Nonperturbative infrared fixed point in sextet QCD

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(YS, BS, & TD, arXiv:0803.1707 [hep-lat])

1. Motivation: Beyond the Standard Model — more gauge groups, more reps
   - β function scenarios
2. Method: Schrödinger Functional (background field method)
3. First Results —

   The β function of the SU(3) gauge theory with $N_f = 2$ fermions in the **6 rep**
4. *Stay for the next talk*

MOTIVATION: Beyond the Standard Model

(A. Nelson, Lattice 2006)

Strong coupling gauge theories:

- Strongly coupled Weinberg–Salam
- Technicolor — the Higgs as a bound state; walking
- Extra dimensions

Unified theories:

- Larger gauge groups
- Tumbling
- Multiple gauge groups, alignment
- Supersymmetry

Why this model?

- Banks–Zaks fixed point (Caswell 1974; Banks & Zaks 1981) — Is it really there?
- Scale separation: $C_2(R) = \frac{10}{3}$ vs. $\frac{4}{3}$ for fund rep (T. DeGrand, following talk)

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Banks–Zaks: Perturbation theory

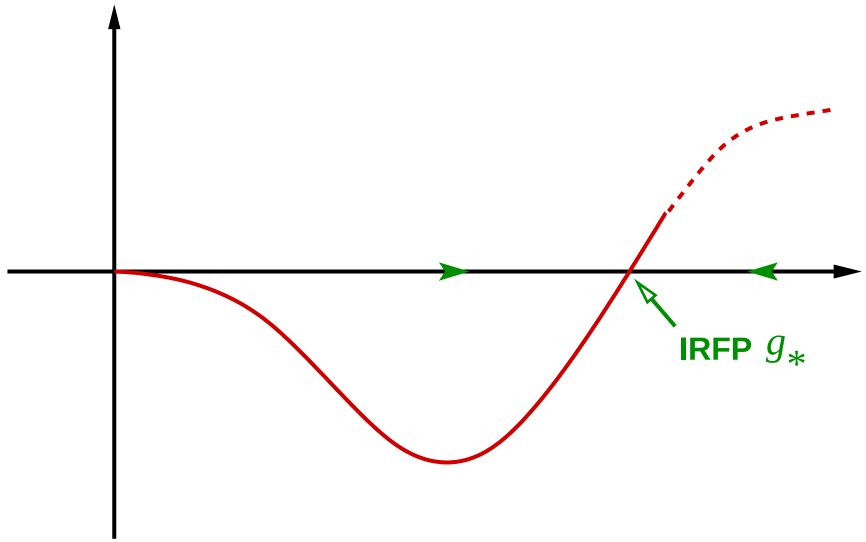
$$\beta(g^2) = -\frac{b_1}{16\pi^2} g^4 - \frac{b_2}{(16\pi^2)^2} g^6 + \cdots$$

Here $b_1 > 0$, $b_2 < 0$ [as in QCD with $8.05 < N_f < 16\frac{1}{2}$]

$\Longrightarrow$ IR-attractive fixed point at $g_*^2 \simeq 10.4$ — a strong coupling

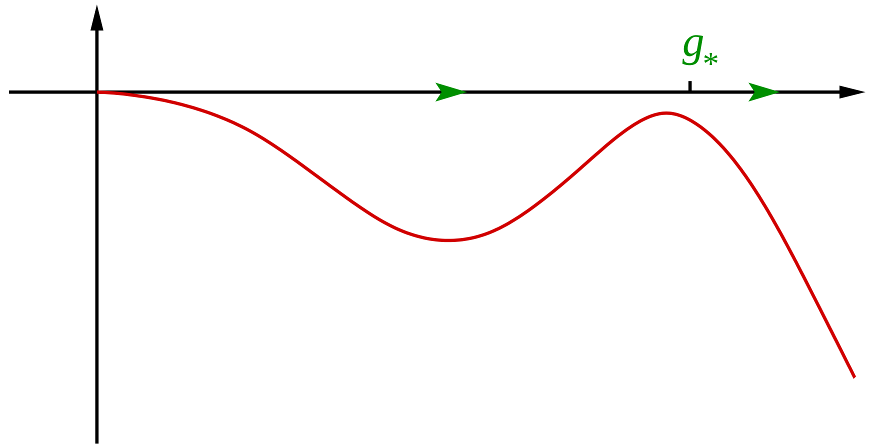
What can happen NONPERTURBATIVELY?

g_*^2 weak
IRFP $\Rightarrow$ conformal dynamics at large distances
$\Rightarrow$ no confinement, no χSB,
no particles!



