Lattice Studies of Hyperon Spectroscopy

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KL2016, Jefferson Lab, Feb 2016

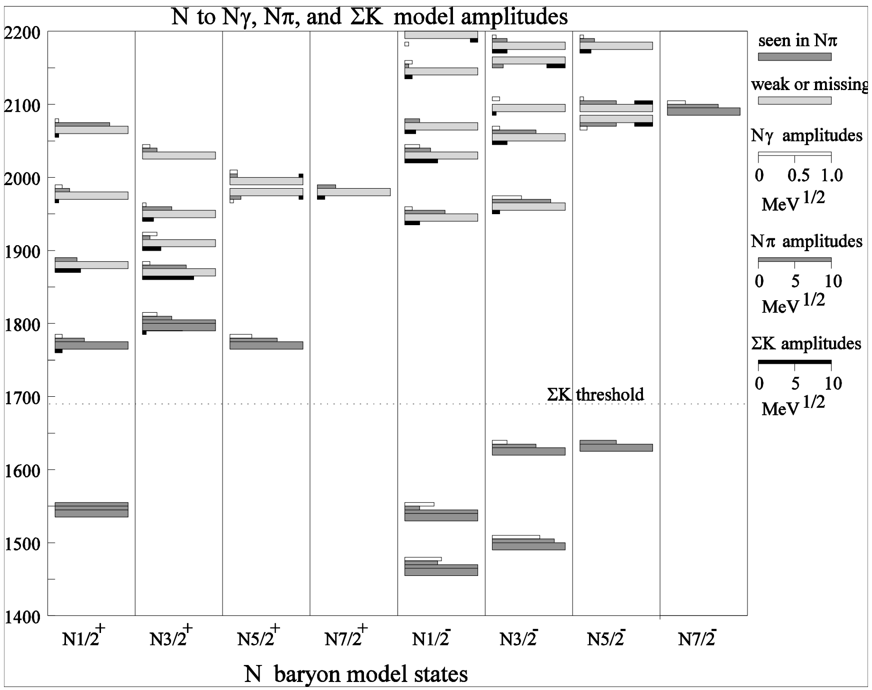
Outline

- Spectroscopy: theory and experiment
- Quantum Chromodynamics on the lattice
- Recent Highlights
- Resonances
  - phenomenology
  - strong decays

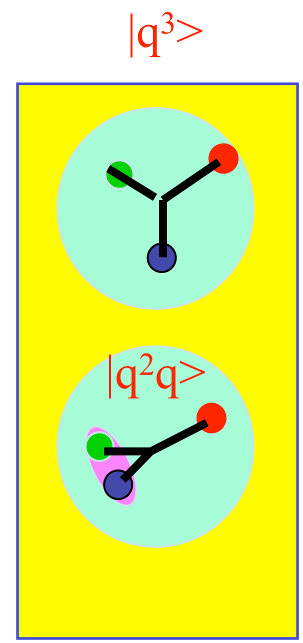
- Summary and prospects

Baryon Spectroscopy

- No baryon **"exotics"**, ie quantum numbers not accessible with simple quark model; but may be **hybrids**!
- **Nucleon Spectroscopy: Quark model masses and amplitudes – states classified by isospin, parity and spin.**



- Missing, because our pictures do not capture correct degrees of freedom?
- Do they just not couple to probes?



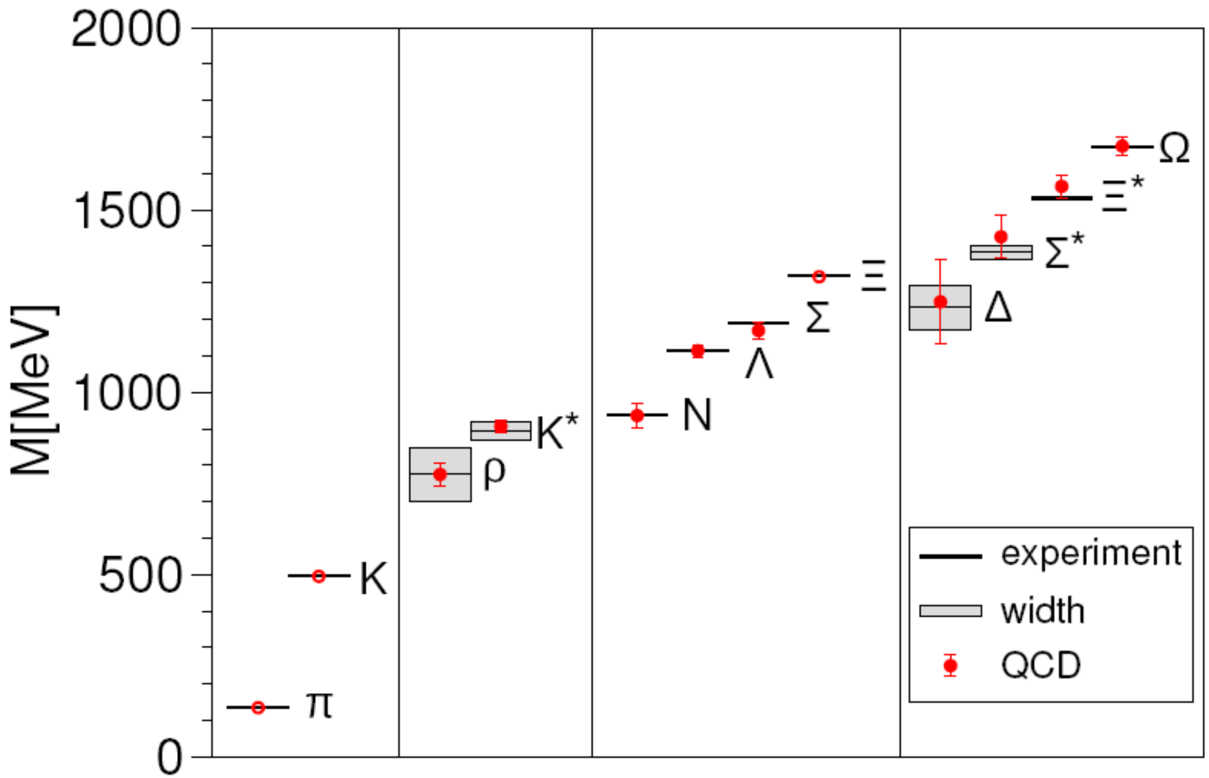
Capstick and Roberts, PRD58 (1998) 074011

Low-lying Hadron Spectrum

Benchmark of LQCD

$$C(t) = \sum_{\vec{x}} \langle 0 \mid N(\vec{x}, t) \bar{N}(0) \mid 0 \rangle = \sum_{n,\vec{x}} \langle 0 \mid e^{ip\cdot x} N(0) e^{-ip\cdot x} \mid n \rangle \langle n \mid \bar{N}(0) \mid 0 \rangle$$

$$= \mid \langle n \mid N(0) \mid 0 \rangle \mid^2 e^{-E_n t} = \sum_n A_n e^{-E_n t}$$



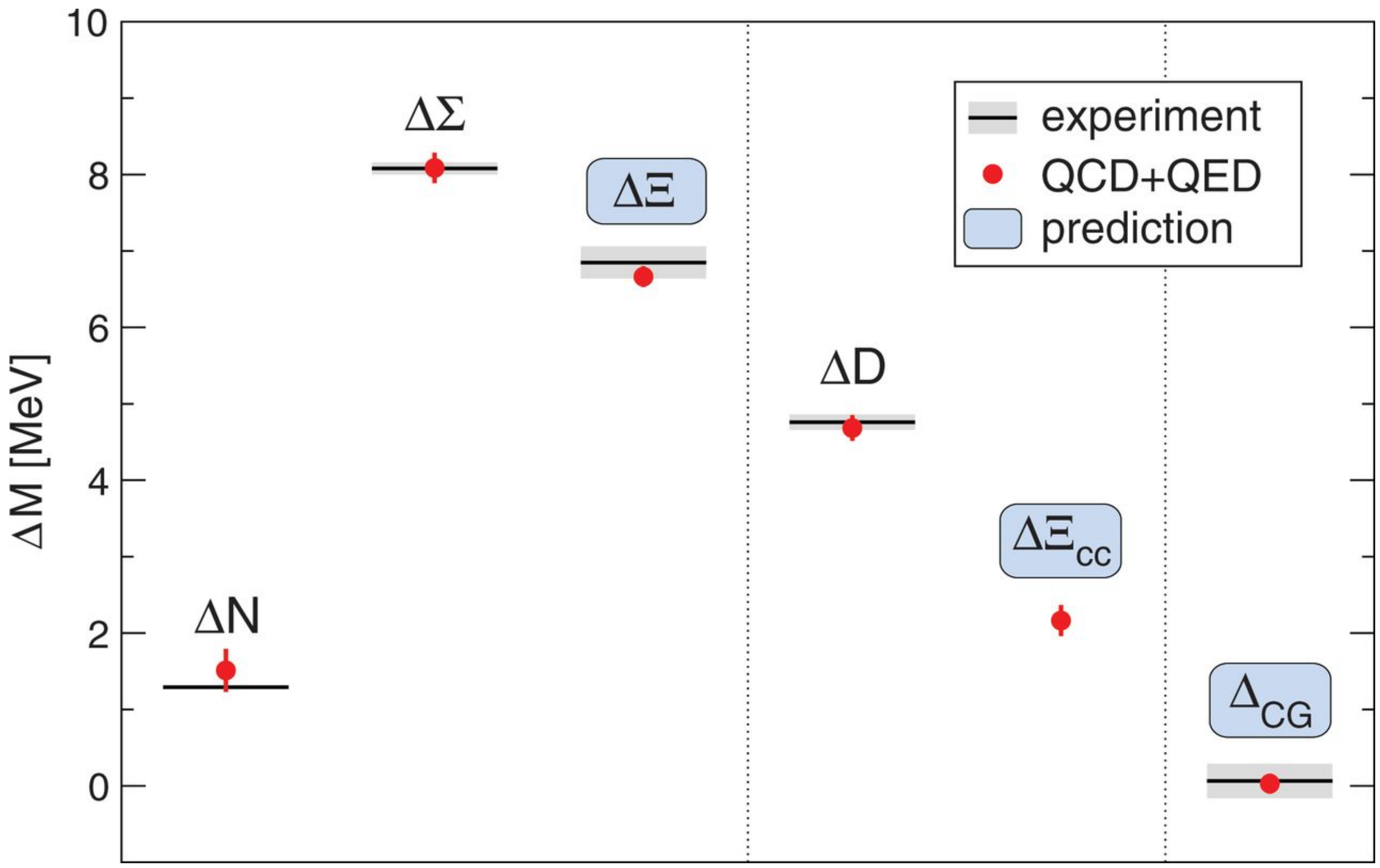
Durr et al., BMW Collaboration

Science 2008

Control over:

- ***Quark-mass dependence***
- ***Continuum extrapolation***
- ***finite-volume effects (pions, resonances)***

QCD + QED



BMW Collaboration, Science 2014

Variational Method

- Construct matrix of correlators

$$C_{\alpha\beta}(t, t_0) = \langle 0 \mid \mathcal{O}_\alpha(t)\mathcal{O}^\dagger_\beta(t_0) \mid 0\rangle \longrightarrow \sum_n Z^n_\alpha Z^{n\dagger}_\beta e^{-M_n(t-t_0)}$$

where $\{\mathcal{O}_\alpha\}$ are basis of operators of definite symmetry: P, C and J?

Delineate contributions using variational method: solve

$$C(t)u(t, t_0) = \lambda(t, t_0)C(t_0)u(t, t_0)$$

$$\lambda_i(t, t_0) \rightarrow e^{-E_i(t-t_0)}\left(1 + O(e^{-\Delta E(t-t_0)})\right)$$

Eigenvectors, with metric $C(t_0)$, are orthonormal and project onto the respective states

Baryon Operators

Aim: interpolating operators of *definite* (continuum) JM: O^{JM}

- **Lattice does not respect symmetries of continuum: *cubic symmetry for states at rest***

$$\langle 0 \mid O^{JM} \mid J', M' \rangle = Z^J \delta_{J,J'} \delta_{M,M'}$$

Starting point

$$B = \left(\mathcal{F}_{\Sigma_F} \otimes \mathcal{S}_{\Sigma_S} \otimes \mathcal{D}_{\Sigma_D}\right) \{\psi_1 \psi_2 \psi_3\}$$

Introduce circular basis:

$$\overleftrightarrow{D}_{m=-1} = \frac{i}{\sqrt{2}} \left(\overleftrightarrow{D}_x - i \overleftrightarrow{D}_y \right)$$

$$\overleftrightarrow{D}_{m=0} = i \overleftrightarrow{D}_z$$

$$\overleftrightarrow{D}_{m=+1} = -\frac{i}{\sqrt{2}} \left(\overleftrightarrow{D}_x + i \overleftrightarrow{D}_y \right).$$

Straighforward to project to definite spin: $J = 1/2, 3/2, 5/2$

$$\big| [J, M] \big\rangle = \sum_{m_1, m_2} \big| [J_1, m_1] \big\rangle \otimes \big| [J_2, m_2] \big\rangle \big\langle J_1 m_1; J_2 m_2 \big| J M \big\rangle$$

