$K$-meson vector decay constant and $B$-parameter from $N_f=2$ tmQCD

Lattice 2008

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Generalities

- ETMC is performing state-of-the-art lattice QCD simulations with $N_f = 2$ dynamical flavours (sea quarks), with “lightish” masses ($300 \text{ MeV} \leq m_{PS} \leq 550 \text{ MeV}$).

- Several quantities are being analyzed for a couple of $\beta$’s.

- With $N_f = 2$ sea quarks, strangeness enters the game in a partially quenched context.

- In this talk we will show preliminary results on the following quantities:
  
  - $m_{K^*}$
  - $f_{K^*}$
  - $[f_T/f_V]_{K^*}$
  - $B_K$

- In parallel, other ETMC subgroups have been working on decay constants in the light and strange quark sector (see talks by C. McNeile and C. Tarantino).

Theory

- ETMC simulations are performed with the tree-level Symanzik improved gauge action.
- The \( N_f = 2 \) sea quark flavours are regularized by the standard Wilson fermion action with a fully twisted mass term (standard \( tmQCD \)).

\[
\bar{\psi} = \begin{pmatrix} \bar{u} \\ \bar{d} \end{pmatrix}
\]

\[
\mathcal{L}_{tm} = \bar{\psi} \left[ D_W + i \mu_q \tau^3 \gamma_5 \right] \psi
\]

- This has the usual advantages:
  - Renormalization properties are, in many cases of interest (e.g. pseudoscalar decay constants, \( B_K \)) much simpler than with standard Wilson quarks.
  - Improvement is automatic with full twist (i.e. imaginary mass term only).


Theory

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$$\bar{\psi} = \begin{pmatrix} \bar{u} \\ \bar{d} \end{pmatrix}$$

$$\mathcal{L}_{tm} = \bar{\psi} \left[ D_W + i \mu_q \tau^3 \gamma_5 \right] \psi$$

- But this is true for most, not all, quantities of interest.
- In particular, for WMEs of 4-fermion operators (e.g. $B_K$), it is not possible to have standard $tmQCD$ formalism, with all flavours at full twist (i.e. automatic improvement), and multiplicative renormalization.

Theory

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\[
\mathcal{L}_{OS} = \bar{\psi}_f \left[ D_W + i\mu_f \gamma_5 \right] \psi_f \quad f = u, d, s \ldots
\]

- One way out is provided by the Osterwalder-Seiler (OS) variant of tmQCD.
- Valence quarks enter with a distinct action for each flavour, which is **fully twisted**.
  - quark fields are not organized in isospin doublets (i.e. no \( \tau^3 \)).
  - there is a separate mass term for each flavour, \( \mu_f \) may be negative.
Theory

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- One way out is provided by the Osterwalder-Seiler (OS) variant of tmQCD.
- Valence quarks enter with a distinct action for each flavour, which is fully twisted.
- Suitable combinations of $\mu_f$ signs for each flavour ensure automatic improvement and multiplicative renormalization for say, $B_K$.

$$\mathcal{L}_{OS} = \bar{\psi}_f \left[ D_W + i \mu_f \gamma_5 \right] \psi_f \quad f = u, d, s \ldots$$

Theory

- ETMC simulations are performed with the tree-level Symanzik improved gauge action.
- The $N_f = 2$ sea quark flavours are regularized by the standard Wilson fermion action with a fully twisted mass term (standard tmQCD).
- One way out is provided by the Osterwalder-Seiler (OS) variant of tmQCD.
- Valence quarks enter with a distinct action for each flavour, which is fully twisted.
- This is a compromise (unitarity issues arise when sea and valence flavours are treated differently) but in our partially quenched setup ($N_f = 2$ sea quark flavours and a valence strange quark) this is unavoidable for any regularization.

$$L_{OS} = \bar{\psi}_f \left[ D_W + i \mu_f \gamma_5 \right] \psi_f \quad \quad f = u, d, s \cdots$$
The Simulation

