tr>
<td>895.4 ± 0.2 ± 0.2</td>
<td>243k</td>
<td>13 DEL-AMO-SA..11i BABB</td>
<td></td>
<td>$D^+ \to K^- \pi^+ e^+ \nu_e$</td>
</tr>
<tr>
<td>895.7 ± 0.2 ± 0.3</td>
<td>141k</td>
<td>14 BONVICINI 08A CLEO</td>
<td></td>
<td>$D^+ \to K^- \pi^+ \pi^+$</td>
</tr>
<tr>
<td>895.41±0.32±0.35</td>
<td>18k</td>
<td>15 LINK 05i FOCUS</td>
<td></td>
<td>$D^+ \to K^- \pi^+ \mu^+ \nu_\mu$</td>
</tr>
<tr>
<td>896 ± 2</td>
<td></td>
<td>BARBERIS 98e OMEG</td>
<td></td>
<td>450 $pp \to p_f p_s K^* K^*$</td>
</tr>
<tr>
<td>895.9 ± 0.5 ± 0.2</td>
<td></td>
<td>88 ASTON 88 LASS</td>
<td></td>
<td>11 $K^- p \to K^- \pi^+ n$</td>
</tr>
</tbody>
</table>
### 3.2 $K^*(892)$ mass and width

#### $K^*(892)$ Mass

<table>
<thead>
<tr>
<th>CHARGED ONLY, HADROPRODUCED</th>
<th>VALUE (MeV)</th>
<th>EVTS</th>
<th>DOCUMENT ID</th>
<th>TECN</th>
<th>CHG</th>
<th>COMMENT</th>
</tr>
</thead>
</table>
| **891.66 ± 0.26 OUR AVERAGE** | 891.66 ± 0.26 | 5040 | BAUBILLIER 84B | HBC | – | 8.25 $K^- p \rightarrow \bar{K}^0 \pi^- p$
| 892.0 ± 0.5 | 5040 | BAUBILLIER 84B | HBC | – | 8.25 $K^- p \rightarrow \bar{K}^0 \pi^- p$
| 888 ± 3 | 888 | NAPIER 84 | SPEC | + | 200 $\pi^- p \rightarrow 2K^0 S$
| 891 ± 1 | 891 | NAPIER 84 | SPEC | – | 200 $\pi^- p \rightarrow 2K^0 S$
| 891.7 ± 2.1 | 891.7 ± 2.1 | 3700 | BARTH 83 | HBC | + | 70 $K^+ p \rightarrow K^0 \pi^+ X$
| 891 ± 1 | 891 | TOAFF 81 | HBC | – | 6.5 $K^- p \rightarrow \pi^-\pi^+ p$
| 892.8 ± 1.6 | 892.8 ± 1.6 | 1800 | AJINENKO 80 | HBC | + | 32 $K^+ p \rightarrow K^0 \pi^+ X$
| 890.7 ± 0.9 | 890.7 ± 0.9 | 1225 | AGUILAR-... 78B | HBC | ± | 0.76 $\pi p \rightarrow K^+ K^0 S$
| 886.6 ± 2.4 | 886.6 ± 2.4 | 6706 | BALAND 78 | HBC | ± | 12 $\pi p \rightarrow (K\pi)^\pm X$
| 891.7 ± 0.6 | 891.7 ± 0.6 | 9000 | PALER 75 | HBC | ± | 14.3 $K^- p \rightarrow (K\pi)^-\pi^0$
| 891.9 ± 0.7 | 891.9 ± 0.7 | 4404 | AGUILAR-... 71B | HBC | ± | 3.946 $K^- p \rightarrow (K\pi)^-\pi^0$
| 892.2 ± 1.5 | 892.2 ± 1.5 | 1000 | CRENNEll 69D | DBC | – | 3.9 $K^- N \rightarrow K^0 \pi^- X$
| 891 ± 2 | 891 | BARLOW 67 | HBC | ± | 1.2 $\bar{p} p \rightarrow (K\pi)^\pm\pi^\pm$
| 890 ± 3.0 | 890 | BARLOW 67 | HBC | ± | 1.2 $\bar{p} p \rightarrow (K\pi)^\pm\pi^\pm$
| 889 ± 3.0 | 889 | BARLOW 67 | HBC | ± | 1.2 $\bar{p} p \rightarrow (K\pi)^\pm\pi^\pm$
| 891 ± 2.3 | 891 | DEBAERE 67B | HBC | ± | 3.5 $K^+ p \rightarrow K^0 \pi^+ p$
| 891.0 ± 1.2 | 891.0 ± 1.2 | 1700 | WOJCICl 64 | HBC | ± | 1.7 $K^- p \rightarrow \bar{K}^0 \pi^- p$