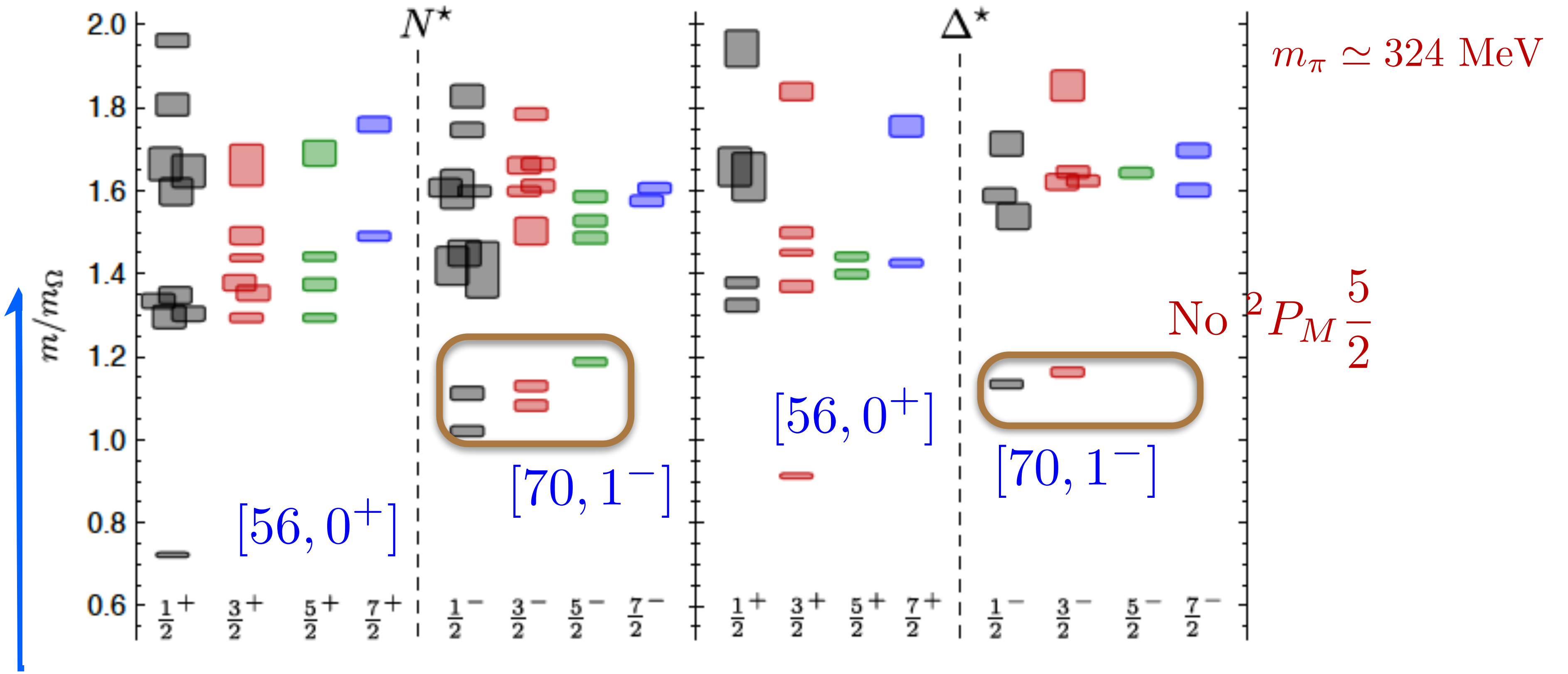
Use projection formula to find subduction under irrep. of cubic group - operators are closed under rotation!

$$O^{[J]}_{\Lambda\lambda}(t, \vec{x}) = \frac{d_\Lambda}{g_{O_h^D}} \sum_{R \in O_h^D} D^{(\Lambda)*}_{\lambda\lambda}(R) U_R O^{J,M}(t, \vec{x}) U_R^\dagger$$

$$= \sum_M S^{J,M}_{\Lambda,\lambda} O^{J,M}$$

Irrep, Row — **Irrep of R in Λ** — **Action of R**

Excited Baryon Spectrum - I



$[70, 0^+], [56, 2^+], [70, 2^+], [20, 1^+]$

N $^{1/2+}$ sector: need for complete basis to faithfully extract states

Broad features of SU(6)xO(3) symmetry.

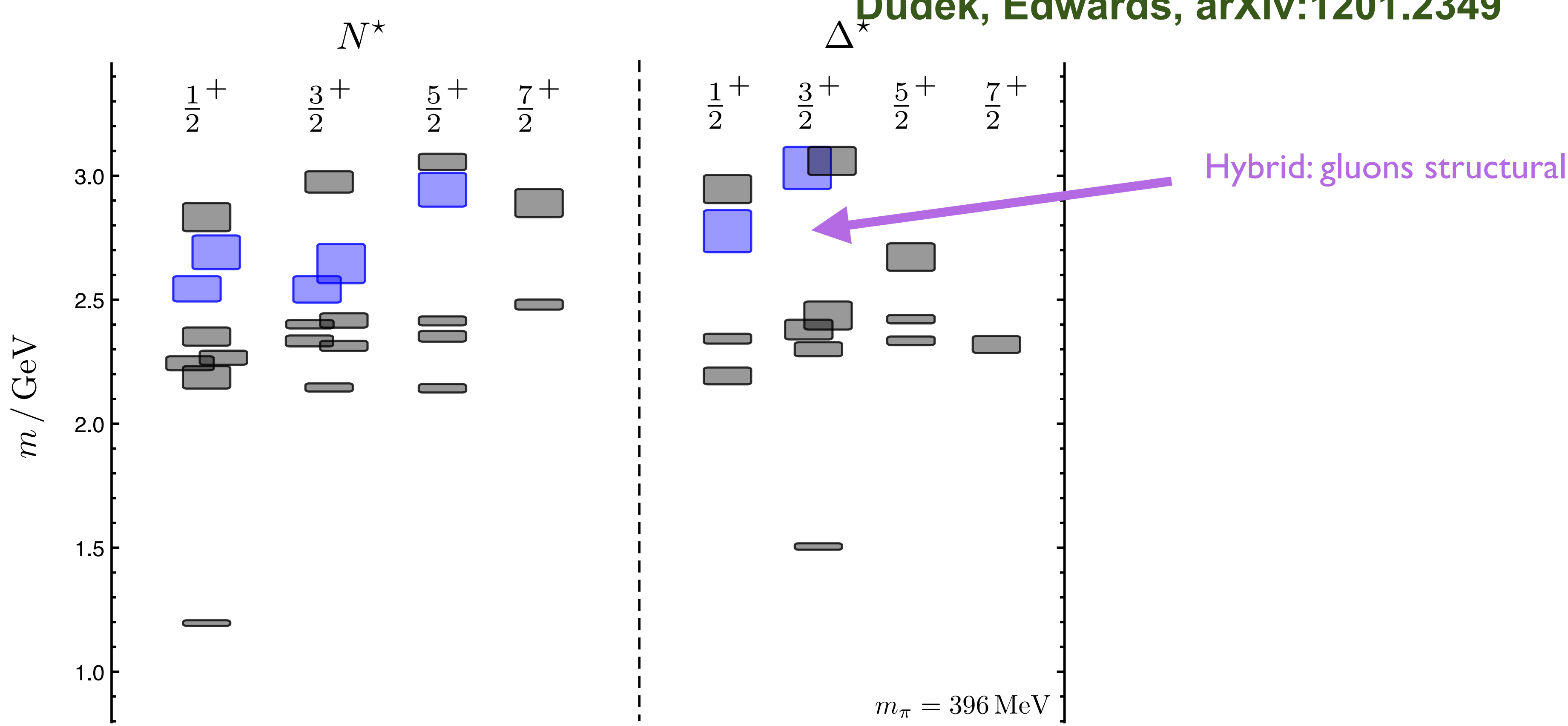
Counting of states consistent with NR quark model.

Inconsistent with quark-diquark picture or parity doubling.

