Controlling Residual Mass in Domain Wall Fermion Simulations

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Motivation

- Perform DWF simulations at strong coupling and with topology change for large volume simulations and thermodynamic simulations near transition
- Achieve selected mres at smaller $\bar{L}_S$ or smaller mres at selected $L_S$
  - Measures chiral symmetry breaking due to overlap in 5’th dimension of chiral fermions nominally bound to walls
- Add weighting function to path integral to steer MD trajectories away from gauge configurations where 5D DWF transfer matrix has near unit eigenvalues
  - Unit eigenvalues of 5D DWF transfer matrix lead to relatively undamped propagation and overlap in 5’th dimension
Background

- 4D Wilson fermion, $D_W(N, m_0)$, zero modes lead to unit eigenvalues of 5D DWF transfer matrix
  - transfer matrix is a function of hermitian Wilson Dirac operator, $H_4(m_0)$
- Add a Wilson fermion to action
  $S_G(\beta, N) + S_{DWF}(N, L_S, m_0, m_f) + S_{PV}(N, L_S, m_0) + S_W(\bar{X}, X; N, m_0)$
- Vranas, 01006v2, 0606014v2: adds Wilson fermions
  - Gap generated for $1 < m_0 < 2$
  - Lowest eigenvalues of $H_4(m_0)$ vs $m_0$
  - Same physical lattice spacings;
  - Quenched DWF sea fermions
Add a Wilson ‘boson’ with ‘twisted’ mass cancel unwanted ‘ultraviolet’ effects of Wilson fermion

\[ S_W \Rightarrow S_W(\bar{X}_f, X_f, U; N, -m_0) - S_W(\bar{X}_b, X_b, U; N, -m_0 + \nu \epsilon_5 \gamma_5) \]

weighting term \( \mathcal{W} \) in path integral becomes

\[
\mathcal{W} = \det (H_W (m_0) H_W^\dagger (m_0)) \quad \Rightarrow \quad \mathcal{W} = \frac{\det (H_W (m_0) H_W^\dagger (m_0))}{\det (H_W (m_0) H_W^\dagger (m_0) + \epsilon_b^2)}
\]

Fukaya et al., 2006, Phys. Rev. D 094505
- demonstrates clear effect on near 0 eigenvalues but little effect on other eigenvalues
Current approach

- Now add a ‘twisted’ mass to Wilson fermion different from ‘twisted’ mass of Wilson ‘boson’ in order to prevent complete suppression of 0 eigenvalues and allow topology change

\[ S_W \Rightarrow S_W(\bar{X}_f, X_f, U; N, -m_0 + \nu_f \gamma_5) - S_W(\bar{X}_b, X_b, U; N, -m_0 + \nu_b \gamma_5) \]

- Weighting term now becomes

\[ W = \frac{\det(H_W(-m_0)H_W^\dagger(-m_0) + \epsilon_f^2)}{\det(H_W(-m_0)H_W^\dagger(-m_0) + \epsilon_b^2)} = \prod_i \frac{\lambda_i^2 + \epsilon_f^2}{\lambda_i^2 + \epsilon_b^2} \]

where \( \lambda_i \) are the eigenvalues of \( H_W \)
Current approach (continued)

- Little or no effect ($\mathcal{W} \simeq 1$) if
  - $0 < \epsilon_f \simeq \epsilon_b$ or
  - $0 < \epsilon_f < \epsilon_b \ll |\lambda_i|$

- Suppression of near 0 eigenvalues if
  - $N$ ‘small’ eigenvalues $|\lambda_i| \simeq 0$; remaining eigenvalues ‘large’ $|\lambda_i| > \epsilon_b$; a low but non-zero weight

\[ 0 < \mathcal{W} = \mathcal{W}_{\text{small}} \mathcal{W}_{\text{large}} \simeq \left( \frac{\epsilon_f}{\epsilon_b} \right)^N \prod_{\text{large}} \frac{\lambda_i^2 + \epsilon_f^2}{\lambda_i^2 + \epsilon_b^2} \simeq \left( \frac{\epsilon_f}{\epsilon_b} \right)^N \mathcal{W}_{\text{large}} \]

- Elimination of near 0 eigenvalues if
  - $\epsilon_f = 0$
Complication because CPS ‘preconditions’ Dirac operator into odd-even blocks

\[
D_{2N \times 2N} = \begin{pmatrix}
  m_\epsilon I_{oo} & W_{eo} \\
  W_{oe} & m_\epsilon I_{ee}
\end{pmatrix}
\]

\[
m_\epsilon = \kappa^{-1} \gamma_5(\theta)
\]

\[
\gamma_5(\theta) = \cos \theta + i \sin \theta \gamma_5
\]

and Dirac determinant becomes

\[
\det(D_{2N \times 2N}) = \det(M_{N \times N}) = \det[\kappa^{-2} I_{oo} - \gamma_5(-\theta)W_{eo} \gamma_5(-\theta)W_{oe}]
\]