#### CHARGED ONLY, PRODUCED IN $\tau$ LEPTON DECAYS

<table>
<thead>
<tr>
<th>VALUE (MeV)</th>
<th>EVTS</th>
<th>DOCUMENT ID</th>
<th>TECN</th>
<th>COMMENT</th>
</tr>
</thead>
</table>
| **895.47 ± 0.20 ± 0.74** | 895.47 ± 0.20 ± 0.74 | 5040 | BELL | $\tau^- \rightarrow K^0 S \pi^- \nu_{\tau}$
| 892.0 ± 0.5 | 892.0 ± 0.5 | 888 | BOITO | RVUE | $\tau^- \rightarrow K^0 S \pi^- \nu_{\tau}$
| 892.0 ± 0.9 | 892.0 ± 0.9 | 888 | BOITO | RVUE | $\tau^- \rightarrow K^0 S \pi^- \nu_{\tau}$
| 895.3 ± 0.2 | 895.3 ± 0.2 | 888 | JAMIN | RVUE | $\tau^- \rightarrow K^0 S \pi^- \nu_{\tau}$
| 896.4 ± 0.9 | 896.4 ± 0.9 | 11970 | BONVICINI | CLEO | $\tau^- \rightarrow K^0 S \pi^- \nu_{\tau}$
| 895 ± 2 | 895 ± 2 | 99R | BARVINNI | ALEP | $\tau^- \rightarrow K^0 S \pi^- \nu_{\tau}$

#### NEUTRAL ONLY

<table>
<thead>
<tr>
<th>VALUE (MeV)</th>
<th>EVTS</th>
<th>DOCUMENT ID</th>
<th>TECN</th>
<th>COMMENT</th>
</tr>
</thead>
</table>
| **895.81 ± 0.19 OUR AVERAGE** | 895.81 ± 0.19 | 5040 | BELL | $\tau^- \rightarrow K^0 S \pi^- \nu_{\tau}$
| 895.4 ± 0.2 | 895.4 ± 0.2 | 243k | DEL-AMO-SA..11i | BARR | $D^+ \rightarrow K^- \pi^+ e^+\nu_e$
| 895.7 ± 0.2 | 895.7 ± 0.2 | 141k | BONVICINI | CLEO | $D^+ \rightarrow K^- \pi^+ \pi^+$
| 895.41 ± 0.32 ± 0.35 | 895.41 ± 0.32 ± 0.35 | 18k | LINK | FOCS | $D^+ \rightarrow K^- \pi^+ \mu^+\nu_{\mu}$
| 896 ± 2 | 896 ± 2 | 98e | BARBERIS | OMEG | $pp \rightarrow pf p_s K^* K^*$
| 895.9 ± 0.5 | 895.9 ± 0.5 | 11 | ASTON | LASS | $11 K^- p \rightarrow K^0 S \pi^-\pi^0$

- *We do not use the following data for averages, fits, limits, etc.*

#### Error includes scale factor of 1.4. See the ideogram below.

Emilie Passemear
3.2 $K^*(892)$ mass and width

**Mass of $K^*(892)$ [MeV]**

- E.g. $K^-p \rightarrow \bar{K}^0\pi^-p$
- $\tau \rightarrow K\pi\nu$
- $D \rightarrow K\pi\nu_e$

<table>
<thead>
<tr>
<th>Hadronic</th>
<th>Tau lepton</th>
<th>D decays</th>
</tr>
</thead>
<tbody>
<tr>
<td>888</td>
<td>889</td>
<td>890-895</td>
</tr>
</tbody>
</table>
3.2 $K^*(892)$ mass and width