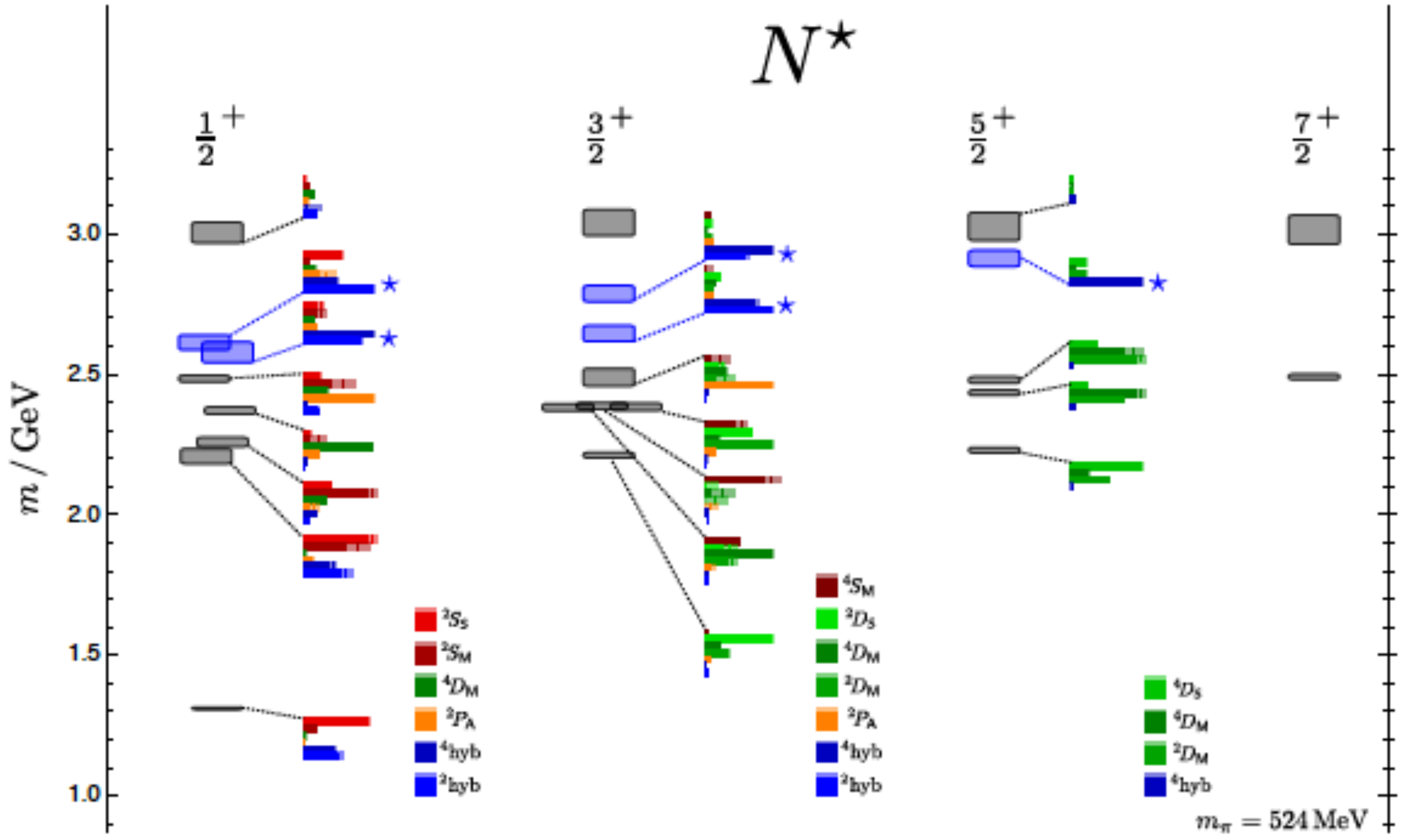
Hybrid Baryon Spectrum

Original analysis ignore hybrid operators of form $D^{[2]}_{l=1,M}$

Dudek, Edwards, arXiv:1201.2349

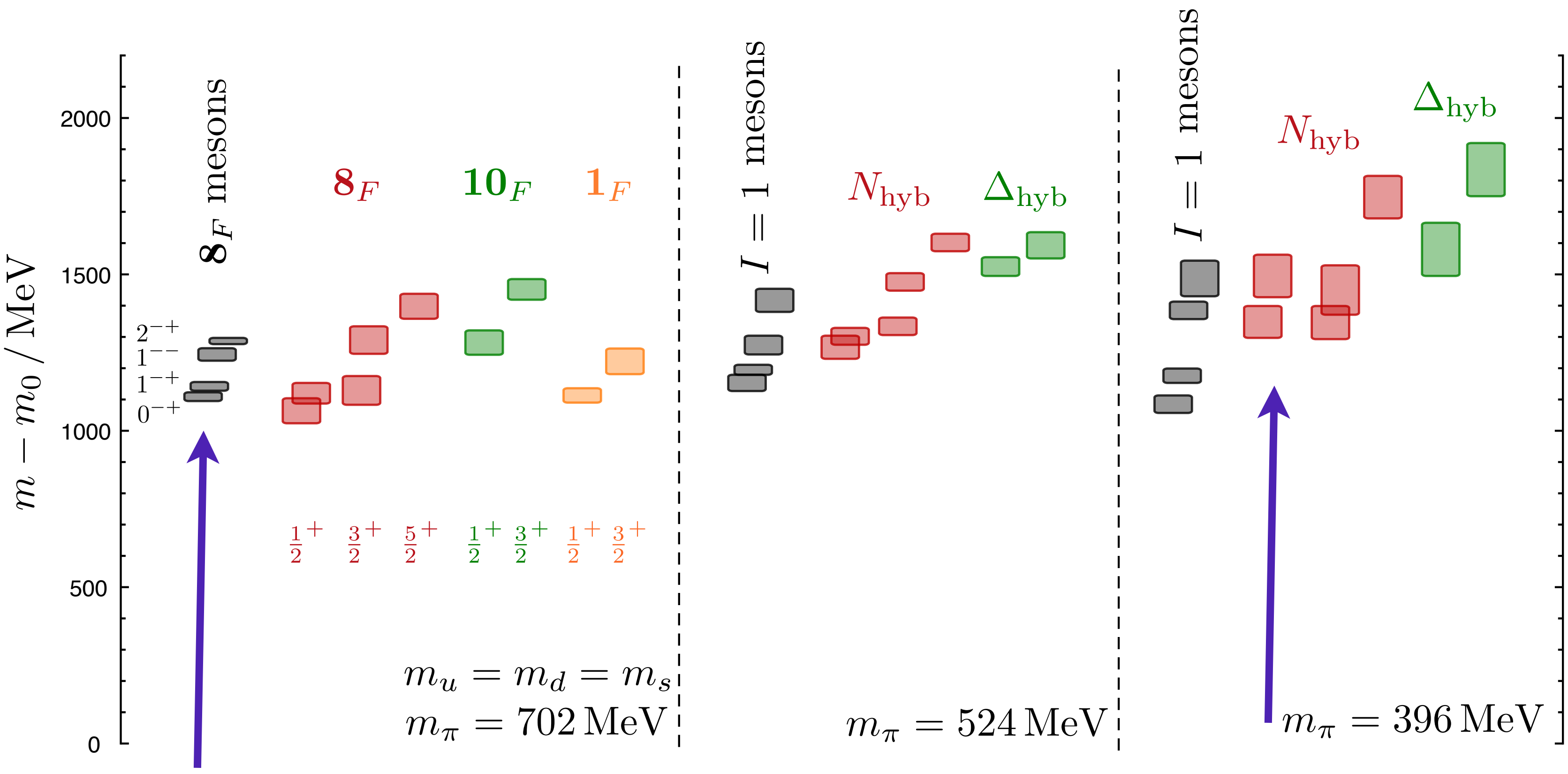


Interpolating Operators



Examine overlaps onto different NR operators, i.e. containing upper components of spinors: *ground state has substantial hybrid component*

Putting it Together



Subtract ρ

Subtract N

Common mechanism in meson and baryon hybrids: chromomagnetic field with *E_g ~ 1.2 - 1.3 GeV*

Setting the strange-quark mass

Tuning performed for three-flavor theory

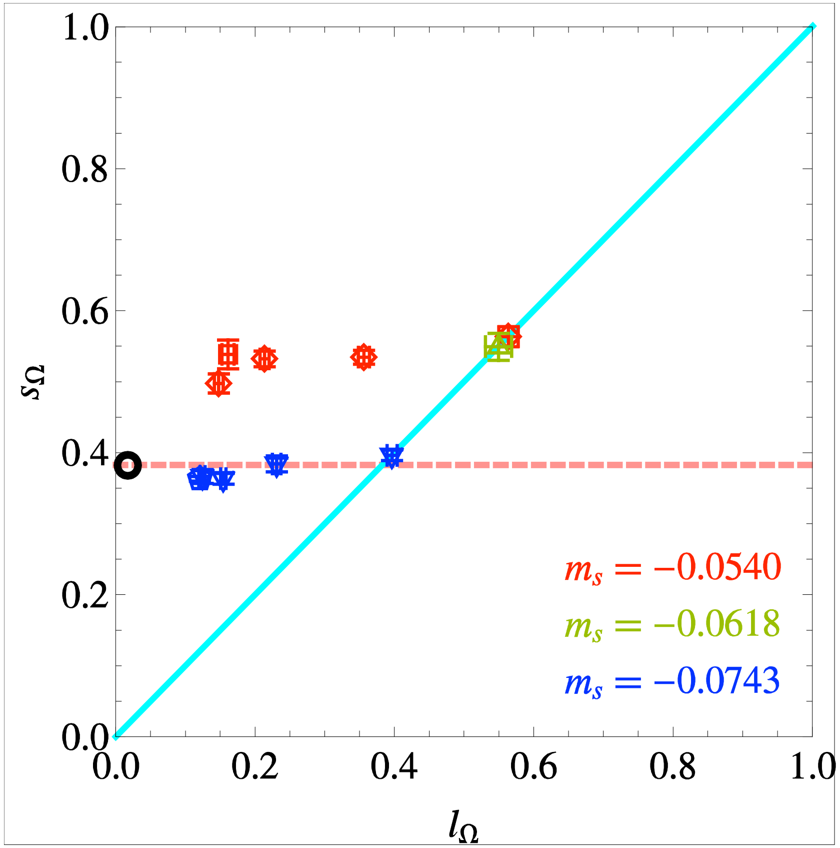
Challenge: setting scale and strange-quark mass

Lattice coupling *fixed*

Proportional to m_s to LO ChPT

$$s_X = (9/4)[2m_K^2 - m_\pi^2]/m_X^2$$

Omega



$$l_X = (9/4)m_\pi^2/m_X^2$$

Express physics in (dimensionless) (l,s) coordinates

H-W Lin et al (Hadron Spectrum Collaboration), PRD79, 034502 (2009)

Proportional to m_l to LO ChPT

Flavor Structure - I

$SU(3)_F$	S	L	J^P		
$\mathbf{8_F}$	$\frac{1}{2}$	1	$\frac{1}{2}^-$	$\frac{3}{2}^-$	
	$\frac{3}{2}$	1	$\frac{1}{2}^-$	$\frac{3}{2}^-$	$\frac{5}{2}^-$
$N_8(J)$			**2**	**2**	**1**
$\mathbf{10_F}$	$\frac{1}{2}$	1	$\frac{1}{2}^-$	$\frac{3}{2}^-$	
$N_{10}(J)$			**1**	**1**	**0**
$\mathbf{1_F}$	$\frac{1}{2}$	1	$\frac{1}{2}^-$	$\frac{3}{2}^-$	
$N_1(J)$			**1**	**1**	**0**

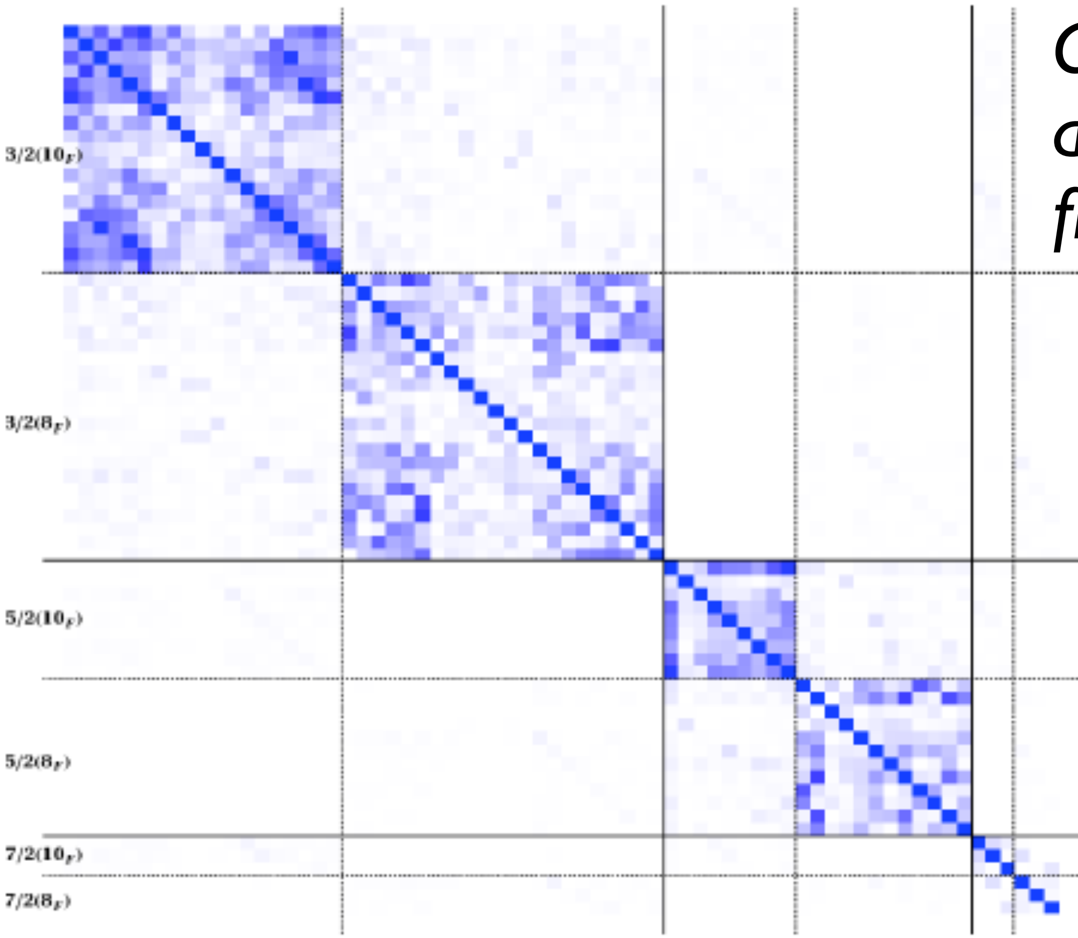
One derivative

$SU(3)_F$	S	L	J^P			
$\mathbf{8_F}$	$\frac{1}{2}$	0	$\frac{1}{2}^+$			
	$\frac{1}{2}$	0	$\frac{1}{2}^+$			
	$\frac{1}{2}$	1	$\frac{1}{2}^+$	$\frac{3}{2}^+$		
	$\frac{1}{2}$	2		$\frac{3}{2}^+$	$\frac{5}{2}^+$	
	$\frac{1}{2}$	2		$\frac{3}{2}^+$	$\frac{5}{2}^+$	
	$\frac{3}{2}$	0		$\frac{3}{2}^+$		
	$\frac{3}{2}$	2	$\frac{1}{2}^+$	$\frac{3}{2}^+$	$\frac{5}{2}^+$	$\frac{7}{2}^+$
$N_8(J)$			**4**	**5**	**3**	**1**
$\mathbf{10_F}$	$\frac{1}{2}$	0	$\frac{1}{2}^+$			
	$\frac{1}{2}$	2		$\frac{3}{2}^+$	$\frac{5}{2}^+$	
	$\frac{3}{2}$	0		$\frac{3}{2}^+$		
	$\frac{3}{2}$	2	$\frac{1}{2}^+$	$\frac{3}{2}^+$	$\frac{5}{2}^+$	$\frac{7}{2}^+$
$N_{10}(J)$			**2**	**3**	**2**	**1**
$\mathbf{1_F}$	$\frac{1}{2}$	0	$\frac{1}{2}^+$			
	$\frac{1}{2}$	2		$\frac{3}{2}^+$	$\frac{5}{2}^+$	
	$\frac{3}{2}$	1	$\frac{1}{2}^+$	$\frac{3}{2}^+$	$\frac{5}{2}^+$	
$N_1(J)$			**2**	**2**	**2**	**0**

Two derivative

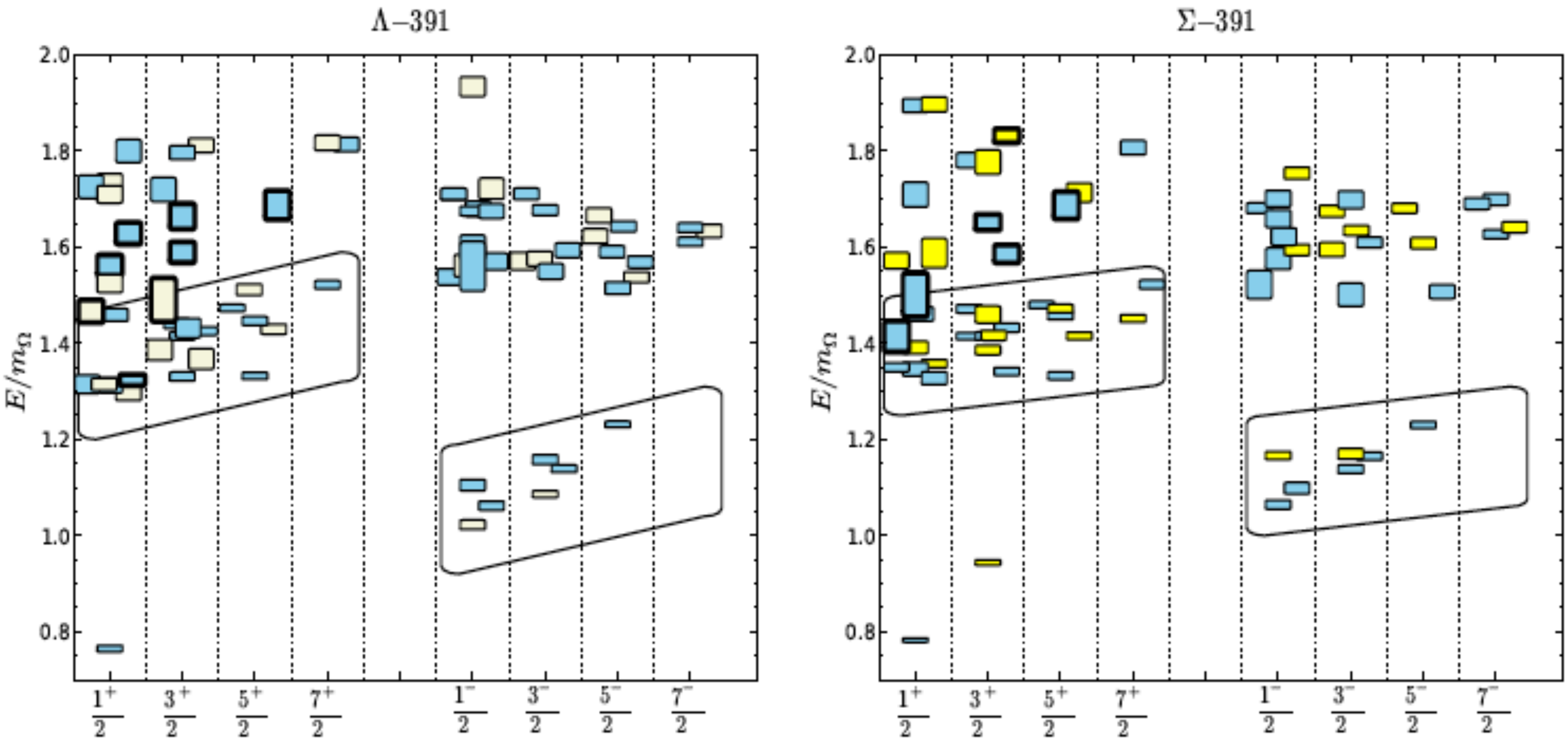
R. Edwards et al., Phys. Rev. D87 (2013) 054506