- The ETMC runs are performed at three gauge couplings $\beta$.
- The master run: 240 measurements at $\beta = 3.90$, corresponding to $a \approx 0.086(1)$ fm [i.e. $1/a \approx 2.3$ GeV ] and volume $V = 24^3 \times 48$
- 5 sea quark masses: $\mu = 0.0040, 0.0064, 0.0085, 0.0100, 0.0150$
  $(300$ MeV $\leq m_{PS} \leq 550$ MeV)$
- 7 valence quark masses; the extra ones are: $\mu = 0.0220, 0.0270$ ($\sim m_{\text{strange}}$)
  ETMC, B. Blossier et al., JHEP 04 (2008) 020
- use existing calibrations: $a\mu_d = a\mu(m\pi) = 0.00079$ and $a\mu_s = a\mu(m_K) = 0.0217(10)$
- For $B_K$ only, at $\beta = 3.90$, we did 200 measurements so far.
- For $B_K$ only, we checked for finite volume effects at $V = 32^3 \times 64$ for $\mu = 0.0040$.
- For $B_K$ only, we did a rough scaling test at $\beta = 4.05$, $\mu = 0.0030$, $V = 32^3 \times 64$ and 100 measurements.
K* meson mass and decay constant

- **P. Dimopoulos, S. Simula, A.V.**

Caveats

- We encountered low quality signals in two cases:
  - 1: For all sea quark masses, when the valence quark masses are in the lightest range (say $\mu_{val} = 0.0040$)

\[
\mu_{val} = 0.0040
\]

\[
\mu_{val} = \mu_{sea} = 0.0040
\]

\[
\mu_{val} = 0.0064
\]

\[
\mu_{val} = \mu_{sea} = 0.0064
\]
K* meson mass and decay constant

- **P. Dimopoulos**, S. Simula, A.V.

**Caveats**

- We encountered low quality signals in two cases:
- Nevertheless, since the signal-to-noise ratio is as expected $\sim \exp[-(m_V - m_{PS})t]$; $\rho$-meson mass and decay constant may be extracted (C. McNeile, this conference).

\[
\mu_{val} = 0.0040
\]

\[
\mu_{val} = \mu_{sea} = 0.0040
\]

\[
\mu_{val} = 0.0064
\]

\[
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\]
K* meson mass and decay constant

• **P. Dimopoulos**, S. Simula, A.V.

Caveats

• We encountered low quality signals in two cases:

• II: For valence quarks lighter than the sea quarks ($\mu_{\text{val}} \leq \mu_{\text{sea}}$) (NB: unlike pseudoscalar case, where everything seems OK)

\[
\mu_{\text{val}} = 0.0040 - 0.0270
\]

\[
\mu_{\text{sea}} = 0.0040
\]

\[
\mu_{\text{sea}} = 0.0150
\]
K* meson mass and decay constant

- In all other cases the signal is satisfactory, so we analyze correlation functions consisting of:
  - one “light” valence quark ($\mu_l = \mu_{\text{sea}} = 0.0040, 0.0064, 0.0085, 0.0100, 0.0150$);
  - one “heavy” valence quark ($\mu_h = 0.0150, 0.0220, 0.0270$).
- Plateau: $11 \leq t \leq 16$
**K* meson mass and decay constant**

- The vector meson mass and the observables of interest:

\[
< 0 | V_k | V; \lambda > = f_V m_V \epsilon_k^\lambda \\
< 0 | T_{0k} | V; \lambda > = -i f_T m_V \epsilon_k^\lambda 
\]

- are obtained from the correlation functions

\[
C_{VV} = \sum_{\vec{x},k} < V_k(x) V_k^\dagger(0) > \quad k = 1, 2, 3 \\
C_{TT} = \sum_{\vec{x},k} < T_{0k}(x) T_{0k}^\dagger(0) > \quad k = 1, 2, 3 
\]

- and the ratio

\[
\frac{f_T}{f_V} \sim \left[ \frac{C_{TT}(t)}{C_{VV}(t)} \right]^{1/2}
\]

- NB: valence quark propagators (also for $B_K$) are not computed from standard inversions of the Dirac operator (i.e. point-like sources), but from stochastic sources of the so-called extended one-end trick.


**K* meson mass and decay constant**

- The vector meson mass and the observables of interest:

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\frac{f_T}{f_V} \sim \left[ \frac{C_{TT}(t)}{C_{VV}(t)} \right]^{1/2}
\]

- NB: The required (re)normalization factors \((Z_A, Z_T)\) are computed non-perturbatively in the RI/MOM scheme at a scale \(\mu = 1/a \approx 2.3\) GeV

\[
Z_A = 0.771 (4) \quad Z_T(1/a) = 0.769 (4)
\]

