STRONG COUPLING CONSTANT AND FOUR QUARK CONDENSATES FROM VACUUM POLARIZATION FUNCTIONS WITH DYNAMICAL OVERLAP FERMIONS

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Calculation of strong coupling constant $\alpha_s$
Fundamental constant of QCD, and provides a high precision test of QCD.

**Phenomenological determination (short distance physics)**
- Deep-inelastic scattering,
- tau decay (OPAL, ALEPH), $e^+e^-$ annihilation.
- Operator Product Expansion (OPE)

**Lattice calculation**
- (Heavy) hadron spectroscopy, [SESAM(1999), HP/UKQCD-Flab(2004)]
- Heavy (static) quark potential, [HPQCD(2008)]
- Schrödinger functional scheme, [ALPHA(2005)]
METHODOLOGY

- Matching the OPE with the lattice data of vector (V) and axial-vector (A) vacuum polarization functions in dynamical overlap fermion.

- Exact chiral symmetry of overlap fermion
  - No additive renomalization terms (in chiral condensate)
  - No $O(a)$ lattice artifacts due to the violation of chiral symmetry.

- Dynamical overlap fermion configurations
  - $N_f=2$, $16^3 \times 32$ lattice, $a^{-1}=1.67$ GeV, quark mass: $m_s/6 \sim m_s/2$
  - Non-perturbative renormalization factor
  - Topology is fixed in $Q=0$

See also [HPQCD(2008)]

Talk by S. Hashimoto
VACUUM POLARIZATION FUNCTIONS

Current correlator in the continuum

Different spin (S=0, 1) components

\[ i \int d^4 x \langle T\{ J_\mu(x), J_\nu^\dagger(0) \} \rangle e^{iqx} = -(g_{\mu\nu}q^2 - q_\mu q_\nu) \Pi_{J}^{(1)}(q^2) + q_\mu q_\nu \Pi_{J}^{(0)}(q^2) \]

focus on \( \Pi_{J}^{(0+1)}(Q^2) = \Pi_{J}^{(0)}(Q^2) + \Pi_{J}^{(1)}(Q^2), \ Q^2 = -q^2 > 0 \)

OPE

\[
\Pi_{J}^{(0+1)}(Q^2) = C_0(Q^2, \mu^2) + \frac{m^2 C_m^J(Q^2)}{Q^2} + C_{qq}^J(Q^2) \frac{\langle m\bar{q}q \rangle}{Q^4} + C_{GG}(Q^2) \frac{\langle \alpha_s/\pi GG \rangle}{Q^4} + \cdots
\]

\( C_0 \) and \( C_m \) are known at 4-loop, \( C_{qq} \) and \( C_{GG} \) are known at 3-loop.
CURRENT CORRELATOR ON THE LATTICE

- Local (non-conserved) current
  \[ V_\mu = Z_V \bar{q} \gamma_\mu \left(1 - \frac{D}{2m_0}\right) q, \quad A_\mu = Z_A \bar{q} \gamma_\mu \gamma_5 \left(1 - \frac{D}{2m_0}\right) q \]

- Correlator
  \[
  \int d^4 x \langle T\{J_\mu(x), J_\nu(0)\}\rangle^{\text{lat}} e^{iQx} = \delta_{\mu\nu} Q^2 \Pi_J^{(1)}(Q) - Q_\mu Q_\nu \Pi_J^{(0+1)}(Q) \\
  - B_0(Q) \delta_{\mu\nu} - \sum_{n=1} B_n(Q) Q_\mu^{2n} \delta_{\mu\nu} - \sum_{m,n=1} C_{mn}(Q) (Q_\mu^{2m+1} Q_\nu^{2n+1} + Q_\nu^{2m+1} Q_\mu^{2n+1})
  \]
  