Determinant ratio was represented as symmetrized quotient (alg_quotient integrator)

\[
S = \phi^\dagger M_b (M_f^\dagger M_f)^{-1} M_b^\dagger \phi \quad \phi = M_b (M_b^\dagger M_b)^{-1} M_f^\dagger \eta \quad \chi = [(M_f^\dagger M_f)^{-1}] M_f^\dagger \eta
\]

The quotient force (derivative of action w.r.t. MD time)

\[
\partial_t S = \chi^\dagger \partial_t [M_f^\dagger M_f] \chi + \phi^\dagger \partial_t [M_b] \chi + \chi^\dagger \partial_t [M_b^\dagger] \phi
\]
CPS is object structured; actions and lattices are objects

Add subclass of the Wilson fermion action class
- pre/post multiply Dirac operator by $\gamma_5(\theta)$ (since $\gamma_5(\theta)$ does not commute with $W$)
- Provide forces in new subclass that take proper account of non-commutativity of $\gamma_5(\theta)$

Modify quotient ‘integrator’ class to be aware of new force type
Data - introduction

- Data without weighting factor ("old data")
  - QCDOC; Columbia (M. Cheng)
    - $16^3 \times 8 \times 32$; $m_0 = 1.8; m_l = 0.003; m_s = 0.037$; Iwasaki action; $\beta = 1.95, 2.00, 2.0375, 2.05, 2.08, 2.11, 2.14$

- Data with weighting factor ("new data")
  - ‘New York Blue’; Brookhaven National Laboratory
    - $16^3 \times 8 \times 16,32$; $m_0 = 1.8; m_l = 0.003; m_s = 0.037$; Iwasaki action; $\beta = 1.95, 2.00; \epsilon_b = 0.10; \epsilon_f = 0.01, 0.25, 0.05, 0.075$

- Initial experimentation with weighting factor
  - find optimum simulation parameters first
Data I – raw data

- Scaled $\bar{\Psi}\Psi$, plaquette, mres
- Lattice scale changing

- Expanded plaquette
- Gauge fields smoother at higher $\epsilon_f$
Data I (continued)

\[ \beta = 2.00 \]

\[ \bar{\Psi} \Psi \]

\[ 5 \times 10^{-3} \]

- \( s_f = 0.010 \)
- \( s_f = 0.075 \)
Data II

- Question: how much of change in mres, $\bar{\Psi}\Psi$ is due to lattice scale change and how much due to specific effects of suppressing 0 eigenvalues

- Obtain qualitative indications by comparing mres at equal $\bar{\Psi}\Psi$ values or at equal plaquette values
  - Assuming that $\bar{\Psi}\Psi$, plaquette reflect lattice scales to some degree
    - Note different $L_S$ values
  - Scaled beta comparison
Data III – equal comparison

'new' data at $L_S = 24$ is equal to or better than 'old' data at $L_S = 32$
Data IV – equal plaquette comparison

- ‘new’ data at $L_S = 24$ is better than ‘old’ data at $L_S = 32$
Data V – scaled $\beta$ comparison

- determine ‘effective’ beta for each $\epsilon_f$ using relation of plaquette and beta from old data
- interpolate mres & $\bar{\Psi}\Psi$ to ‘effective’ beta using relations of mres & $\bar{\Psi}\Psi$ and beta from old data
- scale for visibility
- admittedly ad hoc

- ‘effective’ beta vs $\epsilon_f$
- $\Delta \beta \sim 0.01/2.00$
Data VI (continued)

- observed mres vs. mres at ‘effective’ beta
- observed $\bar{\psi}\bar{\psi}$ vs. $\bar{\psi}\bar{\psi}$ at ‘effective’ beta

![Graphs showing observed mres vs. mres and observed $\bar{\psi}\bar{\psi}$ vs. $\bar{\psi}\bar{\psi}$ at ‘effective’ beta.](image)
Conclusions

- Implemented weighting factor with ratio of ‘twisted’ mass Wilson fermions
  - Characterization on going
    - current emphasis on finding optimum simulation parameters
- Achieved 2 fold reduction of mres (most recent data)
  - Further improvements expected
- Future directions

\[
\left\{ \frac{\det(H_W(-m_0)H_W^+(m_0 + \epsilon_f^2))}{\det(H_W(-m_0)H_W^+(m_0 + \epsilon_b^2))} \right\}^\gamma
\]