Flavor Structure - II

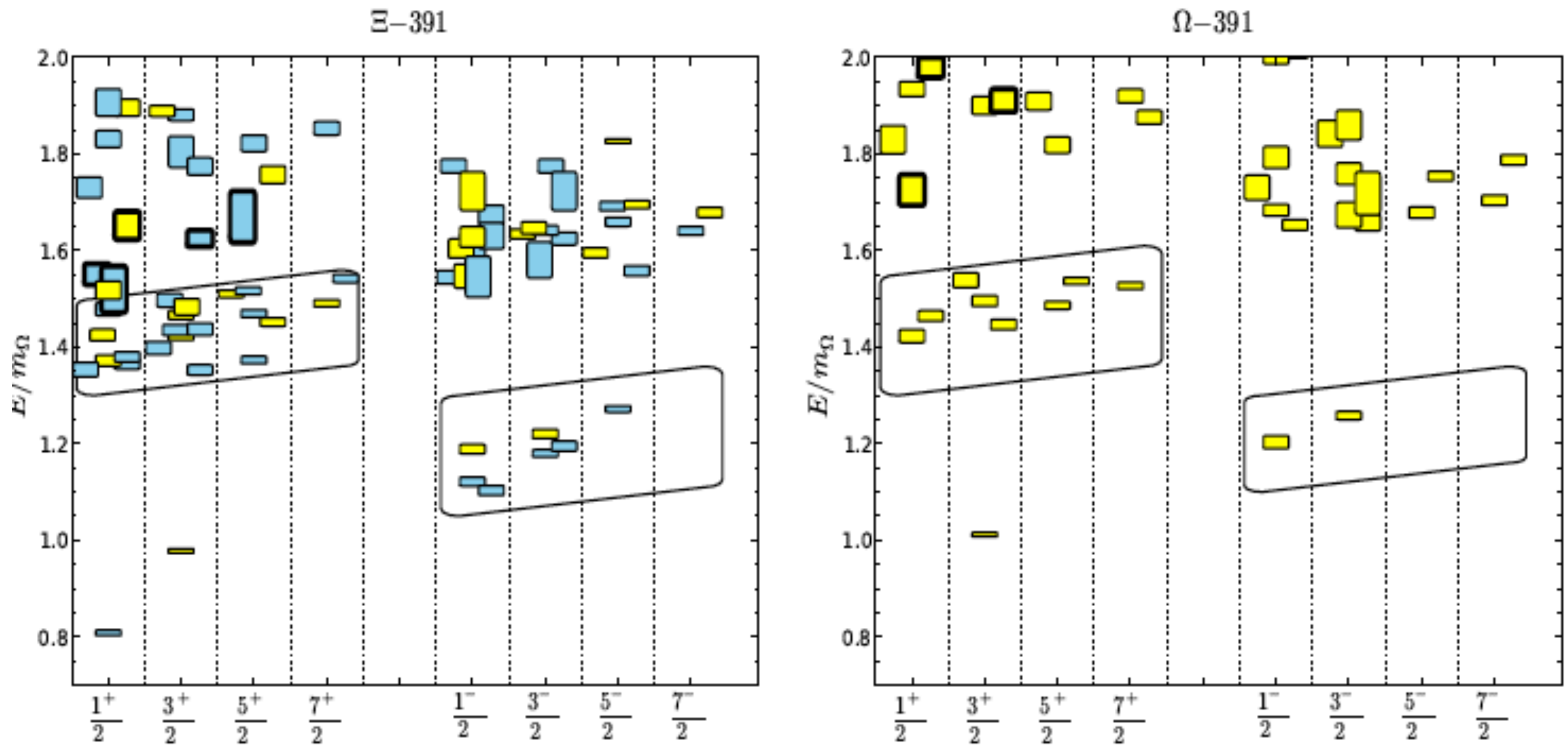


Correlation matrix diagonal in spin and flavor

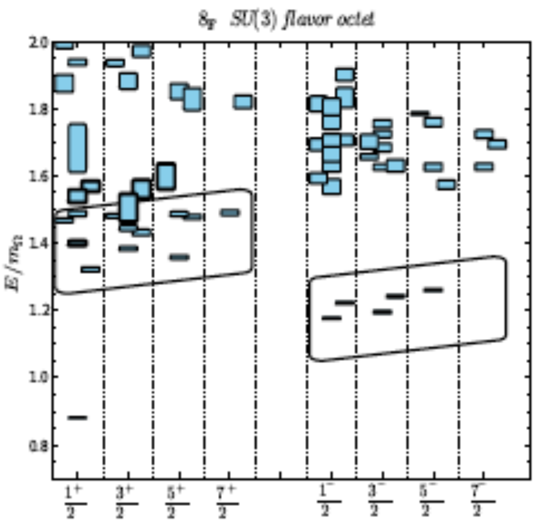
$m_\pi \simeq 400$ MeV



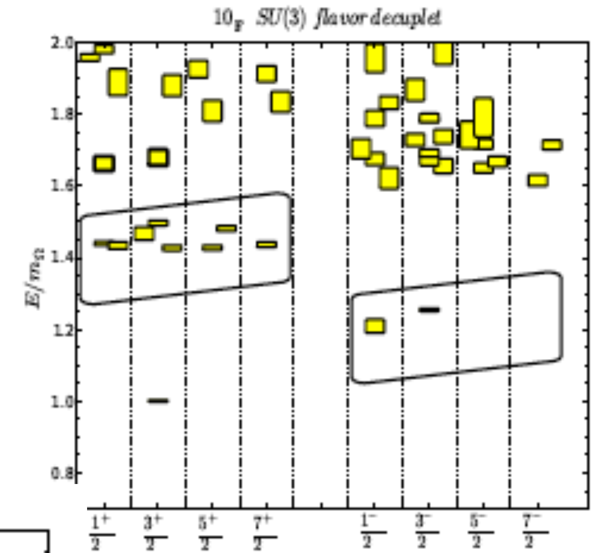
• Can identify predominant flavor for each state: Yellow (10F), Blue (8F), Beige (1F).
• SU(6) x O(3) Counting
• Presence of “hybrids” characteristic across all *+ve parity* channels: **BOLD Outline**

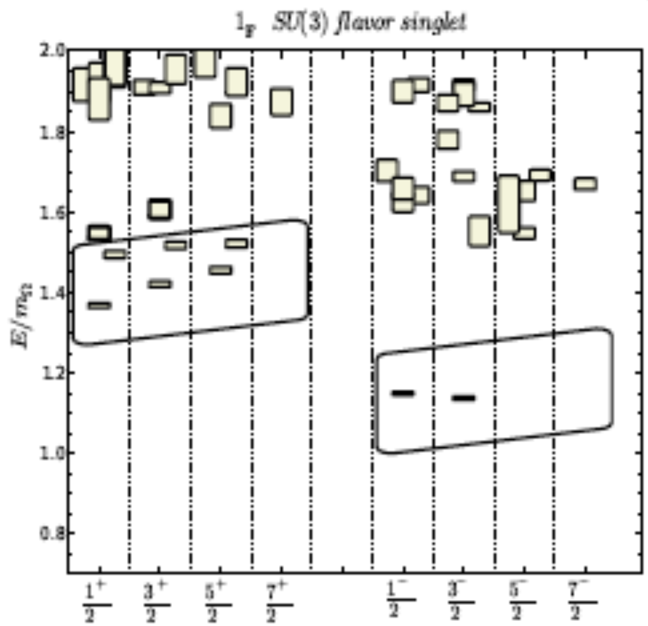


Spectrum superposition of flavor structure

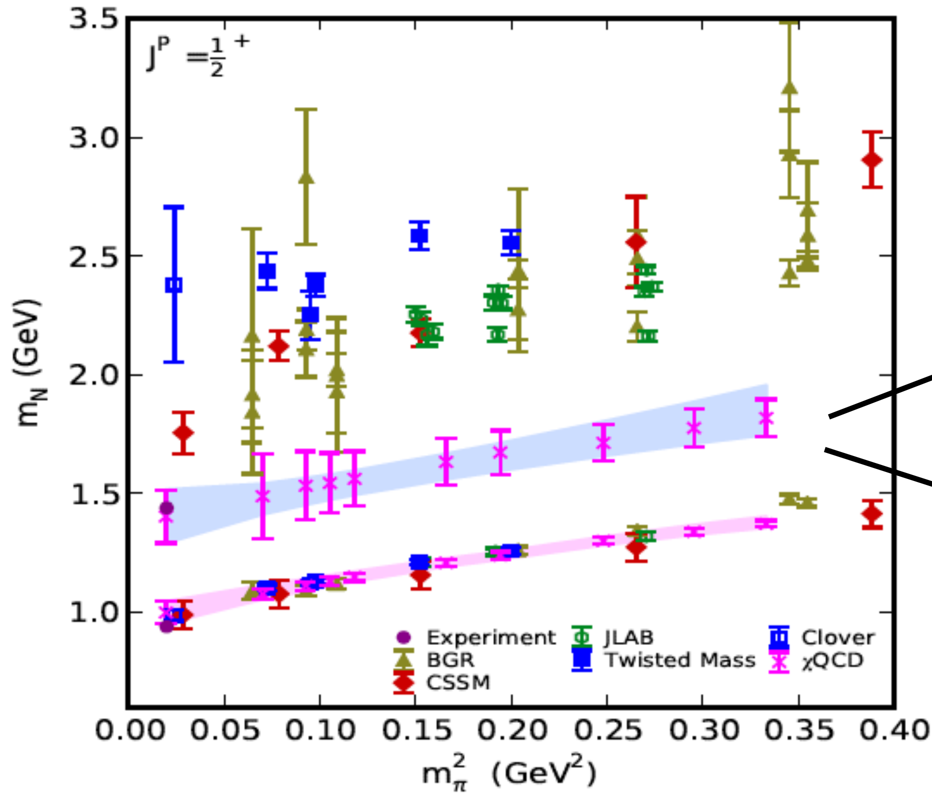


SU(3) Symmetric





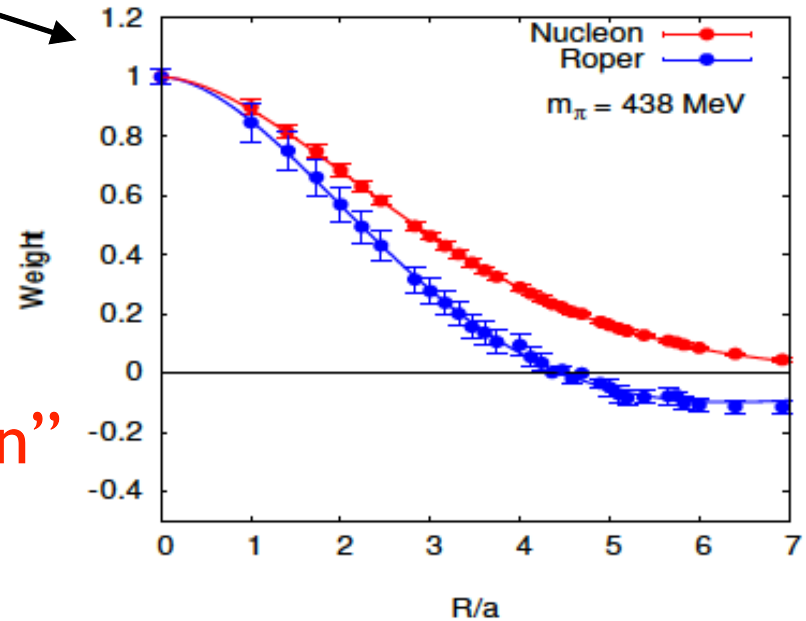
Roper Resonance



Compendium of Roper results

χQCD, arXiv:1403.6847

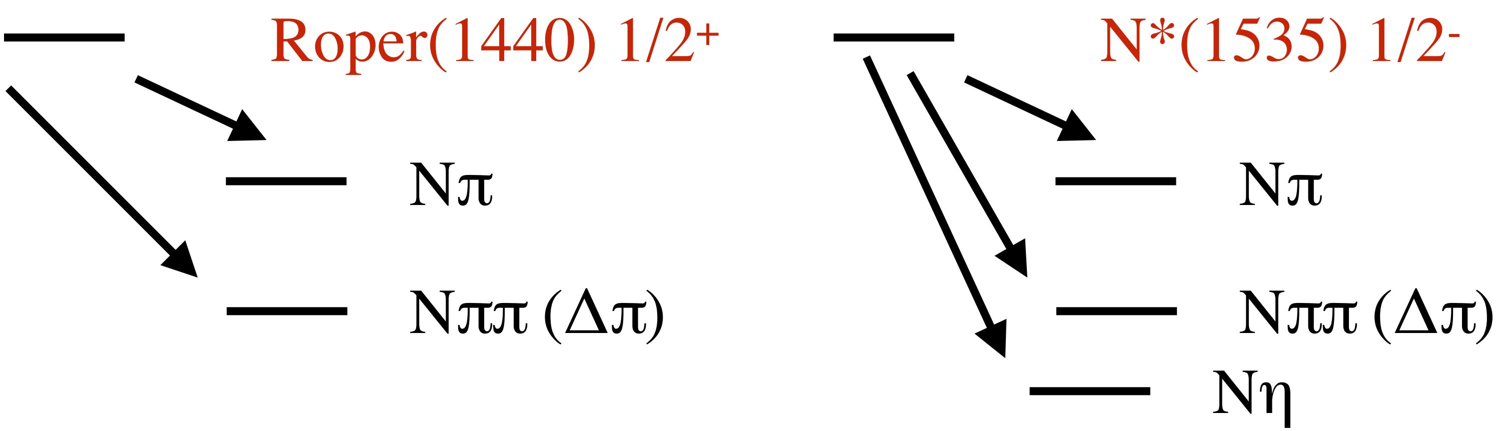
Overlap Fermions,
Sequential Bayesian Method