Decay constant $f_V$ and ratio $f_T/f_V$

- $\mu_{\text{val}} = 0.0040$ - $0.0150$
- $\mu_{\text{sea}} = 0.0040$
- $\mu_{\text{val}} = 0.0040 \pm 0.0150$
- $\mu_{\text{sea}} = 0.0040$
- $\mu_{\text{val}} = 0.0150$ - $0.0150$
- $\mu_{\text{sea}} = 0.0040$
- $\mu_{\text{val}} = 0.0150 \pm 0.0150$
- $\mu_{\text{sea}} = 0.0040$

Graphs show the decay constant $f_V$ and ratio $f_T/f_V$ for different values of $\mu_{\text{val}}$ and $\mu_{\text{sea}}$. The graphs illustrate the relationship between $f_V$ and $f_T/f_V$ as $x_0$ varies from 2 to 20.
At each fixed $\mu_h$, we extrapolate linearly in $\mu_l \to \mu_d$.

We subsequently interpolate the $\mu_h$ results in $\mu_h \to \mu_s$. 
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We subsequently interpolate the $\mu_h$ results in $\mu_h \to \mu_s$. 

**Mass extrapolations for $f_V$**

![Graph showing mass extrapolations for $f_V$](image-url)
• At each fixed $\mu_h$, we extrapolate linearly in $\mu_l \to \mu_d$

• We subsequently interpolate the $\mu_h$ results in $\mu_h \to \mu_s$
Results for $m_V$, $f_V$ and $f_T/f_V$

\[
a M_{V}^{K^*} = 0.437(08)(04)
\]
\[
a f_{V}^{K^*} = 0.117(03)(01)
\]
\[
f_T/f_V|_{K^*} = 0.759(19)(03)
\]

NB: analysis repeated with OS valence quarks

\[r_0/a = 5.22\]
Results for $m_V$, $f_V$ and $f_T/f_V$

\[
\begin{align*}
    aM_V^{K^*} &= 0.437(08)(04) \\
    af_V^{K^*} &= 0.117(03)(01) \\
    f_T/f_V|_{K^*} &= 0.759(19)(03)
\end{align*}
\]

Doing the RG running from $1/a = 2.3$ GeV to 2 GeV we find:

\[
[f_T/f_V]_{K^*} = 0.764(19)(03)
\]

D. Becirevic, V. Lubicz, F. Mescia C. Tarantino, JHEP05 (2003) 007

NB: continuum quenched result

\[
[f_T/f_V]_{K^*} = 0.74(2)
\]
**B_K: a progress report**

- Recall that we require both automatic improvement and multiplicative renormalization; thus the setup is that of OS valence quarks.

- We have two walls with noise sources at fixed times and a moving 4-fermion operator.

\[ \mu_h \]
\[ -\mu_h \]

\[ \mu_l = \mu_v \]
\[ \mu_l = \mu_v \]

\( x_0 = 0 \quad x_0 = t \quad x_0 = T/2 \)

\[ \bar{O}_{VA+AV} = \lim_{a \to 0} Z_{VA+AV}(g_0^2, a\mu) O_{VA+AV}(a) \]

- **P. Dimopoulos**, R. Frezzotti, V. Gimenez, V. Lubicz, **F. Mescia**, G.C. Rossi, A.V.
$B_K$: finite volume effects

$\beta=3.90 \quad \mu_{\text{sea}}=0.0040$

$R_{B_K(\text{bare})}$ vs. $2x_0/T$

$L=24$ (red squares)
$L=32$ (green squares)
$B_K$: scaling effects (VERY ROUGH!!)

$\beta=3.90$, $\mu_{\text{sea}}=0.0040$, $M_K(\text{deg})=480$ MeV: $B_K(\text{bare}) = 0.590(06)$

$\beta=4.05$, $\mu_{\text{sea}}=0.0030$, $M_K(\text{deg})=490$ MeV: $B_K(\text{bare}) = 0.571(08)$
B_K: chiral fits

• At fixed $\mu_h$, we fit the light mass behaviour in $\mu_l = \mu_v$, using PQ-ChPT

• in general $\mu_v \neq \mu_{\text{sea}}$ gives rise to chiral logs

RBC/UKQCD C. Allton et al., 0804.0473 [hep-lat]

\[
B(\mu_h) = B_{\chi}(\mu_h) \left[ 1 + b_1(\mu_h) \frac{2B_0}{f^2} \mu_{\text{sea}} + b_2(\mu_h) \frac{2B_0}{f^2} \mu_v - \frac{2B_0}{32\pi^2 f^2 \mu_{\text{sea}}} \ln \left( \frac{2B_0 \mu_v}{\Lambda^2_{\chi}} \right) \right]
\]

