The Hyperon Spectrum and Other JPAC Projects
Part II

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Joint Physics Analysis Center

KL2016-JLab
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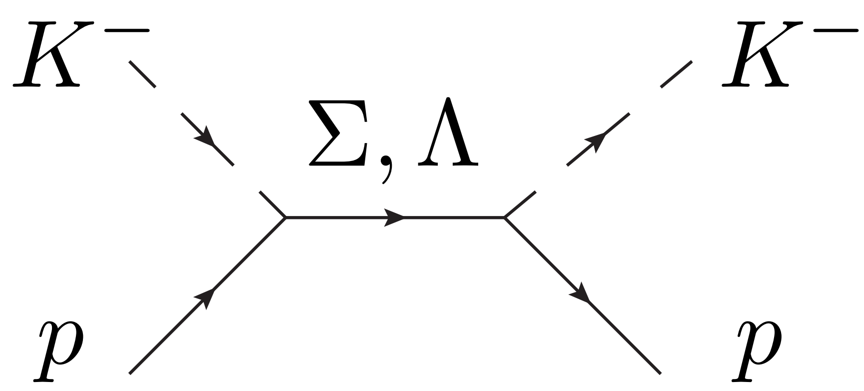
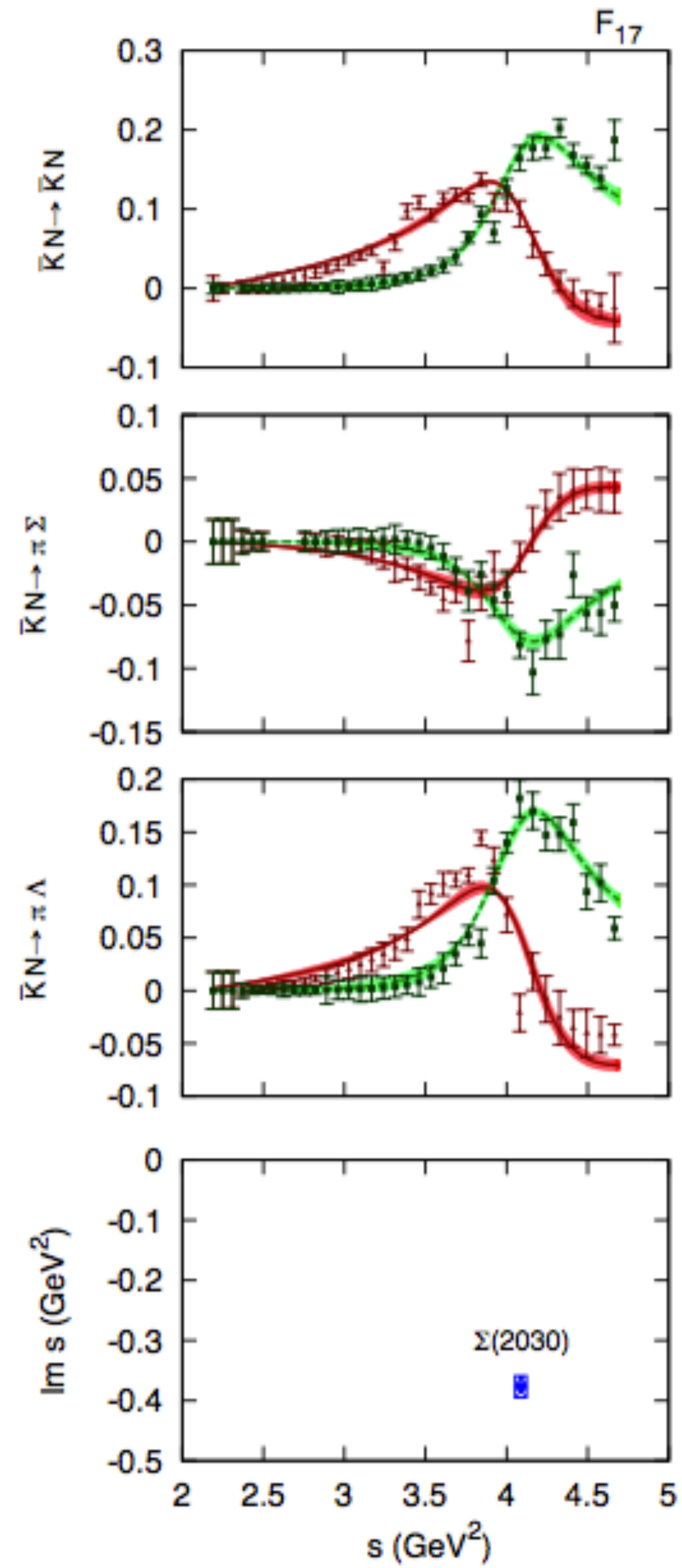
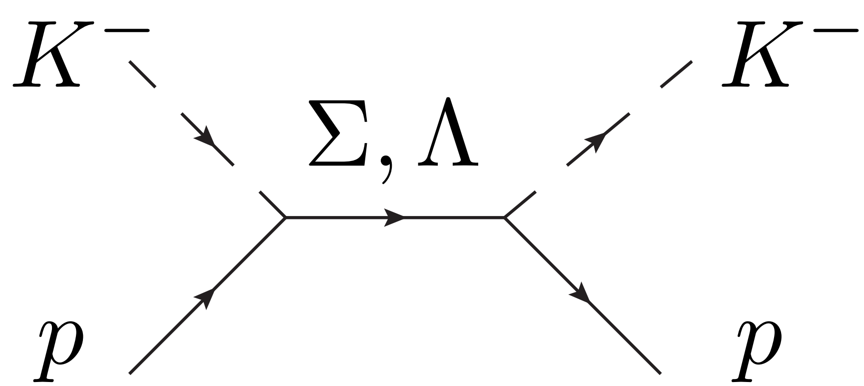
Σ *and* Λ *Baryon Spectrum*

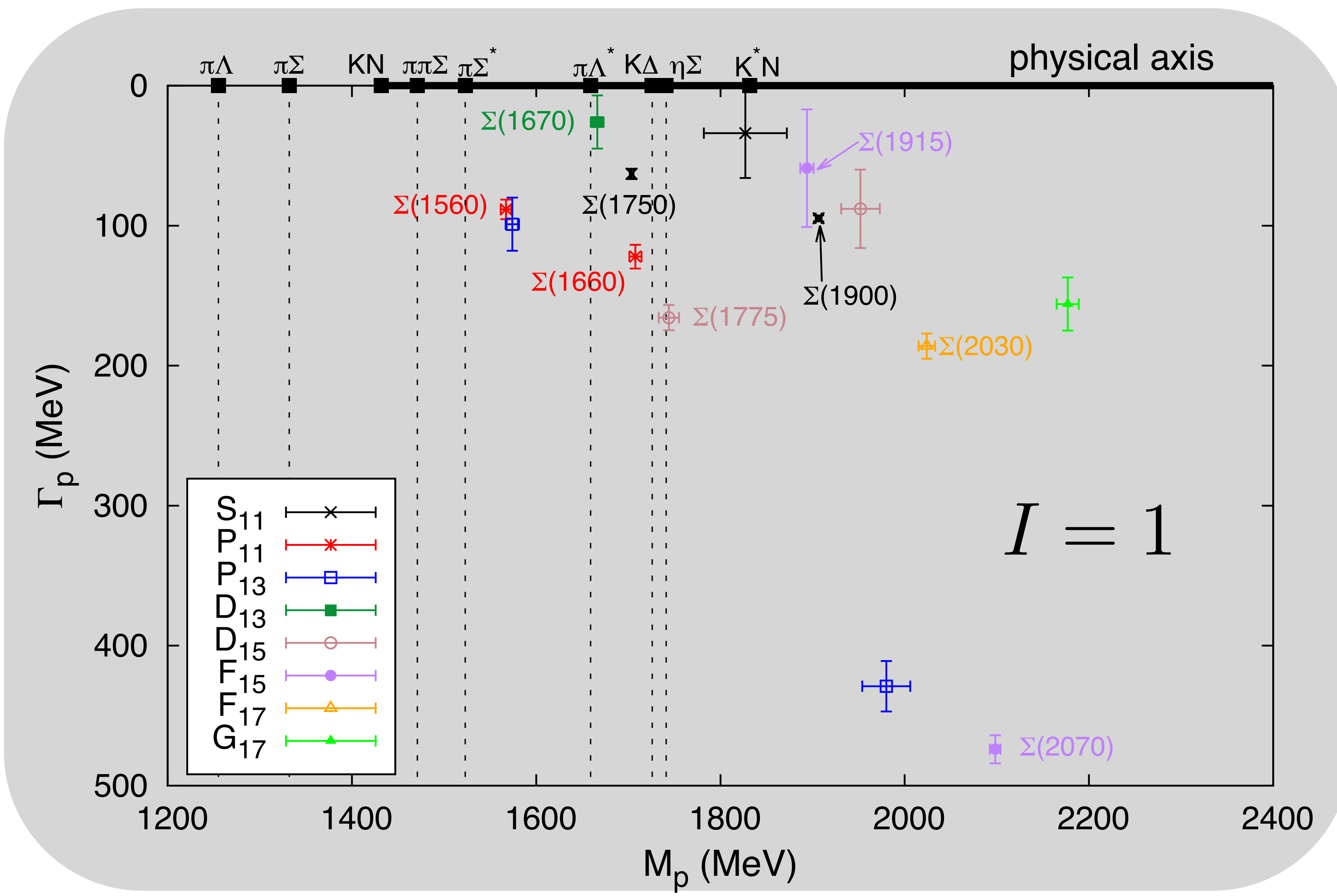


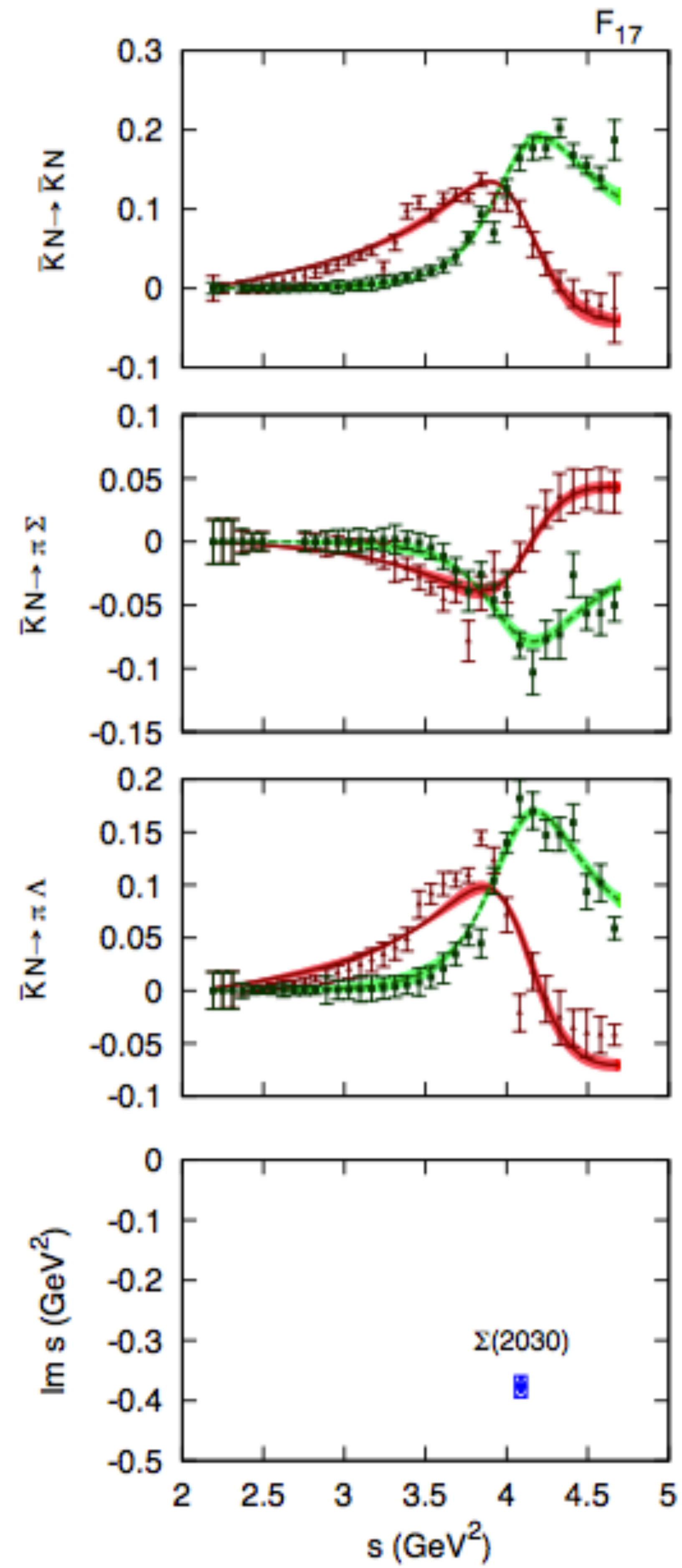


C. Fernandez-Ramirez et al. (JPAC) ArXiv:1510:07065

Σ and Λ Baryon Spectrum







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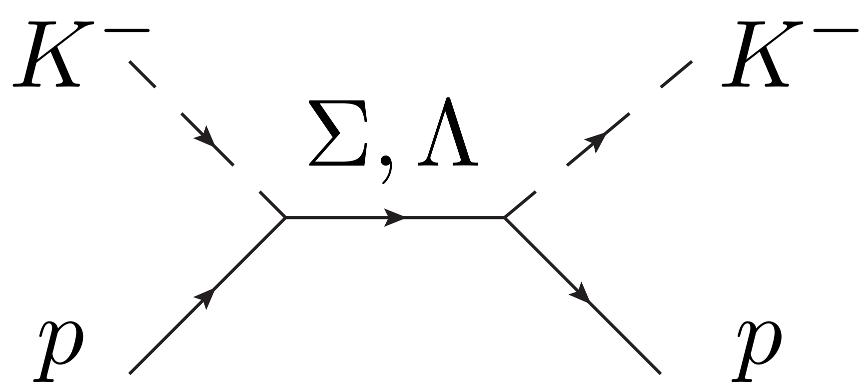
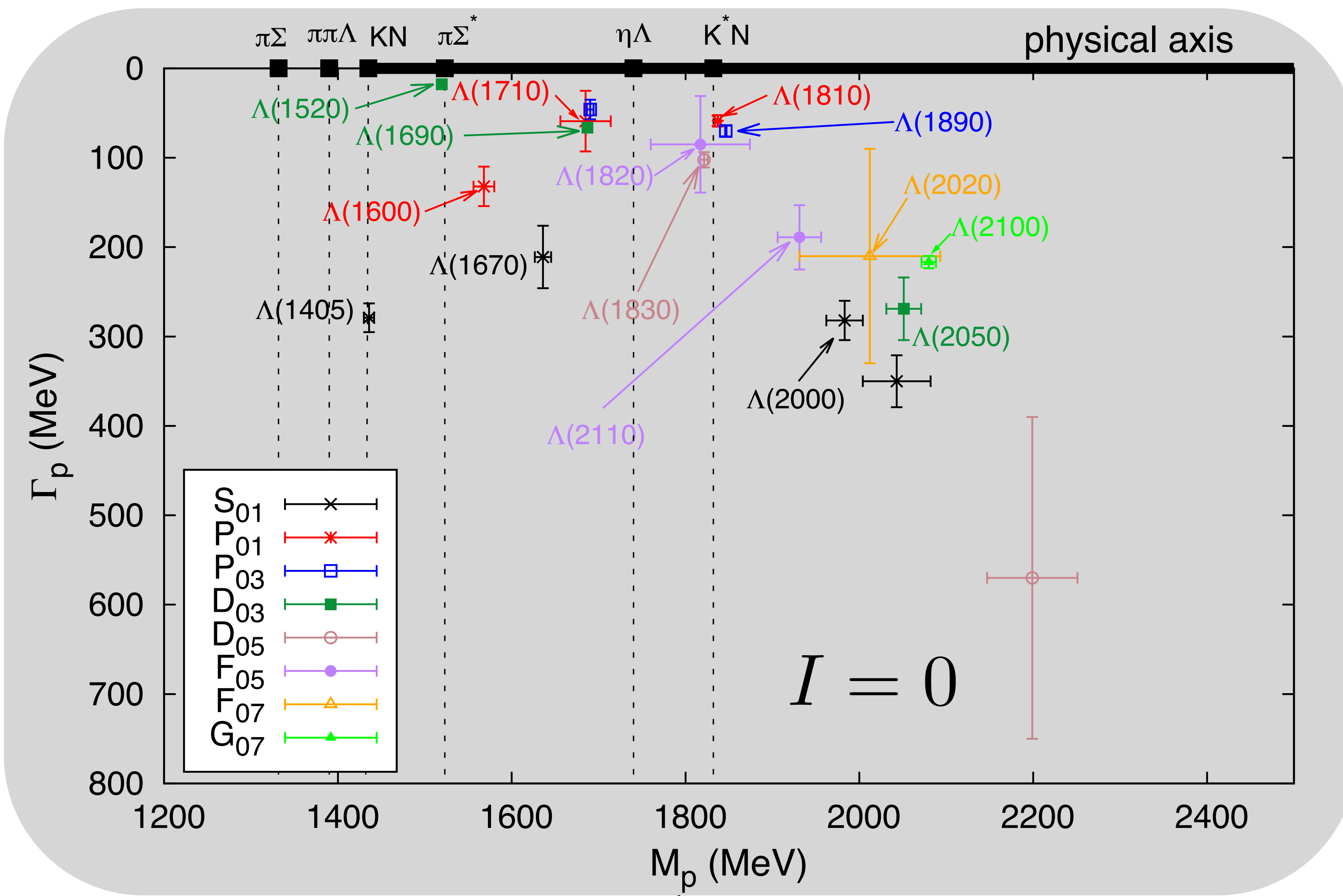
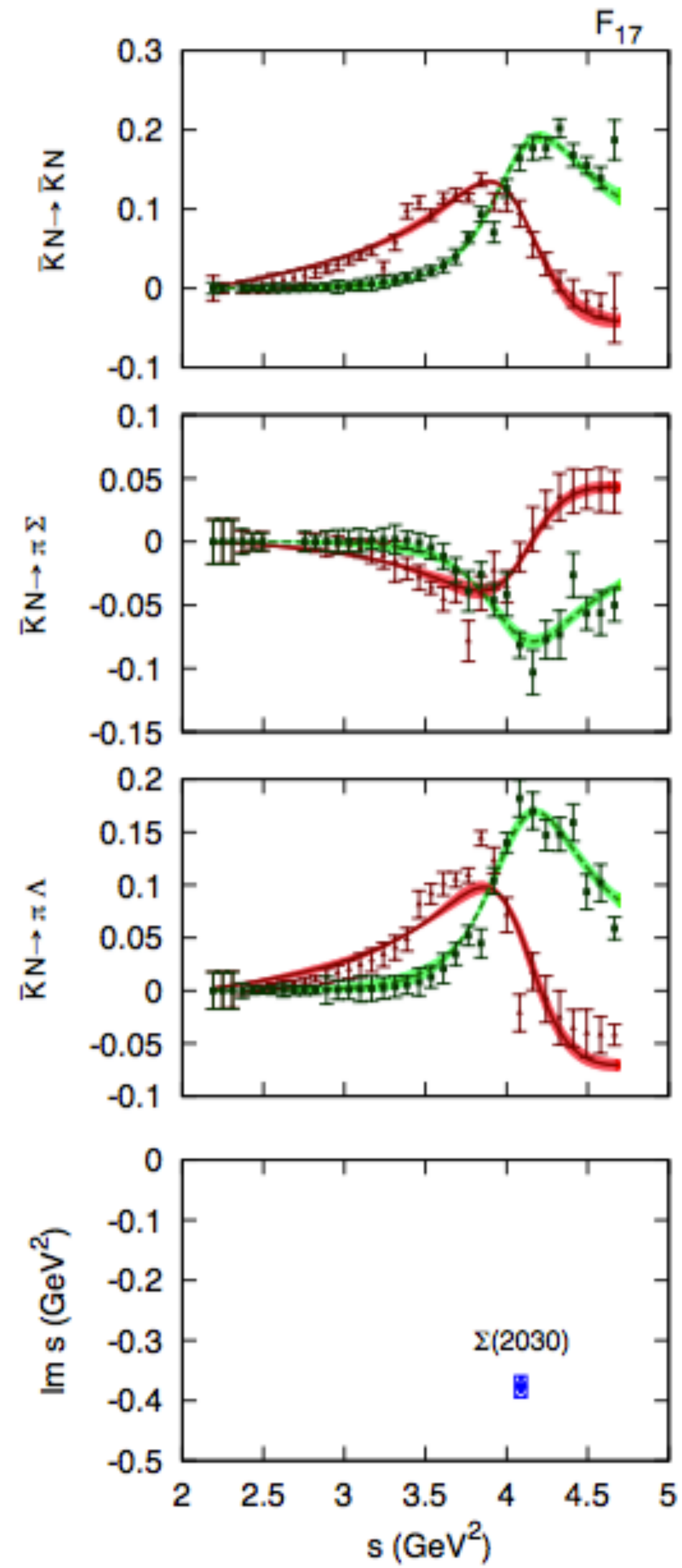
Σ and Λ Baryon Spectrum







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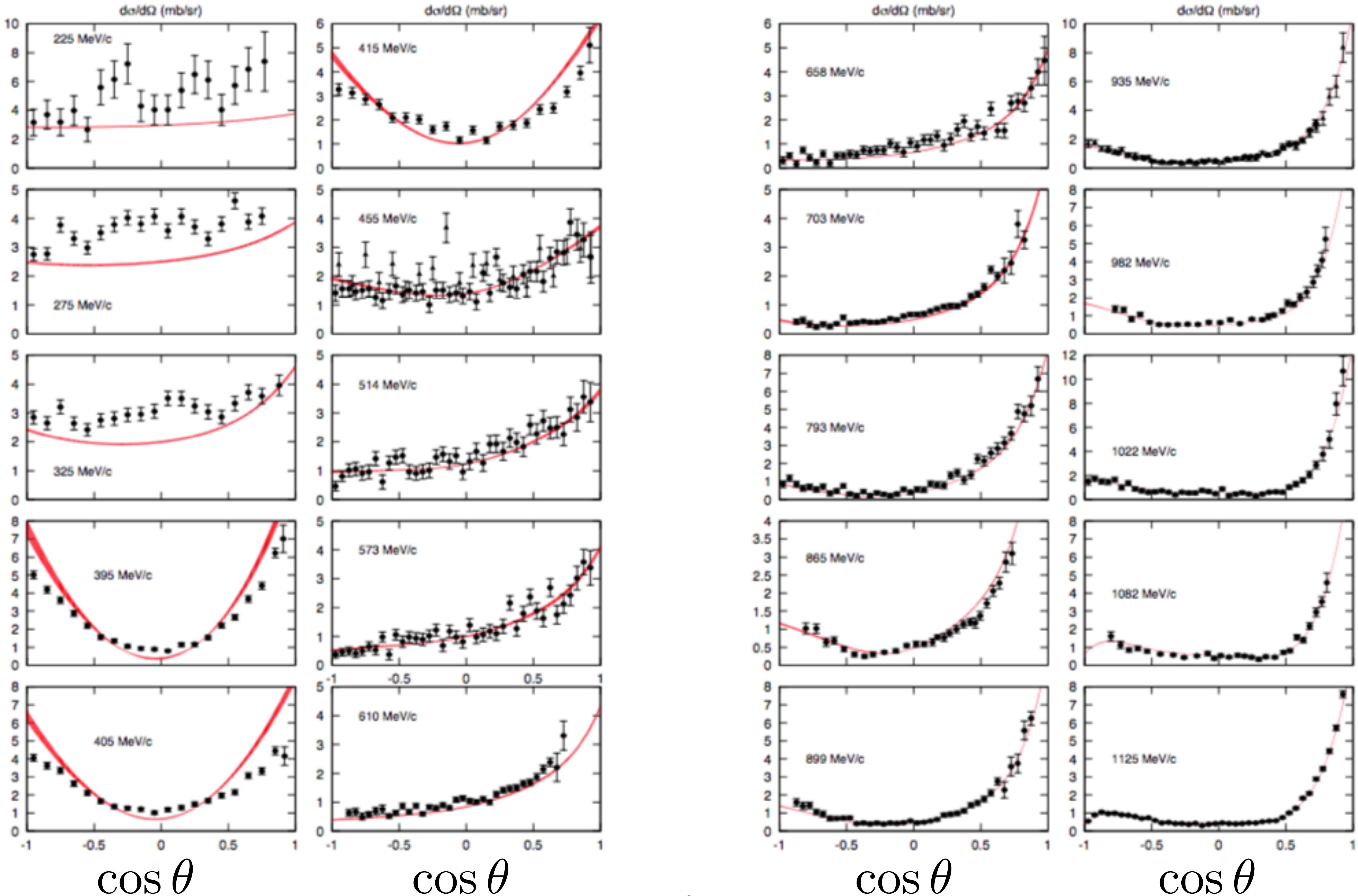
$K^- p \to K^- p$ ***Energy Evolution***