"radial wave function"

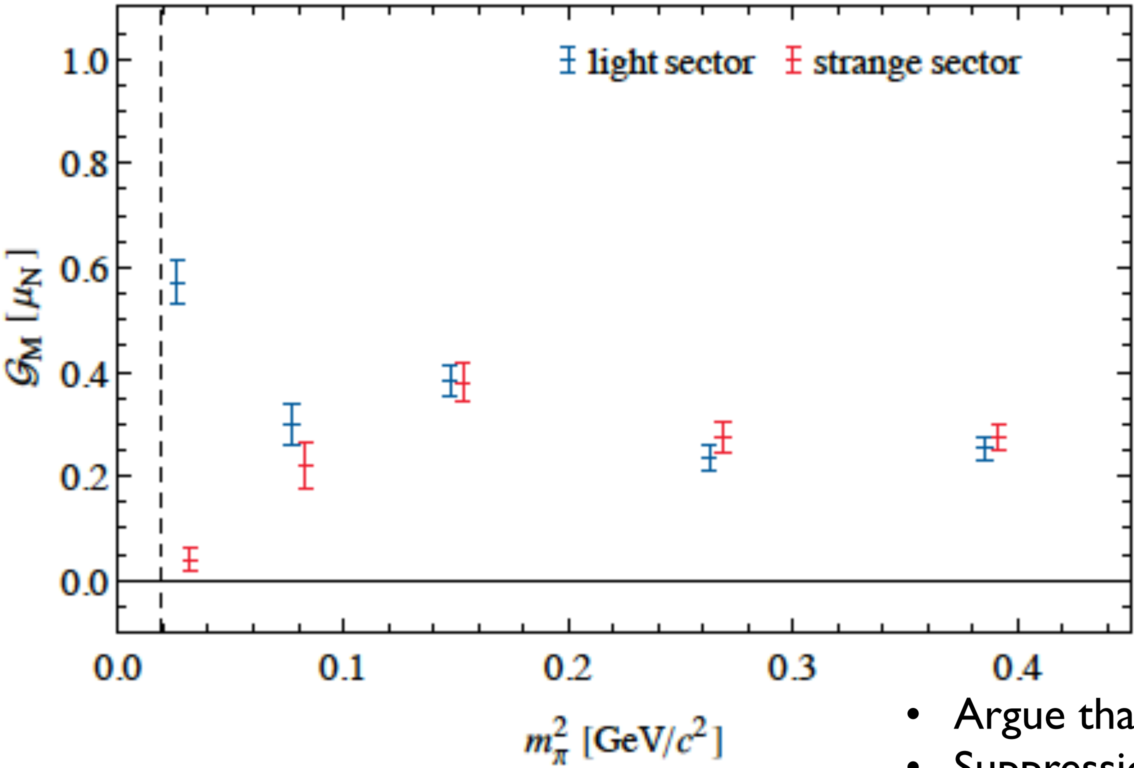
Some of our states are missing…

Partial decay widths



Momenta are quantised → discrete spectrum of energies. *Even above threshold at our quark masses we should see (close-to?) these energies in spectrum*

Lambda (1405)



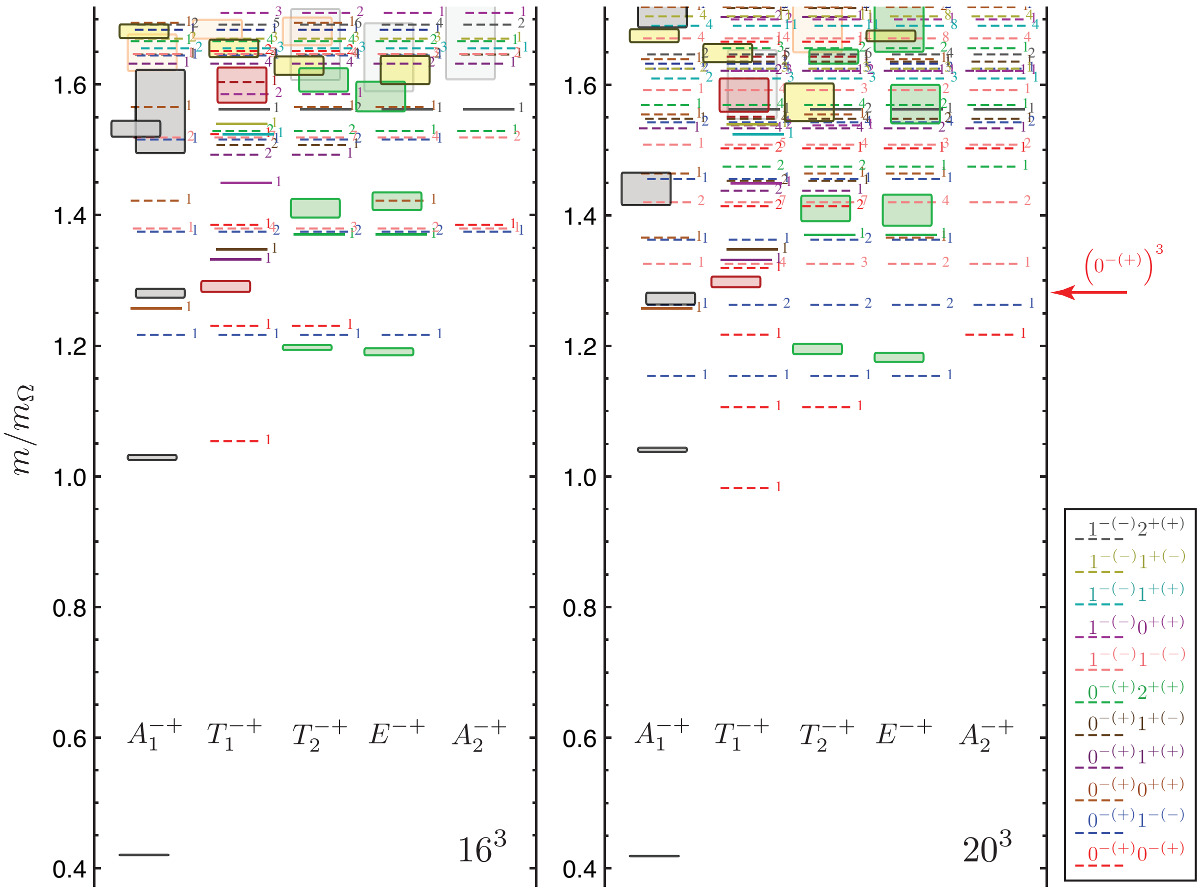
Hall et al, arXiv;1411.3402, PRL

- Argue that is molecular state
- Suppression of strangeness contribution to magnetic moment consistent with KN molecule
- Strong caveat - *interpretation in terms of infinite-volume matrix element requires two-body analysis at finite volume*

Two poles below KN threshold?

Isovector meson spectrum

States unstable under strong interactions



Meson spectrum on two volumes: dashed lines denote expected (non-interacting) multi-particle energies.

Allowed two-particle contributions - momenta - governed by cubic symmetry of volume

Calculation is incomplete.

Momentum-dependent I = 2 ππ Phase Shift

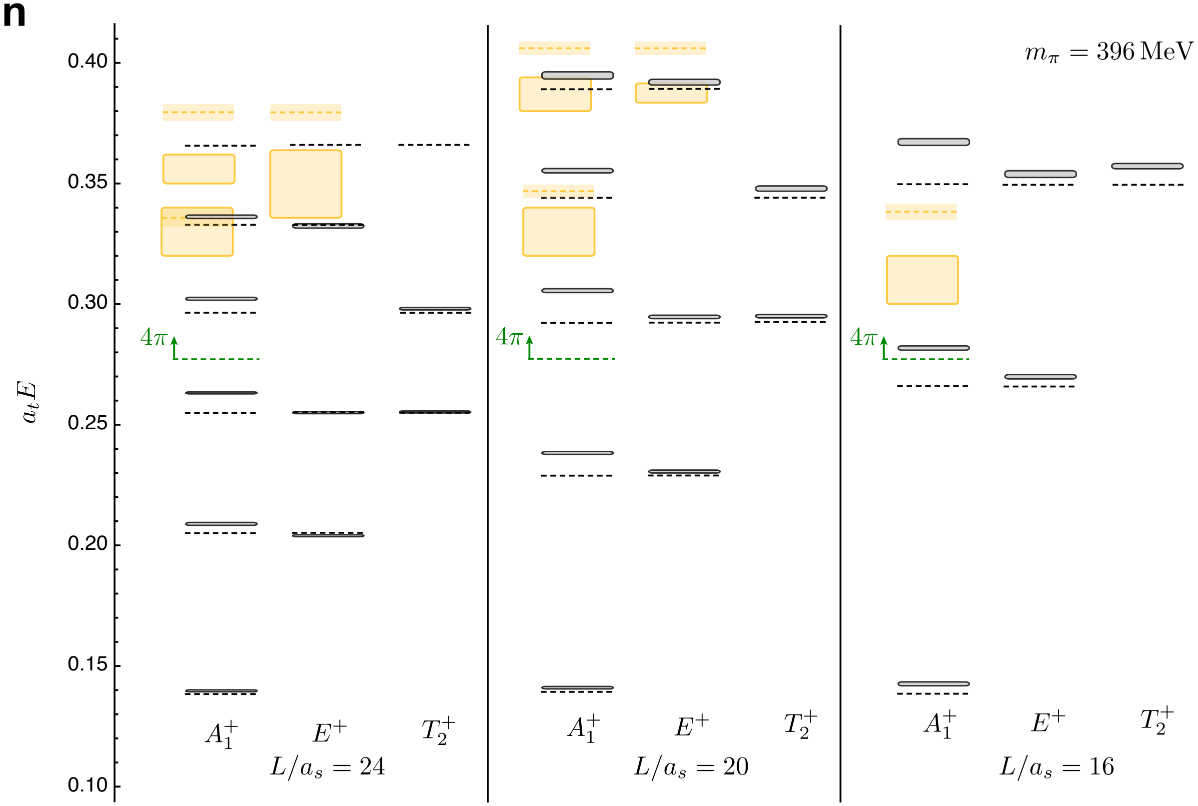
Dudek *et al.*, Phys Rev D83, 071504 (2011)

Include two-body operators

Operator basis $\mathcal{O}^{\Gamma,\gamma}_{\pi\pi}(|\vec{p}|) = \sum_m \mathcal{S}^{\ell,m}_{\Gamma,\gamma} \sum_{\hat{p}} Y^m_\ell(\hat{p})\, \mathcal{O}_\pi(\vec{p})\mathcal{O}_\pi(-\vec{p})$

Total momentum zero - pion momentum ±*p*

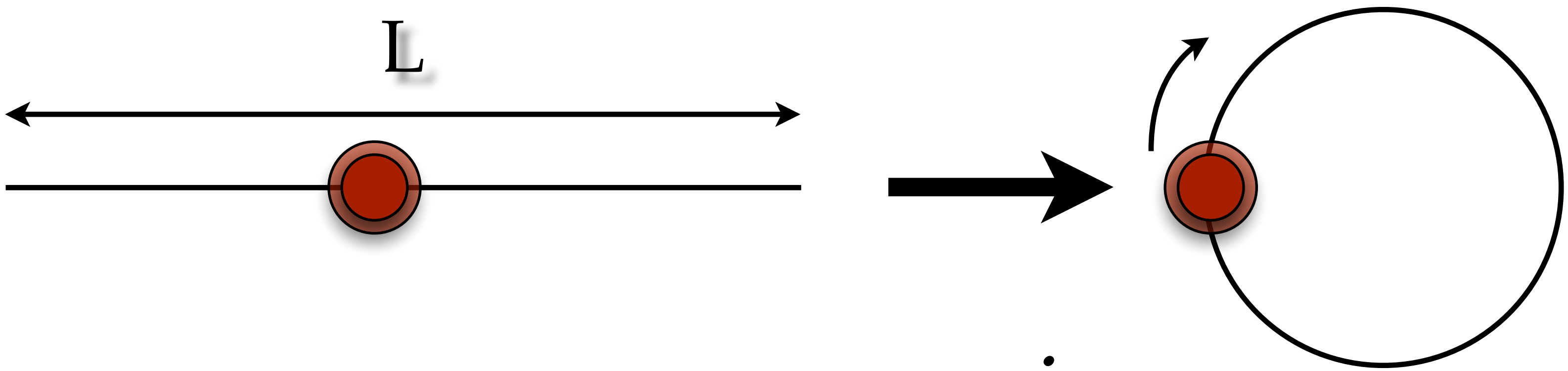
Luescher: energy levels at finite volume ↔ phase shift at corresponding *k*



