A Fitting Robot for Variational Analysis

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CSSM

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Introduction

- All-to-all propagators
- The folly of effective mass plots
- Variational analysis
- Need to minimise work *and uncertainty*
The quark propagator is broken up into two subspaces:

\[ Q^{-1} = \bar{Q}_0 + \bar{Q}_1, \]

- \( \bar{Q}_0 \) is given by truncated spectral decomposition.
- \( \bar{Q}_1 \) is estimated stochastically.
All-to-all Propagators - Hybrid Method

The quark propagator is broken up into two subspaces: \( Q^{-1} = \tilde{Q}_0 + \tilde{Q}_1 \),
- \( \tilde{Q}_0 \) is given by truncated spectral decomposition.
- \( \tilde{Q}_1 \) is estimated stochastically.

Construct the hybrid list

\[
\begin{align*}
 w(i) &= \left\{ \frac{\nu(1)}{\lambda_1}, \ldots, \frac{\nu(N_{ev})}{\lambda_{N_{ev}}}, \eta(1), \ldots, \eta(N_d) \right\} \\
 u(i) &= \left\{ \nu(1), \ldots, \nu(N_{ev}), \psi(1), \ldots, \psi(N_d) \right\}
\end{align*}
\]

The hybrid formula for the all-to-all quark propagator (where \( Q = \gamma_5 M \)) is given by

\[
M^{-1} = \sum_{i=1}^{N_{HL}} u(i)(\vec{x}, x_4) \otimes w(i)(\vec{y}, y_4)^{\dagger} \gamma_5
\]
Lattice Parameters

- $N_f = 2$ dynamical background
- $12^3 \times 80$ anisotropic lattice with $\xi = 6$ and $a_s = 0.2\text{fm}$.
- 96 gauge configurations
- Operators - quark bilinears, extended, smeared.
- Light quark mass comparable to strange.
- 20 eigenvectors, time and colour dilution
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Motivation

All-to-all Propagators - Hybrid Method
Lattice Parameters
Conclusions and Further Work
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$0^{-+}$

![Graph of $a_tM_{\text{eff}}$ vs. Timeslice](image)

State 0

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**All-to-all Propagators - Hybrid Method**

Lattice Parameters

**Motivation**

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All-to-all Propagators - Hybrid Method

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- Identify data limitations
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- Identify data limitations
- $\chi_{\text{PDOF}}^2$ - measure of fit
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- Identify data limitations
- $\chi^2_{PDOF}$ - measure of fit
- Search for maximum sized fit window
Identify data limitations

$\chi^2_{\text{PDOF}}$ - measure of fit

Search for maximum sized fit window

$t_{\text{min}}$ constrains fit and $\chi^2_{\text{PDOF}}$, search in $t_{\text{max}}$ direction first
Identify data limitations

$\chi^2_{PDOF}$ - measure of fit

Search for maximum sized fit window

$t_{min}$ constrains fit and $\chi^2_{PDOF}$, search in $t_{max}$ direction first

Resulting bias - largest fit window beginning at lowest $t_{min}$ with acceptable $\chi^2_{PDOF}$ value
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The graph shows a plot of \( I^{-+} \) versus timeslice. The x-axis represents the timeslice, ranging from 0 to 80, while the y-axis represents a logarithmic scale from \( 10^{-7} \) to 10, indicating the magnitude of \( I^{-+} \). The data points are marked with error bars, suggesting variability or uncertainty in the measurements. The graph highlights the state of the system as it evolves over time, possibly indicating a decay or growth process depending on the context of the variational analysis.
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\[ \theta^{++} \]

\[ \theta^{-} \]

State 0

\[ a_{\text{eff}} \]

\text{Timeslice}

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$0^{--}$

State 0

$\alpha_{\text{Meff}}$

Timeslice

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Extracting excited-state energies requires matrix of correlators