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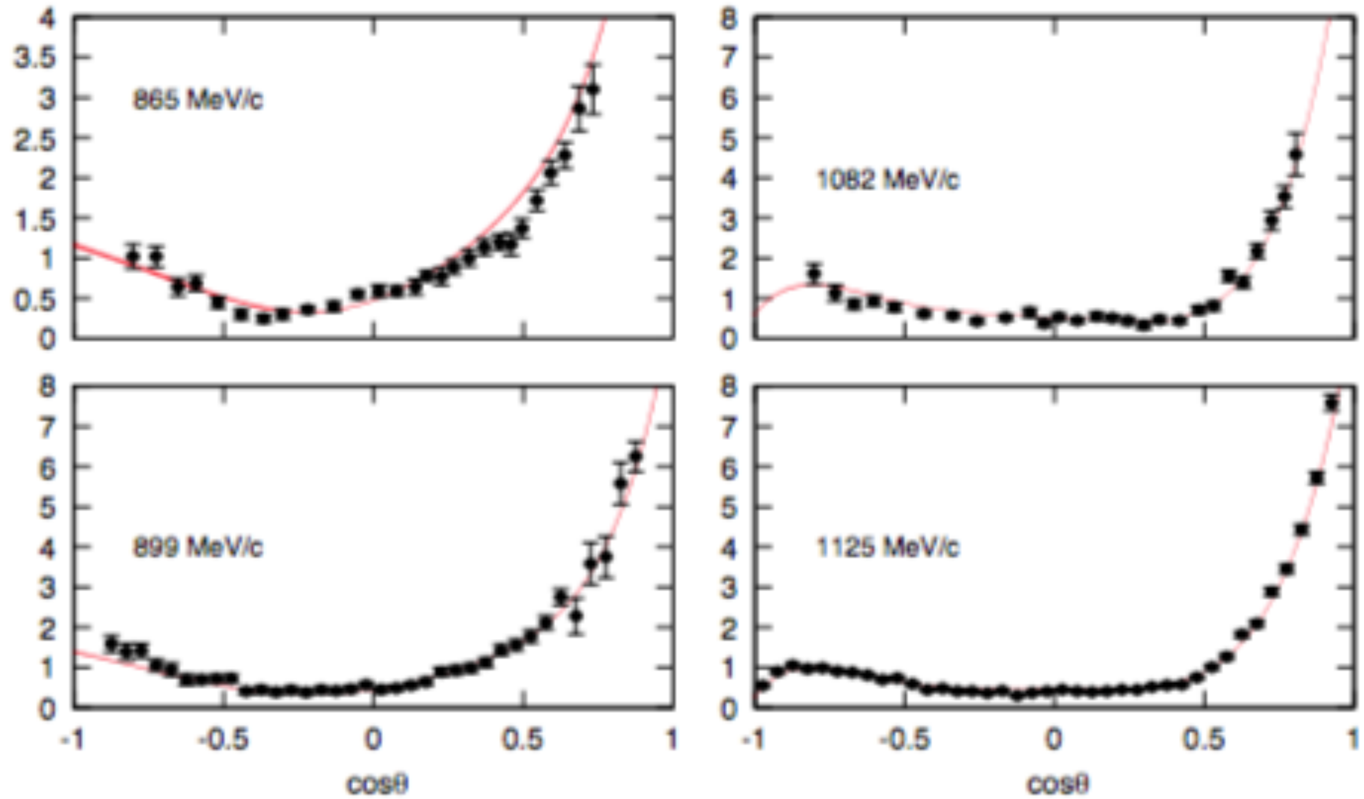
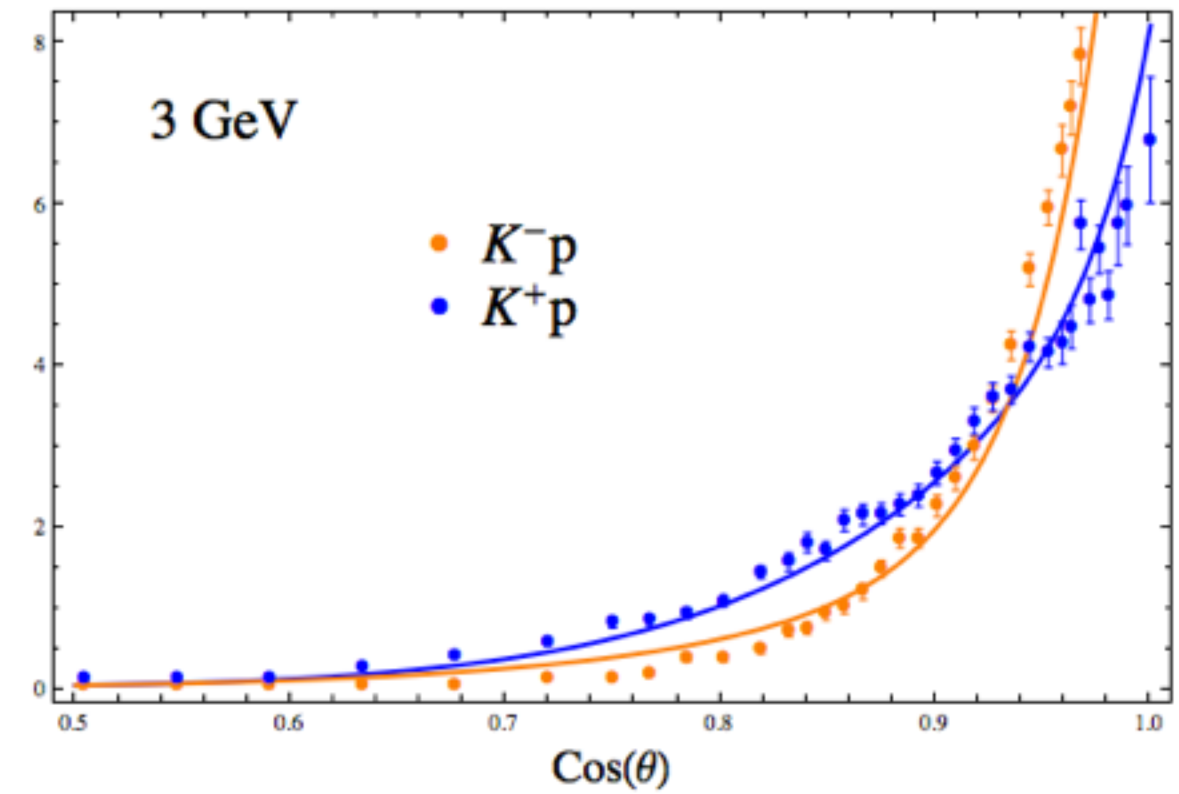
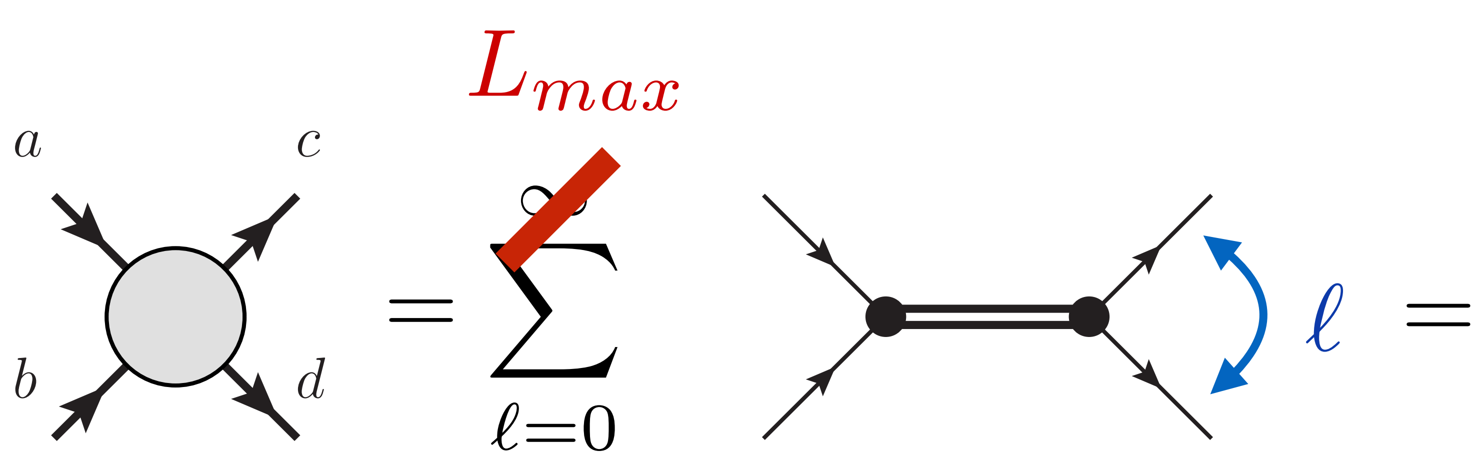


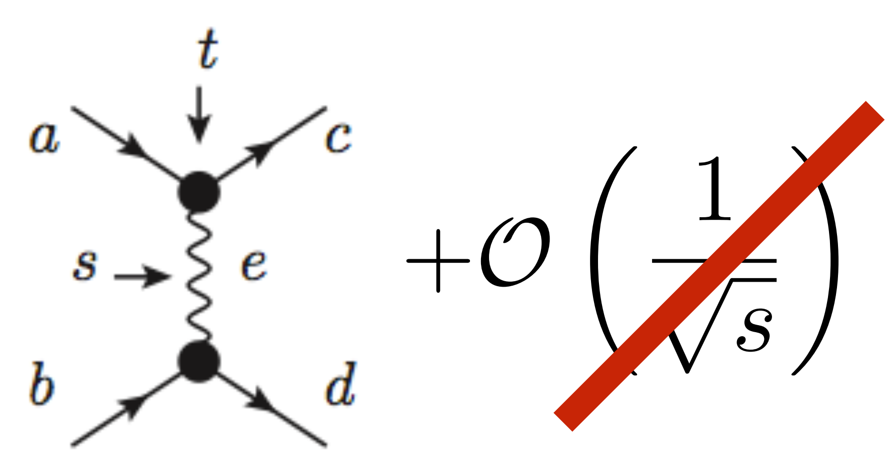
VM (unpublished)



