Numerical Investigation of the 2-D $\mathcal{N}=2$ Wess-Zumino Model

Christian Wozar

Theoretisch-Physikalisches Institut
FSU Jena

with Georg Bergner, Tobias Kästner,
Sebastian Uhlmann and Andreas Wipf

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Outline

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2. The model
3. Limitations of improvement
4. Weak coupling results
5. Intermediate coupling results
6. Summary

Introduction

Motivation (Physics)

- The lattice breaks supersymmetry explicitly.
- No spontaneous supersymmetry breaking of the continuum model expected.
  $\Rightarrow$ Supersymmetry restoration in continuum limit can be analyzed.
- In former works (M. Beccaria et al. (1998), S. Catterall and S. Karamov (2003)) only Wilson fermions with Nicolai improved action were used. Problems at stronger couplings.
- Effects of Nicolai improvement?

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2. Motivation (Algorithms)

- Explicit investigation and improvement of the used algorithms, cf. e.g. Bergner et al. (2007) for WZ model in $1d$ with different discretizations.
- High precision measurements available in lower dimensions.

The model

- The continuum action

$$S_{\text{cont}} = \int d^2x \left(2\bar{\partial}\bar{\varphi}\partial\varphi + \tfrac{1}{2}|W'(\varphi)|^2 + \bar{\psi}M\psi\right),$$

$$M = \gamma^z\partial + \gamma^{\bar{z}}\bar{\partial} + W''P_+ + \overline{W}''P_-$$

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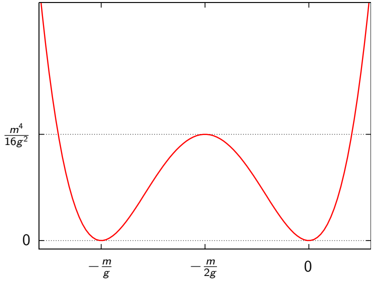
  allows for 4 real supersymmetries, $\varphi = \varphi_1 + i\varphi_2$.
- We use $W(\varphi) = \frac{1}{2}m\varphi^2 + \frac{1}{3}g\varphi^3$ with dimensionless coupling $\lambda = \frac{g}{m}$.

Classical potential $|W'(\varphi_1)|^2$:



- $\lambda = 0$ corresponds to free theory
  $\Rightarrow$ perturbative expansion in λ possible.

The model

Using the Nicolai variable $\xi_x = 2(\bar{\partial}\bar{\varphi})_x + W_x$ an action on the lattice preserving one supersymmetry is given by

$$S = \frac{1}{2}\sum_x \bar{\xi}_x \xi_x + \sum_{xy} \bar{\psi}_x M_{xy} \psi_y$$

with $W_x = W'(\varphi_x)$, $W_{xy} := \partial W_x / \partial \varphi_y$ and

$$M_{xy} = \begin{pmatrix} W_{xy} & 2\bar{\partial}_{xy} \\ 2\partial_{xy} & \overline{W}_{xy} \end{pmatrix} = \begin{pmatrix} \frac{\partial \xi_x}{\partial \varphi_y} & \frac{\partial \xi_x}{\partial \bar{\varphi}_y} \\ \frac{\partial \bar{\xi}_x}{\partial \varphi_y} & \frac{\partial \bar{\xi}_x}{\partial \bar{\varphi}_y} \end{pmatrix}.$$

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In terms of the original fields the action reads

$$S = \sum_x \Big(2\left(\bar\partial\bar\varphi\right)_x (\partial\varphi)_x + \frac{1}{2}\left|W_x\right|^2 + W_x(\partial\varphi)_x + \overline{W}_x(\bar\partial\bar\varphi)_x\Big) + \sum_{xy} \bar\psi_x M_{xy} \psi_y.$$

The difference to a straightforward discretization is given by surface terms

$$\Delta S = \sum_x \Big(W_x(\partial\varphi)_x + \overline{W_x}(\bar\partial\bar\varphi)_x\Big).$$

We use different lattice derivatives *(the same for bosonic and fermionic degrees of freedom)*:

- Symmetric derivative $\left(\partial^S_\mu\right)_{xy} = \frac{1}{2}(\delta_{x+\hat{\mu},y} - \delta_{x-\hat{\mu},y})$ with standard Wilson term $W_x = W'(\varphi_x) - \frac{r}{2}(\Delta\varphi)_x$ using $(r = 1)$.

$$M_{xy} = \begin{pmatrix} W''(\phi_x)\delta_{xy} & 2\bar{\partial}_{xy} \\ 2\partial_{xy} & \overline{W''(\phi_x)}\delta_{xy} \end{pmatrix} - \frac{r}{2}\Delta_{xy}$$

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- SLAC derivative $\partial_{x \neq y} = (-1)^{x-y} \frac{\pi/N}{\sin(\pi(x-y)/N)}$, $\partial_{xx} = 0$ with

  M_{xy} unchanged.