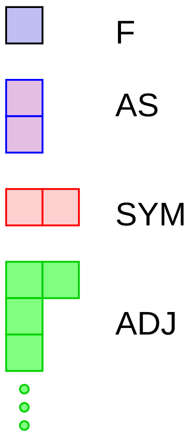
g_*^2 strong
χSB $\Rightarrow$ fermions decouple, back to β fn of pure gauge theory

(maybe WALKING)



GAUGE GROUPS, REPs, and N_f

(Dietrich & Sannino, PRD 2007)



Ladder approx, Bethe-Salpeter, etc. $\Longrightarrow$ ($N = 3$, REP=SYM=$\mathbf{6}$, $N_f = 2$) — lies below conformal window

CALCULATING THE β FUNCTION: the Schrödinger Functional

- Wilson fermions — because
  1. boundary values (background field) can be set on a single time slice
  2. control over N_f

  $+$ clover term $\Rightarrow$ removes tree-level $O(a)$ discretization errors
  $+$ tadpole improvement $\Rightarrow$ reduces leading $O(g^2 a)$ errors (known to be large)

- SF: fix spatial links U_i on time boundaries $t = 0, L$
  $+$ give fermions a spatial twist

- χS explicitly broken $\Rightarrow$ additive renormalization of m_q $\Rightarrow$ must fix $\kappa = \kappa_c$
  1. Define m_q via AWI

  $$m_q \equiv \frac{1}{2} \left. \frac{\partial_4 \left\langle A_4^b(t)\, \mathcal{O}^b(t'=0, \vec{p}=0)\right\rangle}{\langle P^b(t)\, \mathcal{O}^b(t'=0, \vec{p}=0)\rangle} \right|_{t=L/2}$$

  2. Find $\kappa_c(\beta)$ by setting $m_q = 0$. Work directly at κ_c: stabilized by SF BC's!

CALCULATING THE β FUNCTION: the Schrödinger Functional

We want $\Gamma \equiv -\log Z$ since $\Gamma \equiv \frac{1}{g^2(L)} S^{cl}_{YM}$. But we can't calculate Γ directly ...

Choose boundary values U_i to depend on a parameter η. Then

$$\frac{\partial \Gamma}{\partial \eta} = \left\langle \frac{\partial S_{YM}}{\partial \eta} - \mathrm{tr}\left(\frac{1}{D_F^\dagger} \frac{\partial (D_F^\dagger D_F)}{\partial \eta} \frac{1}{D_F} \right) \right\rangle = \frac{K}{g^2(L)}\,, \qquad K \equiv \frac{\partial S^{cl}_{YM}}{\partial \eta} = 37.7\ldots$$