Partial wave expansion

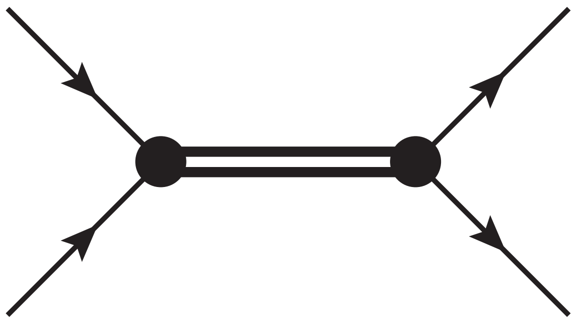


Regge pole expansion

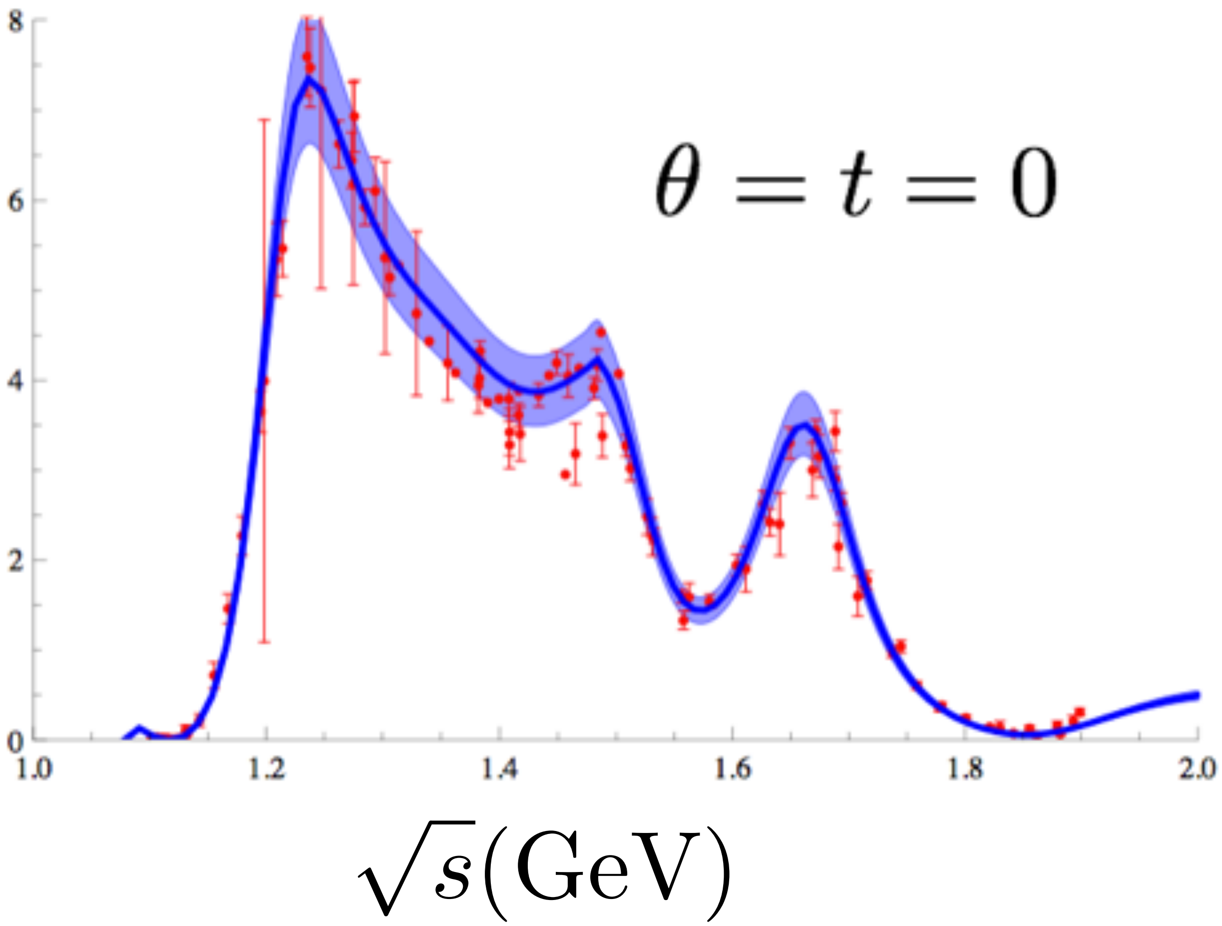


$\pi^- p \to \pi^0 n$

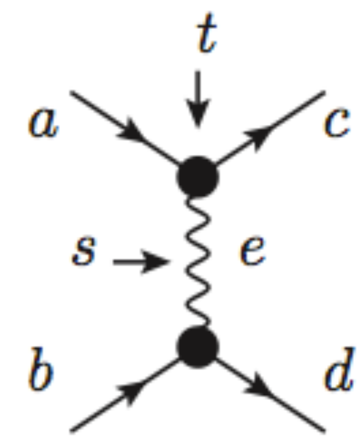
Low energy: baryon resonances

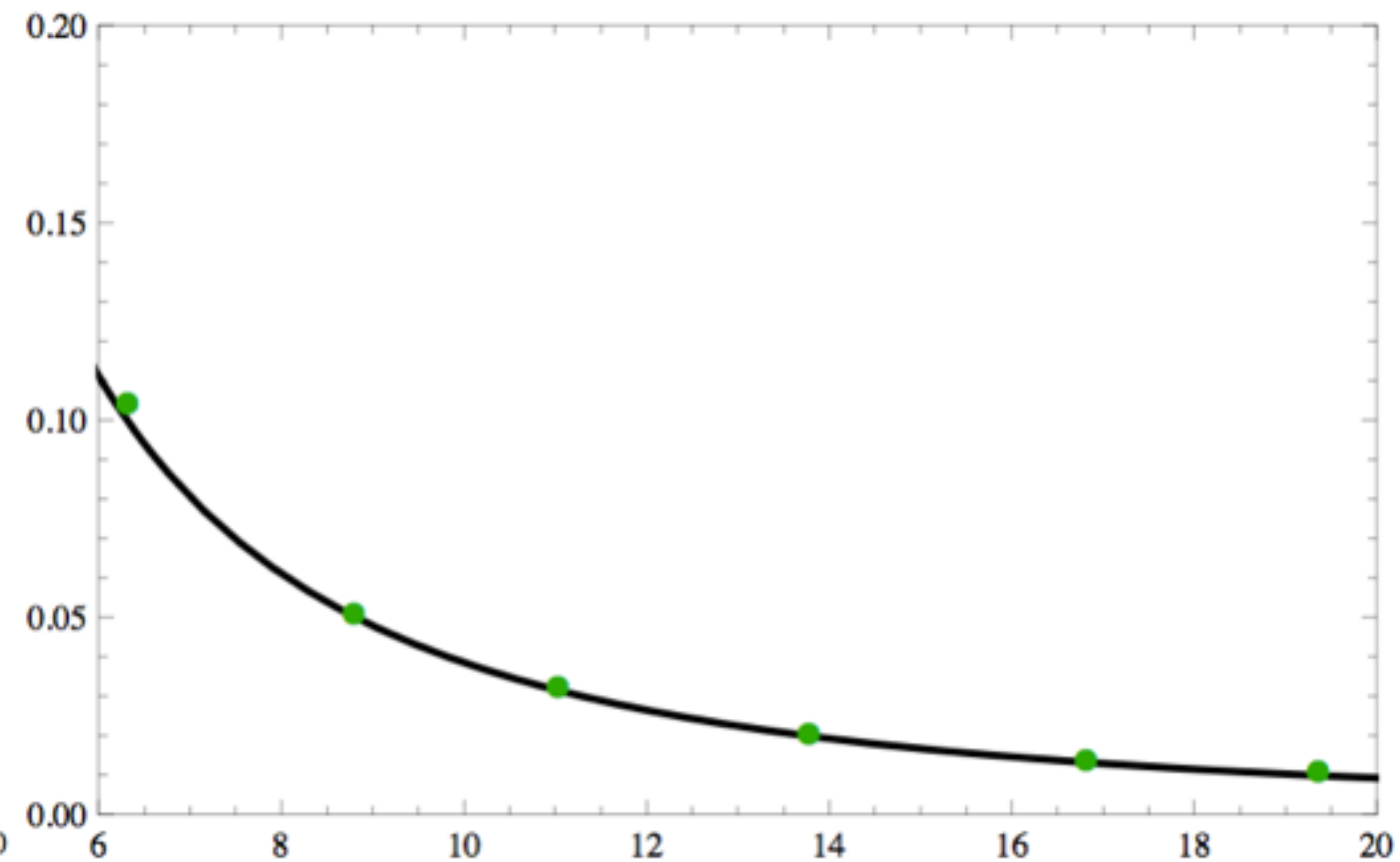


Total cross section



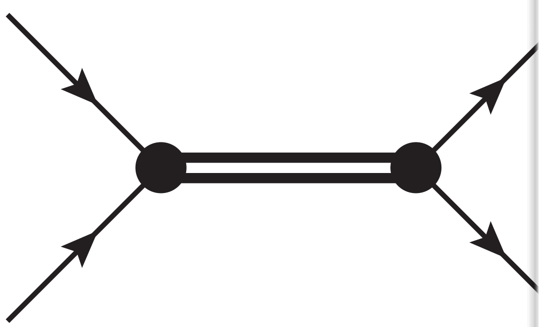
High energy: Regge exchange

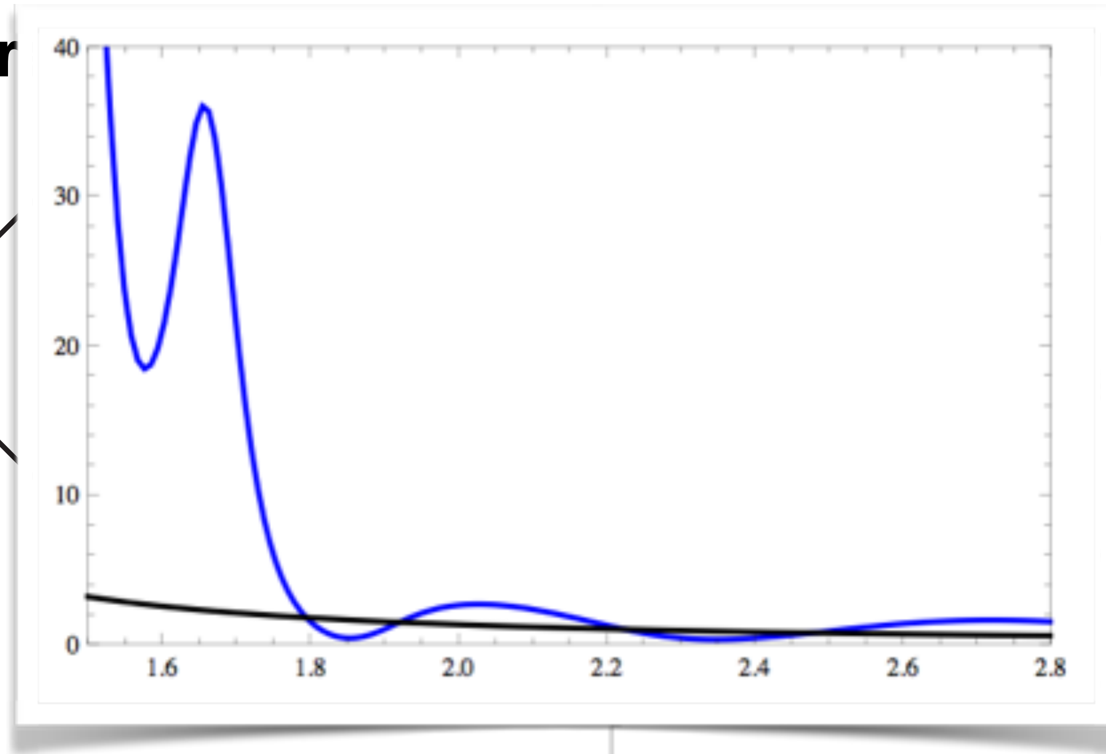




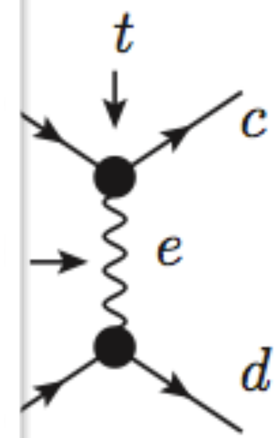
$\pi^- p \to \pi^0 n$

Low energy: **baryon r**

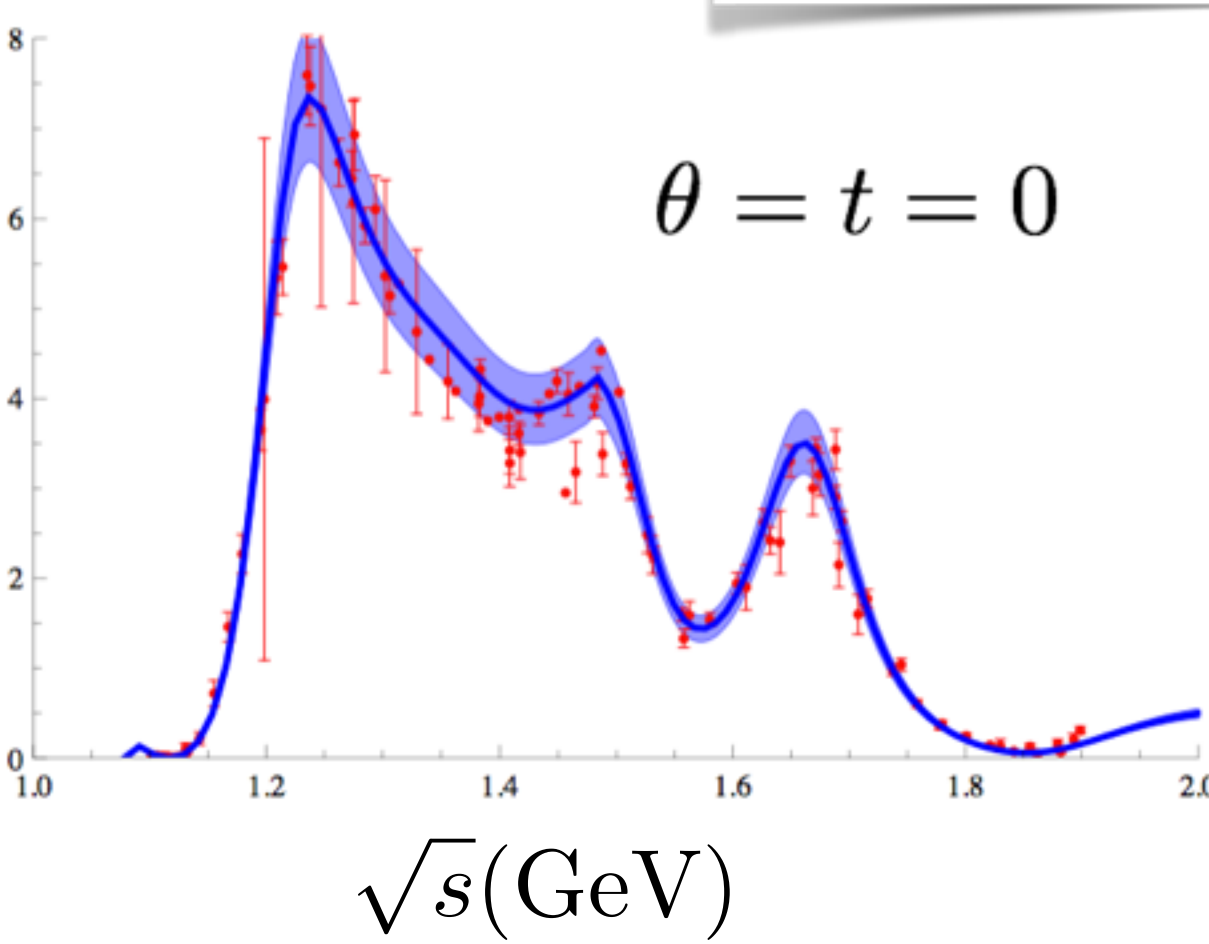


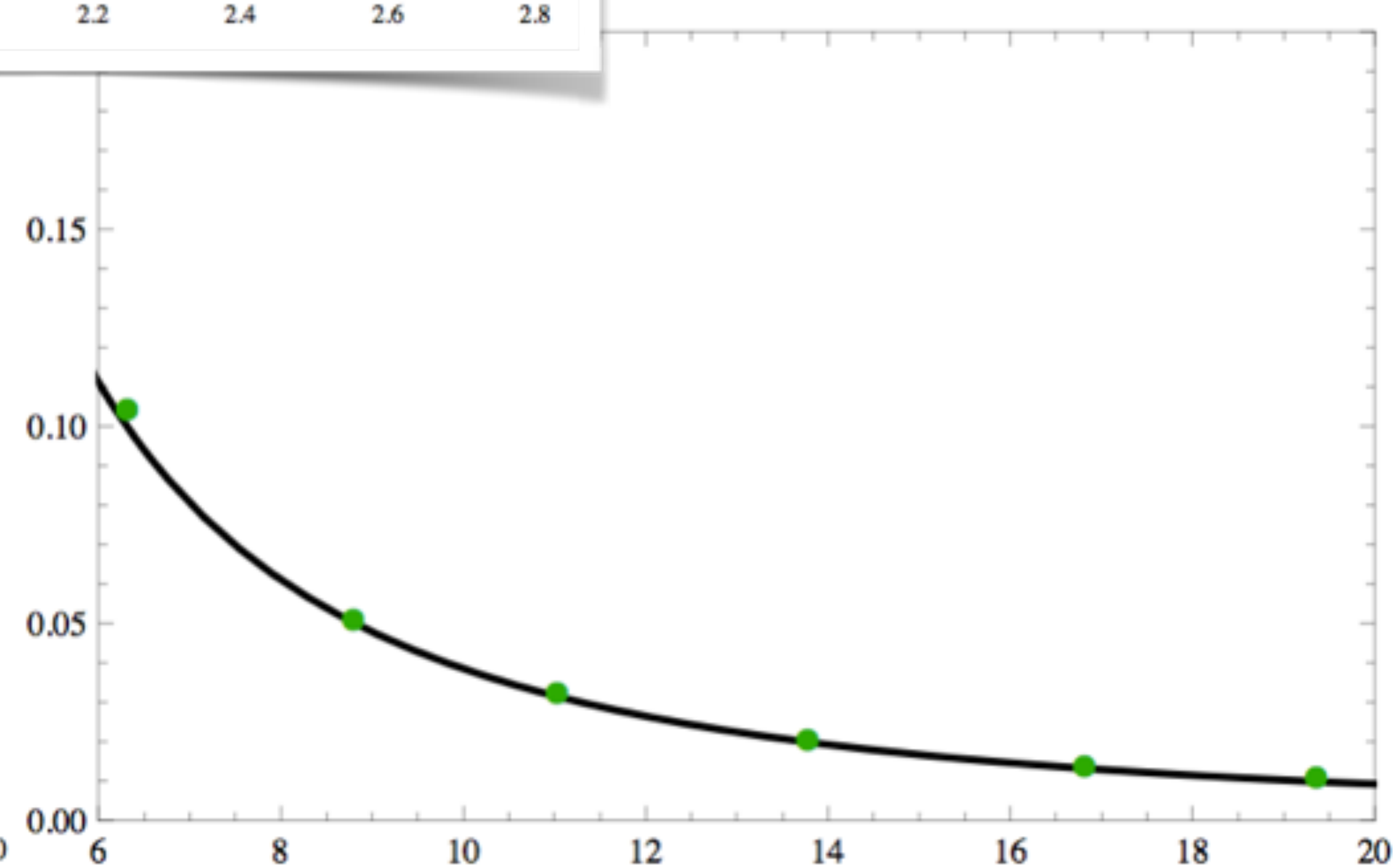


egge exchange



Total cross section





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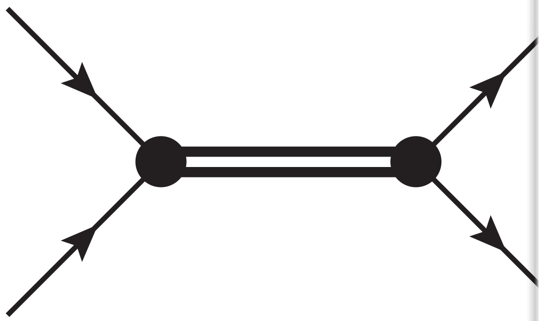
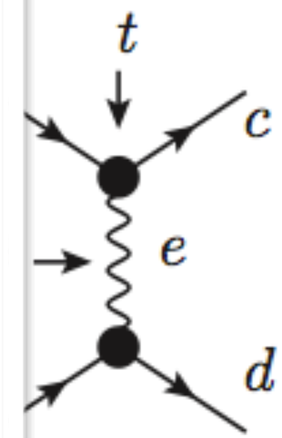
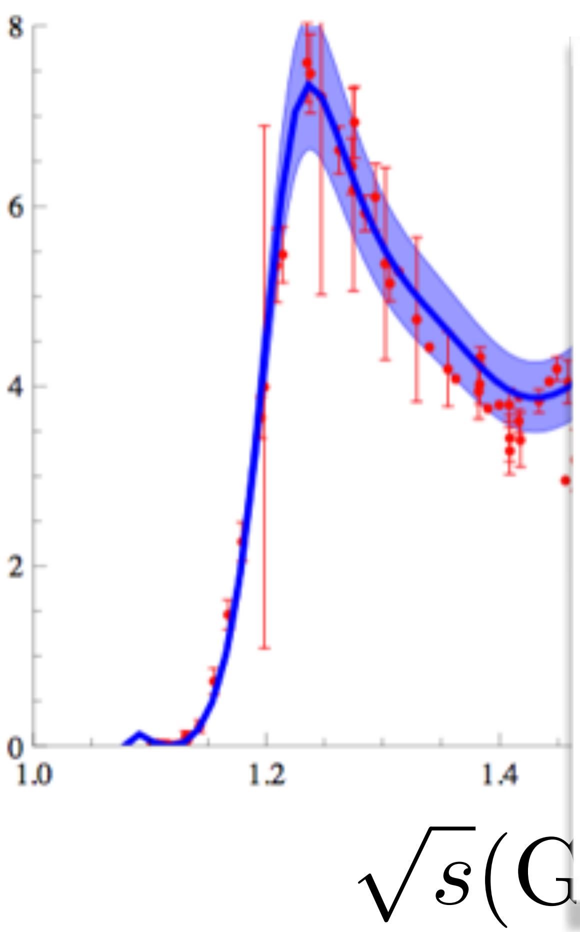
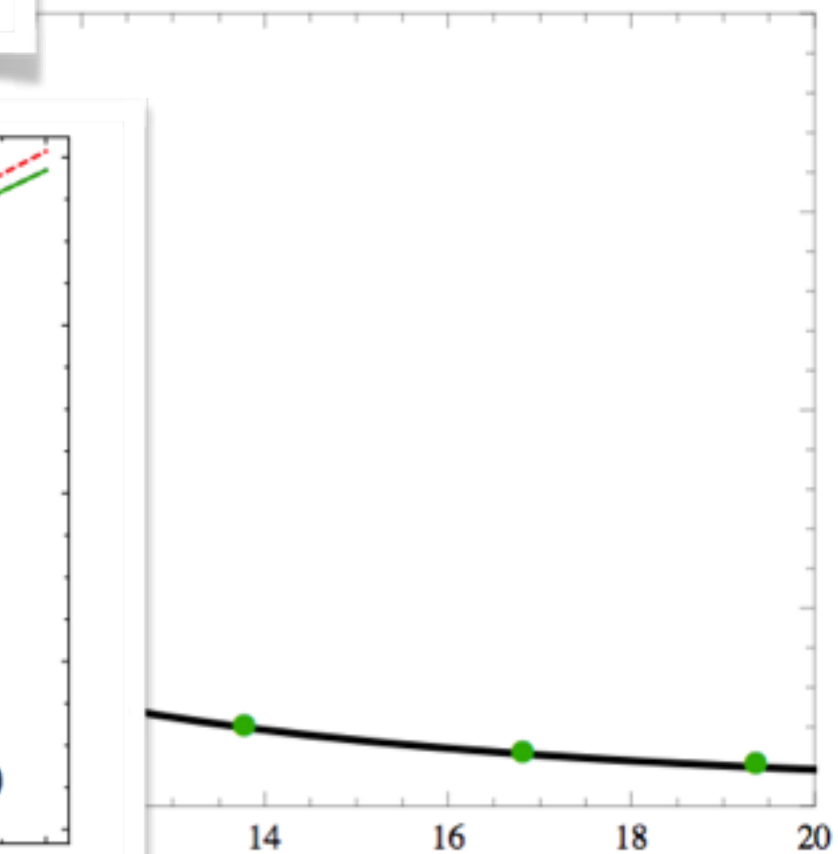


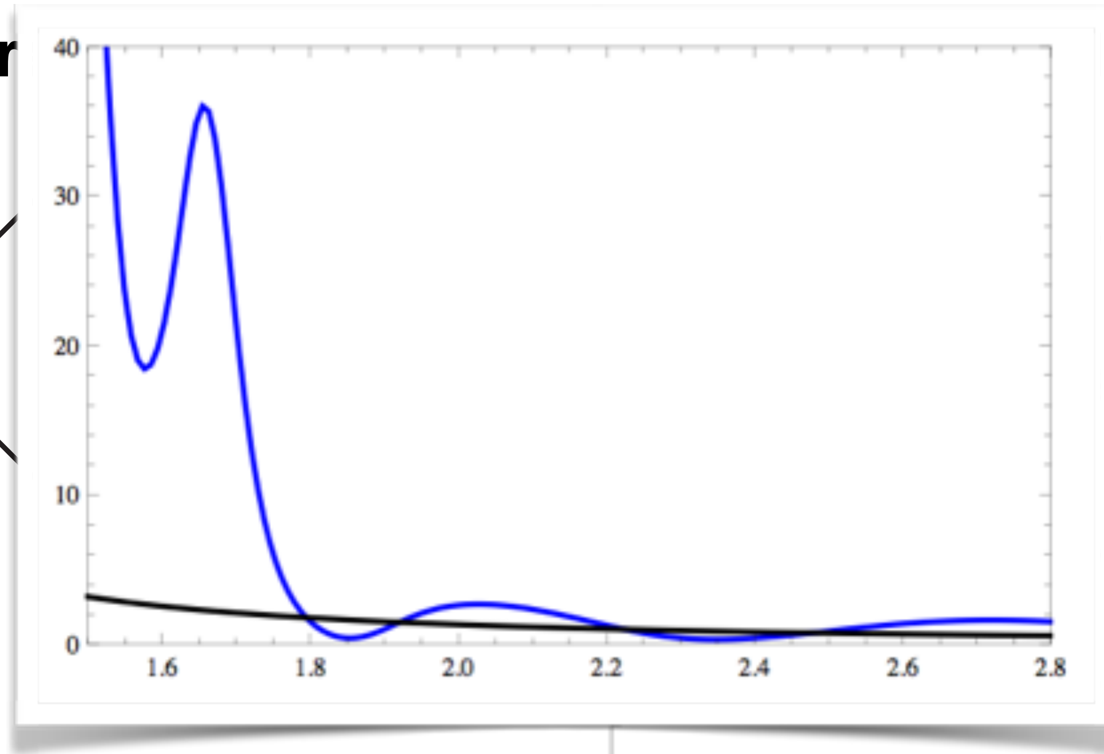
Regge exchange

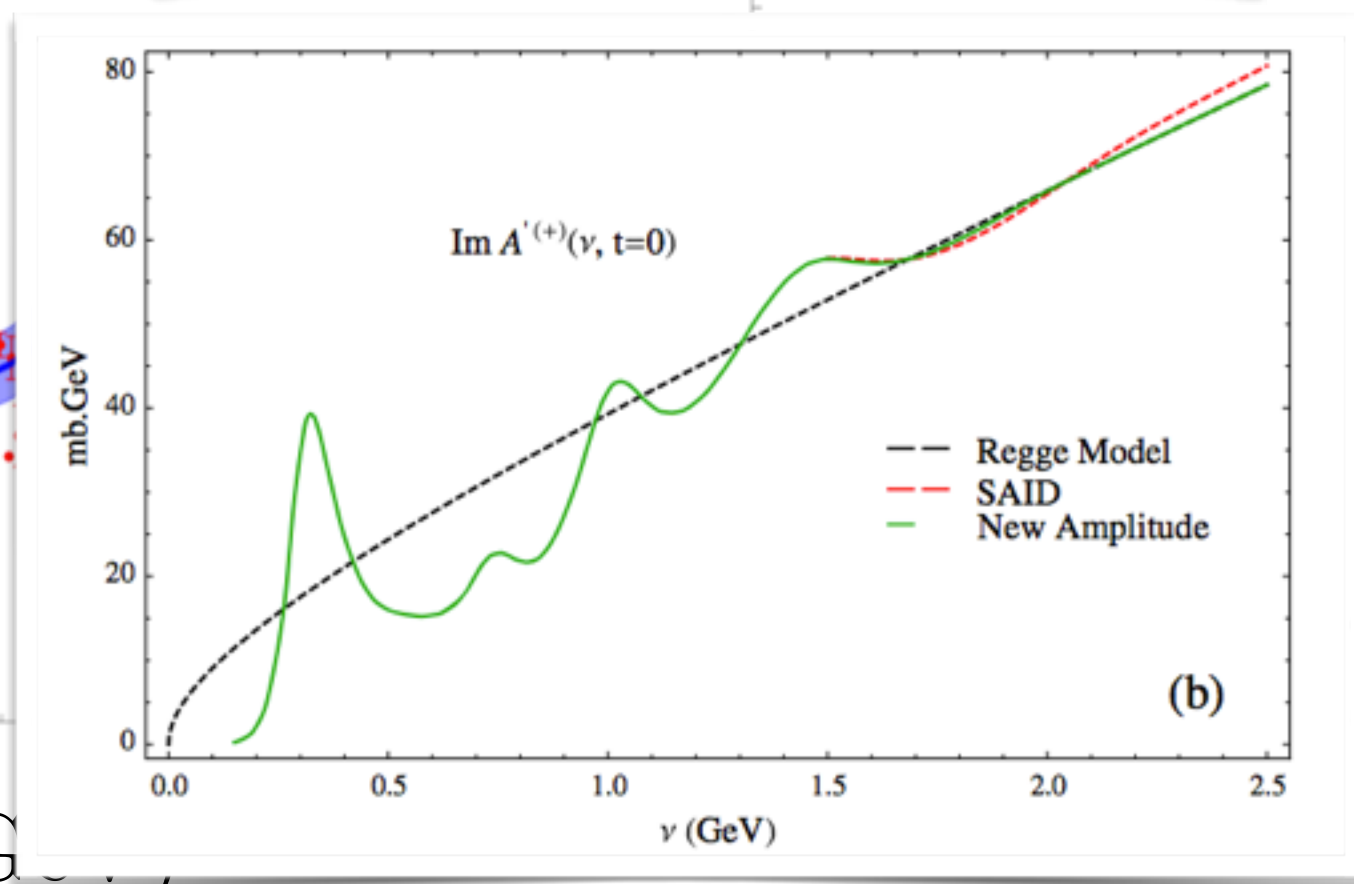


Total cross section

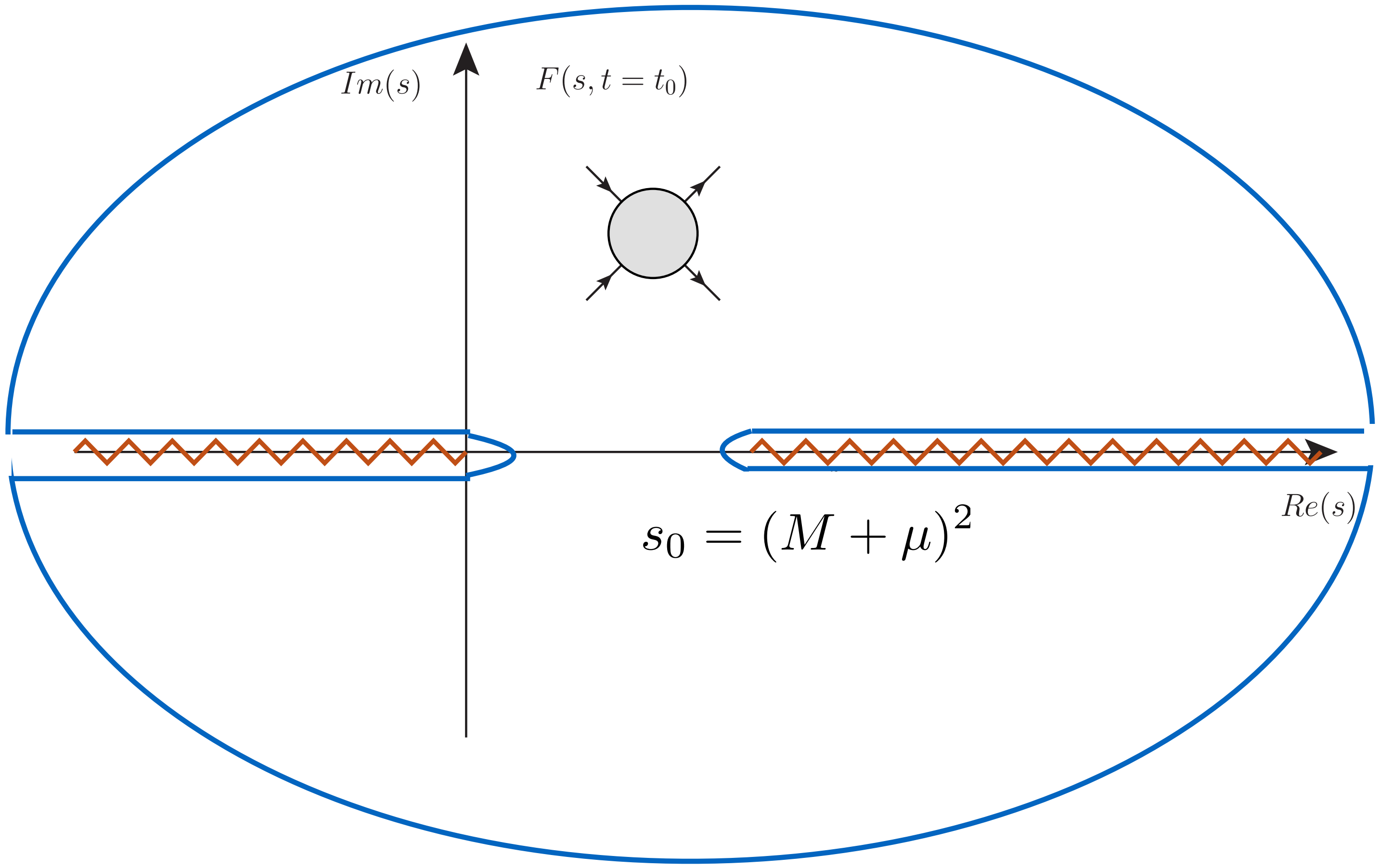








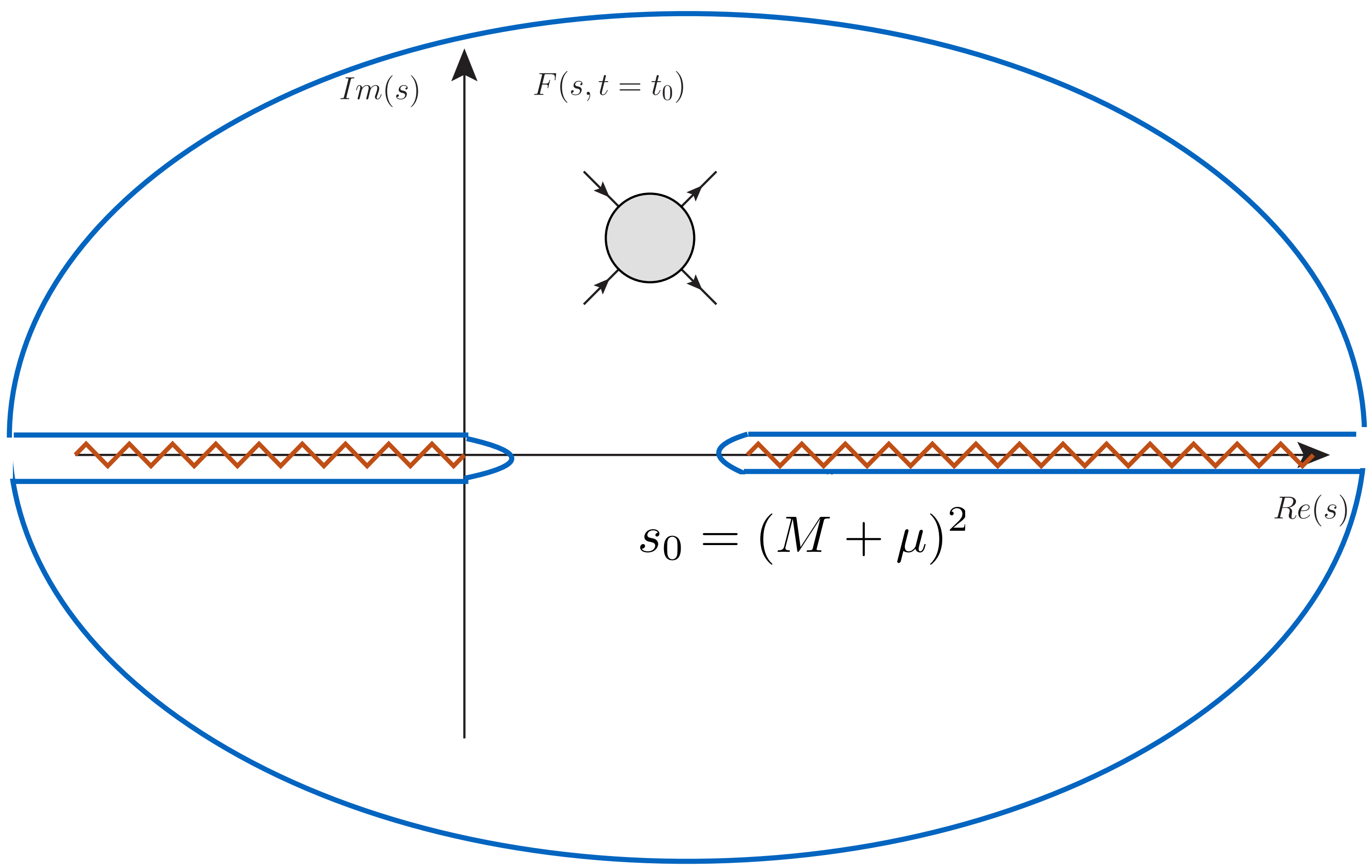
Dispersion Relation



Dispersion Relation

$$A(s,t) = \frac{1}{\pi}\int_{s_0}^{\infty} \frac{\mathrm{Im}\, A(s',t)}{s'-s} ds' + \frac{1}{\pi}\int_{u_0}^{\infty} \frac{\mathrm{Im}\, A(u',t)}{u'-u} du'$$

$$u(s,t) = -s - t + 2M^2 + 2\mu^2$$



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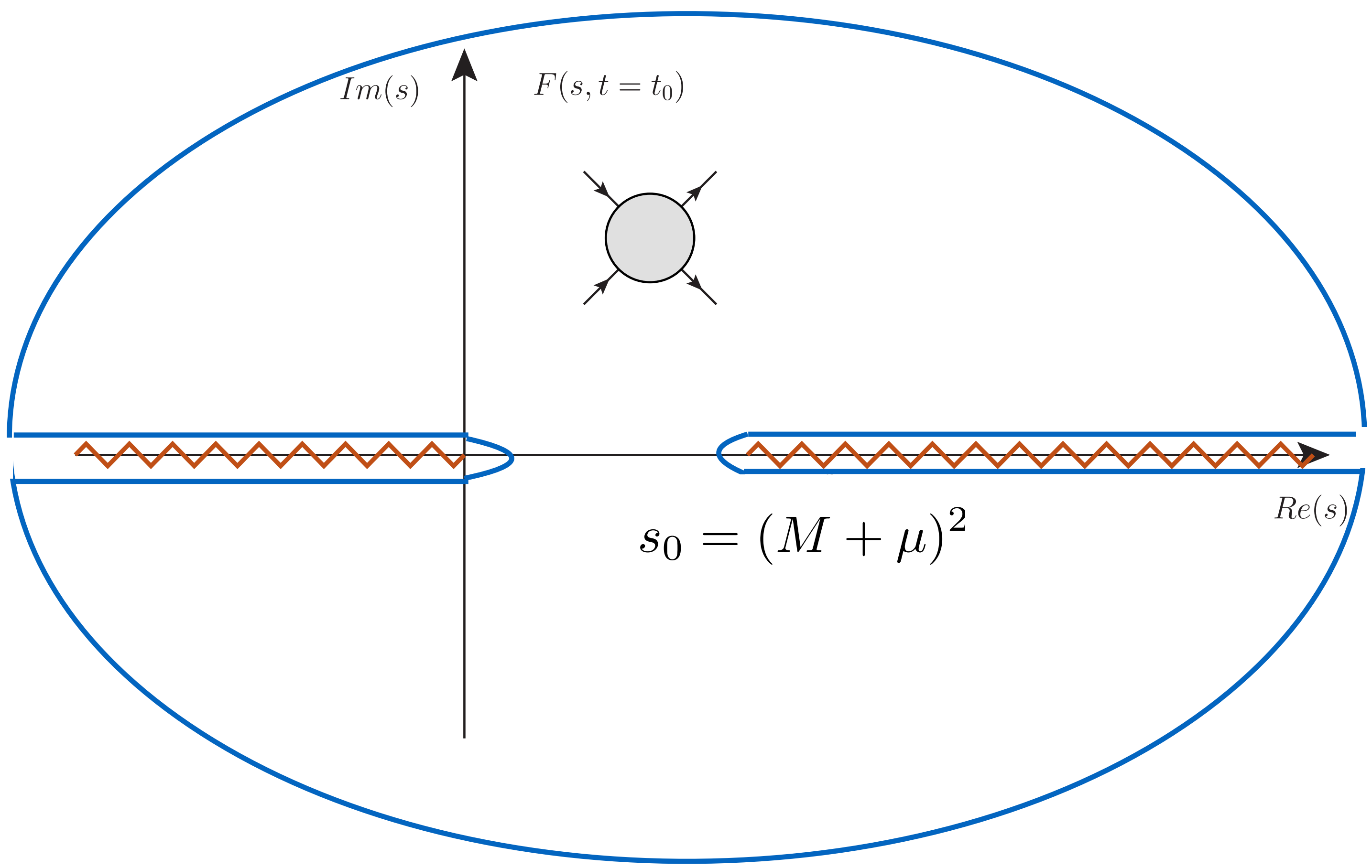


Introduce the crossing variable

$$\nu = \frac{s-u}{2}$$

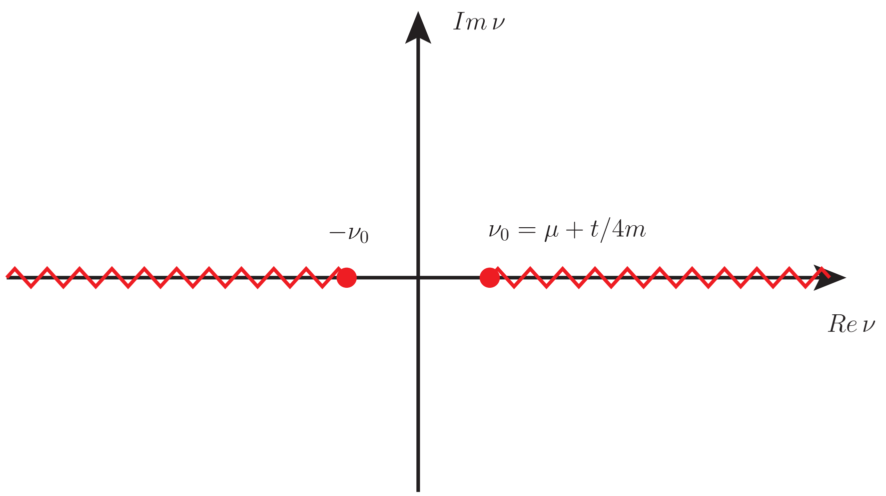
and combine the two cuts

$$A(\nu,t) = \frac{2}{\pi}\int_{\nu_0}^{\infty} \frac{\mathrm{Im}\,A(\nu',t)}{\nu'^2-\nu^2}\nu' d\nu'$$

Finite Energy Sum Rules

Satisfy dispersion relations

$$A(\nu, t) = \frac{2}{\pi} \int_{\nu_0}^{\infty} \frac{\text{Im}\, A(\nu', t)}{\nu'^2 - \nu^2} \nu' d\nu'$$