$\Rightarrow$ Simulate the (un)improved model with these different discretizations!
We use a combination of fourier accelerated HMC with higher-order integrators.

Limitations of improvement

For dynamical simulations of the improved model the bosonic action is fixed to $\langle S_{\mathrm{B}} \rangle = N = \#$ lattice points.
With SLAC fermions at different coupling strenghts we observe the improvement term $\Delta S = \sum_x \left(W_x(\partial\varphi)_x + \overline{W}_x(\bar\partial\bar\varphi)_x \right)$:

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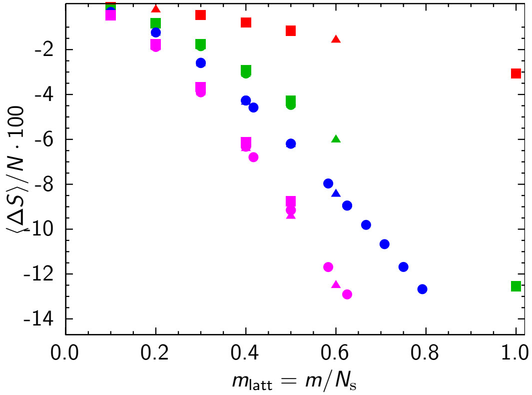


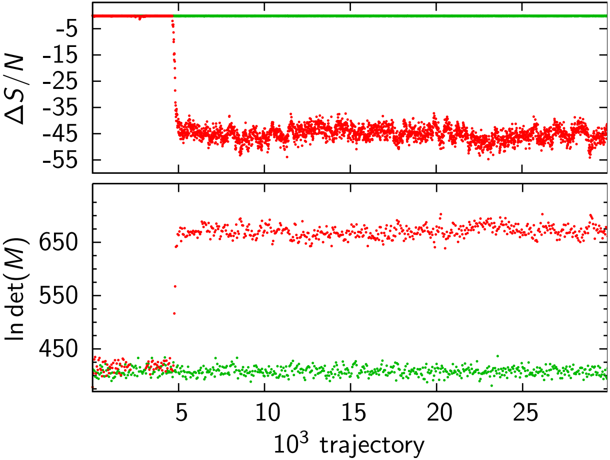
$N \in \{9 \times 9, 15 \times 15, 25 \times 25\}$

$\lambda = 0.8, 1.0, 1.2, 1.5$

Simulations break down when $\langle \Delta S \rangle / N$ exceeds 14%.

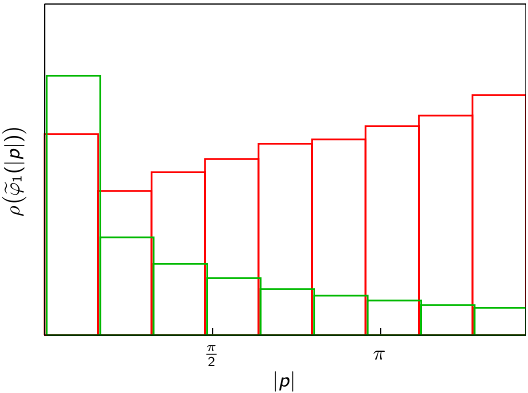
Limitations of improvement

MC history of the improvement term and the fermion determinant at $\lambda = 1.4$ and $\lambda = 1.7$ ($m_{\text{latt}} = 0.6$, $N = 15 \times 15$), $\langle S_B \rangle \approx N$ in each run:



Limitations of improvement

Analyzing the distribution of the fields in momentum space at $\lambda = 1.4$ and $\lambda = 1.7$:



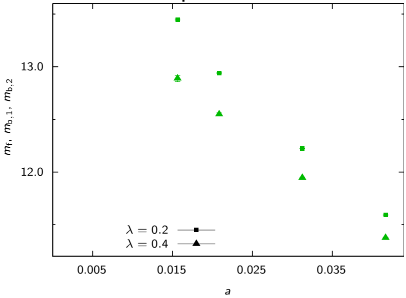
⇒ For too large couplings λ (or lattice masses m_{latt}) the simulation samples only unphysical UV dominated configurations.

⇒ At larger couplings a careful analysis of the improvement term during the simulation must be ensured.

With Wilson fermions we test for supersymmetry breaking effects on the lattice at different lattice spacings for $\lambda \in \{0.2, 0.4\}$, $m = 15$.