**Decay width of $K^*(892)$ [MeV]**

- Hadronic: $e.g. K^- p \rightarrow \bar{K}^0 \pi^- p$
- Tau lepton: $\tau \rightarrow K \pi \nu$
- D decays: $D \rightarrow K \pi \nu_e$

**PDG'15**

Emilie Passemar
3.3 Kappa(800)

- The results coming from Roy-Steiner and data at higher energy not in agreement with low energy experimental data need improvement! Problem: no other precise data

<table>
<thead>
<tr>
<th></th>
<th>$M_\kappa$ (MeV)</th>
<th>$\Gamma_\kappa$ (MeV)</th>
</tr>
</thead>
<tbody>
<tr>
<td>This work</td>
<td>$658 \pm 13$</td>
<td>$557 \pm 24$</td>
</tr>
<tr>
<td>Zhou, Zheng</td>
<td>$694 \pm 53$</td>
<td>$606 \pm 89$</td>
</tr>
<tr>
<td>Jamin et al.</td>
<td>$708$</td>
<td>$610$</td>
</tr>
<tr>
<td>Aitala et al.</td>
<td>$721 \pm 19 \pm 43$</td>
<td>$584 \pm 43 \pm 87$</td>
</tr>
<tr>
<td>Pelaez</td>
<td>$750 \pm 18$</td>
<td>$452 \pm 22$</td>
</tr>
<tr>
<td>Bugg</td>
<td>$750_{-55}^{+30}$</td>
<td>$684 \pm 120$</td>
</tr>
<tr>
<td>Ablikim et al.</td>
<td>$841_{-55}^{+65}$</td>
<td>$618 \pm 52_{-87}^{+55}$</td>
</tr>
<tr>
<td>Ishida et al.</td>
<td>$877_{-30}^{+65}$</td>
<td>$668_{-110}^{+235}$</td>
</tr>
</tbody>
</table>

- Existence would suggest $\kappa$ not a glueball
3.3 Kappa(800)

- Inputs for S wave in Roy-Steiner analysis from LASS

*Buettik, Descotes-Genon, Moussallam’04*

![Graph showing S-wave behavior](image)
3.3 Kappa(800)

- The results coming from Roy-Steiner and data at higher energy not in agreement with low energy experimental data need improvement!

![Graph showing S-wave phase shifts with data points and curves for I=1/2 and I=3/2.](image)

Buettiker, Descotes-Genon, Moussallam’04
4. Test sof the SM and new physics
K\pi scattering lengths: P-wave

\[ I = \frac{1}{2} \text{P-wave scattering phase} \]

- \( \mathcal{K} \) and Passemar

- \( \tau \rightarrow K\pi\nu_\tau \) with restrictions from \( K^*\pi \) threshold

- \( K\pi \) scattering lengths

- Boito, Escribano & Jamin’10

- Tau data

- Estabrooks et al.

- LASS

- Threshold parameters

- \( m_{K\pi} \) [GeV]

- \( \delta \) [deg]
4.1 Determination of fundamental parameters: $V_{us}$

- Master formula for $K \rightarrow \pi l \nu$:

\[
\Gamma(K \rightarrow \pi l \nu \{\gamma]\) = \frac{G_F^2 m_K^5}{192\pi^3} C_K S_{EW}^K |V_{us}|^2 |f_+^{K_0 \pi^-}(0)|^2 I_K^l \left(1 + \delta_{EM}^{Kl} + \delta_{SU(2)}^{K\pi}\right)^2
\]
4.1 Determination of fundamental parameters: $V_{us}$

- Master formula for $K \rightarrow \pi l \nu$: 

$$
\Gamma(K \rightarrow \pi l \nu [\gamma]) = \frac{G_F^2 m_K^5}{192\pi^3} C_K^2 S_{EW}^K \left| V_{us} \right|^2 f_+^{K^0\pi^-}(0)^2 I_K^l \left( 1 + \delta_{EM}^{Kl} + \delta_{SU(2)}^{K\pi} \right)^2
$$

$$
\langle \pi(p_{\pi}) | \bar{s} \gamma_\mu u | K(p_K) \rangle = \left[ (p_K + p_{\pi})_\mu - \frac{\Delta_{K\pi}}{t} (p_K - p_{\pi})_\mu \right] f_+(t) + \frac{\Delta_{K\pi}}{t} (p_K - p_{\pi})_\mu f_0(t)
$$