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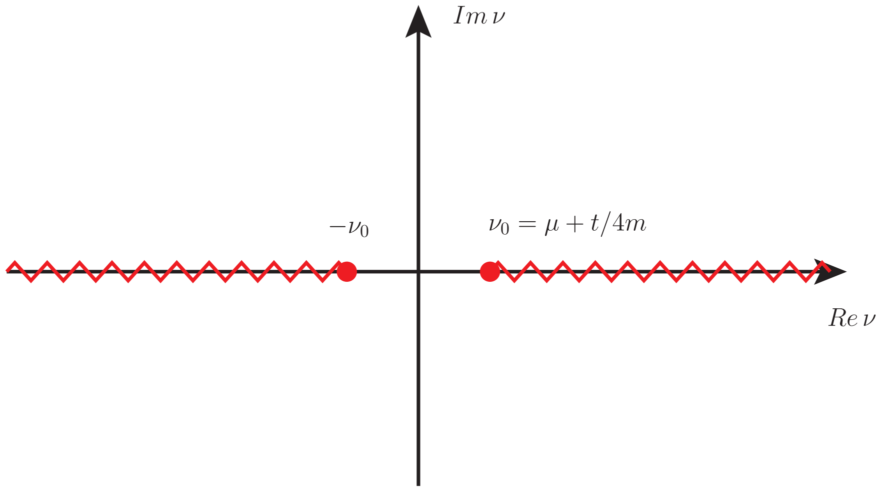


$$\nu \to \infty$$

$$\operatorname{Im} A(\nu, t) \longrightarrow \beta(t) \nu^{\alpha(t)}$$

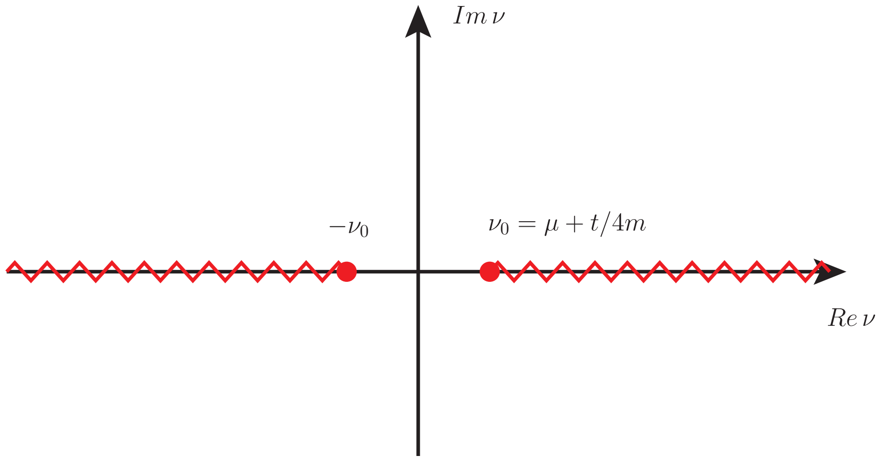
Finite Energy Sum Rules

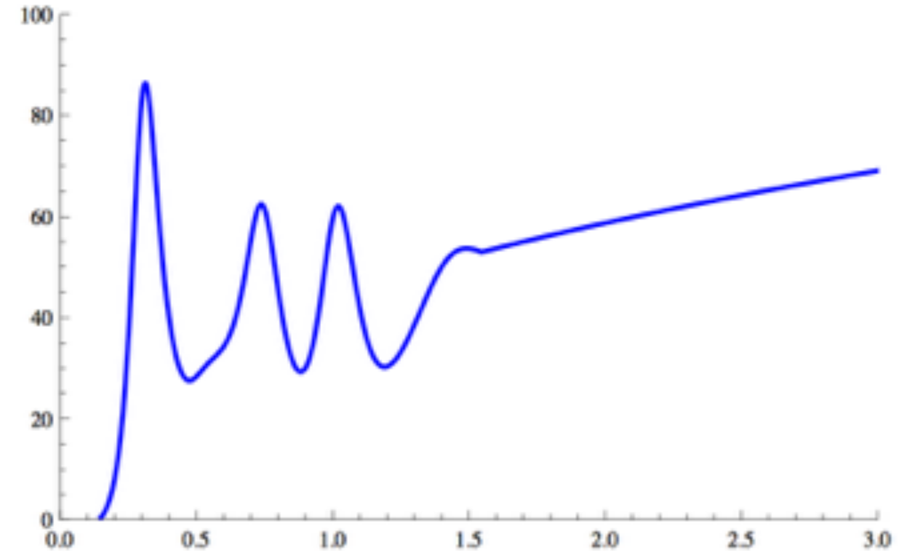
Satisfy dispersion relations

$$A(\nu,t) = \frac{2}{\pi}\int_{\nu_0}^{\infty} \frac{\operatorname{Im} A(\nu',t)}{\nu'^2-\nu^2}\nu' d\nu'$$

$$\nu > \Lambda$$

$$\operatorname{Im} A(\nu,t) \longrightarrow \beta(t)\nu^{\alpha(t)}$$





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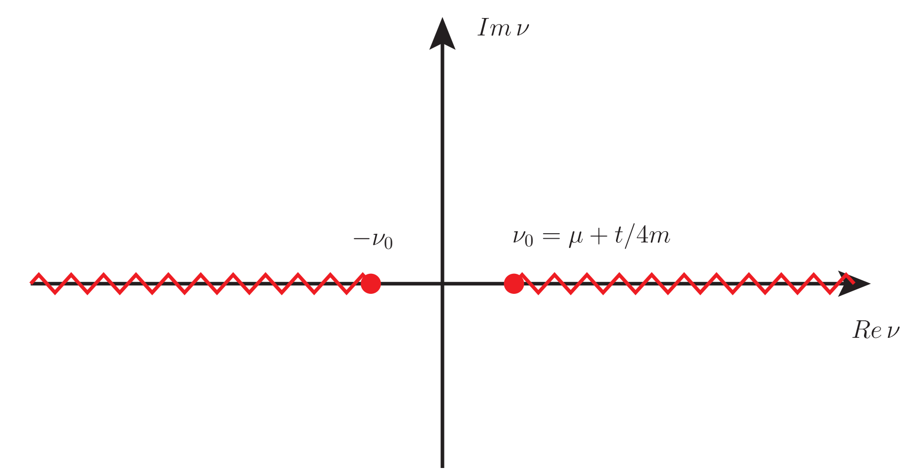

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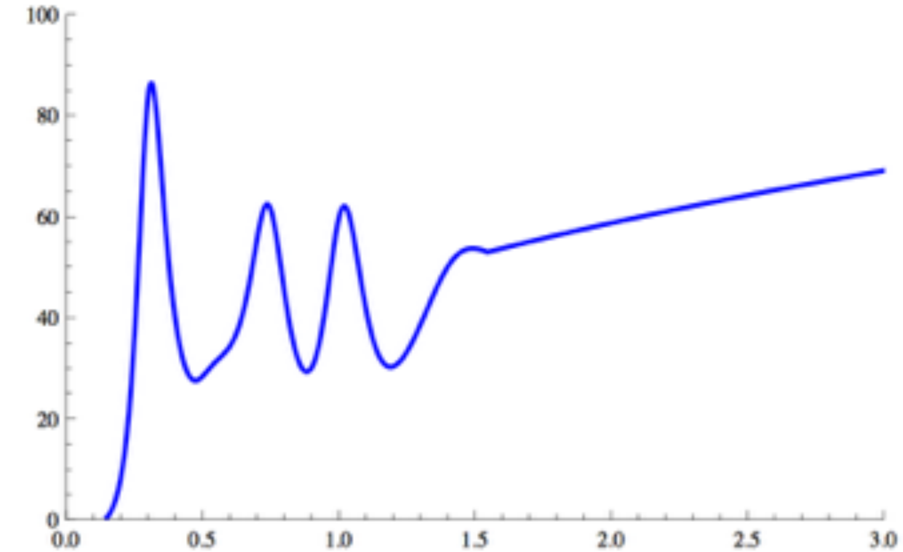
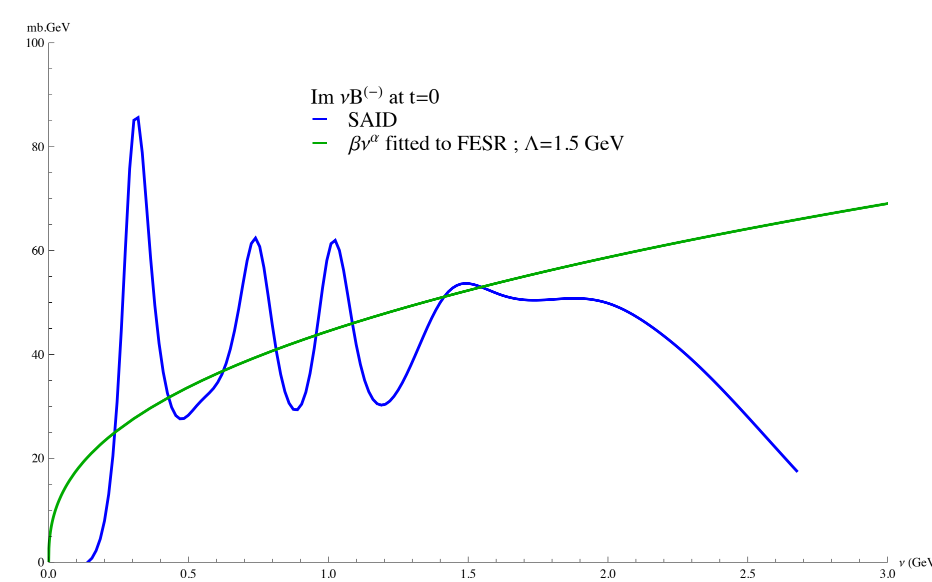


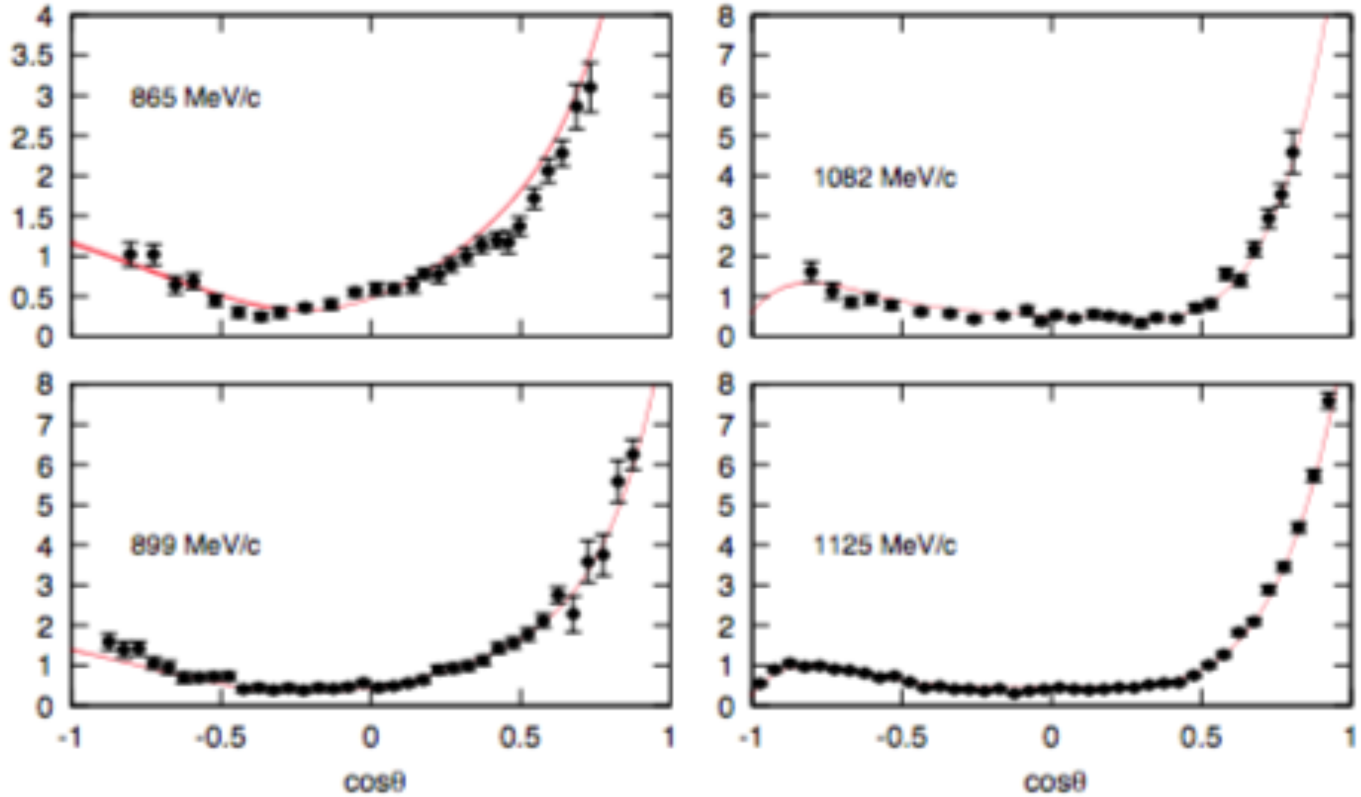
Analyticity implies FESR

$$\frac{1}{\Lambda^k} \int_{\nu_0}^{\Lambda} \text{Im}\ A(\nu, t)\nu^k d\nu = \frac{\beta(t)\Lambda^{\alpha(t)+1}}{\alpha(t) + k + 1}$$

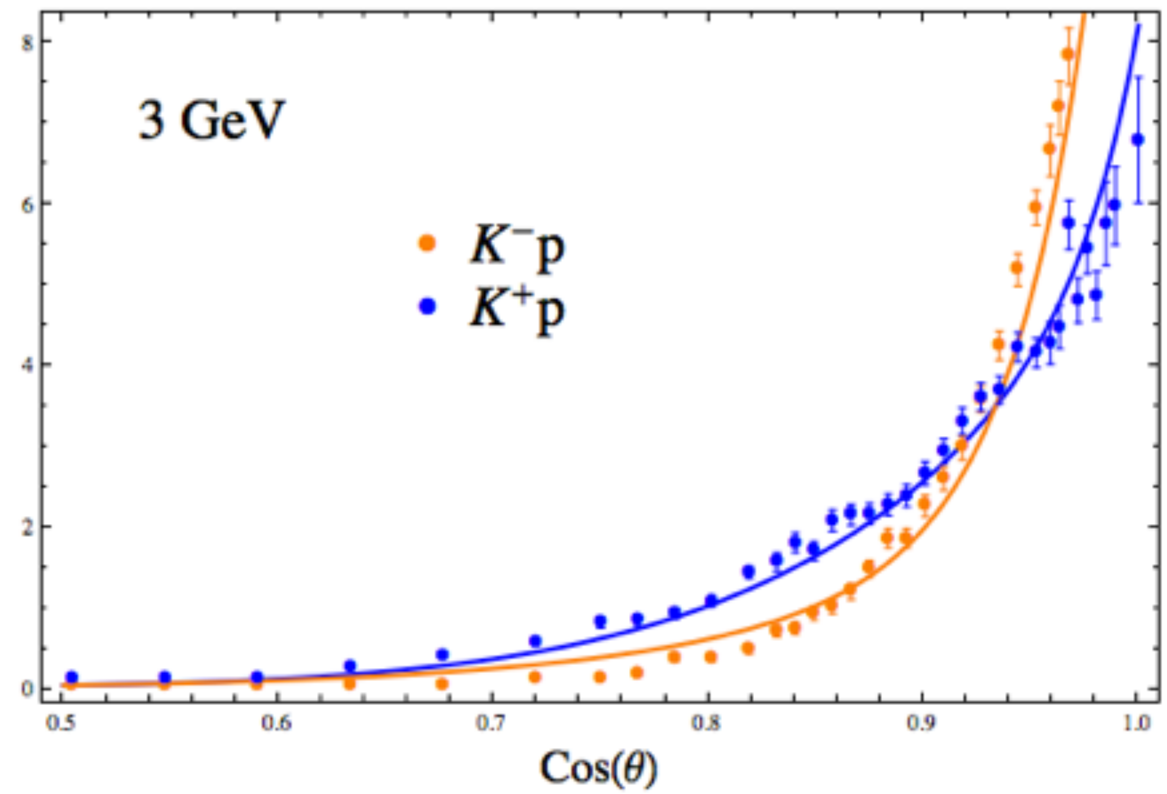


How to Use the FESR ?

C. Fernandez-Ramirez et al. (JPAC) ArXiv:1510:07065



VM (unpublished)



$$\frac{1}{\Lambda^k}\int_{\nu_0}^{\Lambda} \mathrm{Im}\ A(\nu,t)\nu^k d\nu = \frac{\beta(t)\Lambda^{\alpha(t)+1}}{\alpha(t)+k+1}$$

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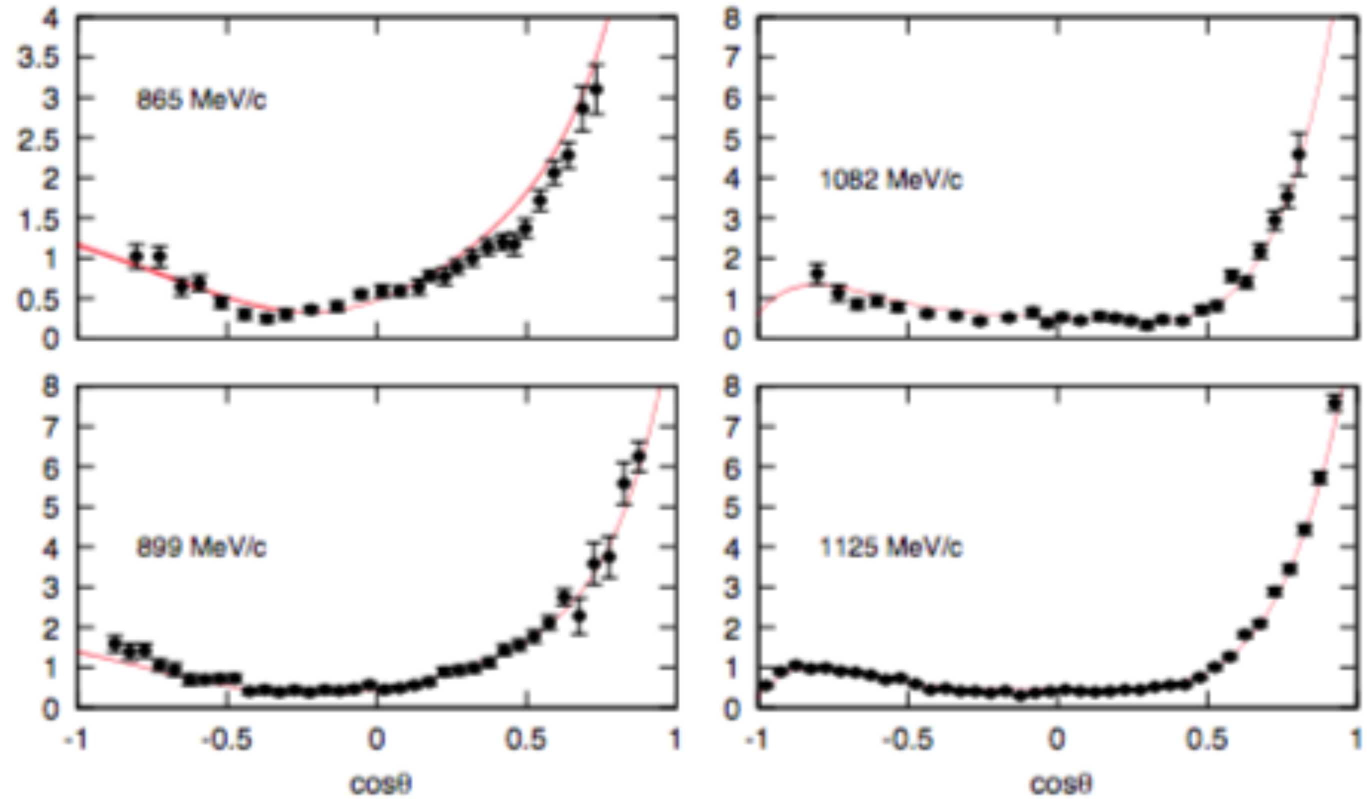

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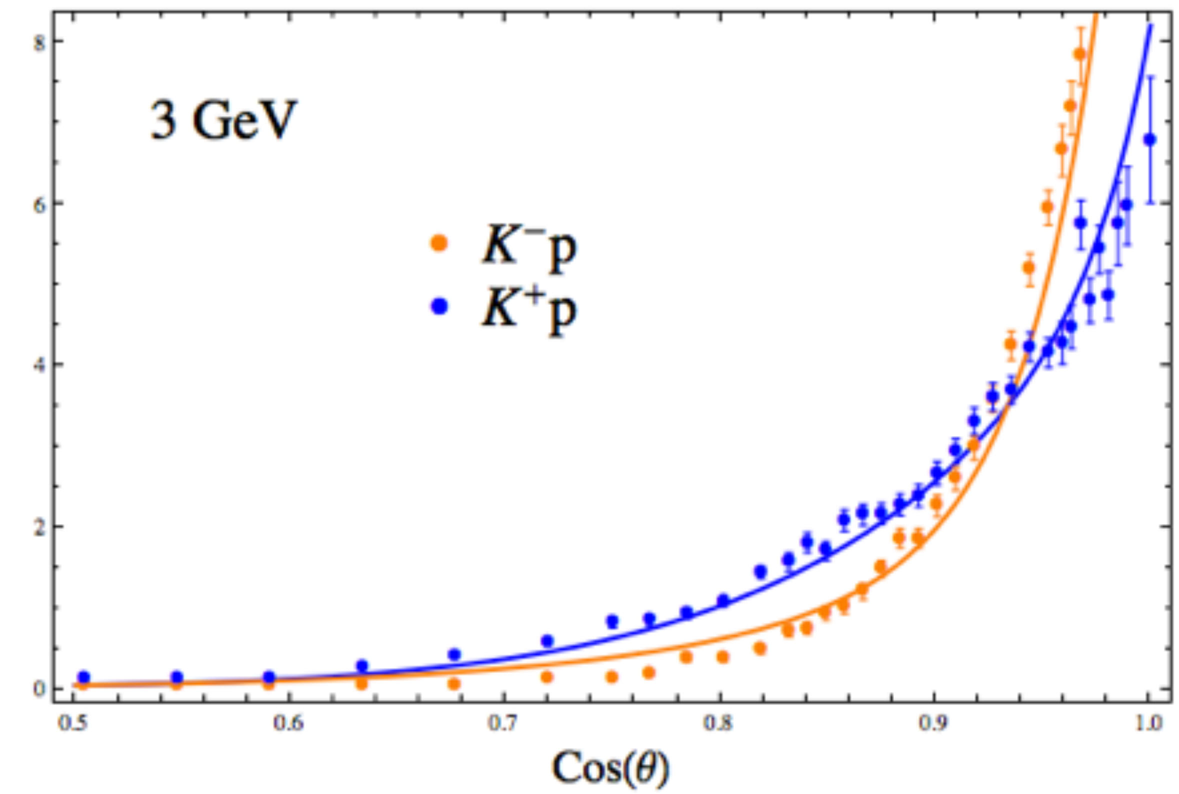


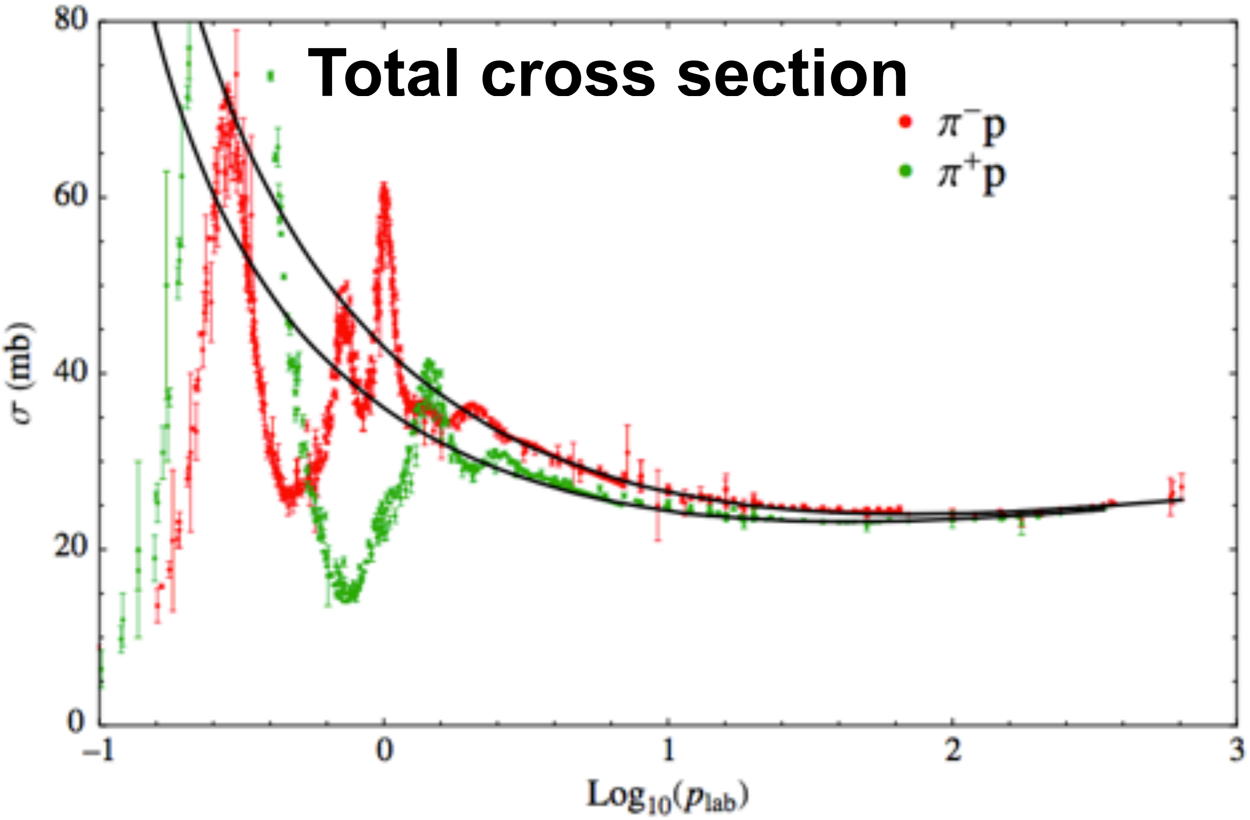
$$\frac{1}{\Lambda^k}\int_{\nu_0}^{\Lambda} \text{Im } A(\nu,t)\nu^k d\nu = \frac{\beta(t)\Lambda^{\alpha(t)+1}}{\alpha(t)+k+1}$$

FESR provides constraint on the low energy fit (that determines resonances parameters)