  - 1\textsuperscript{st} and 2\textsuperscript{nd} term \([\Pi_J]\): Vacuum polarization function
  - 3\textsuperscript{rd} term: \([B_0]\)
    Same Lorentz structure as \(\Pi_J^{(1)}\) and contains contact term which is divergent as \(1/a^2\).
  - 4\textsuperscript{th} and 5\textsuperscript{th} term: \([B_n, C_{mn}]\)
    Violation of the Lorentz symmetry (lattice artifacts),
CURRENT CORRELATOR ON THE LATTICE

\[ \int d^4x \langle T\{J_\mu(x), J_\nu(0)\}\rangle^{\text{lat}} e^{iQx} = \delta_{\mu\nu} Q^2 \Pi^{(1)}_J(Q) - Q_\mu Q_\nu \Pi^{(0+1)}_J(Q) \]

\[-B_0(Q) \delta_{\mu\nu} - \sum_{n=1} B_n(Q) Q^{2n}_\mu \delta_{\mu\nu} - \sum_{m,n=1} C_{mn}(Q) (Q^{2m+1}_\mu Q^{2n+1}_\nu + Q^{2m+1}_\nu Q^{2n+1}_\mu) \]

**Our method:**
- Focus on \( \Pi^{(0+1)}_J \), then \( Q^2 \Pi^{(1)}_J + B_0 \) can be ignored.
- Truncate the terms of \( O(Q^6) \) and higher, we only consider \( B_{1,2} \) and \( C_{11} \)
- Off-diagonal part \( (\mu \neq \nu) \)
  - extract \( \Pi^{(0+1)}_J \) and \( C_{11} \)
- Diagonal part \( (\mu = \nu) \)
  - extract \( \Pi^{(0+1)}_J \) and \( B_{1,2} \) using \( C_{11} \) from off-diagonal part.
- Comparison of \( \Pi^{(0+1)}_J \) obtained from diagonal \( (\mu = \nu) \) and off-diagonal \( (\mu \neq \nu) \) provides a good check of consistency.
NUMERICAL RESULTS: LATTICE ARTIFACTS

Subtraction coefficients (lightest quark mass)

- Solid line: fit function (polynomial), Dashed line: one-loop in lat. PT.
- Dominated by the perturbative contribution
- $B_1$ (in diagonal part), is much larger than others,
- These coefficients mostly cancel in V-A.

[arXive: 0806.4222] and Yamada’s talk
NUMERICAL RESULTS: SUBTRACTION

$\Pi_V^{(0+1)}$ and subtraction factor

blue filled circle and green filled triangle reasonably agree in $(aQ)^2 < 1.4$

- Higher order is small
- Truncation at $O(Q^6)$ is enough to reduce the Lorentz violating terms
Analysis of two forms of $\Pi_J$ using the OPE

**V+A**

Coupling constant $\alpha_s (\Lambda_{\text{MS}})$ and gluon condensate

**V - A**

Four quark condensate, $a_6(\mu)$, $b_6(\mu)$:

$$a_6(\mu) = 2 \left[ 2\pi \langle \alpha_s O_8 \rangle + A_8 \langle \alpha_s^2 O_8 \rangle + A_1 \langle \alpha_s^2 O_1 \rangle \right]$$

$$b_6(\mu) = 2 \left[ B_8 \langle \alpha_s^2 O_8 \rangle + B_1 \langle \alpha_s^2 O_1 \rangle \right]$$

with

$$\langle O_1 \rangle = \left\langle \bar{q} \gamma_\mu \frac{\tau^3}{2} q \bar{q} \gamma^\mu \gamma_5 \frac{\tau^3}{2} q - \bar{q} \gamma_\mu \gamma_5 \frac{\tau^3}{2} q \bar{q} \gamma^\mu \gamma_5 \frac{\tau^3}{2} q \right\rangle$$

$$\langle O_8 \rangle = \left\langle \bar{q} \gamma_\mu \lambda_a \frac{\tau^3}{2} q \bar{q} \gamma^\mu \lambda_a \frac{\tau^3}{2} q - \bar{q} \gamma_\mu \gamma_5 \lambda_a \frac{\tau^3}{2} q \bar{q} \gamma_5 \gamma^\mu \lambda_a \frac{\tau^3}{2} q \right\rangle$$

which corresponds to $K \rightarrow \pi \pi$ (I=2) matrix element.