$\delta_{EM}^{Kl}$: vector

$\delta_{SU(2)}^{K\pi}$: scalar
Dispersive representation for the form factors

- Omnès representation:

\[ f_{+,0}(s) = \exp \left( \frac{s}{\pi} \int_{s_{th}}^{\infty} ds' \frac{\phi_{+,0}(s')}{s'-s-i\varepsilon} \right) \]

\[ \phi_{+,0}(s): \text{phase of the form factor} \]

\[ s < s_{in} : \phi_{+,0}(s) = \delta_{K\pi}(s) \]

Kπ scattering phase

\[ s \geq s_{in} : \phi_{+,0}(s) \]

\[ \phi_{+,0}(s) = \phi_{+,0as}(s) = \pi \pm \pi \left( \frac{f_{+,0}(s)}{1/s} \right) \]

[Brodyk&Lepage]

- Subtract dispersion relation to weaken the high energy contribution of the phase. Improve the convergence but sum rules to be satisfied.

Emilie Passemar
Global fit to $V_{us}$ & $V_{ud}$

Fit results, no constraint

$V_{ud} = 0.97416(21)$
$V_{us} = 0.2248(7)$
$\chi^2/ndf = 1.16/1 (28.1\%)$
$\Delta_{CKM} = -0.0005(5)$
$-1.0\sigma$

Fit results, unitarity constraint

$V_{ud} = 0.97432(12)$
$V_{us} = 0.2251(5)$
$\chi^2/ndf = 2.06/2 (36\%)$

FlaviaNet KaonWG’10
Updated by
Moulson@CKM2014

1σ contours

fit with unitarity
4.2 FSI in the quest for New Physics

- Ex: CP violating asymmetries: $B \to K^* \pi$

LHCb at EPS13: 2.9 $\sigma$ discrepancy in $P_2$, 4.0 $\sigma$ in $P'_5$!

[blue: SM unbinned, purple: SM binned, crosses: LHCb]
4.2 FSI in the quest for New Physics

- Ex: CP violation in $D \rightarrow K\pi\pi$

Ex: Dalitz plot
4.2 FSI in the quest for New Physics

- Ex: CP violation in $D \to K\pi\pi$

Full set of equations

\[
S^2_{\pi\pi}(u) = \Omega^2_0(u) \left\{ u^2 \int_{4M^2_{\pi}}^{\infty} \frac{\hat{S^2_{\pi\pi}}(u')}{u'^2(u' - u)} d\mu^2_0 \right\}
\]

\[
P^1_{\pi\pi}(u) = \Omega^1_1(u) \left\{ c_0 + c_1 u + u^2 \int_{4M^2_{\pi}}^{\infty} \frac{\hat{P^1_{\pi\pi}}(u')}{u'^2(u' - u)} d\mu^1_1 \right\}
\]

\[
S^{1/2}_{\pi K}(s) = \Omega^{1/2}_0(s) \left\{ c_2 + c_3 s + c_4 s^2 + c_5 s^3 + s^4 \int_{(M_K+M_\pi)^2}^{\infty} \frac{\hat{S^{1/2}_{\pi K}}(s')}{s'^4(s' - s)} d\mu^{1/2}_0 \right\}
\]

\[
S^{3/2}_{\pi K}(s) = \Omega^{3/2}_0(s) \left\{ s^2 \int_{(M_K+M_\pi)^2}^{\infty} \frac{\hat{S^{3/2}_{\pi K}}(s')}{s'^2(s' - s)} d\mu^{3/2}_0 \right\}
\]

\[
P^{1/2}_{\pi K}(s) = \Omega^{1/2}_1(s) \left\{ c_6 + s \int_{(M_K+M_\pi)^2}^{\infty} \frac{\hat{P^{1/2}_{\pi K}}(s')}{s'(s' - s)} d\mu^{1/2}_1 \right\}
\]

\[
D^{1/2}_{\pi K}(s) = \Omega^{1/2}_2(s) \left\{ \int_{(M_K+M_\pi)^2}^{\infty} \frac{\hat{D^{1/2}_{\pi K}}(s')}{(s' - s)} d\mu^{1/2}_2 \right\}
\]
4.2 FSI in the quest for New Physics