• This simplifies to a 2-parameter fit with a well defined chiral limit when $\mu_v = \mu_{\text{sea}}$.
**B_K: chiral fits**

- At fixed $\mu_h$, we fit the light mass behaviour in $\mu_l = \mu_{val}$, using PQ-ChPT

- in general $\mu_{val} \neq \mu_{sea}$ gives rise to chiral logs

RBC/UKQCD C. Allton et al., 0804.0473 [hep-lat]


\[
B(\mu_h) = B_\chi(\mu_h) \left[ 1 + b_1(\mu_h) \frac{2B_0}{f^2} \mu_{sea} + b_2(\mu_h) \frac{2B_0}{f^2} \mu_v - \frac{2B_0}{32\pi^2 f^2 \mu_{sea}} \ln \left( \frac{2B_0 \mu_v}{\Lambda^2_\chi} \right) \right]
\]

- This simplifies to a 2-parameter fit with a well defined chiral limit when $\mu_{val} = \mu_{sea}$.

**Fixed by earlier ETMC chiral fit in the light sector**

- Polynomial fitting alternatives are in the works!
**B_K: chiral fits**

- At fixed $\mu_s$, close to the physical strange mass we fit for $\mu_d = \mu_{\text{sea}}$

$$B_K(\mu_h, \mu_v) \_\mu_{\text{sea}} \text{ for } \mu_v = \mu_{\text{sea}} \text{ @ } \mu_h = 0.0220 \quad -- \beta = 3.9$$
**B_K: chiral fits**

- Now interpolate the previous (physical $\mu_d$ result) in $\mu_s$
$B_K$: renormalization

- RI/MOM scheme implemented

![Renormalization constants of four quark operators](image)

Twisted mass $\beta=3.9$ $V=24^3 \times 48$ TlSym

$\mu_{\text{sea}} = 0.0040$
B_K: renormalization

- RI/MOM scheme implemented

Renormalization constants of four quark operators
Twisted mass $\beta=3.9$ V=$24^3 \times 48$ TlSym $\mu_{\text{sea}}=0.0040$

V. Gimenez, V. Lubicz
**B_K: caveat**

- At fixed $\beta$, the two Kaon states, obtained with different regularizations (i.e. standard tm and OS) are not degenerate, differing by $O(a^2)$ terms.

- The two different exponential decays cancel in the $B_K$ ratio.

- We are left with a matrix element

$$< \bar{K}^0(m_K^{tm}) | O_{VA+AV} | K^0(m_K^{OS}) > \propto m_K^{tm} m_K^{OS}$$

\[ a_{\mu \text{ sea}} = 0.0040 \]
\[ a_{\mu \text{ val1}} = 0.0040 - a_{\mu \text{ val2}} = 0.0220 \]

<table>
<thead>
<tr>
<th>$a_{\mu}$</th>
<th>$m_K^{tm}$</th>
<th>$m_K^{OS}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>OS</td>
<td>0.2923(16)</td>
<td></td>
</tr>
<tr>
<td>TM</td>
<td>0.2391(07)</td>
<td></td>
</tr>
</tbody>
</table>

\[ am_{PS}^{OS} = 0.2923(16) \]
\[ am_{PS}^{tm} = 0.2391(07) \]
**B_K: a our first VERY ROUGH estimate**

Although a lot is still missing for giving a definitive result, we cannot resist from fooling around with our preliminary numbers:

\[ B_K(\beta=3.9) = 0.581(7) \text{ (bare)} \]

\[ Z_{VA+AV}(\beta=3.9; 2 \text{ GeV}; \text{RI/MOM}) = 0.454 (18) \]

\[ Z_V(\beta=3.9) = 0.771 \]

\[ Z_A(\beta=3.9) = 0.6104 \]

\[ B_K(2.0 \text{ GeV}; \text{RI/MOM}) \approx 0.56(2) \text{ (renormalized)} \]

\[ B_K(2.0 \text{ GeV}; \text{RI/MOM}) \approx 0.77(3) \text{ (RGI)} \]

**NB: this is far from being our definitive, result!!!!!!**

But how does it compare with the results of other groups?
B_K: a “ballpark plot”

- This is not a world data plot! It is a compilation of existing results, in order to confirm that our preliminary B_K is in the right ballpark.

\[ N_f = 2 \] preliminary \[ N_f = 3 \]