[Donaghue(2000)]
ANALYSIS OF V+A

For \[ \Pi_{V+A}^{(0+1)} = \Pi_V^{(0+1)} + \Pi_A^{(0+1)} \]

\[ \Pi_{V+A}^{(0+1)} |_{\text{OPE}} (Q^2) = c + C_0(Q^2, \mu^2) + \frac{m^2 C_{m+V}^{V+A}(Q^2)}{Q^2} \]

\[ + C_{qq}^{V+A}(Q^2) \frac{\langle m\bar{q}q \rangle}{Q^4} + C_{GG}(Q^2) \frac{\langle \alpha_s/\pi GG \rangle}{Q^4} \]

3 free parameters
- \( \alpha_s (\Lambda_{\text{MS}}) \) and gluon condensate \( <\alpha_s/\pi GG> \),
- \( c \): difference of renormalization scheme (lattice and dimensional regularization)
- \( C_0, C_m, C_{qq}, C_{GG} \) from perturbation theory (3-loop)
- Quark condensate is an input, \([0.251 \text{ GeV}]^3 \) \[\text{[Fukaya}(2007)\text{]}\]

↑ No additional renormalization necessary.

Mass dependence is controlled by the 4\(^{th}\) term
**NUMERICAL RESULTS: V+A**

![Graphs showing numerical results](image)

- **Fit range** $[0.58, 1.3]$
- **Systematic error** estimated by replacing $C_0$ by lattice perturbation (one-loop)
- **Gluon condensate** has a large systematic error

\[ \Lambda_{MS}^{(2)} = 0.234(9)(^{+16}_{-0}) \text{ GeV} \]

\[ \langle \alpha_s/\pi GG \rangle = -0.06 \sim 0.1 \text{ GeV}^4 \]

*Statistical* vs. *Systematic* errors, e.g. 0.250(16)(16) GeV

[ALPHA(2005)]
ANALYSIS OF V−A

For $\Pi_{V-A}^{(0+1)} = \Pi_V^{(0+1)} - \Pi_A^{(0+1)}$

$$\Pi_{V-A}^{(0+1)}|_{\text{OPE}}(Q^2) = \frac{m^2 C_m^{V-A}(Q^2)}{Q^2} + C_{\bar{q}q}^{V-A}(Q^2) \frac{\langle m\bar{q}q \rangle}{Q^4}$$

$$+ \left[ a_6(\mu) + b_6(\mu) \ln(Q^2/\mu^2) + m_q c_6 \right] \frac{1}{Q^6} + \frac{a_8}{Q^8}$$

- In the chiral limit, 1st and 2nd terms go to zero.
  ⇒ start from the dimension-six term (leading)
- $C_m^{V-A}(Q^2), C_{\bar{q}q}^{V-A}(Q^2) \sim O(\alpha_s)$
  ⇒ sub-dominant
- Fit with or without the $a_8$ term in order to estimate the truncation effect
- Exact chiral symmetry of overlap fermion is important to remove additional operator mixing.
NUMERICAL RESULTS: \( V - A \)

Fit range \([0.58, 1.3]\]

\[
a_6 = -0.0038(3)(^{16}_{-0}) \text{ GeV}^6, \quad b_6 = +0.0017 \sim -0.0008 \text{ GeV}^6
\]

- Systematic error is determined by the comparison with and without the \( a_8 \) term.
- Phenomenological estimate: \( a_6 = -0.003 \sim -0.009 \text{ GeV}^6, \quad b_6 \sim 0.03a_6 \)
SUMMARY

Calculation of strong coupling $\alpha_s$ and four-quark condensate $a_6$ from vacuum polarization function $\Pi_{V\pm A}$ by matching with OPE.

Dynamical overlap fermions ($N_f=2$)

For $V+A$
- Subtract Lorentz violating terms $B_{1,2}, C_{11}$
- We obtain $\Lambda_{MS}^{(2)} = 0.234(9)(^{+16}_{-0})$, $\langle \alpha_s / \pi G G' \rangle = -0.06 \sim 0.1 \text{GeV}^4$

For $V-A$
- Four-quark condensate is leading term in the chiral limit
- We obtain $a_6 = -0.0038(3)(^{+16}_{-0}) \text{GeV}^6$, $b_6 = +0.0017 \sim -0.0008 \text{GeV}^6$
- Good agreement with phenomenological estimation

On-going project on $N_f=2+1$ configurations