- Ex: CP violation in $D \rightarrow K\pi\pi$

**Dalitz plot**

CLEO’08

- **full fit:** $\chi^2/\text{ndof} \approx 1.1$
4.2 FSI in the quest for New Physics

- Ex: CP violation in $D \to K\pi\pi$

<table>
<thead>
<tr>
<th>fit fractions</th>
<th>slices</th>
</tr>
</thead>
<tbody>
<tr>
<td>$S^{2}_{\pi\pi}$</td>
<td>(8 ± 3)%</td>
</tr>
<tr>
<td>$S^{1/2}_{\pi K}$</td>
<td>(72 ± 12)%</td>
</tr>
<tr>
<td>$P^{1/2}_{\pi K}$</td>
<td>(10 ± 2)%</td>
</tr>
<tr>
<td>$S^{3/2}_{\pi K}$</td>
<td>(16 ± 3)%</td>
</tr>
<tr>
<td>$D^{1/2}_{\pi K}$</td>
<td>(0.15 ± 0.1)%</td>
</tr>
<tr>
<td>$\Sigma$</td>
<td>(106 ± 20)%</td>
</tr>
</tbody>
</table>

- **full fit:** $\chi^2/\text{ndof} \approx 1.1$
- **fit fractions:** hierarchy of partial-wave amplitudes compare to previous analyses

*Niecknig & Kubis’15*
5. Conclusion and Outlook
Conclusion and Outlook

• Determining $K\pi$ scattering reliably very important:
  – Low energy: test of Chiral Dynamics
  – Intermediate energy: Determination of Resonance parameters
  – Very important to help taking into account final state interactions and hunting for new physics  

• Hadronic data on which most of the analyses rely not in good agreement with more recent data coming mainly from tau decays worth remeasuring it.

• Possibility at Jlab with KL?
  Major advantage: pure $I=1/2$ measurement
6. Back-up
2.5 Determination of some low energy constants

<table>
<thead>
<tr>
<th>$10^3 L_1$</th>
<th>$10^3 L_2$</th>
<th>$10^3 L_3$</th>
<th>$10^3 L_4$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\pi K$ Roy-Steiner</td>
<td>$\pi K$ sum-rules</td>
<td>$Kl_4$, $O(p^4)$</td>
<td>$Kl_4$, $O(p^6)$</td>
</tr>
<tr>
<td>1.05 ± 0.12</td>
<td>0.84 ± 0.15</td>
<td>0.46 ± 0.24</td>
<td>0.53 ± 0.25</td>
</tr>
<tr>
<td>1.32 ± 0.03</td>
<td>1.36 ± 0.13</td>
<td>1.49 ± 0.23</td>
<td>0.71 ± 0.27</td>
</tr>
<tr>
<td>−4.53 ± 0.14</td>
<td>−3.65 ± 0.45</td>
<td>−3.18 ± 0.85</td>
<td>−2.72 ± 1.12</td>
</tr>
<tr>
<td>0.53 ± 0.39</td>
<td>0.22 ± 0.30</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

- Significant violation of OZI rule in the scalar sector

  Large values for the condensates!
3.2 Kappa(800)

- Inputs for S wave in Roy-Steiner analysis from LASS

Buettiker, Descotes-Genon, Moussallam’04

![Graph showing S-wave phase vs. energy](image-url)
3.2 Kappa(800)

- The results coming from Roy-Steiner and data at higher energy not in agreement with low energy experimental data need improvement!
Figure 4: Experimental data from ref. [27] for the phase $\Phi_1$ and the fit used in the calculations.

The data of Aston et al. and the fits for both $a_l$ and $\Phi_l$ for $l = 0$ to $l = 5$ and energy up to $E = 2.5$ GeV show significant features. At energies $E \geq 1.8$ GeV, ref. [27] found two different solutions A and B for the phase shifts, between which one chooses solution A (as pointed out in ref. [19] that solution B violates the unitarity bound). These fits allow us to compute the relevant integrals up to $E = 2.5$ GeV. Above this point, we use the Regge model discussed in sec. 4.3.