Masses for bosons (φ_1, φ_2, statistics 10^6–10^7 configs) and fermions (statistics 10^4 configs)

improved:



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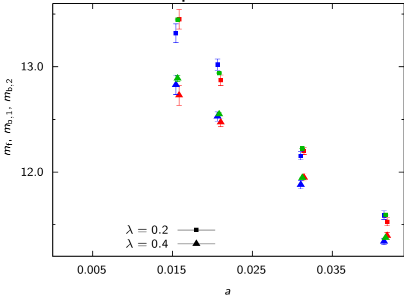

Weak coupling results
Bosons vs. fermions

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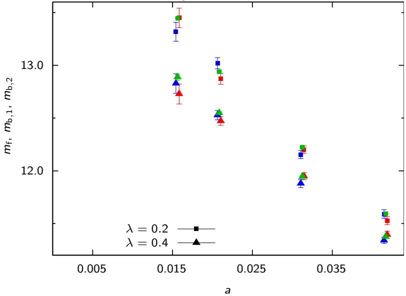
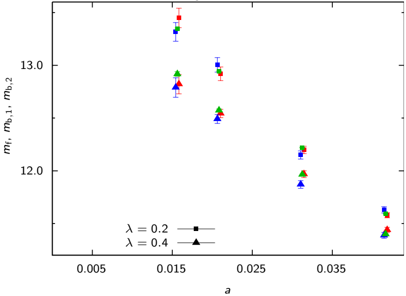


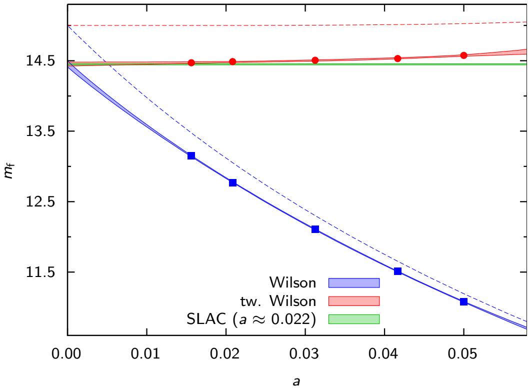
unimproved:



⇒ Improved and unimproved model can not be distinguished even with that high statistics.

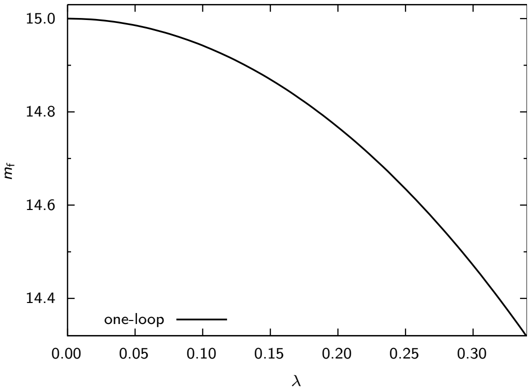
⇒ Bosonic and fermionic masses coincide.

Extrapolation from finite lattice spacing to the continuum using Wilson and twisted Wilson fermions for the improved model ($m = 15$, $\lambda = 0.3$):



$\Rightarrow$ All formulations yield the same continuum result.

The perturbative one-loop result $m_{\text{ren}}^2 = m^2\left(1 - \frac{4\lambda^2}{3\sqrt{3}}\right) + \mathcal{O}(\lambda^4)$ can be compared to the continuum extrapolation of the lattice data:



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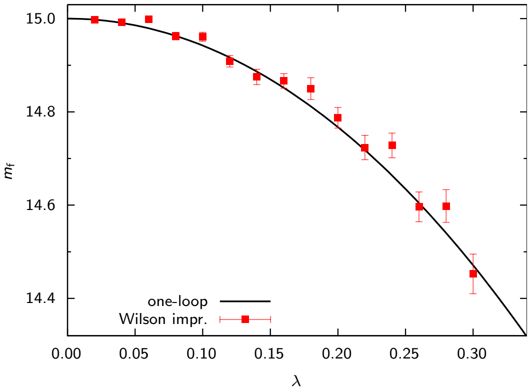

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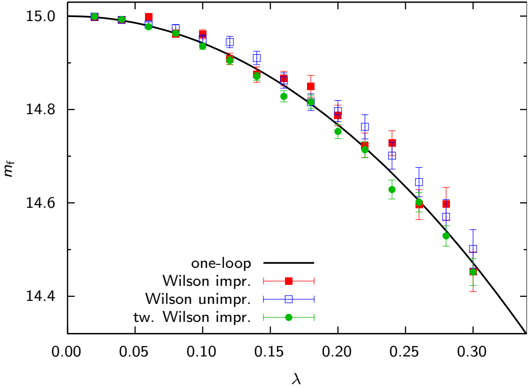
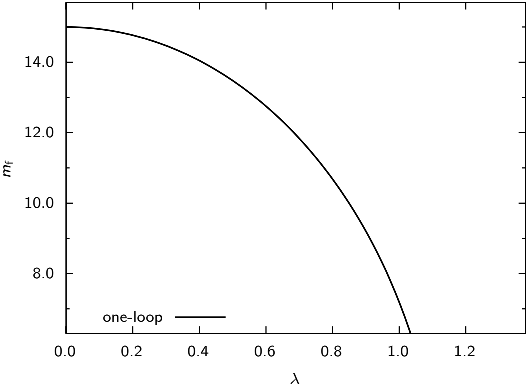


⇒ All different formulations coincide with perturbation theory.