High energy fit determines $\beta(t)$ and $\alpha(t)$

Application to πN : High Energy Fit

VM et al (JPAC) PRD92
arXiv:1506.01764



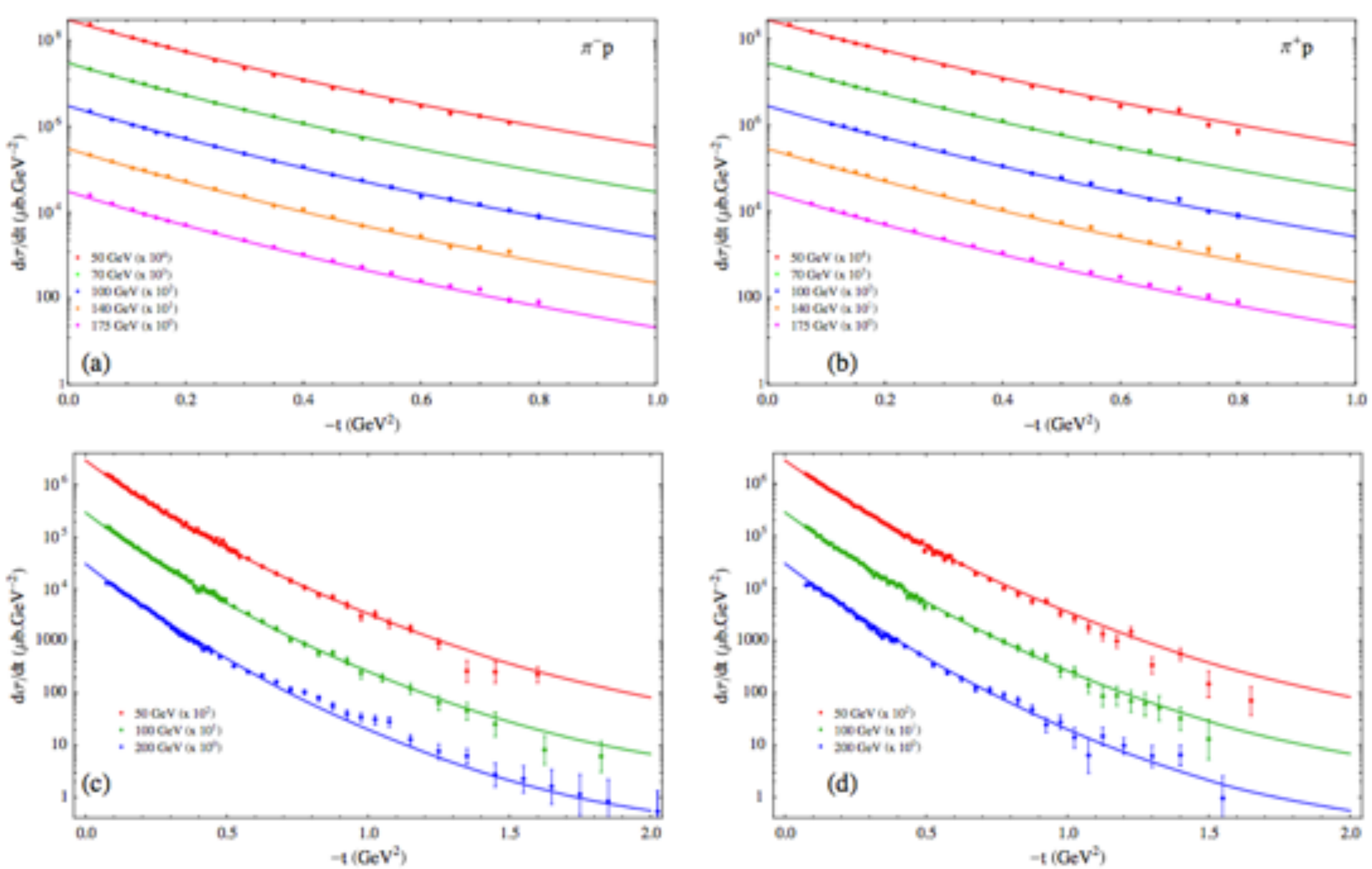
Fit to the world data on

$$\pi^{\pm} p \to \pi^{\pm} p$$

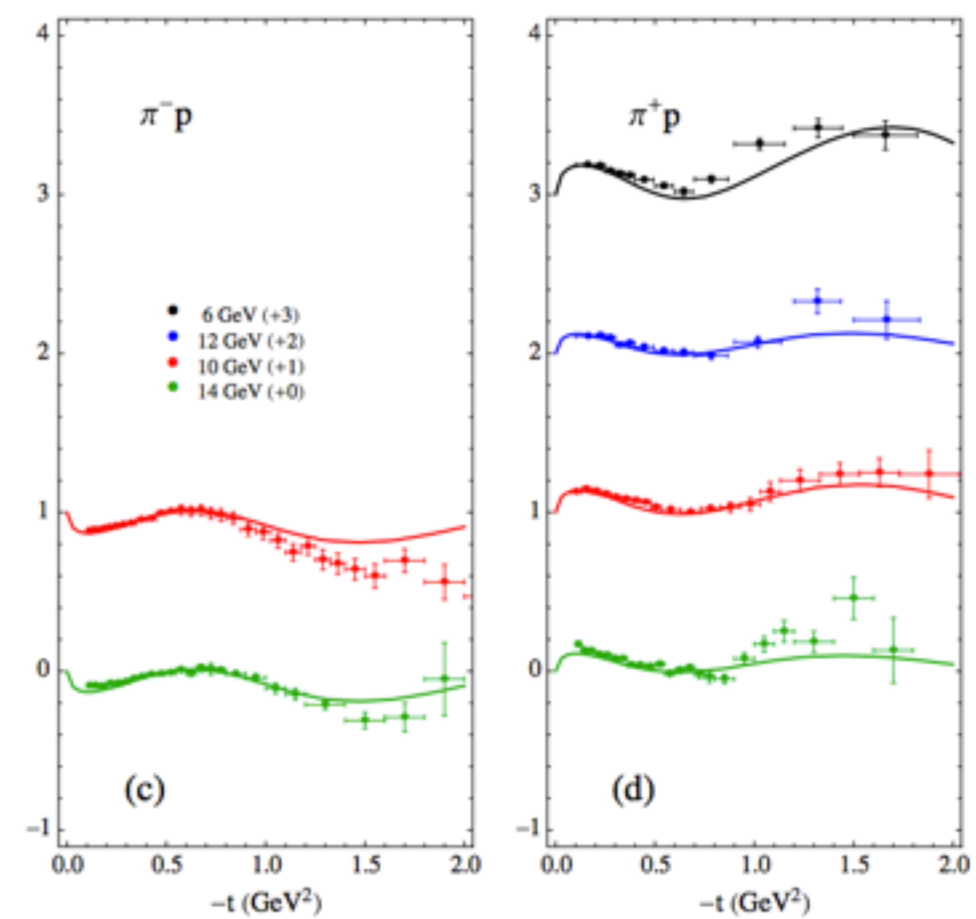
$$\pi^{-} p \to \pi^{0} n$$

for beam energy > 2 GeV

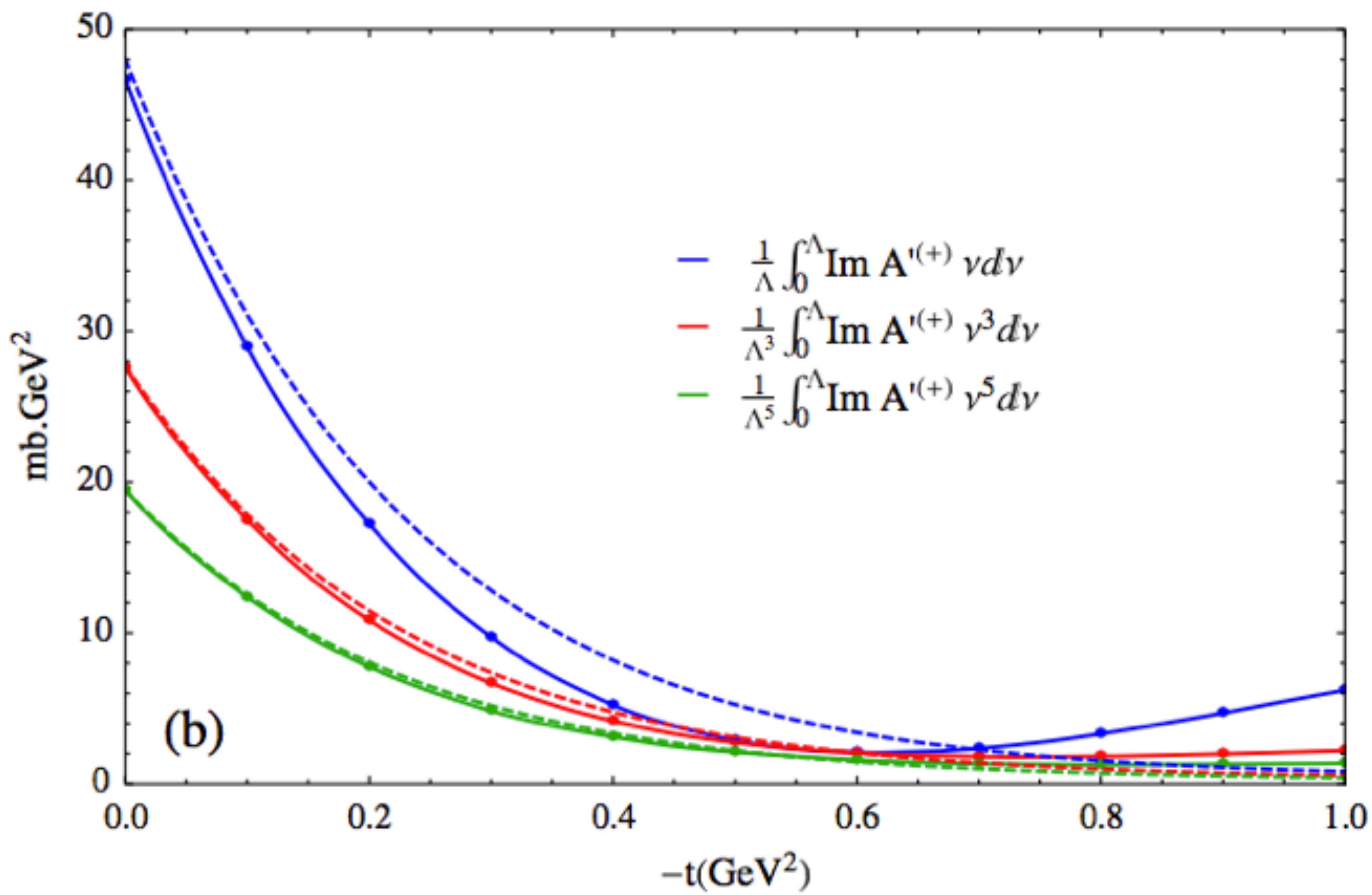
Differential cross section



Polarization observable



Let's compare both side of the sum rule



$$\frac{1}{\Lambda^k}\int_{\nu_0}^{\Lambda} \text{Im } A(\nu,t)\nu^k d\nu = \frac{\beta(t)\Lambda^{\alpha(t)+1}}{\alpha(t)+k+1}$$

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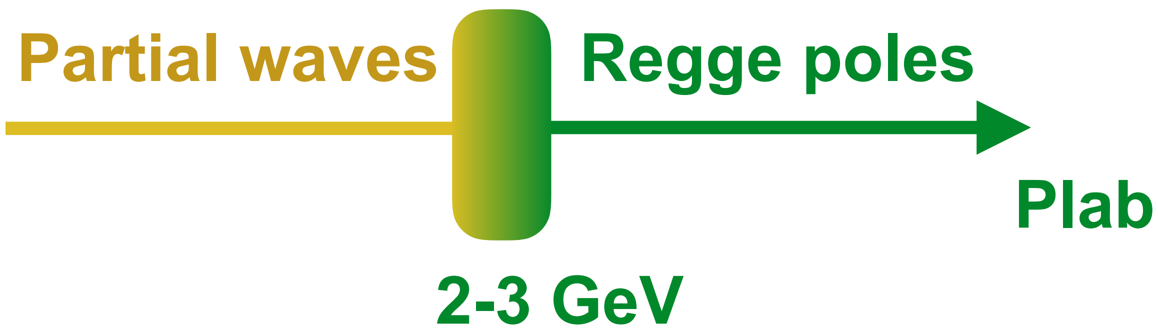
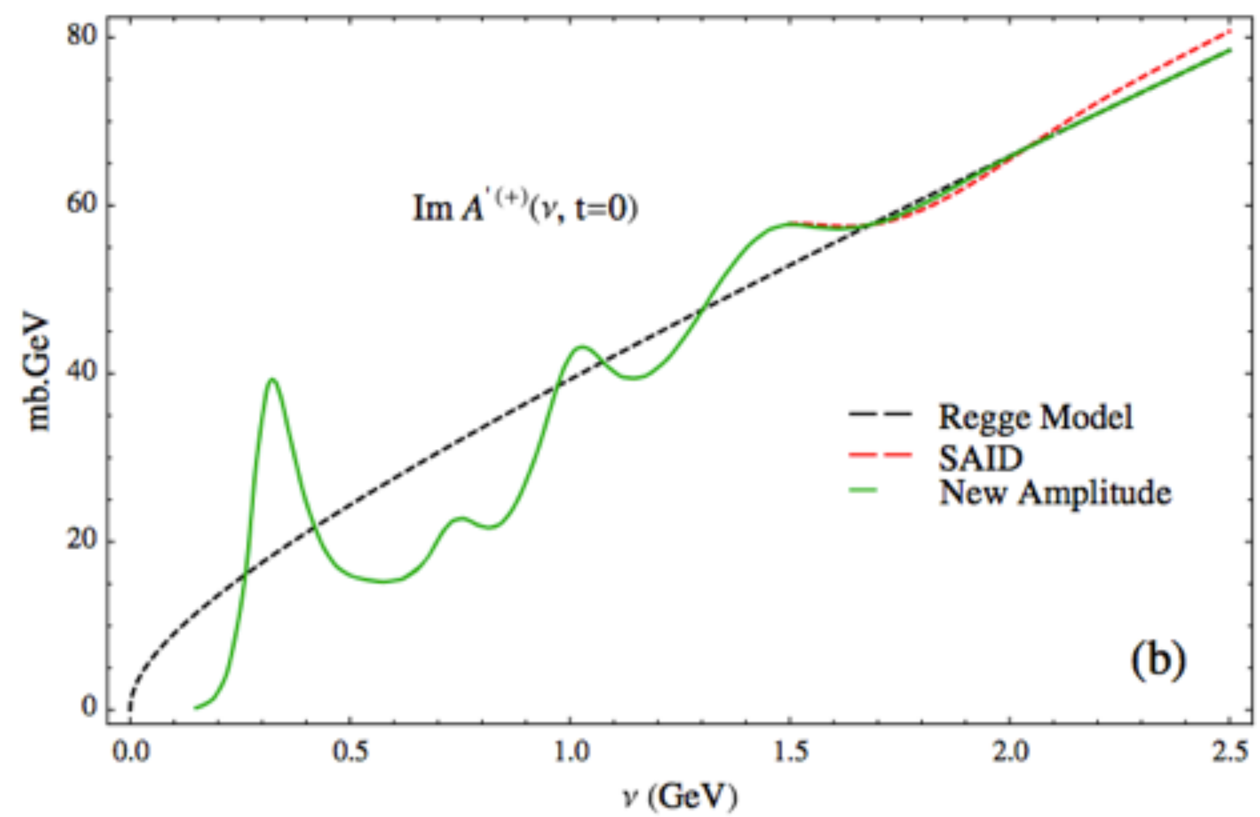
Checking Analyticity

Match low energy (PW) and high energy (Regge) imaginary parts





Reconstruct the real part from the dispersion relation

$$A(\nu, t) = \frac{2}{\pi} \int_{\nu_0}^{\infty} \frac{\mathrm{Im}\, A(\nu', t)}{\nu'^2 - \nu^2} \nu' d\nu'$$

VM et al (JPAC) PRD92 arXiv:1506.01764

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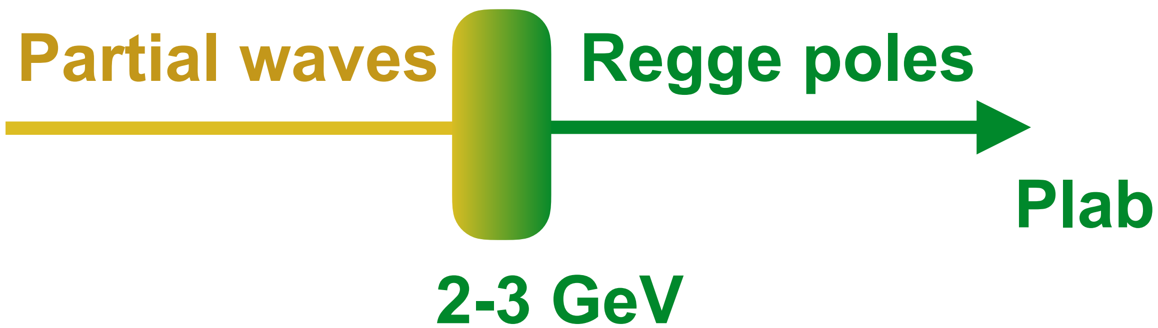

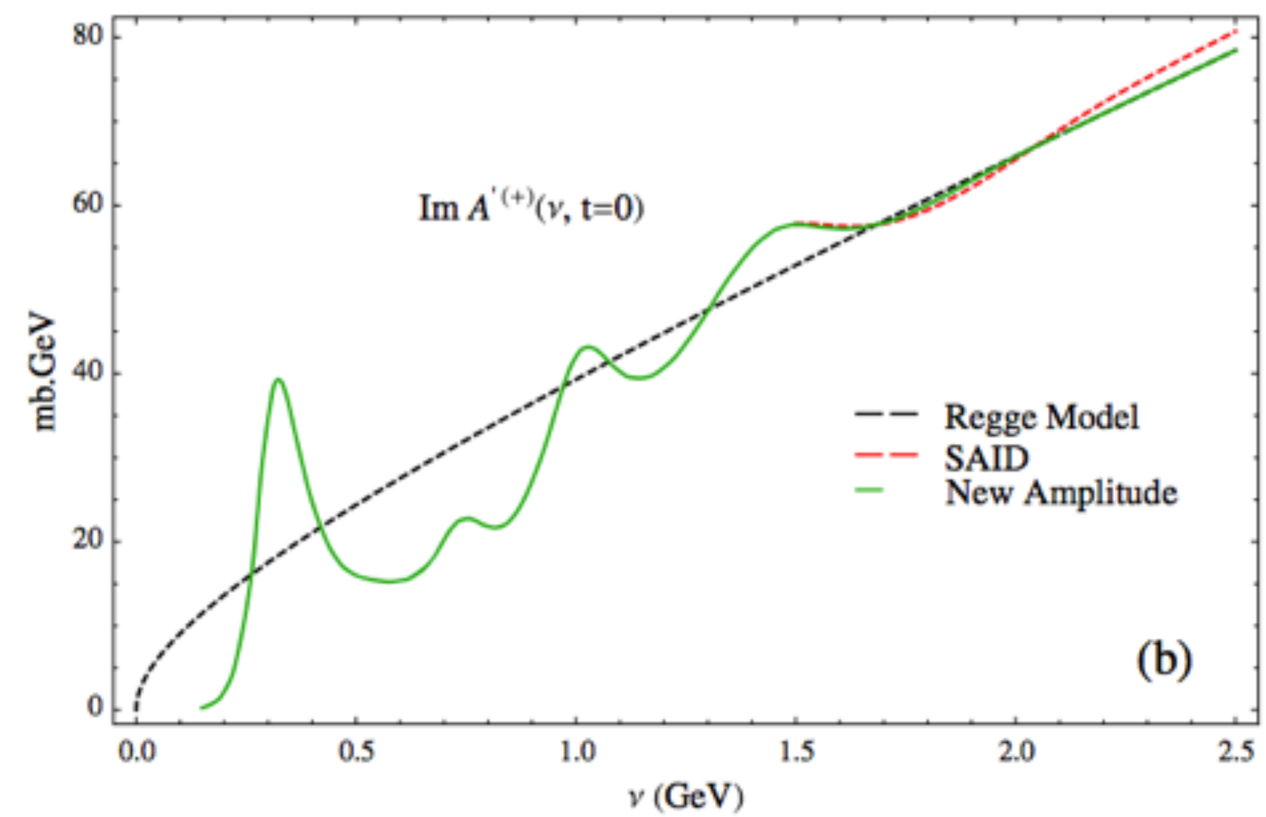

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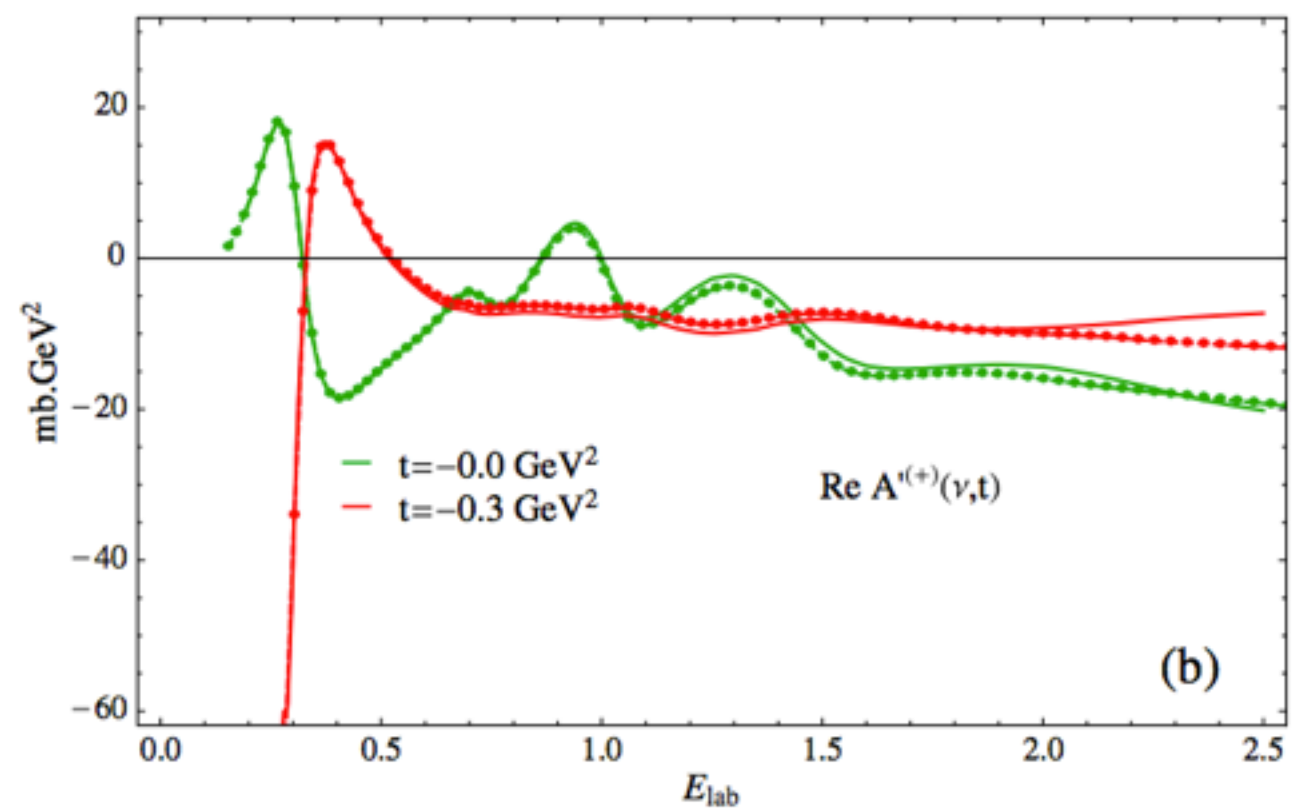
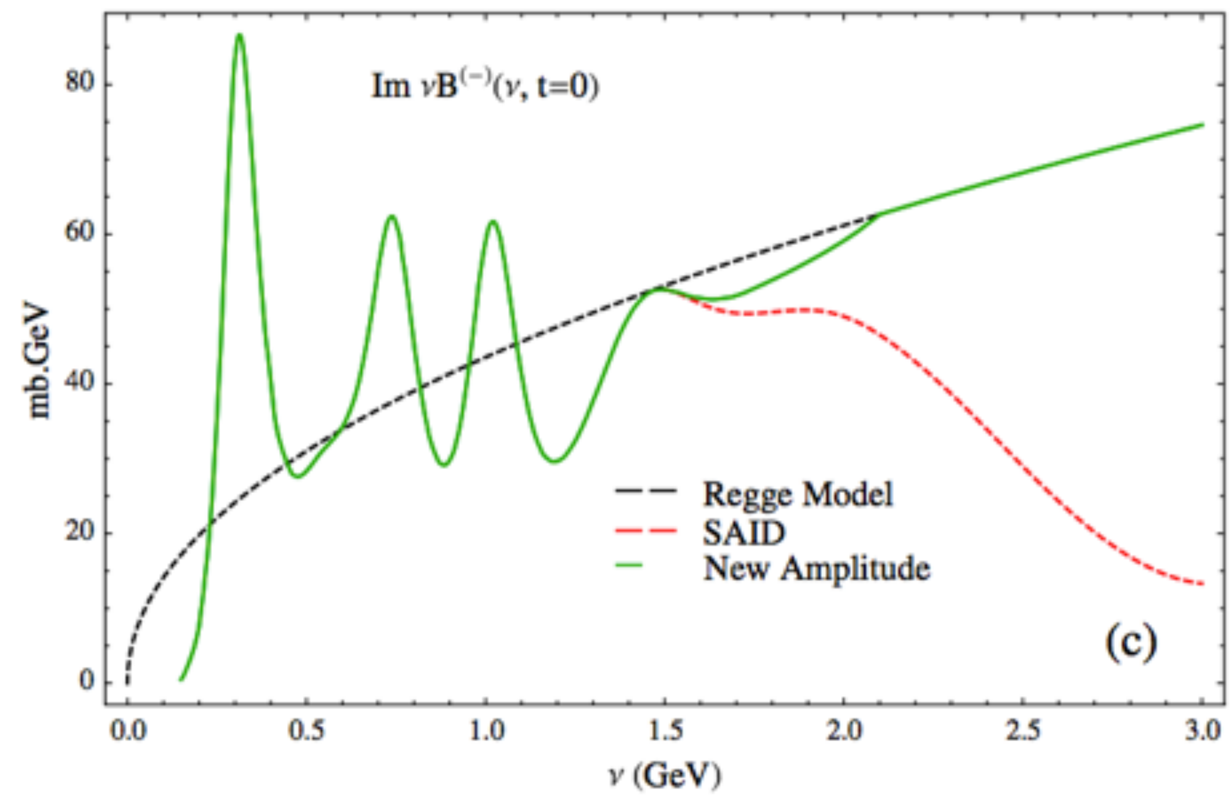
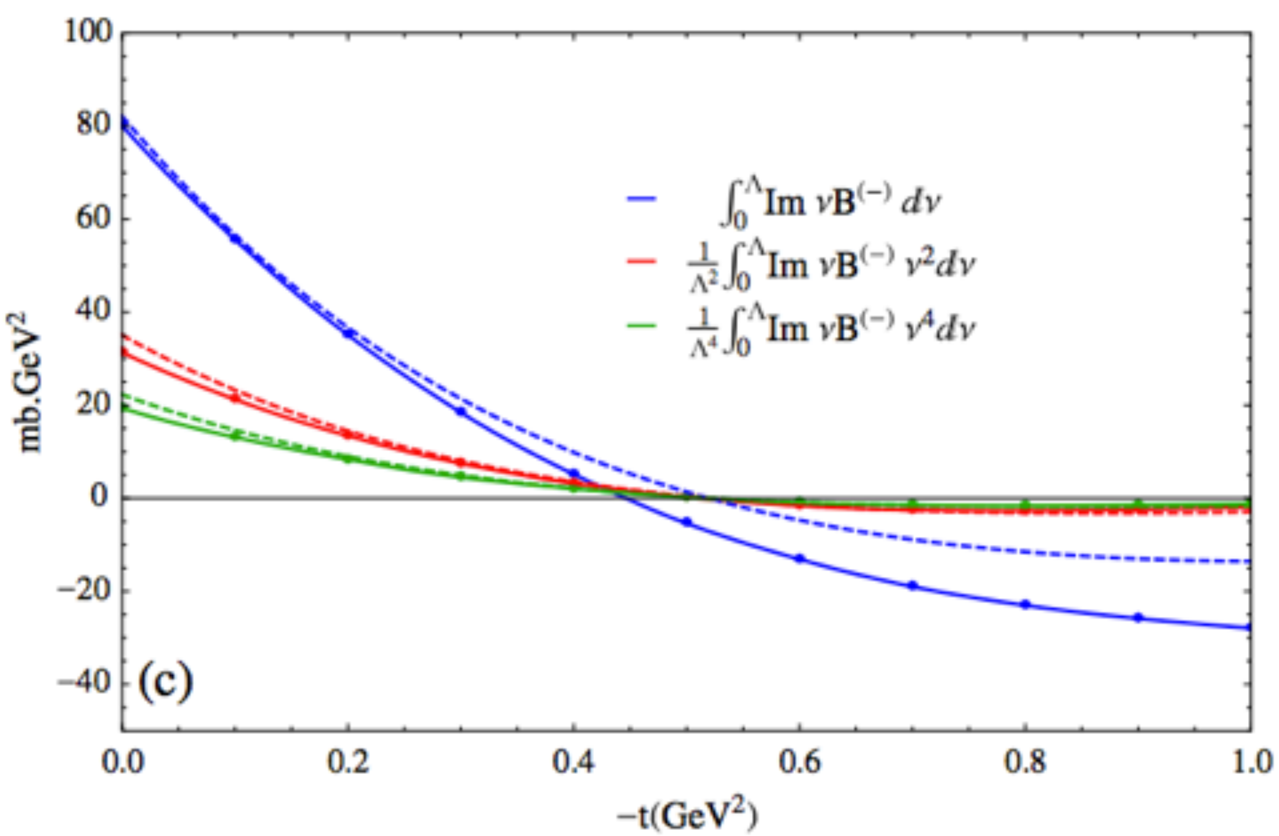


