Nonperturbative infrared fixed point in sextet QCD

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(YS, BS, & TD, arXiv:0803.1707 [hep-lat])

1. Motivation: Beyond the Standard Model — more gauge groups, more reps
   • $\beta$ function scenarios

2. Method: Schrödinger Functional (background field method)

3. First Results —
   The $\beta$ function of the SU(3) gauge theory with $N_f = 2$ fermions in the 6 rep

4. Stay for the next talk
MOTIVATION: Beyond the Standard Model

(A. Nelson, Lattice 2006)

Strong coupling gauge theories:
- Strongly coupled Weinberg–Salam
- Technicolor — the Higgs as a bound state; walking
- Extra dimensions

Unified theories:
- Larger gauge groups
- Tumbling
- Multiple gauge groups, alignment
- Supersymmetry
Why this model?

- Banks–Zaks fixed point (Caswell 1974; Banks & Zaks 1981) — Is it really there?
- Scale separation: $C_2(R) = \frac{10}{3}$ vs. $\frac{4}{3}$ for fund rep (T. DeGrand, following talk)
Why this model?

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Banks–Zaks: Perturbation theory

$$\beta(g^2) = -\frac{b_1}{16\pi^2}g^4 - \frac{b_2}{(16\pi^2)^2}g^6 + \cdots$$

Here $b_1 > 0$, $b_2 < 0$ [as in QCD with $8.05 < N_f < 16\frac{1}{2}$]

$\implies$ IR-attractive fixed point at $g_*^2 \simeq 10.4$ — a strong coupling
What can happen NONPERTURBATIVELY?

$g_*^2$ weak
IRFP $\Rightarrow$ conformal dynamics at large distances
$\Rightarrow$ no confinement, no $\chi_{SB}$, no particles!

$g_*^2$ strong
$\chi_{SB} \Rightarrow$ fermions decouple, back to $\beta$ fn of pure gauge theory
(maybe WALKING)
Ladder approx, Bethe-Salpeter, etc. $\implies (N = 3, \text{REP=SYM=6}, N_f = 2)$ — lies below conformal window
CALCULATING THE $\beta$ FUNCTION: the Schrödinger Functional

- Wilson fermions — because
  1. boundary values (background field) can be set on a single time slice
  2. control over $N_f$
     + clover term ⇒ removes tree-level $O(a)$ discretization errors
     + tadpole improvement ⇒ reduces leading $O(g^2a)$ errors (known to be large)
- SF: fix spatial links $U_i$ on time boundaries $t = 0, L$
  + give fermions a spatial twist
- $\chi_S$ explicitly broken ⇒ additive renormalization of $m_q$ ⇒ must fix $\kappa = \kappa_c$
  1. Define $m_q$ via AWI
     \[
     m_q \equiv \frac{1}{2} \left. \frac{\partial_4 \langle A_A^b(t) \mathcal{O}^b(t' = 0, \vec{p} = 0) \rangle}{\langle P^b(t) \mathcal{O}^b(t' = 0, \vec{p} = 0) \rangle} \right|_{t=L/2}
     \]
  2. Find $\kappa_c(\beta)$ by setting $m_q = 0$. Work directly at $\kappa_c$: stabilized by SF BC’s!
CALCULATING THE $\beta$ FUNCTION: the Schrödinger Functional

We want $\Gamma \equiv -\log Z$ since $\Gamma \equiv \frac{1}{g^2(L)} S_{YM}^{\text{cl}}$. But we can’t calculate $\Gamma$ directly . . .

Choose boundary values $U_i$ to depend on a parameter $\eta$. Then

$$\frac{\partial \Gamma}{\partial \eta} = \left\langle \frac{\partial S_{YM}}{\partial \eta} - \text{tr} \left( \frac{1}{D_F^\dagger} \frac{\partial (D_F^\dagger D_F)}{\partial \eta} \frac{1}{D_F} \right) \right\rangle = \frac{K}{g^2(L)}, \quad K \equiv \frac{\partial S_{YM}^{\text{cl}}}{\partial \eta} = 37.7 . . .$$

EXTRACTING PHYSICS

1. Fix lattice size $L$, couplings $\beta$, $\kappa = \kappa_c(\beta)$
2. Calculate $K/g^2(L)$ and $K/g^2(2L)$. Use common lattice spacing (= UV cutoff) $a = L/4$.
3. Result: Discrete Beta Function

$$B(u, 2) = \frac{K}{g^2(2L)} - \frac{K}{g^2(L)},$$

a function of $u \equiv K/g^2(L)$. 

The DISCRETE BETA FUNCTION

\[ B(u, 2) \] crosses zero at \( g^2 \simeq 2.0 \)

\( 4^4 \rightarrow 8^4 \)

\( B(u, 2) \) crosses zero at \( g^2 \simeq 2.0 \)

not at \( g^2 \simeq 10! \)

\[ u = K/g^2 (4^4) \]

IR theory is CONFORMAL
The DISCRETE BETA FUNCTION

\[ u = \frac{K}{g^2} (6^4) \]

Cf. $6^4 \rightarrow 8^4$
Caveat cursor

- Is there only one, unique running coupling?
  - Perturbatively, yes.
  - If the $q\bar{q}$ potential is almost Coulombic: $V(r) \simeq g^2(r)/r$

- Is it really an IRFP?

- Can we extend the picture off the $\kappa_c(\beta)$ curve?
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"PHASE DIAGRAM" in finite volume

(T. DeGrand, following talk)

$L = 8a$

Note weak coupling at IRFP
Caveat cursor

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**ANSWERS** will come from:

- More knowledge of phase diagram
- Checking beta function with more volumes
- Eventually: scaling towards the continuum limit

**MORE QUESTIONS**

- Properties of (near-) conformal theory