Reinventing the *quantum-mechanical* wheel

Thanks to Raul Briceno

(in 1+1 dimensions)

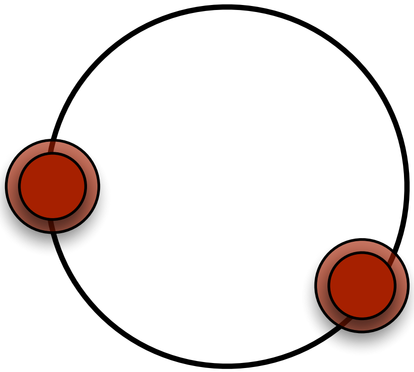


$$\phi(x) \sim e^{ipx}$$

Periodicity: $L\, p_n = 2\pi n$

Reinventing the *quantum-mechanical* wheel

Two particles:



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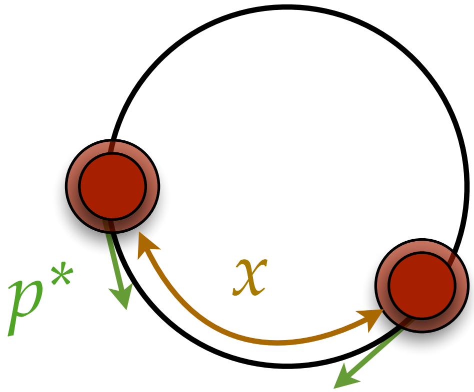

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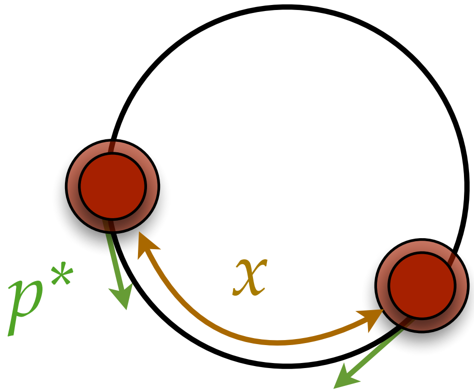
Two particles:



$$\psi(x) \sim e^{ip^*|x| + i2\delta(p^*)}$$

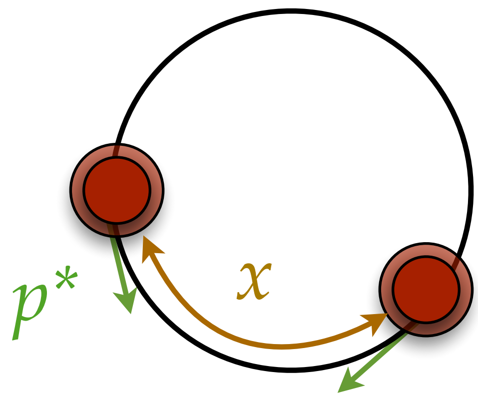
Asymptotic wavefunction

infinite volume scattering phase shift

Reinventing the *quantum-mechanical* wheel

Two particles:

infinite volume scattering phase shift



$$\psi(x) \sim e^{ip^*|x| + i2\delta(p^*)}$$

Asymptotic wavefunction

Periodicity:

$$L\, p_n^* + 2\delta(p_n^*) = 2\pi n$$

Reinventing the *quantum-mechanical* wheel

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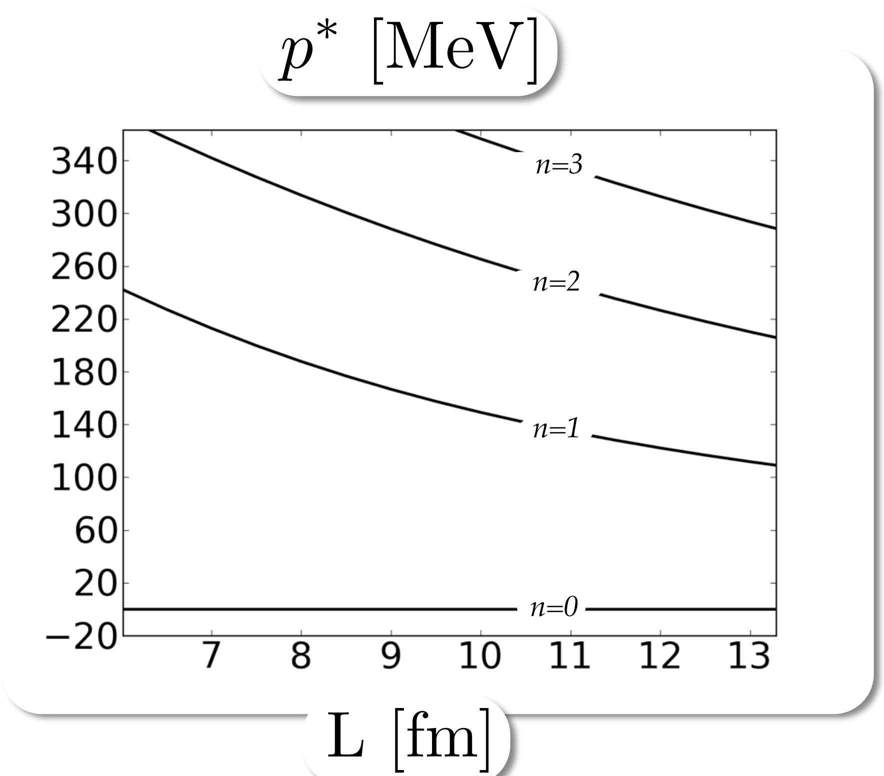

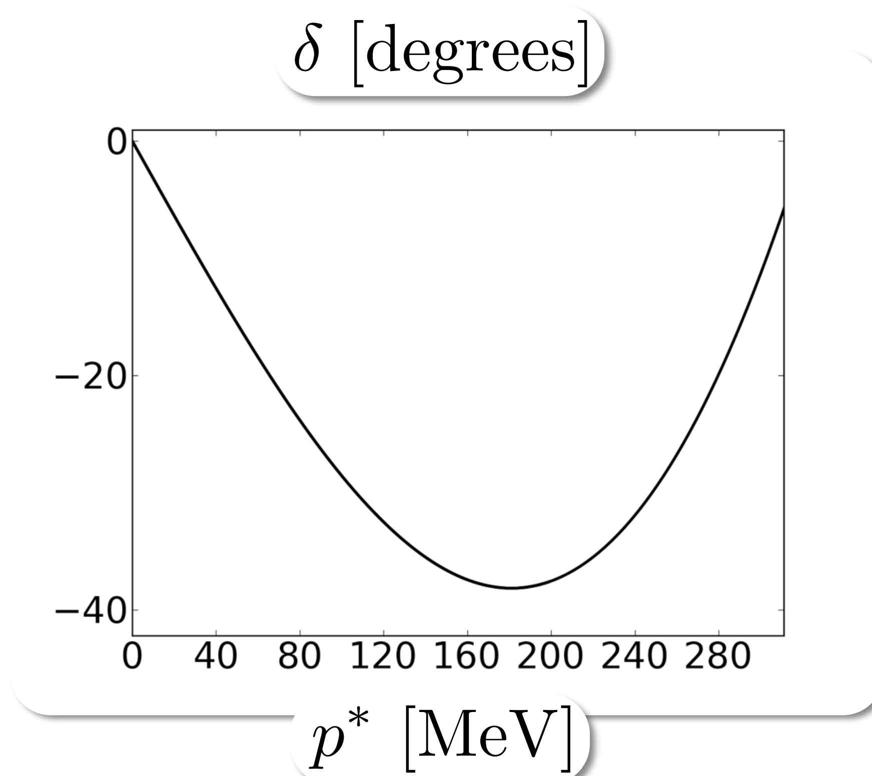

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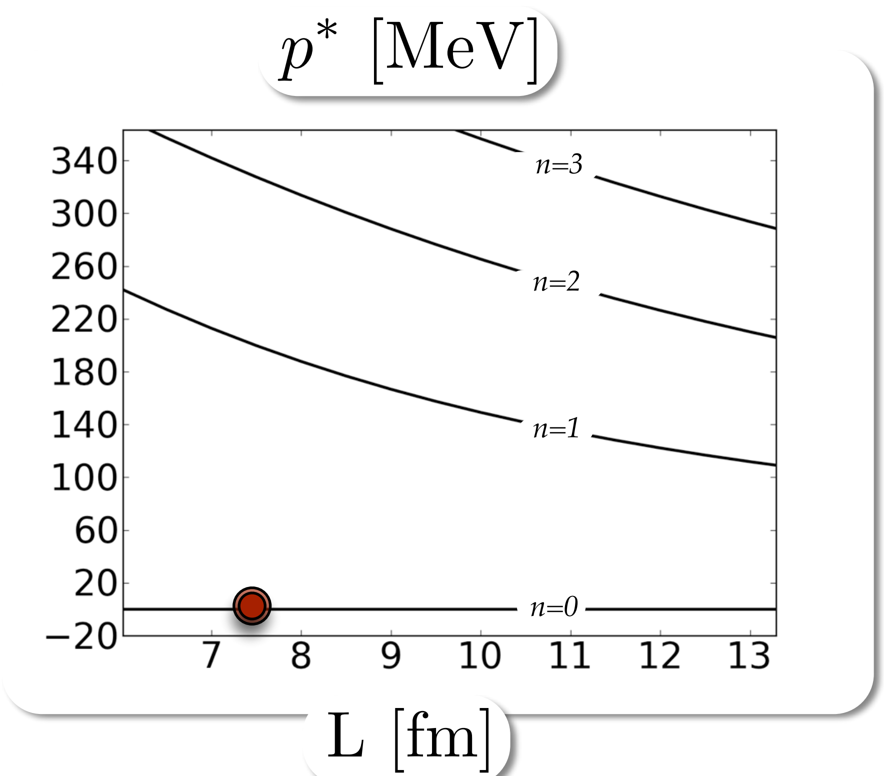

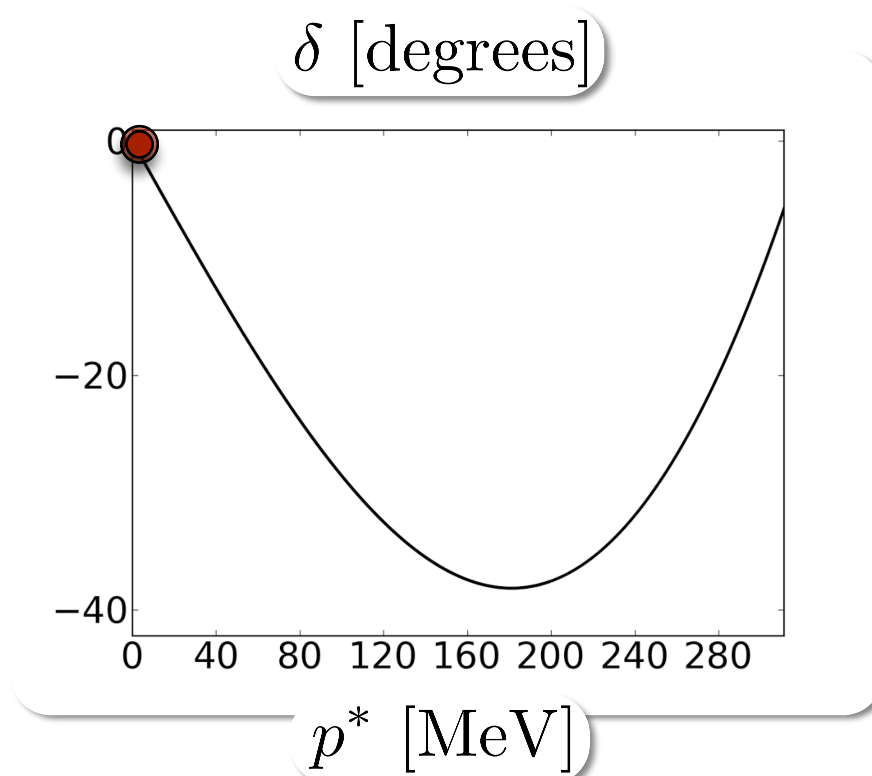

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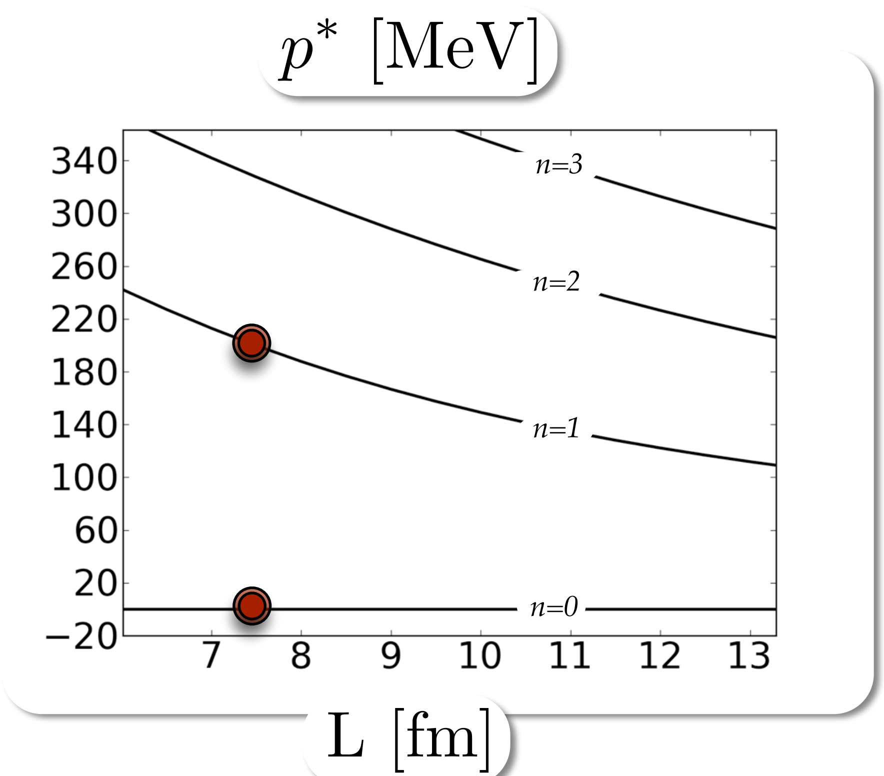