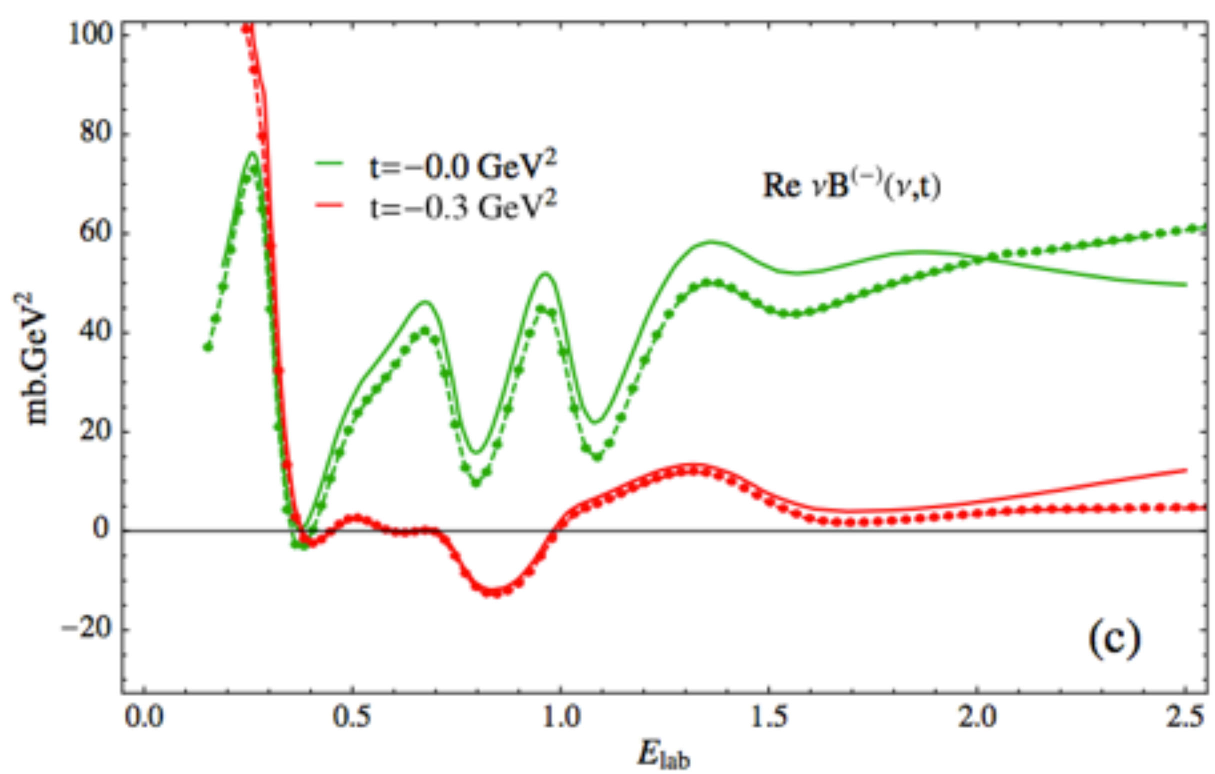
VM et al (JPAC) PRD92 arXiv:1506.01764

Similar results for the other amplitude

$$T = \bar{u}(p_4, \lambda_4)\left(A + \frac{1}{2}(\not{p}_1 + \not{p}_3)\, B\right) u(p_2, \lambda_2)$$







Summary: Methodology

VM et al (JPAC) PRD92
arXiv:1506.01764

Going beyond partial waves truncation

Use FESR to extrapolate to high energy

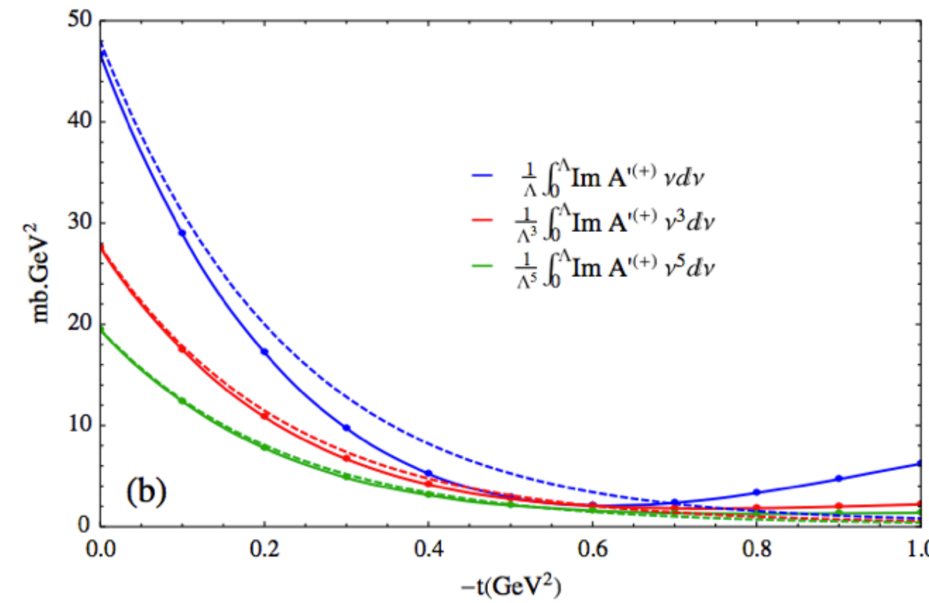
$$\frac{1}{\Lambda^k}\int_{\nu_0}^{\Lambda} \text{Im } A(\nu,t)\nu^k d\nu = \frac{\beta(t)\Lambda^{\alpha(t)+1}}{\alpha(t)+k+1}$$

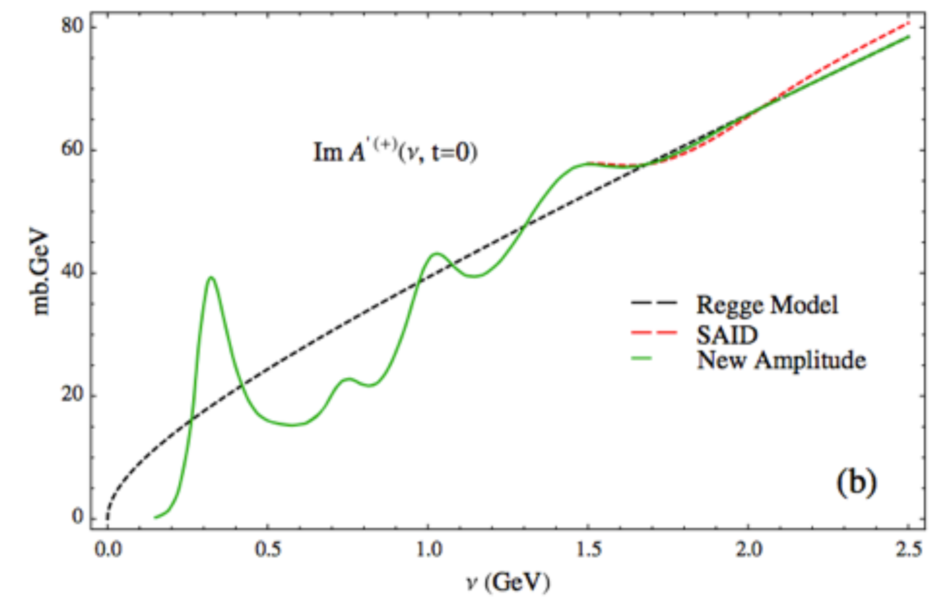
**Reconstruct imaginary part
from threshold to infinity**

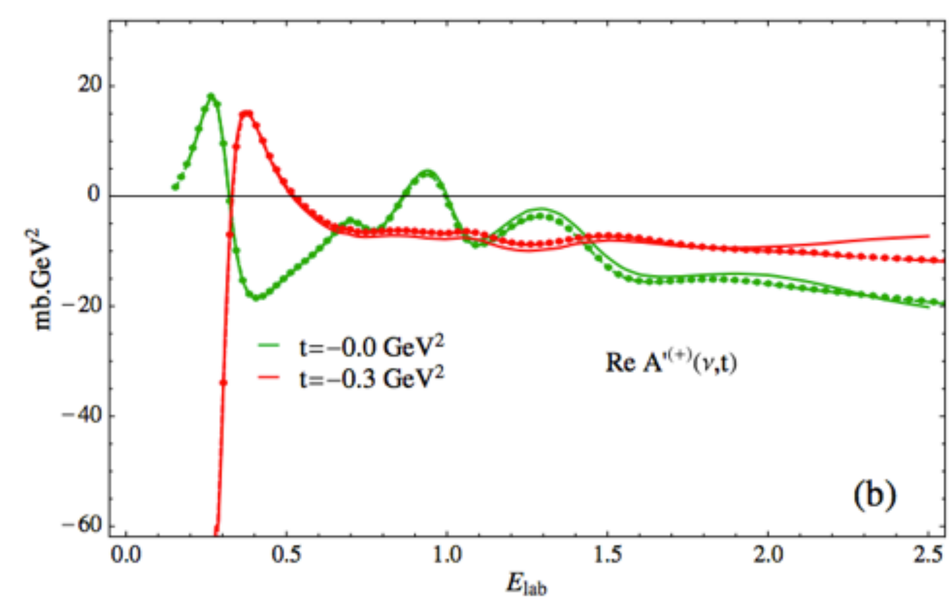
Impose dispersion relation

$$A(\nu,t) = \frac{2}{\pi}\int_{\nu_0}^{\infty} \frac{\text{Im } A(\nu',t)}{\nu'^2-\nu^2}\nu' d\nu'$$

Analytically continue and extract poles

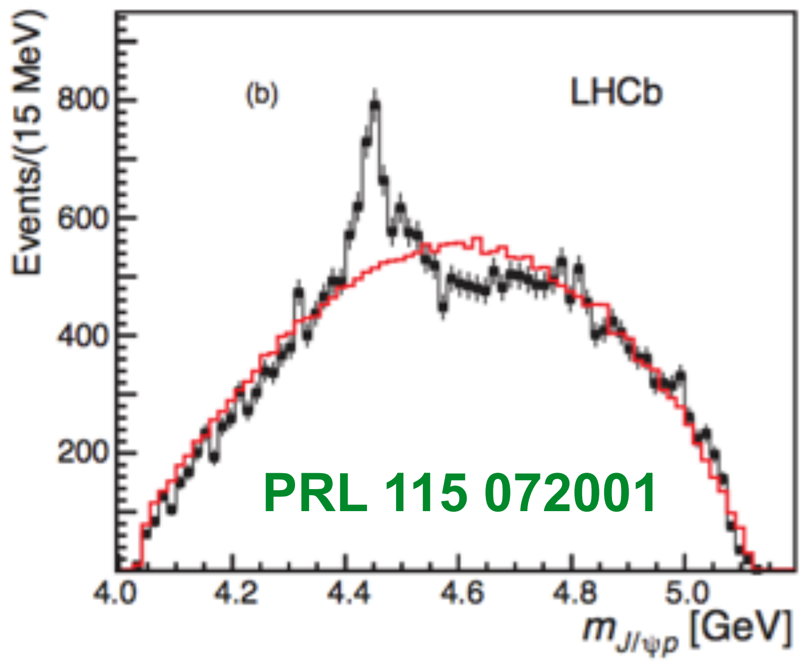




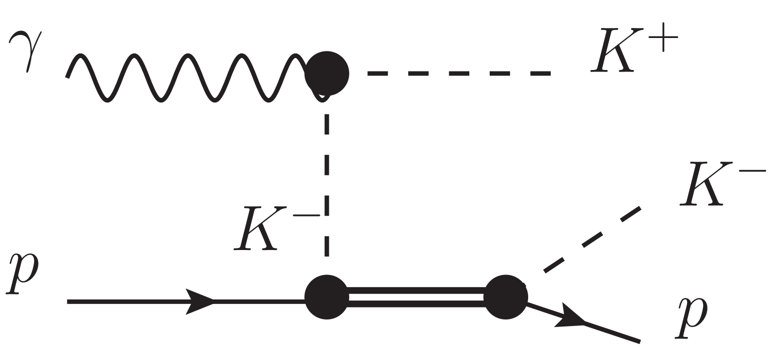


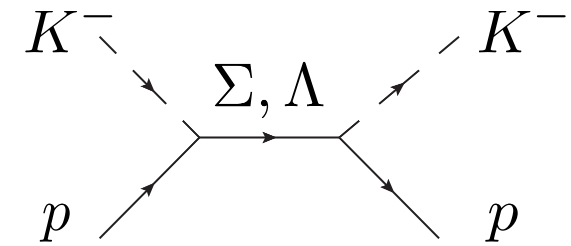
Using KN Amplitudes

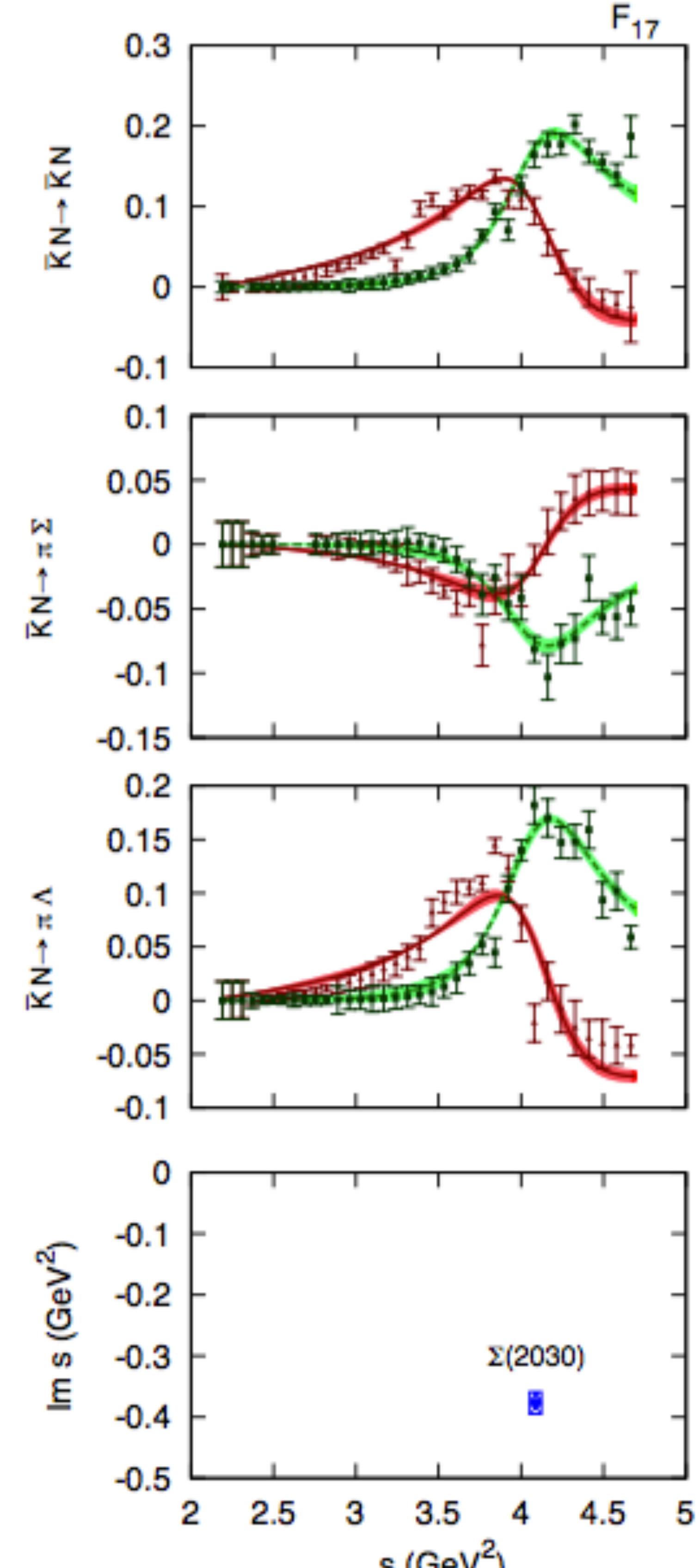
$$\Lambda_b^0 \to J/\psi K^- p$$



$$\gamma p \to K^+ K^- p$$







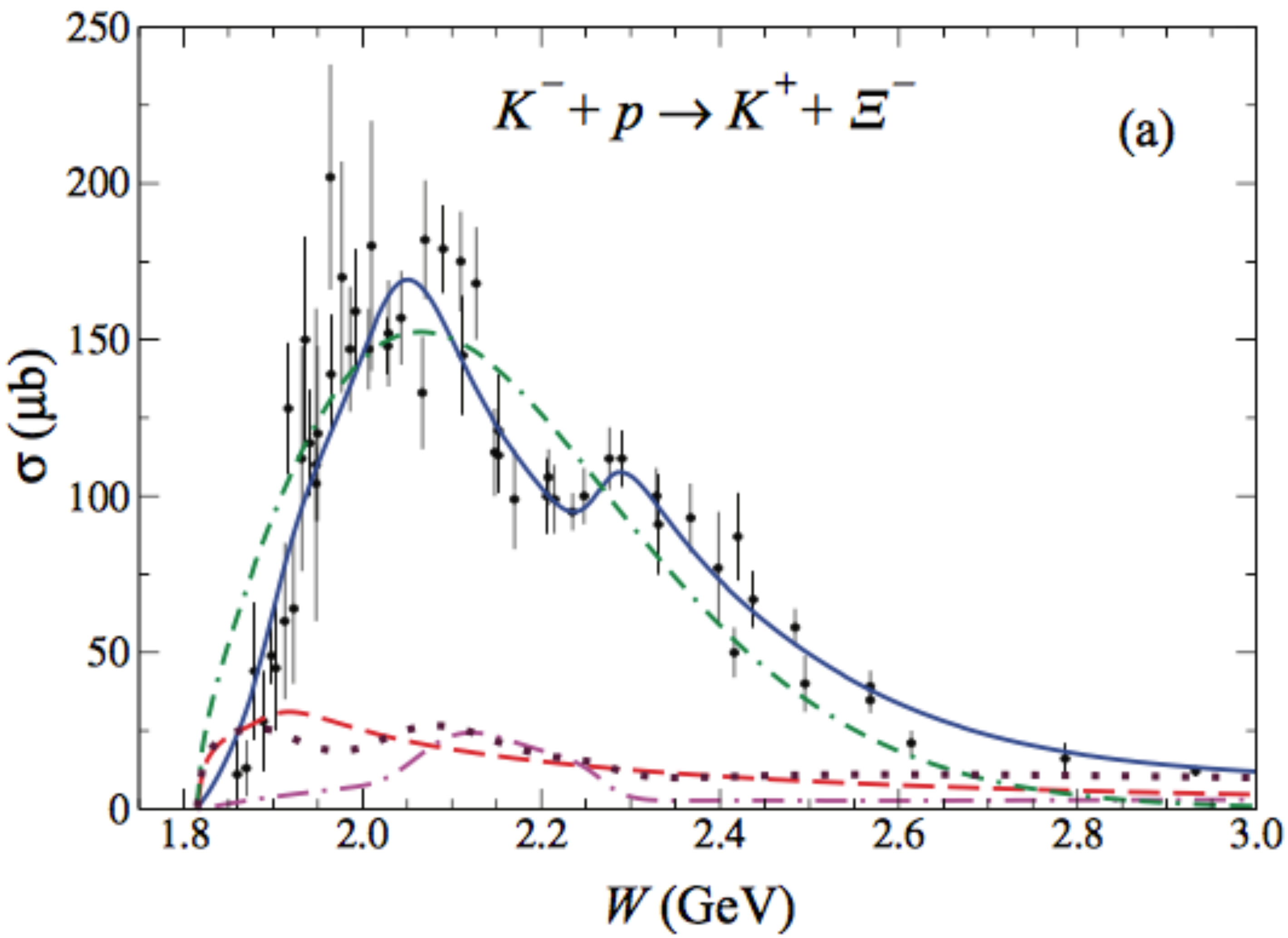
C. Fernandez-Ramirez et al. (JPAC) ArXiv:1510:07065

INDIANA UNIVERSITY

Interactive webpage: http://www.indiana.edu/~jpac/index.html

$\pi N \to \pi N$	VM et al (JPAC)	arXiv:1506.01764	PRD92 7 074004
$\gamma p \to \pi^0 p$	VM et al	arXiv:1505.02321	PRD92 7 074013
$\eta \to \pi^+\pi^-\pi^0$	P. Guo et al (JPAC)	arXiv:1505.01715	PRD92 5 054016
$\omega, \phi \to \pi^+\pi^-\pi^0$ $\to \gamma^*\pi^0$	I. Danilkin et al (JPAC)	arXiv:1409.7708	PRD91 9 094029
$\gamma p \to K^+K^-p$	M. Shi et al (JPAC)	arXiv:1411.6237	PRD91 3 034007
$KN \to KN$	C. Fernandez-Ramirez et al (JPAC)	arXiv:1510.07065	

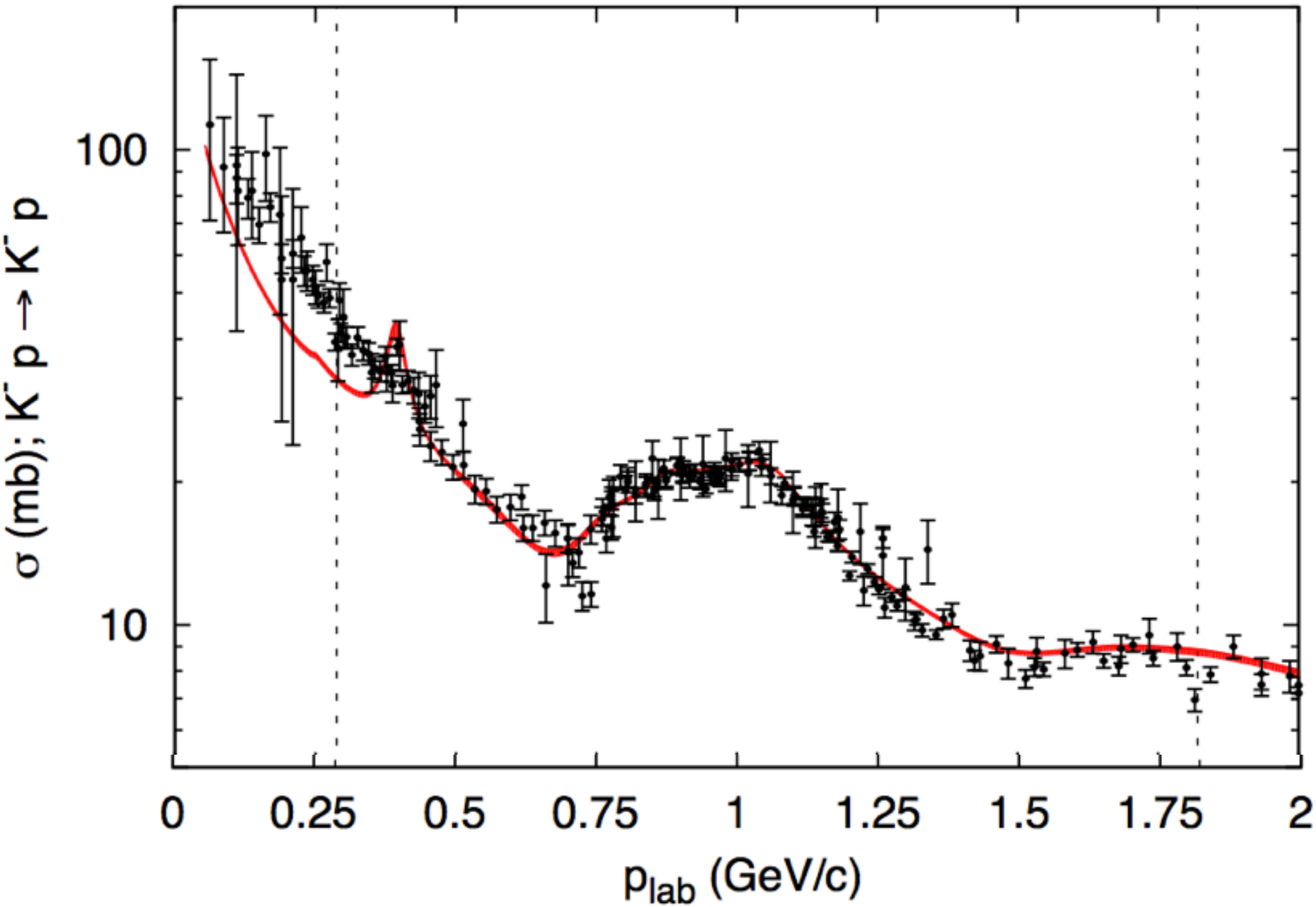
Backup Slides



KL simulation

unitarity -> analyticity

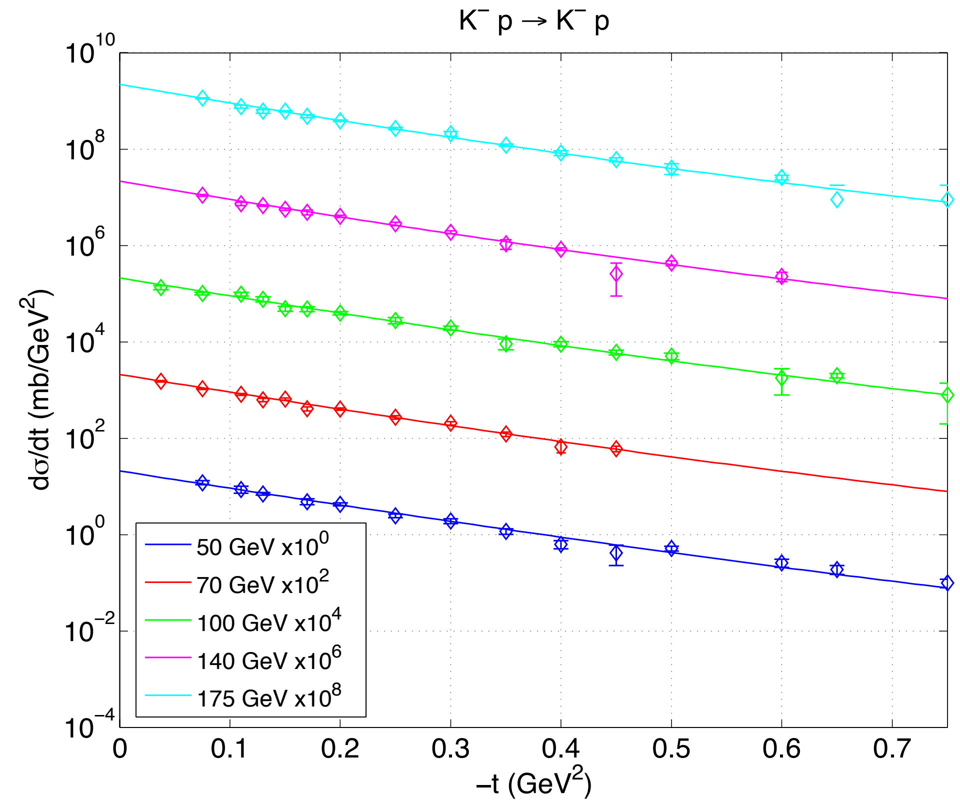
PW -> linked them

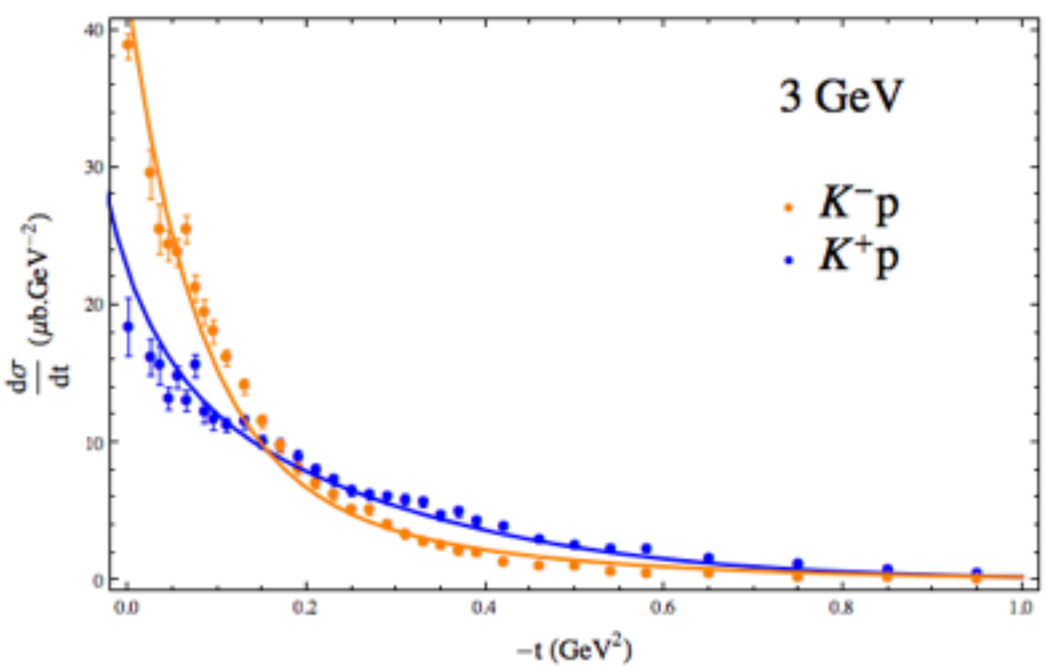


σ (mb); K⁻ p → K⁻ p
100
10
0
0.25
0.5
0.75
1
1.25
1.5
1.75
2
p_lab (GeV/c)