⇒ The supersymmetric continuum limit is reached.

We checked for the limitations of the one-loop calculation using $\lambda \in [0, 1.2]$:



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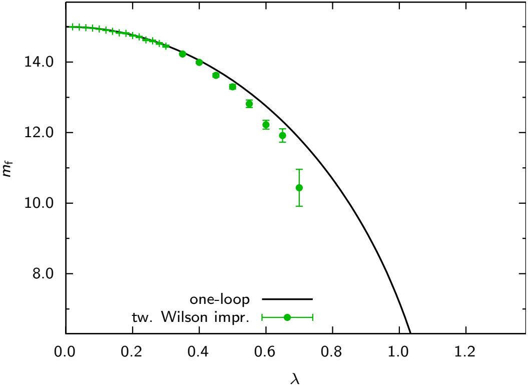

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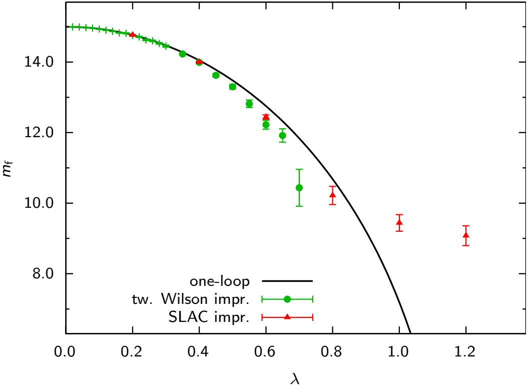

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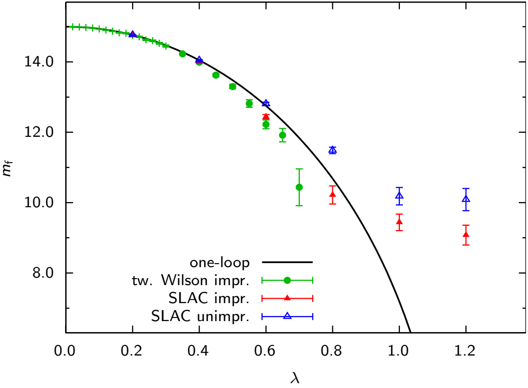

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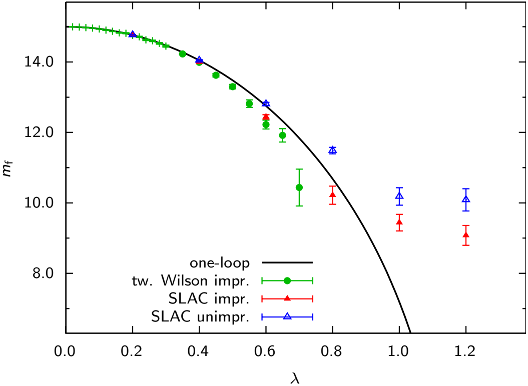

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⇒ For SLAC fermions with $\lambda > 0.6$ improved and unimproved models differ.
⇒ Larger lattice for $\lambda = 0.8$!

N_s	improved	unimproved
45	10.22(26)	11.49(9)
63	10.54(15)	10.70(19)

⇒ The correct continuum limit is reached for both models, where the improved SLAC model is closer to the continuum limit.

Summary

Results

- With very high statistics bosonic and fermionic masses can not be distinguished in the weak to intermediate coupling region for both improved and unimproved formulation.
- For intermediate coupling the improved action in closer to the continuum limit (at least for SLAC fermions).
- The "Nicolai improvement" introduces new problems due to the sampling of unphysical (high-momentum) states. (no real improvement?)
- Even without improvement the correct continuum limit is reached.

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Outlook

- Further algorithmic improvements (PHMC, multiple r.h.s. solvers) to obtain results for strong coupling ($\lambda > 1.5$).
- Use the elaborate algorithms to explore the $\mathcal{N} = 1$ WZ model in $d = 2$ (SUSY breaking expected).

More on this model including

finite size effects, discussion of the sign problem and technical details

can be found under

arXiv:0807.1905 [hep-lat]

Thank you!

... and please be cautious when using Nicolai improved actions.