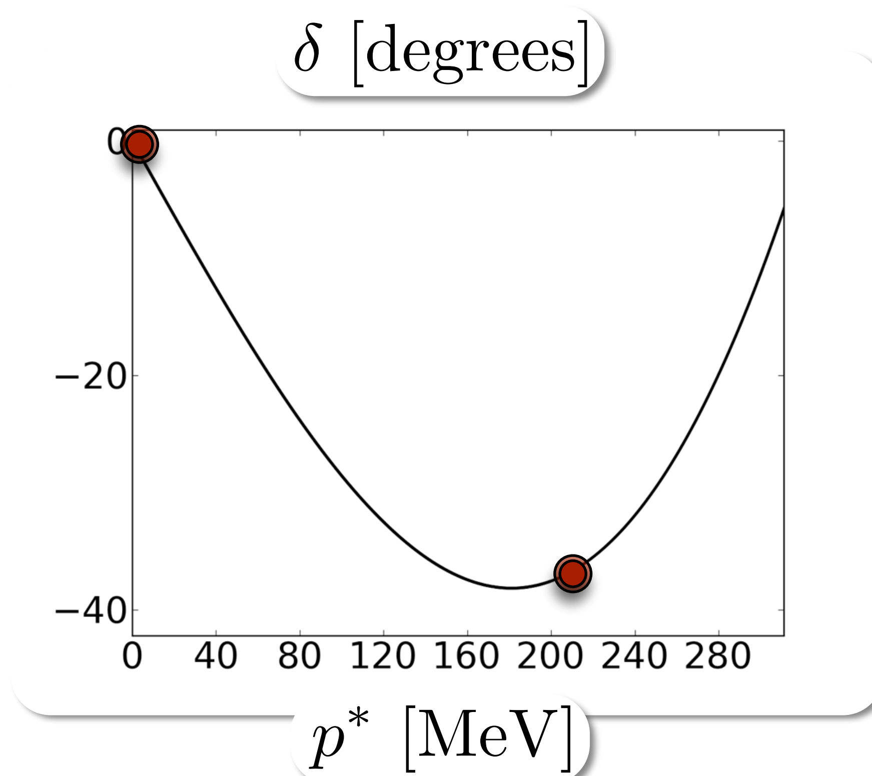

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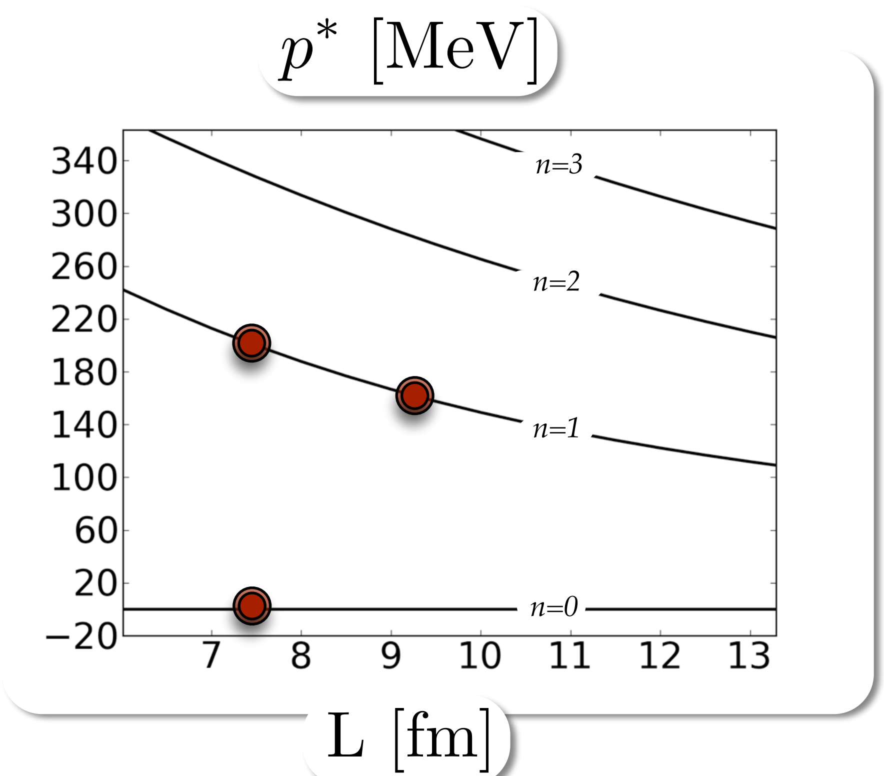

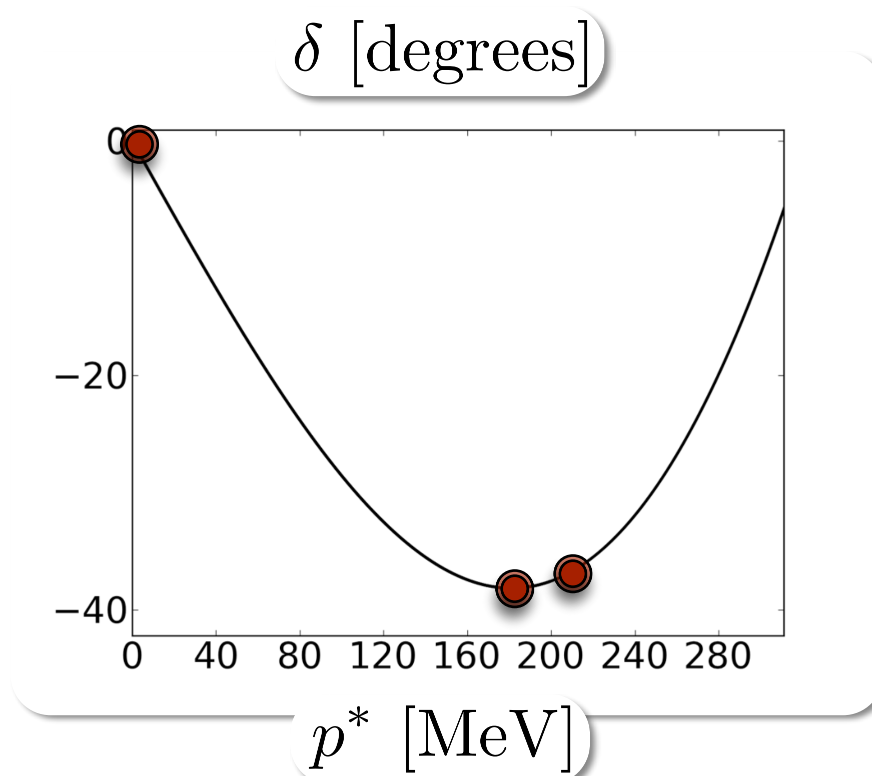

Reinventing the *quantum-mechanical* wheel

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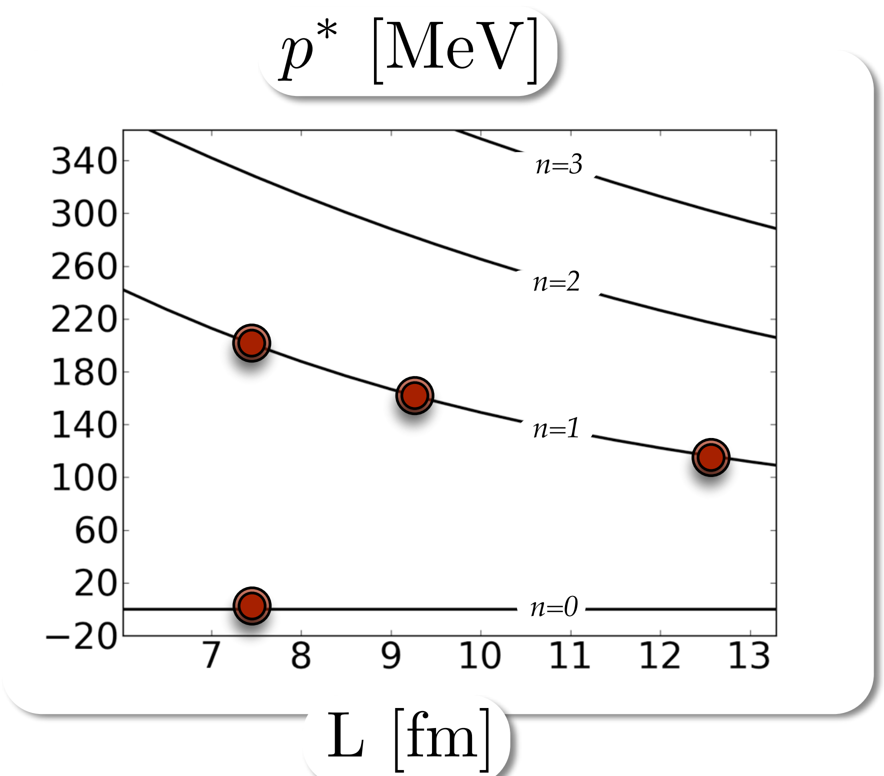
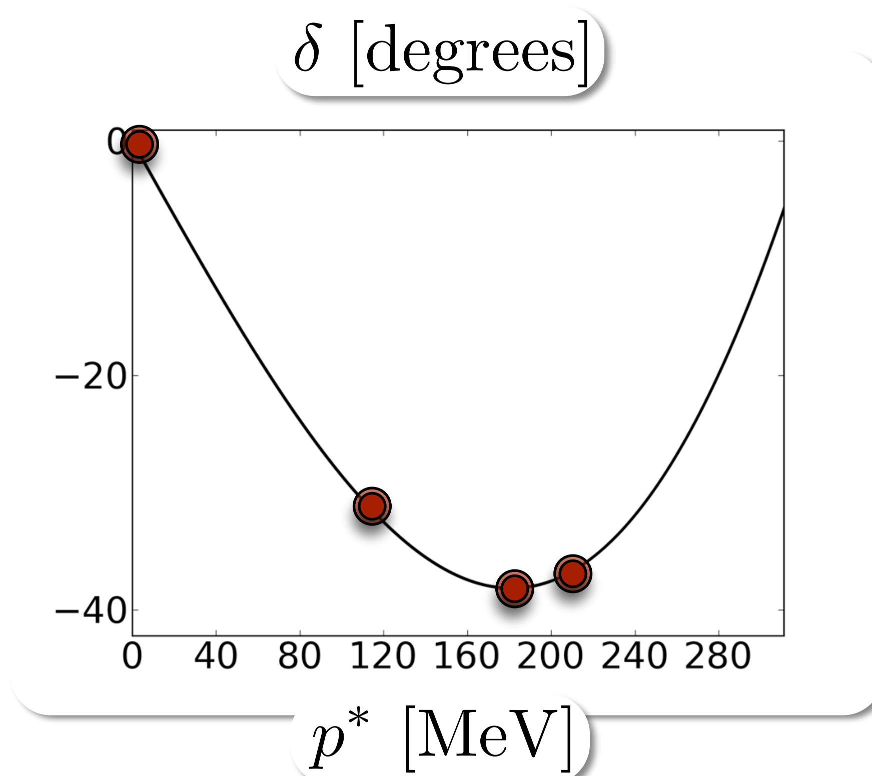