4.2 $\pi\pi \rightarrow KK$ input

For our purposes, a key role is played by the $l = 0$ and $l = 1$ $\pi\pi \rightarrow KK$ amplitudes, which can be determined from $\pi N \rightarrow KK$ production experiments in the range $t \geq 4 m_K^2$. We will make use of the two high-statistics experiments described in Cohen et al. [28] and Etkin et al. [29, 47]. The experiment of Cohen et al. [28] determines the charged amplitude $\pi^+ \pi^- \rightarrow K^+ K^-$, thereby providing results for both $g_0^0$ and $g_1^1$. There are several solutions but physical requirements select a single one, called solution II b in ref. [28]. Close to the $KK$ threshold, the presence of the $l = 1$ phase allows the authors to accurately determine the $l = 0$ phase. The experiment of Etkin et al. concerns the amplitude $\pi^+ \pi^- \rightarrow KS KS$ which is purely $I = 0$. Because of the absence of the $P$-wave in this channel, their determination of the phase of $g_0^0$ close to the threshold (where the $D$-wave phase is very small) is likely to be less reliable than that of ref. [28]. Their determination of the magnitude of $g_0^0$ close to the threshold disagrees with that of Cohen et al. and also with earlier experiments [48]. Consequently, we make the choice to use the results of Etkin et al. only in the range $\sqrt{t} \geq 1.2$ GeV.
The $I=1/2$ P-wave phase shift obtained from solving the RS equations is shown in Figure 15. The central curve corresponds to solving with $a_{1/2}^{1/2}$ and $a_{3/2}^{1/2}$ taken at the center of the ellipse in Figure 14. The upper (lower) curves are obtained by using the points with the maximum (minimal) values for $a_{1/2}^{1/2}$ on this ellipse.

The two S-wave phase shifts predicted by the RS equations are shown in Figure 16. For the isospin $I=1/2$, the RS solution does not exhibit any of the oscillations appearing in the data of Ref. [26]. For the isospin $I=3/2$ phase shift, the experimental data for $E < 0.9$ GeV lie systematically below the RS curve, by 2-3 standard deviations. The RS equations also predict the $I=3/2$ P-wave phase shift, the result is shown in Figure 17. This phase shift displays the unusual feature that it is positive at very low energy and changes sign as energy increases. In the region around 1 GeV the results are in qualitative agreement with the experimental data of Ref. [10].

For instance, the result depends on the input values for $m_{\pi}$ and $m_{K}$ for which we used $m_{\pi} = 0.13957$ GeV, $m_{K} = 0.4957$ GeV.
P-wave

Figure 17: Same as fig. 15 for the \( I = \frac{3}{2} \) P-wave phase shift.

Divergences may appear in this process because derivatives are discontinuous at threshold and it must be specified that the limit is to be taken from above. This problem is easily handled by computing some pieces of the integrals analytically as explained in ref. [6]. The sum rules are evaluated by using RS solutions below the matching points and the fits to the experimental data above. For \( l = 0 \), \( 1 \) we have computed the parameters \( a_l, b_l \) and \( c_l \) in an alternative manner by using our solution for \( \text{Re} I_l(s) \) for three values of \( s \) and solving a linear system of equations. The two methods were in very good agreement and the results for the threshold parameters are summarized in table 3. The values of the P-waves scattering lengths in ChPT at NLO was given in ref. [18].

\[
\delta_{1}^{\frac{3}{2}} \equiv 0.16 \pm 0.003 \quad m \quad 3 \pi a_{1}^{\frac{1}{2}} = 0,
\]

\[
\delta_{1}^{\frac{3}{2}} \equiv 1.13 \pm 0.57 \quad 10^{-3} \quad \text{(ref. [18])}
\]

Within the errors, these values are compatible with our corresponding results displayed in table 3.

ChPT expansions of the amplitude are expected to have best convergence properties in unphysical regions away from any threshold singularity. The isospectral representations derived in sec. 2 allow us to evaluate the amplitude in such regions. A first domain considered in the literature is the neighborhood of the point \( s = u, t = 0 \). The following expansion parameters are conventionally introduced:

\[
F_{+}(s, t) = \sum \tilde{C}_{ij} t^{i} \nu_{2}^{j},
\]

\[
F_{-}(s, t) = \nu \{ \sum \tilde{C}_{ij} t^{i} \nu_{2}^{j} \}
\]

(97)