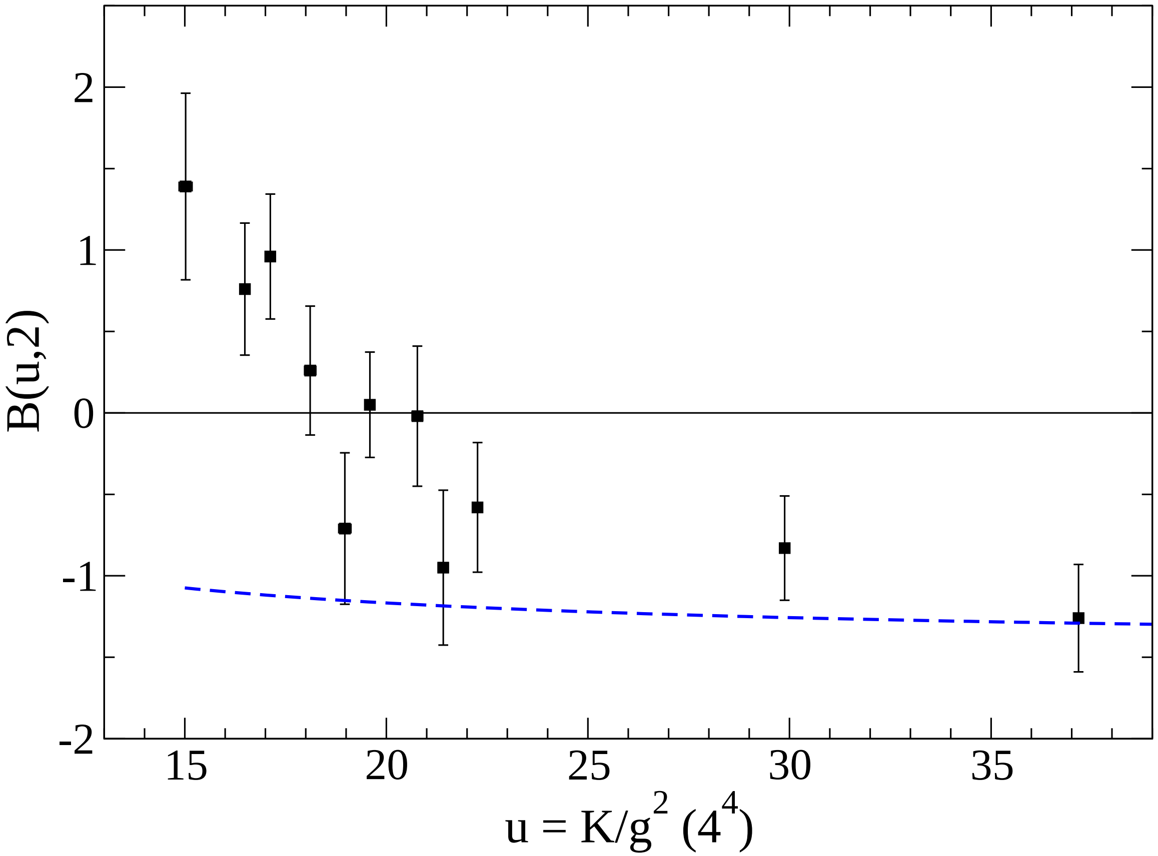
EXTRACTING PHYSICS

1. Fix lattice size L, couplings β, $\kappa = \kappa_c(\beta)$
2. Calculate $K/g^2(L)$ and $K/g^2(2L)$. Use common lattice spacing ($=$ UV cutoff) $a = L/4$.
3. Result: Discrete Beta Function

$$B(u, 2) = \frac{K}{g^2(2L)} - \frac{K}{g^2(L)}\,,$$

a function of $u \equiv K/g^2(L)$.