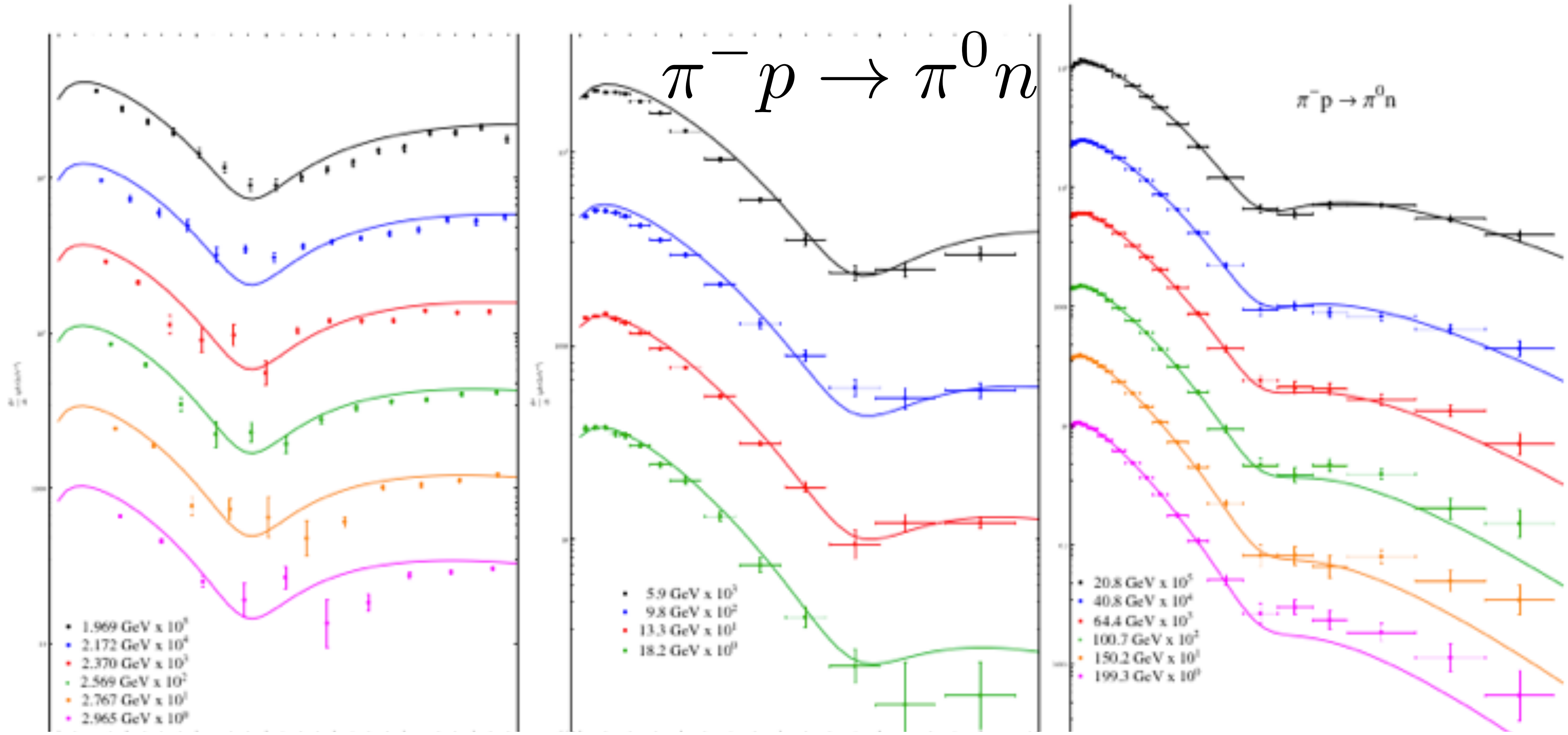
Kaon-Nucleon

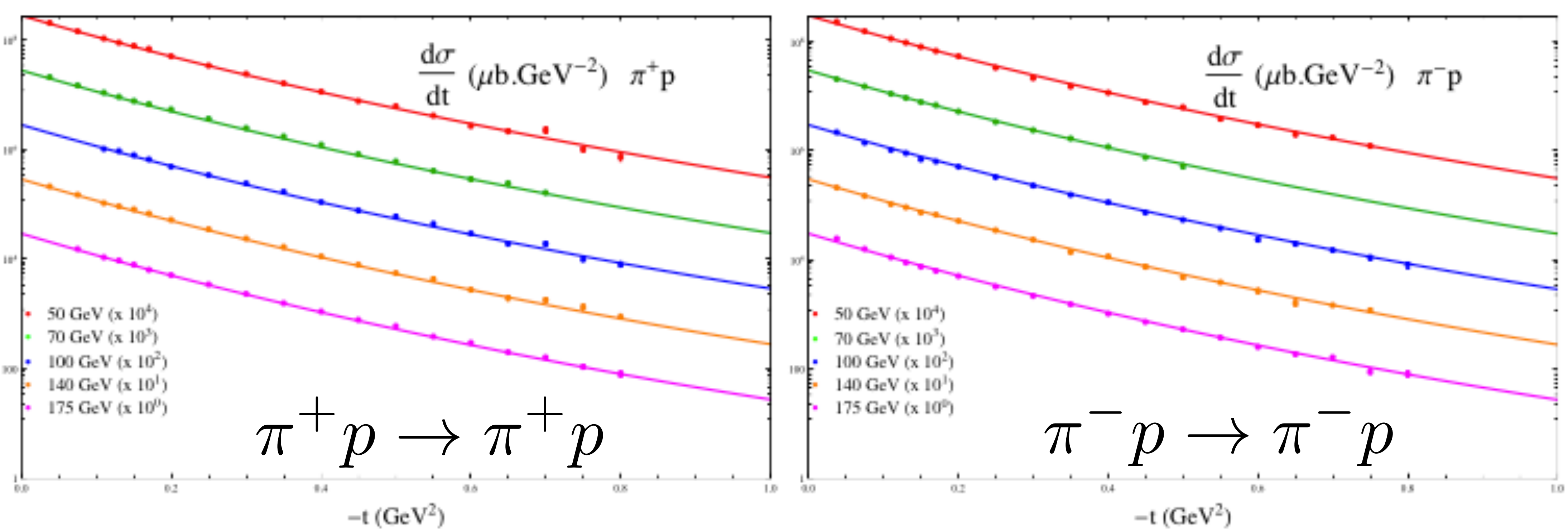
$$K^{\pm}p : \mathbb{P} + f + a \pm \rho \pm \omega$$



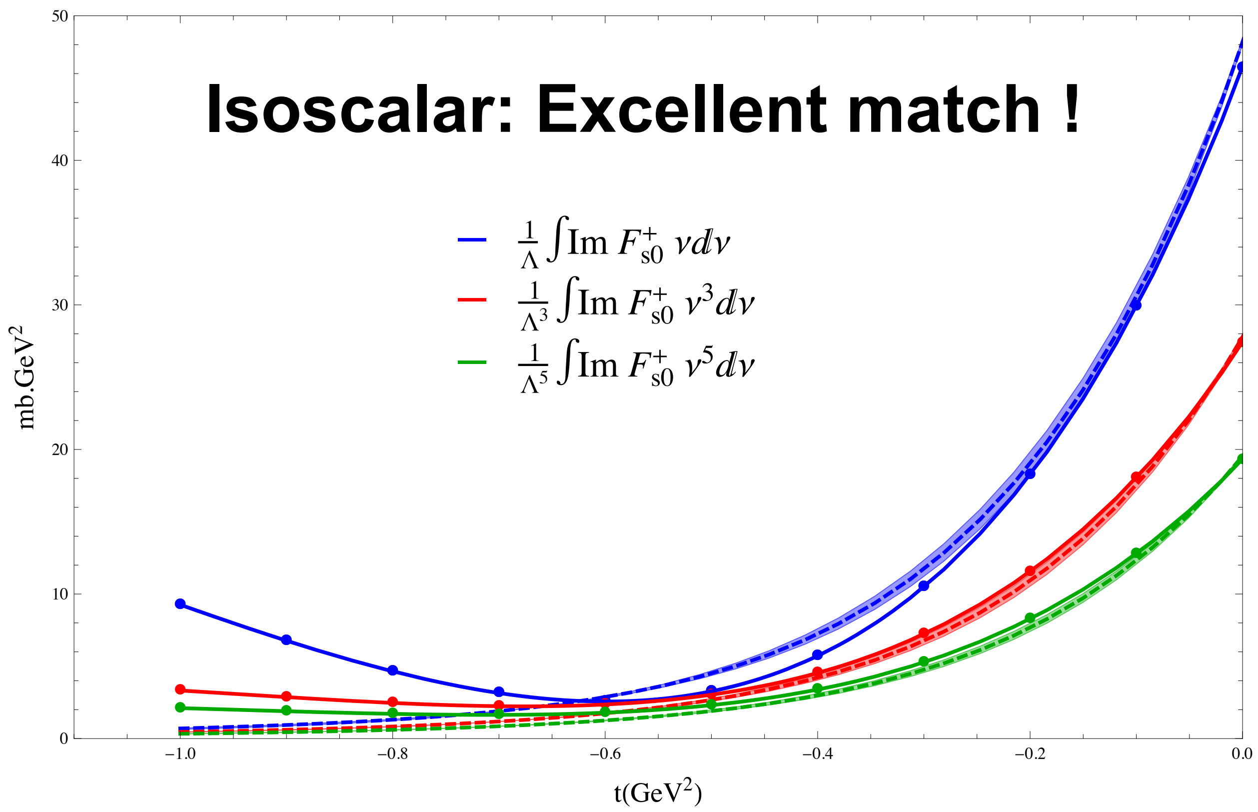


Parametrization of High Energy Data



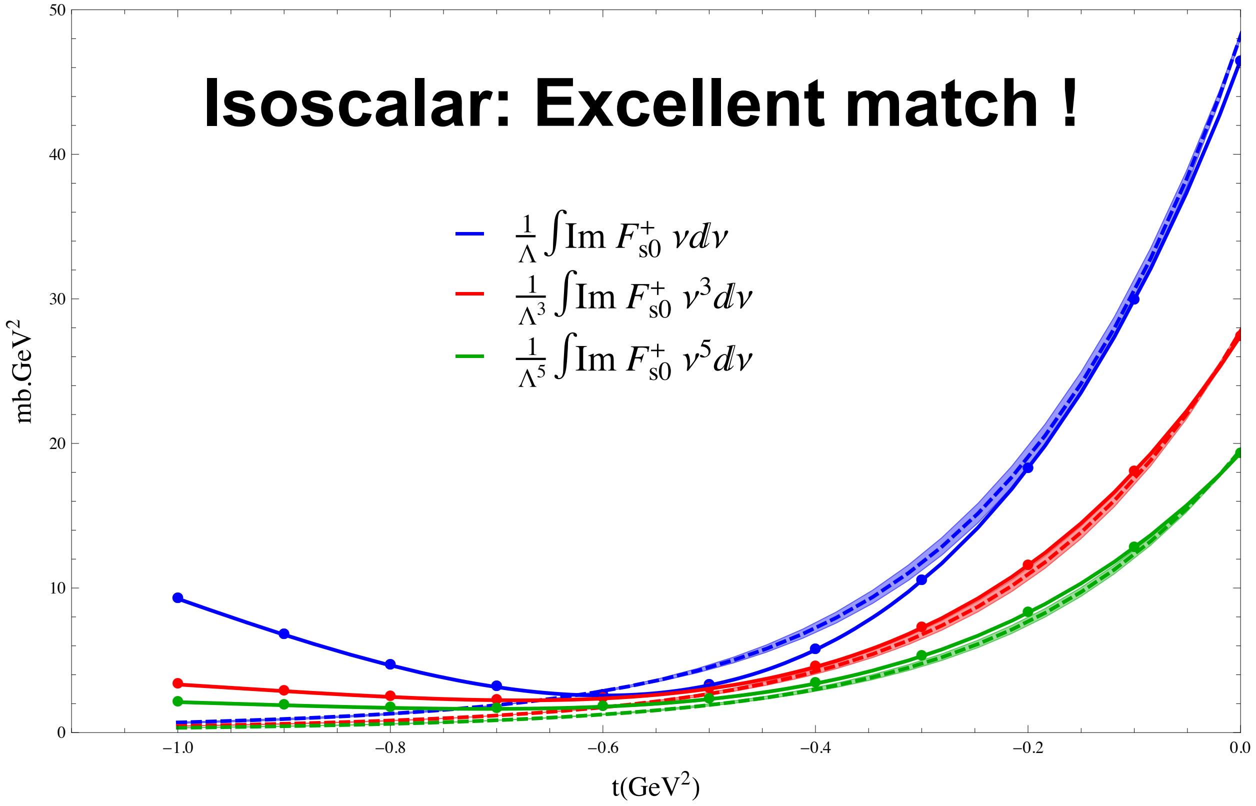


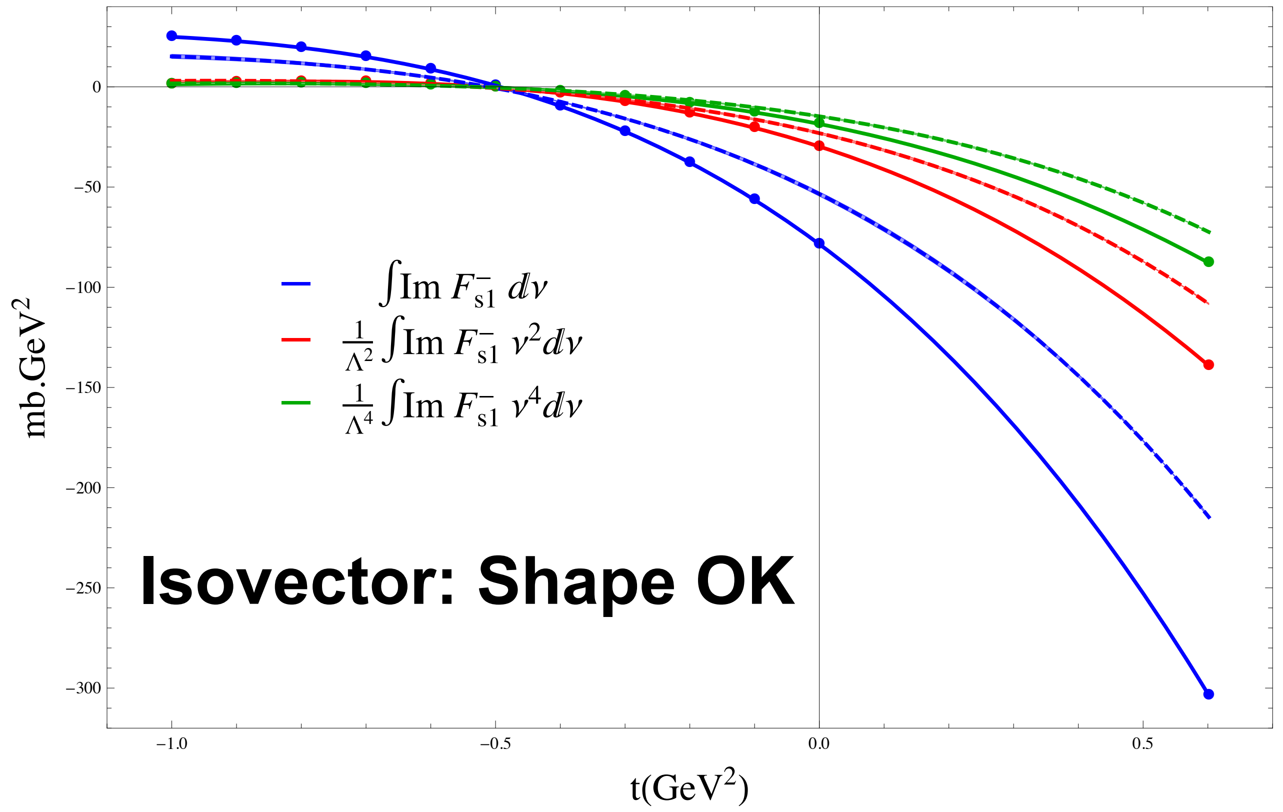
Comparison between Low and High Energy Amplitudes



VM et al. (in preparation)

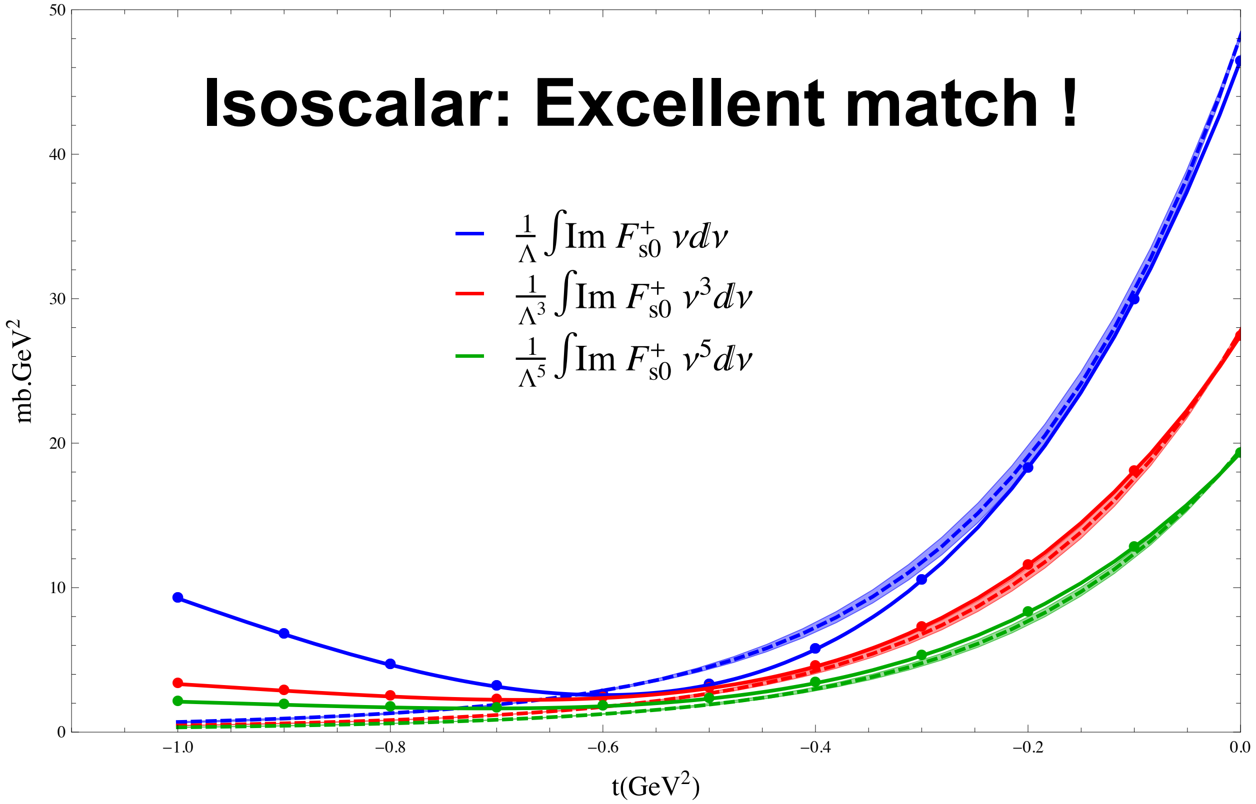
Comparison between Low and High Energy Amplitudes

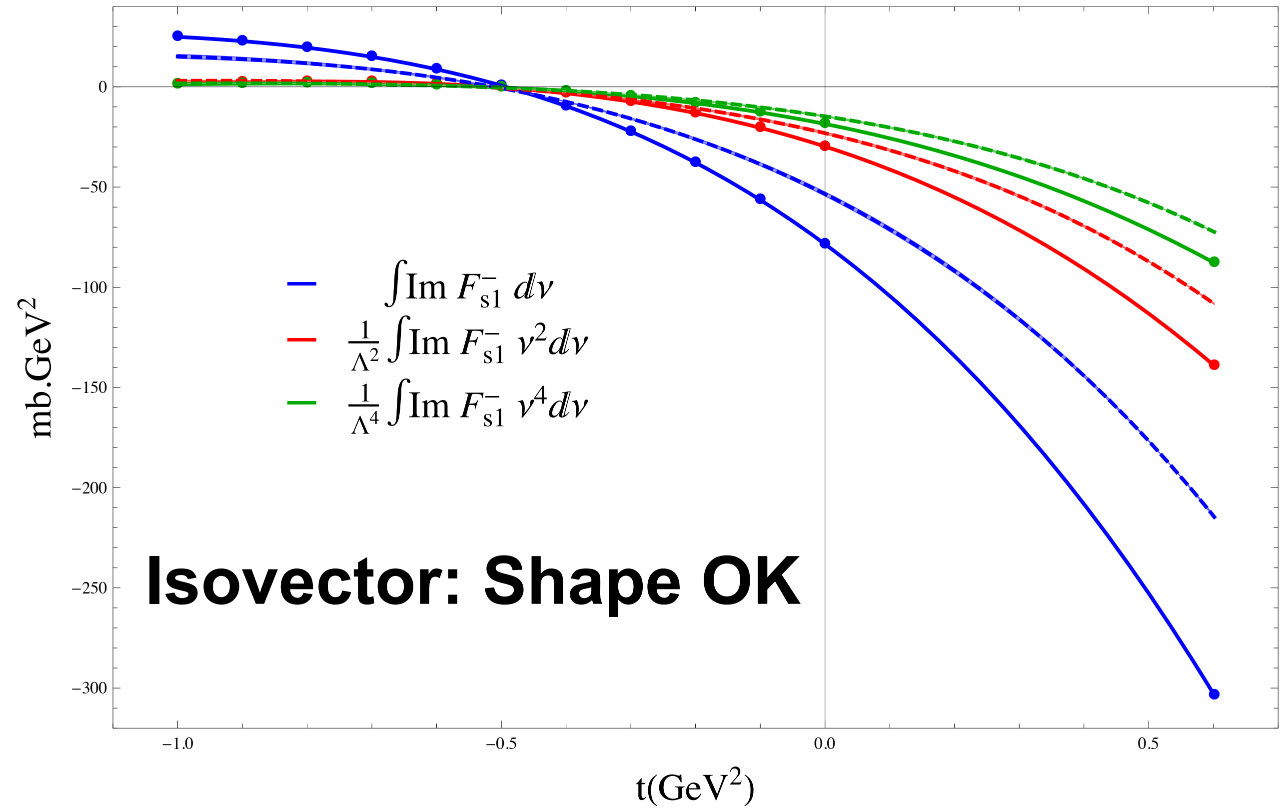


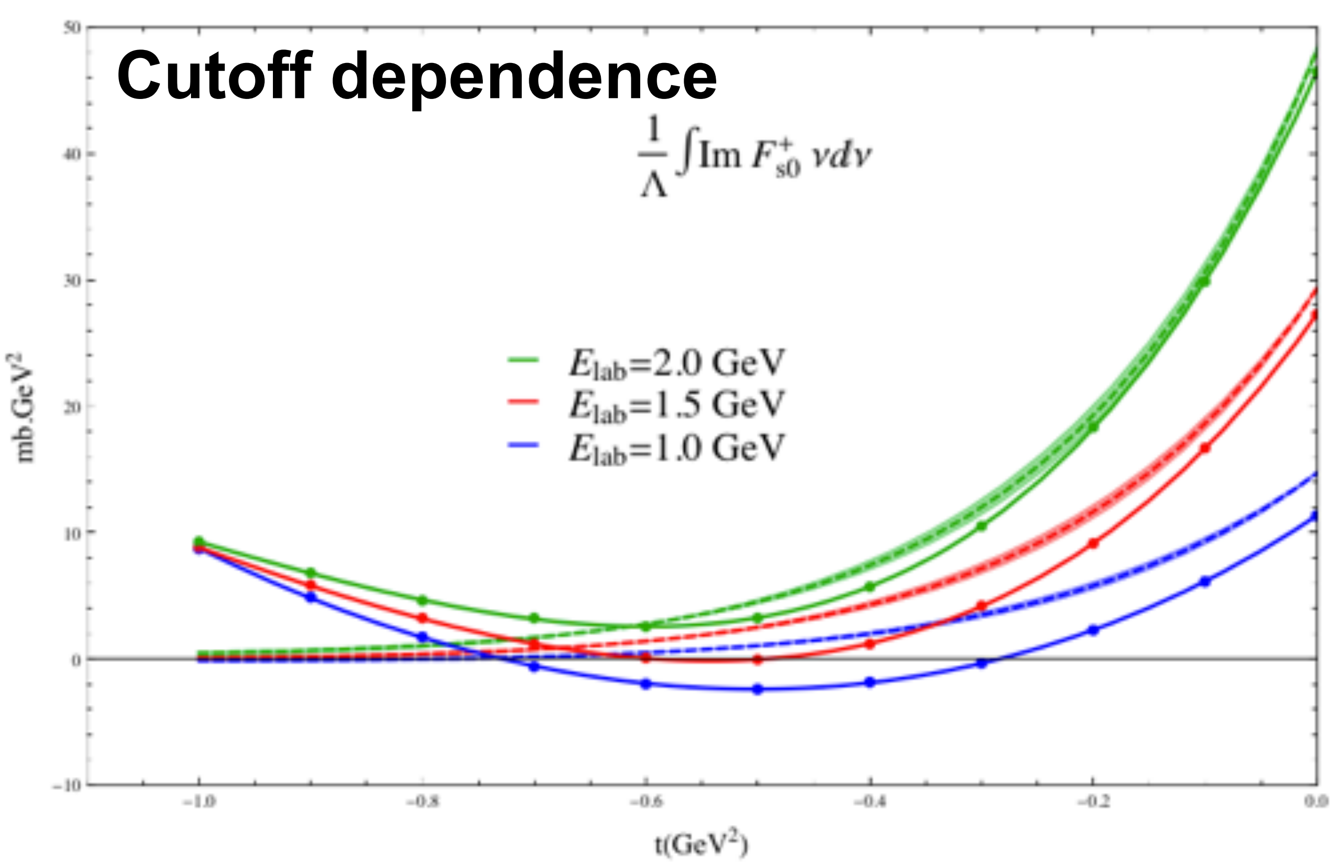


VM et al. (in preparation)