For a given $N \times N$ correlator matrix

$$C_{\alpha\beta}(t) = \langle 0 | O_\alpha(t) O_\beta(0) | 0 \rangle$$

one defines the $N$ principal correlators $\lambda_\alpha(t, t_0)$ as the eigenvalues of

$$C(t_0)^{1/2} C(t) C(t_0)^{1/2}$$

where $t_0$ (the time defining the metric) is small

Can show that $\lim_{t \to \inf} \lambda_\alpha(t, t_0) = e^{-(t-t_0)E_\alpha} (1 + e^{-t\Delta E_\alpha})$

N principal effective masses defined by $m_{\alpha}^{\text{eff}}(t) = \ln\left(\frac{\lambda_\alpha(t, t_0)}{\lambda_\alpha(t+1, t_0)}\right)$ now tend (plateau) to the $N$ lowest-lying stationary-state energies, as do the projected correlation functions
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Effective Mass ($m_a$)

Eigenvalue - State 0

Eigenvalue - State 1

Metric Timeslice

$0^-$
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Effective Mass (mₐₜ)

Metric Timeslice

Fitted - State 0
Fitted - State 1
Eigenvalue - State 0
Eigenvalue - State 1
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Effective Mass ($m_a t$) vs. Metric Timeslice

- Fitted - State 0
- Fitted - State 1
- Eigenvalue - State 0
- Eigenvalue - State 1
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Results

Effective Mass (mₐₜ)

Metric Timeslice

Fitted - State 0
Fitted - State 1
Eigenvalue - State 0
Eigenvalue - State 1

α - +

0 - +
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Effective Mass ($m_{at}$) vs. Metric Timeslice

- Fitted - State 0
- Fitted - State 1
- Fitted - State 2
- Eigenvalue - State 0
- Eigenvalue - State 1
- Eigenvalue - State 2
Conclusions and Further Work

- Fitting robot works!
Conclusions and Further Work

- Fitting robot works!
- Necessary improvements:
  - Consistency check - bootstrapping the fit region
  - Ensure no subsequent plateau after fit region
Fitting robot works!

Necessary improvements:
- Consistency check - bootstrapping the fit region
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Necessary to explore the variational analysis parameter space
Conclusions and Further Work

- Fitting robot works!
- Necessary improvements:
  - Consistency check - bootstrapping the fit region
  - Ensure no subsequent plateau after fit region
- Necessary to explore the variational analysis parameter space
- Need to replace effective mass plots with more informative visual aid
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Other Symmetry Channels
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Other Symmetry Channels
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Other Symmetry Channels

![Graph showing effective mass and metric timeslice with fitted states and eigenvalues]
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Other Symmetry Channels

Metric Timeslice

Effective Mass (m_a)

Eigenvalue - State 0
Eigenvalue - State 1
Eigenvalue - State 2
Eigenvalue - State 3
Eigenvalue - State 4

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Other Symmetry Channels

![Graph](image)

- $I^{++}$
- Effective Mass ($m_a$)
- Metric Timeslice
- Fitted - State 0
- Fitted - State 1
- Fitted - State 2
- Fitted - State 3
- Fitted - State 4

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\[ I^{++} \]

- Fitted - State 0
- Fitted - State 1
- Fitted - State 2
- Fitted - State 3
- Fitted - State 4
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2^+ 

Effective Mass (m_{a_t})

Metric Timeslice

Fitted - State 0
Fitted - State 1
Fitted - State 2
Eigenvalue - State 0
Eigenvalue - State 1
Eigenvalue - State 2
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Other Symmetry Channels
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Other Symmetry Channels

Effective Mass ($m_t$) vs. Metric Timeslice

- Fitted - State 0
- Fitted - State 1
- Eigenvalue - State 0
- Eigenvalue - State 1

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Effective Mass ($m_a$)

Metric Timeslice

Fitted - State 0
Fitted - State 1
Eigenvalue - State 0
Eigenvalue - State 1

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