The DISCRETE BETA FUNCTION



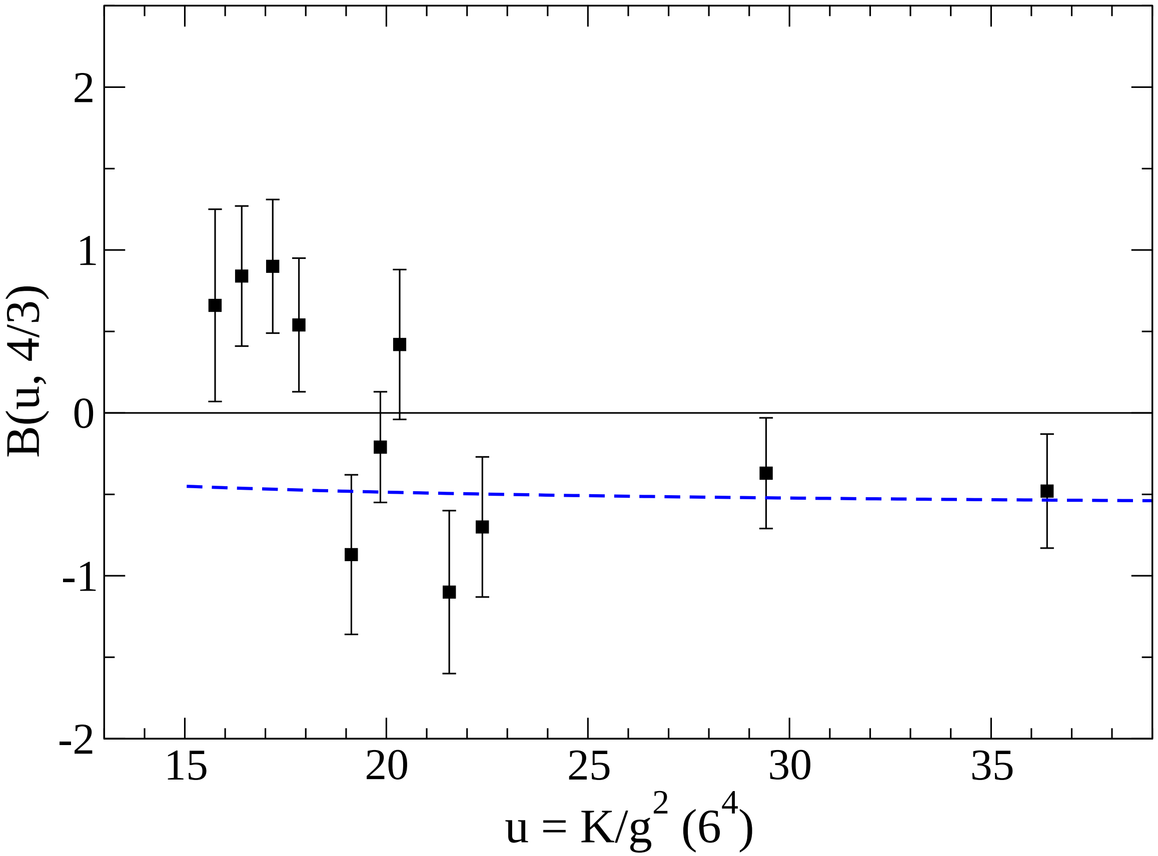
$4^4 \longrightarrow 8^4$

$B(u,2)$ crosses zero at $g^2 \simeq 2.0$

not at $g^2 \simeq 10$!

$\Longrightarrow$ IR theory is CONFORMAL

The DISCRETE BETA FUNCTION



Cf. $6^4 \longrightarrow 8^4$

Caveat cursor

- Is there only one, unique running coupling?
  - – Perturbatively, yes.
  - – If the $q\bar{q}$ potential is *almost* Coulombic: $V(r) \simeq g^2(r)/r$
- Is it really an IRFP?
- Can we extend the picture off the $\kappa_c(\beta)$ curve?

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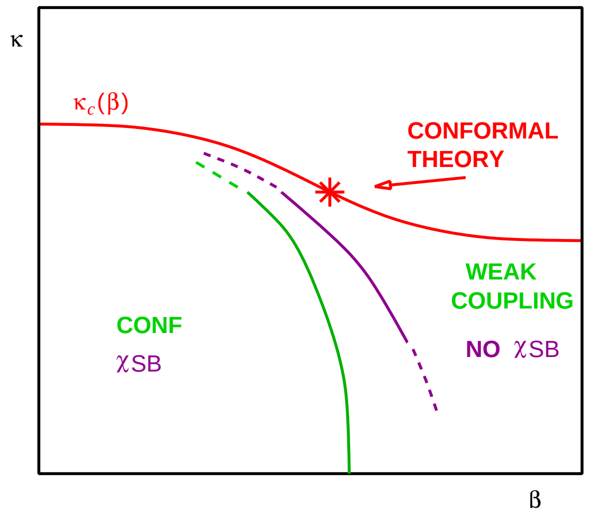
"PHASE DIAGRAM" in finite volume

(T. DeGrand, following talk)

$L = 8a$

Note *weak coupling* at IRFP



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ANSWERS will come from:

- More knowledge of phase diagram
- Checking beta function with more volumes
- Eventually: scaling towards the continuum limit

MORE QUESTIONS

- Properties of (near-) conformal theory