Comparison between Low and High Energy Amplitudes

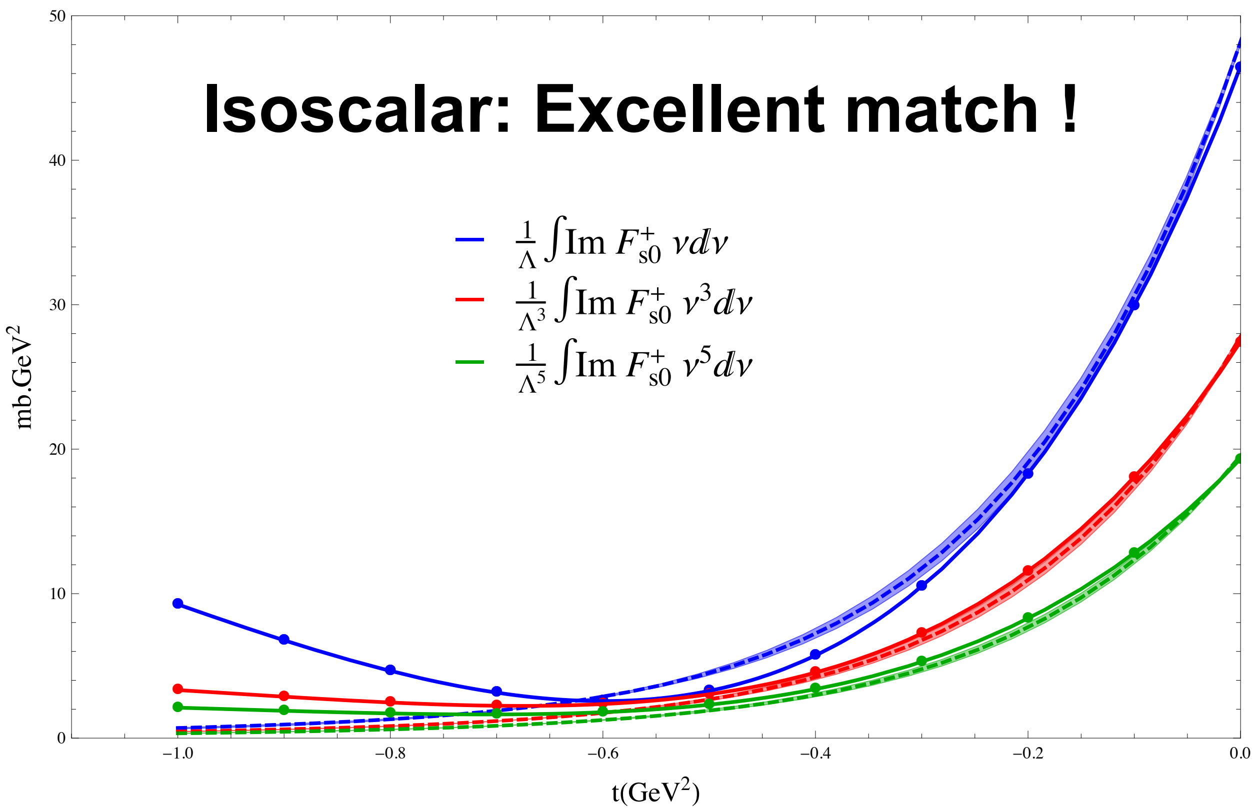


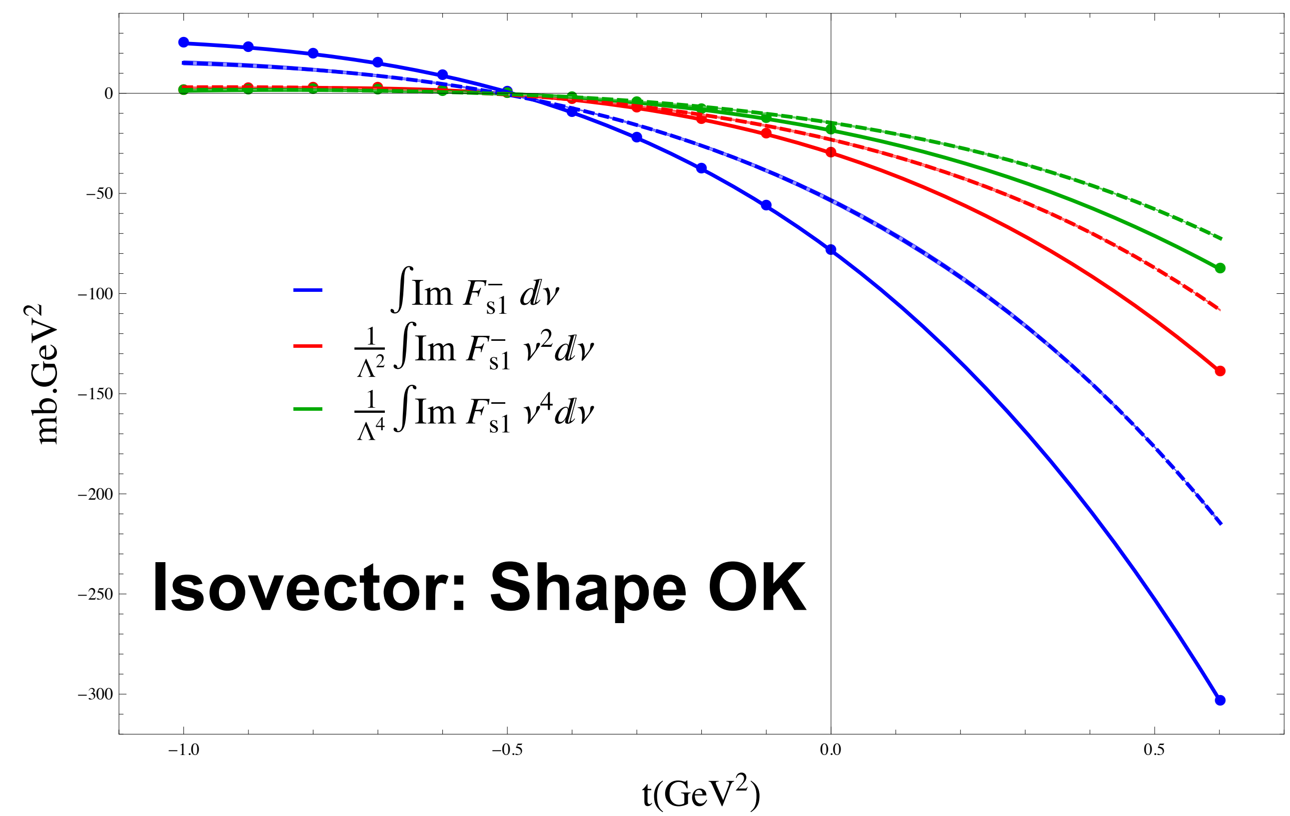


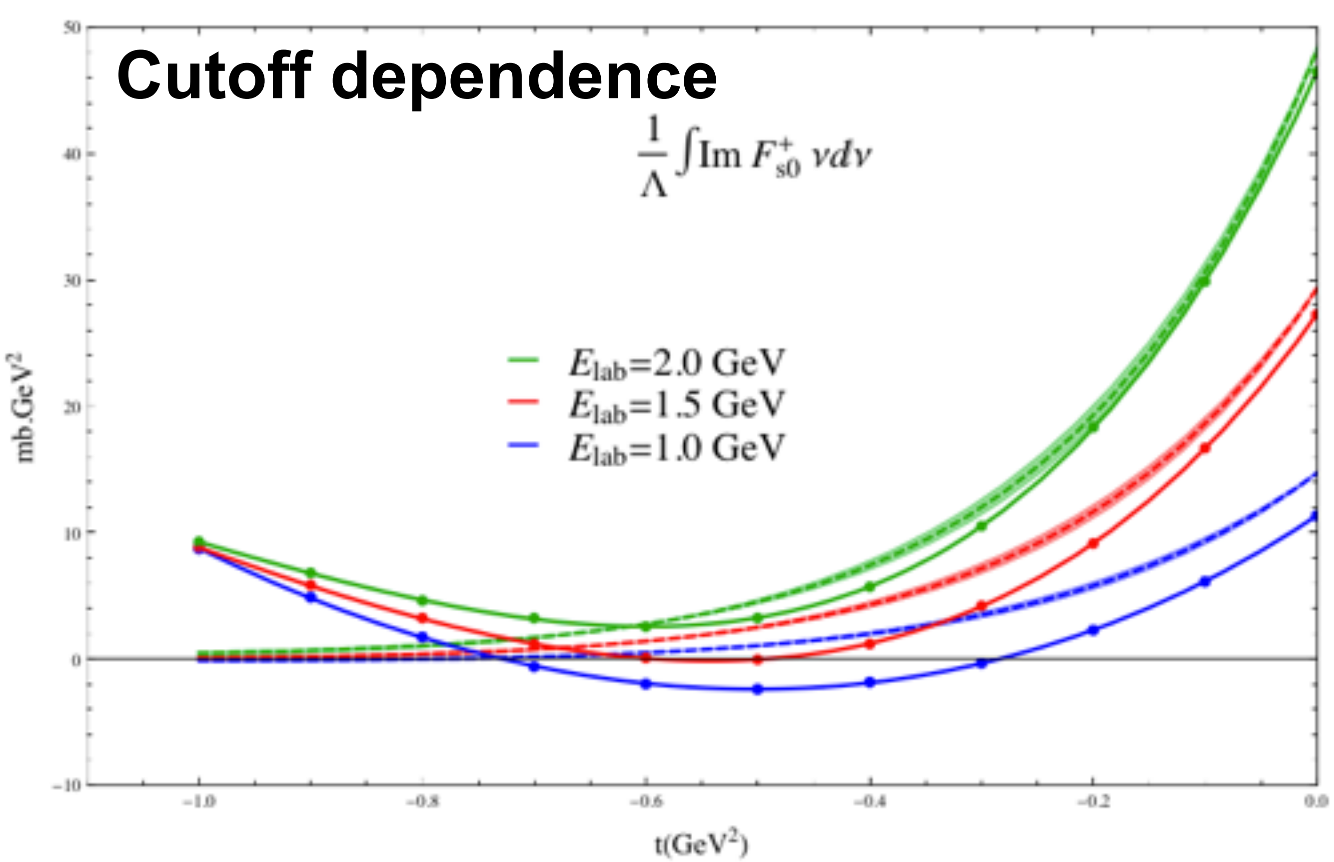


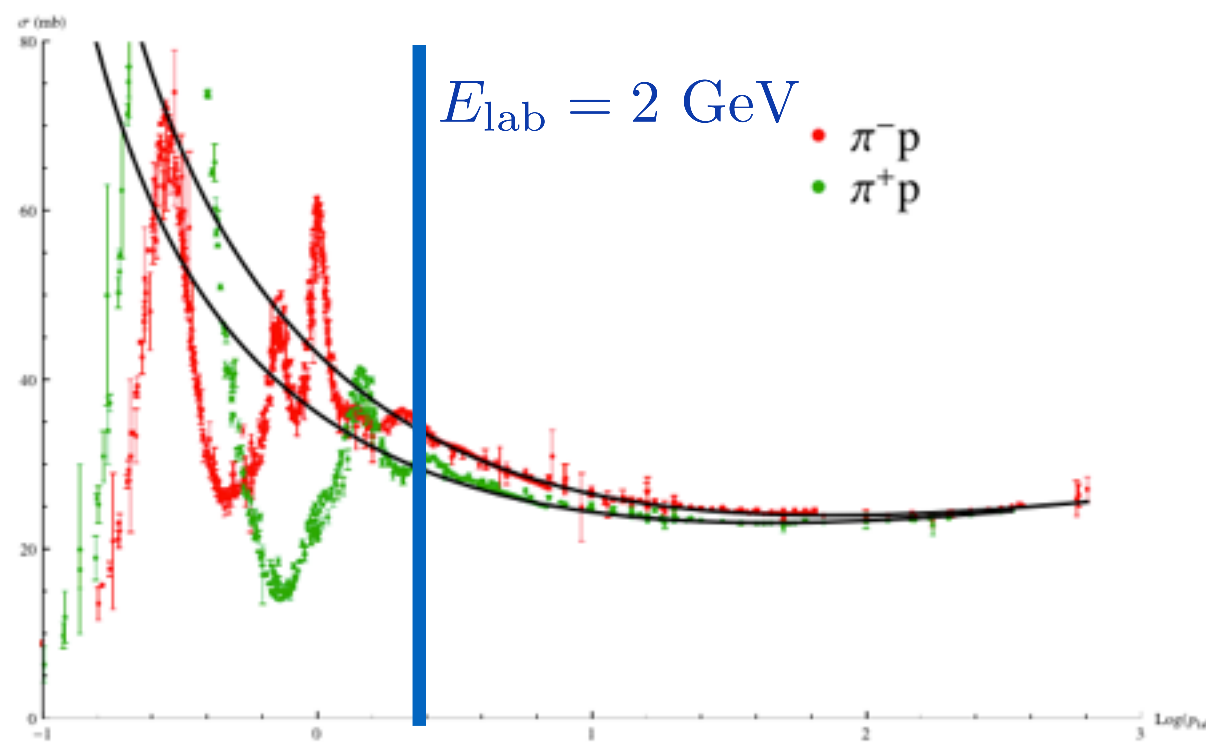
VM et al. (in preparation)

Comparison between Low and High Energy Amplitudes



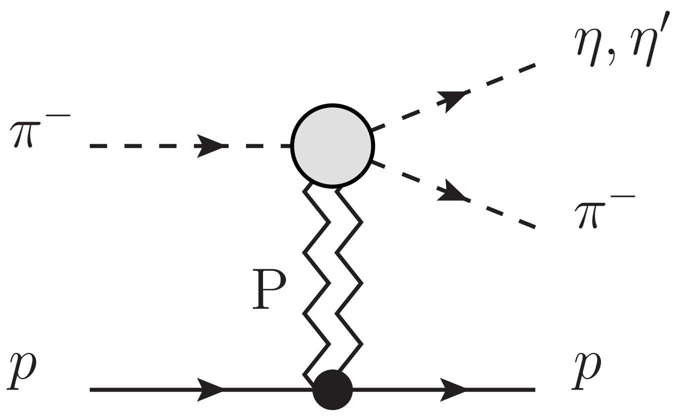


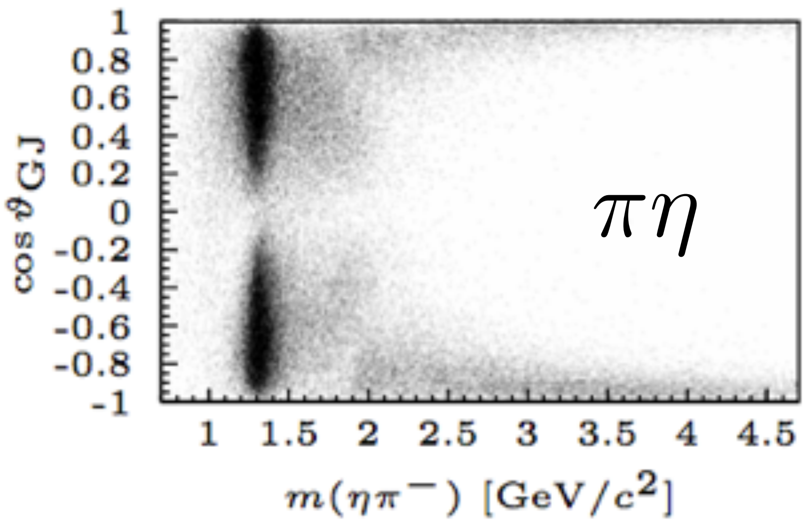


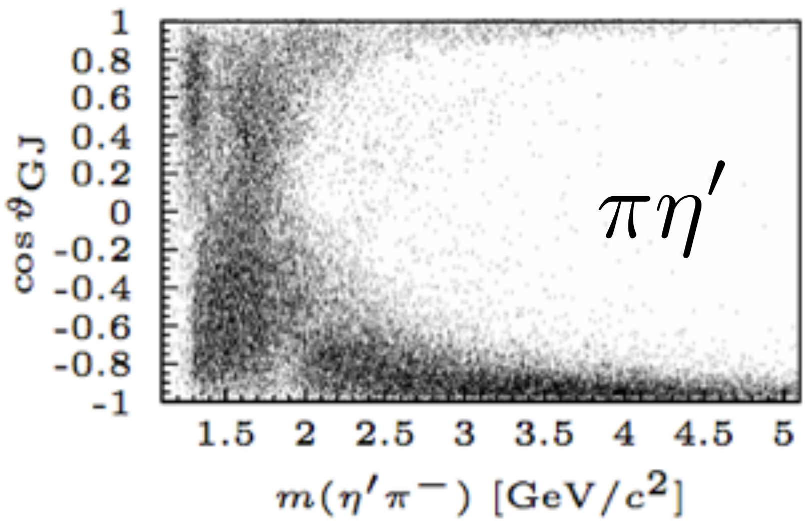


VM et al. (in preparation)

3b. Discovering (?) New Resonances: Eta(')-Pi @COMPASS

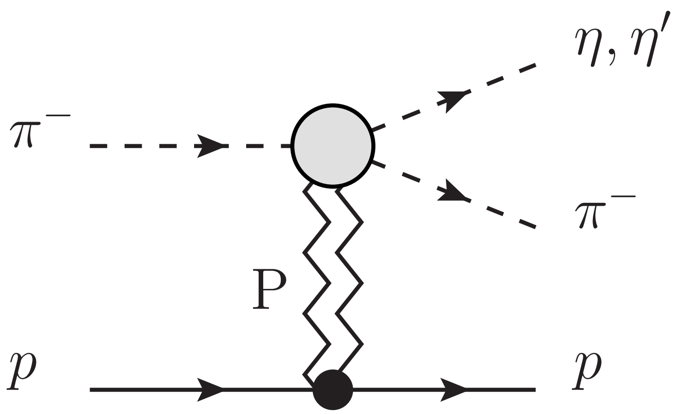


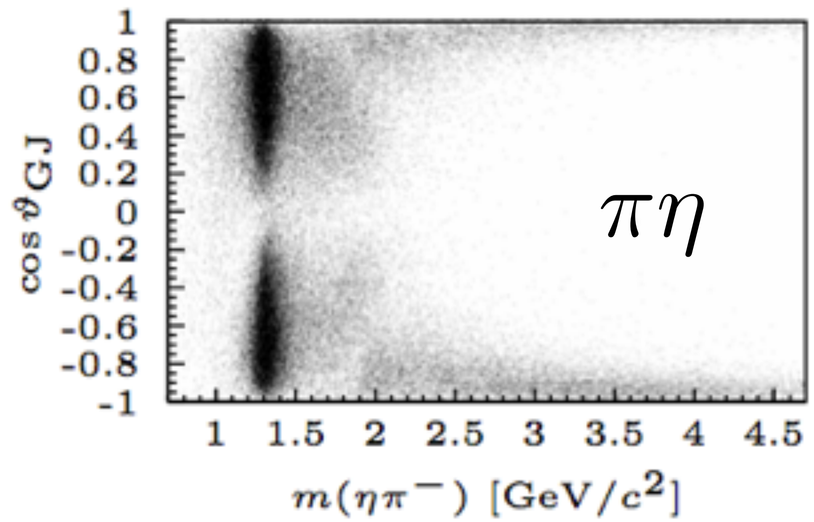


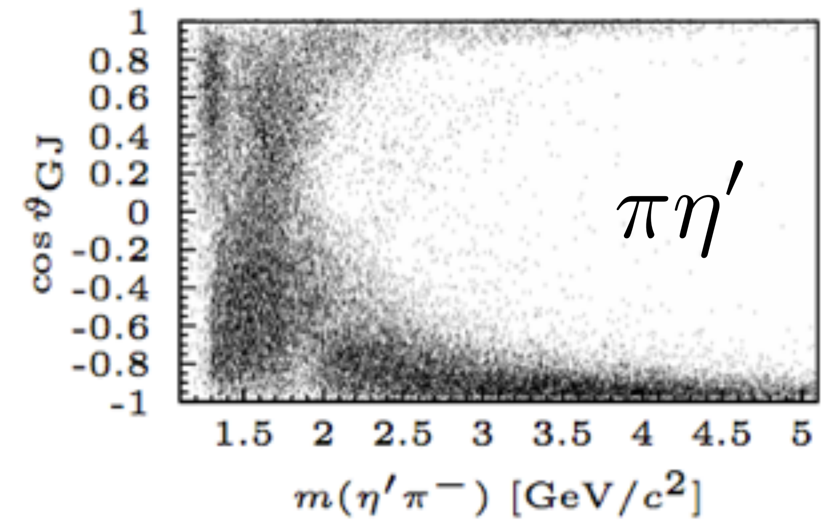


COMPASS ArXiv:1408.4286 PLB740 303

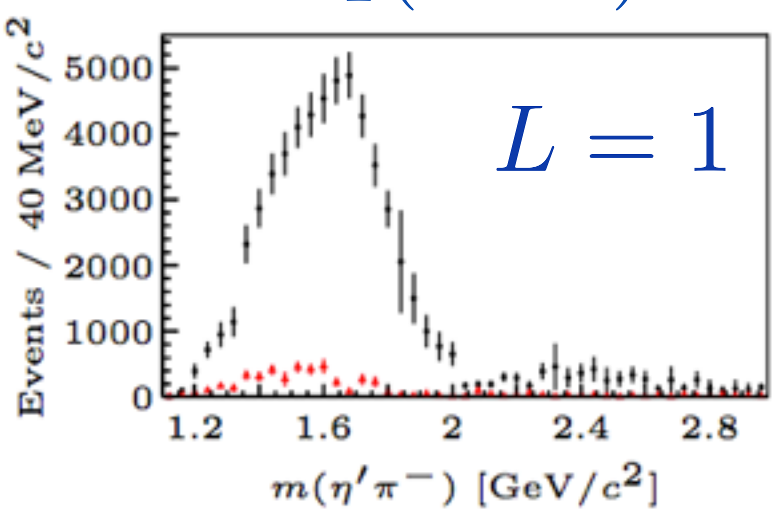
3b. Discovering (?) New Resonances: Eta(')-Pi @COMPASS



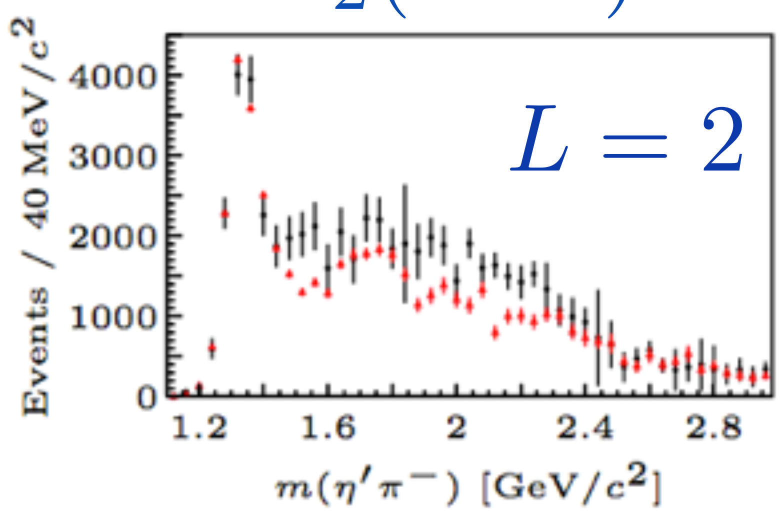




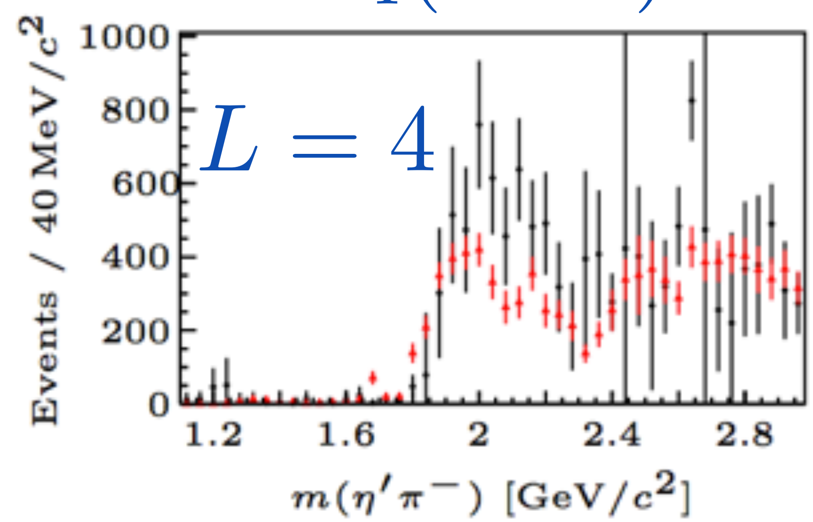
$\pi_1(1600)$?

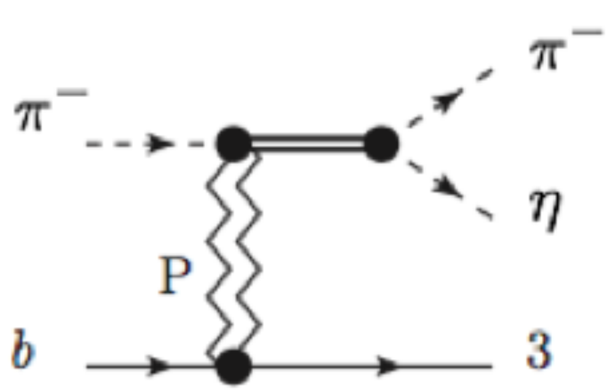


$a_2(1320)$



$a_4(2040)$





black: $\pi\eta'$

red: $\pi\eta$ **(scaled)**

Resonance in angular mom. L = 1 ?

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