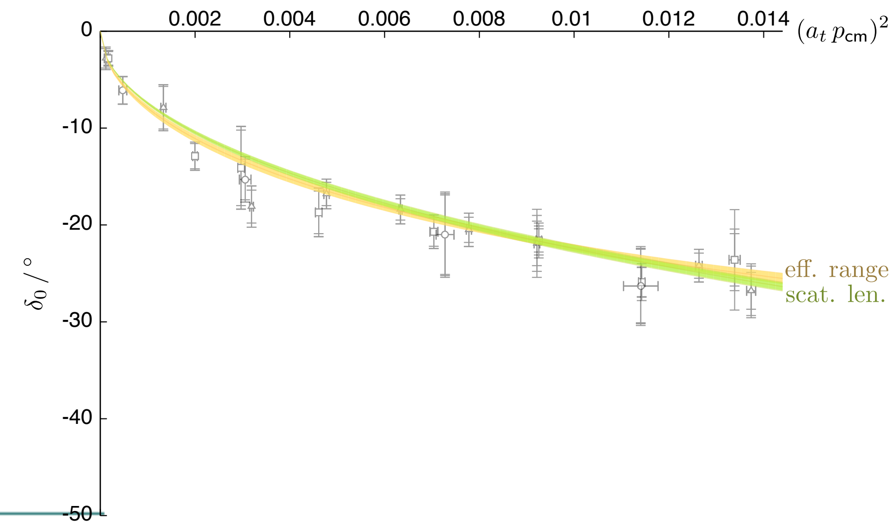


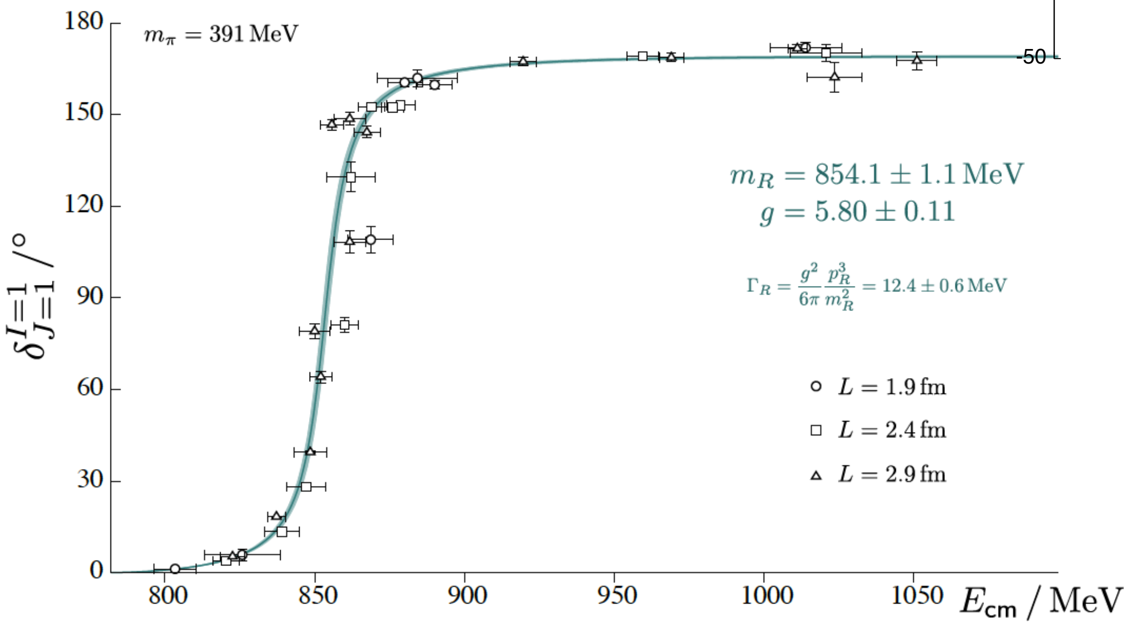
I=2 and Resonant I = 1 ππ Phase Shift

$$\det\left[e^{2i\boldsymbol{\delta}(k)} - \mathbf{U}_\Gamma\left(k\frac{L}{2\pi}\right)\right] = 0$$

Matrix in l **lattice irrep**

Dudek *et al.*, Phys Rev D83, 071504 (2011); arXiv:1203.6041





Feng, Renner, Jansen, PRD83, 094505

PACS-CS, PRD84, 094505

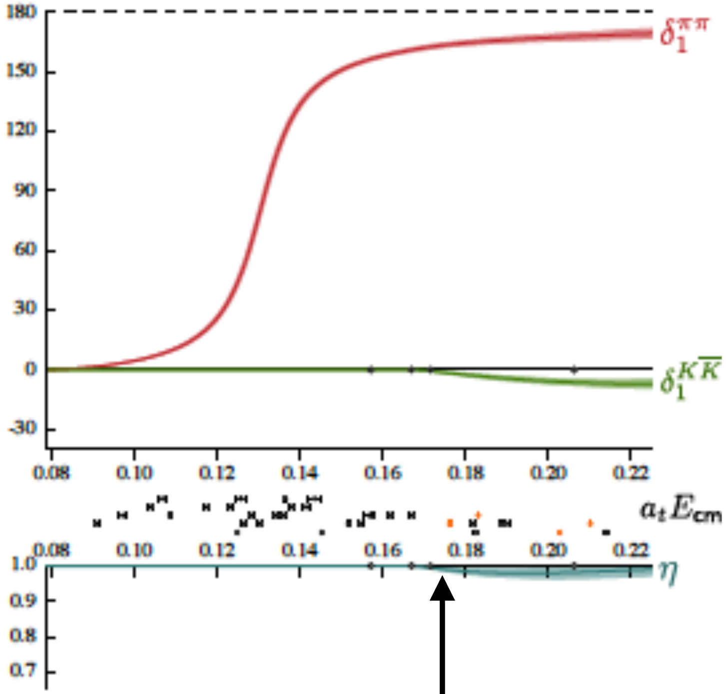
Alexandru et al

Lang et al., PRD84, 054503

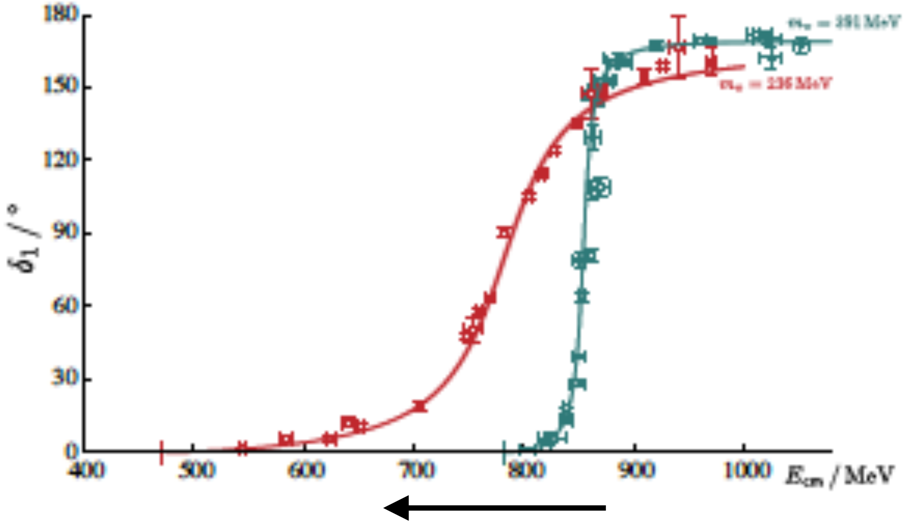
Dudek, Edwards, Thomas, Phys. Rev. D 87, 034505 (2013)

Inelastic in ππ KK channel

Wilson, Briceno, Dudek, Edwards, Thomas, arXiv:1507.02599



Inelastic Threshold



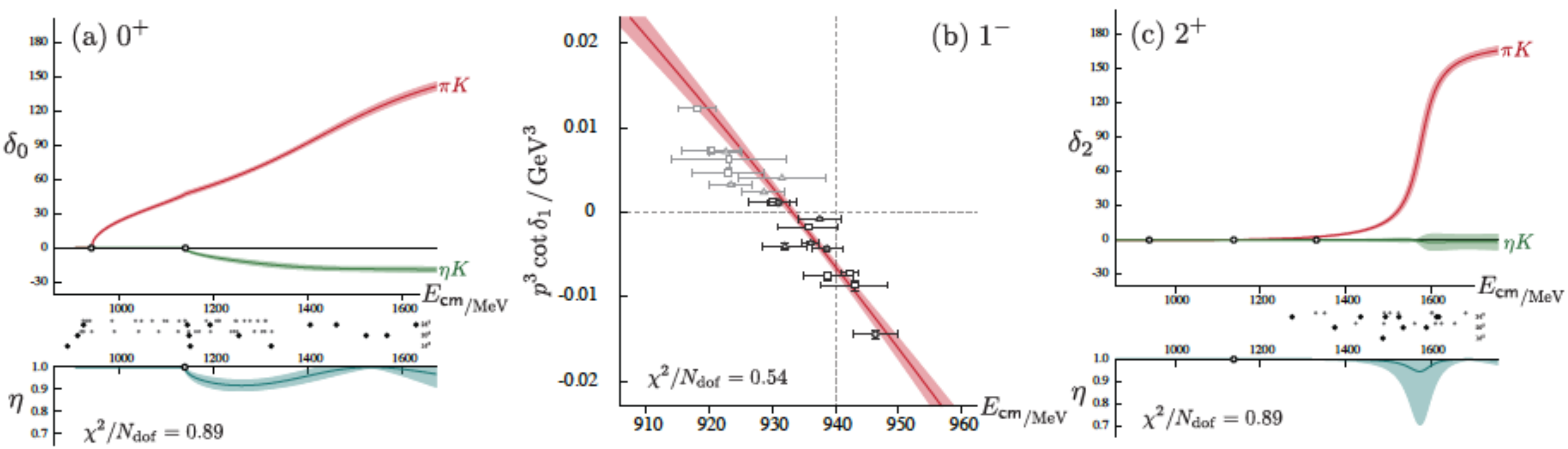
Decreasing Pion Mass

First - and Successful - inelastic

$$\det\left[\delta_{ij}\delta_{JJ'} + i\rho_i\, t_{ij}^{(J)}(E_{\mathsf{cm}})\left(\delta_{JJ'} + i\mathcal{M}_{JJ'}^{\vec{P}\Lambda}(p_i L)\right)\right] = 0$$

Parametrized as phase shift + inelasticity

$$t_{ii} = \frac{(\eta e^{2i\delta_i} - 1)}{2i\rho_i}, \quad t_{ij} = \frac{\sqrt{1-\eta^2}\, e^{i(\delta_i+\delta_j)}}{2\sqrt{\rho_i\,\rho_j}}$$



Dudek, Edwards, Thomas, Wilson, PRL, PRD

Lattices for Hadron Physics

- Calculations at physical light-quark masses: *direct comparison with experiment*
- Several fine lattice spacings: *controlled extrapolation to continuum, and to reach high Q2*
- Hypercube symmetry: *simplified operator mixing*
- *Variational method, to control and extract excited states*

$$\mathrm{Cost}_{\mathrm{traj}} = C\xi^{1.25}\left(\frac{\mathrm{fm}}{a_s}\right)^6 \cdot \left[\left(\frac{L_s}{\mathrm{fm}}\right)^3\left(\frac{L_t}{\mathrm{fm}}\right)\right]^{5/4}$$

Major Effort by USQCD

Summary

- Determining the quantum numbers and the study of the “single-hadron” states a solved problem
- Lattice calculations used to construct new “phenomenology” of QCD
  - Quark-model like spectrum, *common mechanism for gluonic excitations in mesons and baryons. LOW ENERGY GLUONIC DOF*
- ***“Prediction”*** *- Additional states in baryon spectrum associated with hybrid dof, including for hyperons! Spectrum at least a rich as quark model - flavor structure a superposition!*
- Formalism for extracting scattering amplitudes, including inelastic channels, developed - applied for first time to meson sector
- COUPLED-CHANNEL METHODS ARE KEY

Efficient Correlation fns:

- Use the new "distillation" method.
- Observe $L^{(J)} \equiv (1 - \frac{\kappa}{n}\Delta)^n = \sum_{i=1} f(\lambda_i) v^{(i)} \otimes v^{*(i)}$

Eigenvectors of Laplacian

- Truncate sum at sufficient i to capture relevant physics modes – we use 64: set "weights" f to be unity
- *Baryon* correlation function

$$
\begin{aligned}
C_{ij}(t) = \; & \Phi^{i,(p,q,r)}_{\alpha\beta\gamma}(t)\Phi^{j,(\bar{p},\bar{q},\bar{r})\dagger}_{\bar{\alpha}\bar{\beta}\bar{\gamma}}(0) \\
& \times \Big[\tau^{p\bar{p}}_{\alpha\bar{\alpha}}(t,0)\tau^{q\bar{q}}_{\beta\bar{\beta}}(t,0)\tau^{r\bar{r}}_{\gamma\bar{\gamma}}(t,0) \\
& \quad - \tau^{p\bar{p}}_{\alpha\bar{\alpha}}(t,0)\tau^{q\bar{r}}_{\beta\bar{\gamma}}(t,0)\tau^{r\bar{q}}_{\gamma\bar{\beta}}(t,0)\Big]
\end{aligned}
$$

M. Peardon *et al.*, PRD80,0 (2009)

where

$$
\Phi^{i,(p,q,r)}_{\alpha\beta\gamma} = \epsilon^{abc} S^i_{\alpha\beta\gamma} (\mathbf{\Gamma}_1 \xi^{(p)})^a (\mathbf{\Gamma}_2 \xi^{(q)})^b (\mathbf{\Gamma}_3 \xi^{(r)})^c
$$

Perambulators

$$
\tau^{p\bar{p}}_{\alpha\bar{\alpha}}(t,0) = \xi^{\dagger(p)}(t) M^{-1}_{\alpha\bar{\alpha}}(t,0)\xi^{(\bar{p})}(0)
$$