Determination of Nucleon Excited States

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University of Maryland

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LHPC Baryon Spectroscopy

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- K.J. Juge, University of the Pacific
- A. Lichtl, Brookhaven National Laboratory
- N. Mathur, Tata Institute of Fundamental Research
Introduction

- Lattice provides first principle calculations of spectrum
- Understand pattern of excited states, QCD
- Challenges: ordering of masses $N'$ and $N^{*}$ (Roper)
Quenched Results

Develop group theory, optimize operators, and refine analysis techniques.


$16^3 \times 64$ and $24^3 \times 64$ lattices, $m_\pi = 490$ MeV

Lichtl, hep-lat/0609019

$12^3 \times 48$ lattice, $m_\pi = 700$ MeV
Calculation Overview

- Form a set of baryon operators \( \{ \bar{O}_1, \bar{O}_2, \ldots, \bar{O}_n \} \)
- Diagonalize matrix of correlators

\[
C_{IJ}(t) = \sum_{\vec{x}} \langle 0 | T \bar{O}_I(\vec{x}, t) \bar{O}_J(0, 0) | 0 \rangle = \sum_n c_n e^{-E_n t}
\]

- Principle correlator - diagonalize correlator matrix on each time slice, \( \lambda_n(t) \propto e^{-E_n t} \)
- Fixed coefficient - diagonalize at an early time slice, rotate each time slice into basis of eigenvectors. \( C_{nn}(t) \propto e^{-E_n t} \)
Group Theory

- Lattice breaks full rotational symmetry
- Construct operators that transform as irreducible representations of $O^D_h$: definite lattice spin and parity:

<table>
<thead>
<tr>
<th>irreps</th>
<th>dim.</th>
</tr>
</thead>
<tbody>
<tr>
<td>$G_{1g}$, $G_{1u}$</td>
<td>2</td>
</tr>
<tr>
<td>$G_{2g}$, $G_{2u}$</td>
<td>2</td>
</tr>
<tr>
<td>$H_g$, $H_u$</td>
<td>4</td>
</tr>
</tbody>
</table>

- Identify continuum spin via patterns of degenerate states in irreps:

<table>
<thead>
<tr>
<th>J</th>
<th>irreps</th>
</tr>
</thead>
<tbody>
<tr>
<td>1/2</td>
<td>$G_1$</td>
</tr>
<tr>
<td>3/2</td>
<td>$H$</td>
</tr>
<tr>
<td>5/2</td>
<td>$H,G_2$</td>
</tr>
<tr>
<td>7/2</td>
<td>$G_1,G_2,H$</td>
</tr>
<tr>
<td>9/2</td>
<td>$G_1$, $H(\times 2)$</td>
</tr>
</tbody>
</table>
Gauge invariant displacements: capture radial and orbital structure of baryons:

<table>
<thead>
<tr>
<th>Operator type</th>
<th>Displacement indices</th>
</tr>
</thead>
<tbody>
<tr>
<td>Single-Site</td>
<td>$i = j = k = 0$</td>
</tr>
<tr>
<td>Singly-Displaced</td>
<td>$i = j = 0, k \neq 0$</td>
</tr>
<tr>
<td>Doubly-Displaced-I</td>
<td>$i = 0, j = -k, k \neq 0$</td>
</tr>
<tr>
<td>Doubly-Displaced-L</td>
<td>$i = 0,</td>
</tr>
<tr>
<td>Triply-Displaced-T</td>
<td>$i = -j,</td>
</tr>
</tbody>
</table>

Pruning: Choose 16 operators in each channel for use with the variational method: low noise, linear independence

Smearing: Enhances coupling to low lying states - Gaussian quark smearing with stout smeared links.
Anisotropic lattices - finer temporal spacing for better measurement of excited states

- 860 configurations: $24^3 \times 64$ $N_f = 2$ Wilson with $m_\pi = 360$ MeV, $a_s = 0.13$ fm, $a_s/a_t = 3$,

- Ratio of spatial and temporal Wilson loops to measure gauge anisotropy, relativistic energy dispersion relation to measure fermion anisotropy

- $r_0$ to set scale
$G_{1g}$ Preliminary Results

<table>
<thead>
<tr>
<th>$M_{ct}$</th>
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<tbody>
<tr>
<td>0.2048(25)</td>
</tr>
<tr>
<td>0.3967(94)</td>
</tr>
<tr>
<td>0.4079(94)</td>
</tr>
<tr>
<td>0.4237(69)</td>
</tr>
</tbody>
</table>

Ground State

![Graph showing data points and error bars for $M_{ct}$ against $t/a_t$. The graph includes a trend line indicating the decrease in $M_{ct}$ over time.](graph.png)
$G_{1g}$ Preliminary Results

<table>
<thead>
<tr>
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1st Excited State

![Graph showing $M_{at}$ vs. $t/a_t$]
$G_{1g}$ Preliminary Results

$M_{at}$

0.2048(25)
0.3967(94)
0.4079(94)
0.4237(69)

![Graph showing the 2nd Excited State with data points and error bars.]
$G_{1g}$ Preliminary Results

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3rd Excited State

![Graph showing $M_{at}/t$ vs. $t/a_t$]
$G_{1u}$ Channel

- Baryon creation operators of parity $P$ create backward propagating baryons of parity $-P$
- Short temporal dimension: interference between forward and backward states
- $G_{1u}$ channel: backward propagating $G_{1g}$ ground state has a lower energy
$G_{1u}$ Channel

$G_{1u}$ ground state effective mass:
Filtering

Filter out the backwards propagating state prior to diagonalization

\[ C(t) = \sum_n c_ne^{-Et} + be^{-E_0'(T-t)} \]

\[ E_0' \int_t^{t_1} dt' C(t') = \sum_n \frac{E_n'}{E_n} c_n \left( e^{-En} - e^{-E_nt_1} \right) - b \left( e^{-E_0'(T-t)} + e^{-E_0'(T-t_1)} \right) \]

\[ C_{filt}(t, t_1) = C(t) - C(t_1) + (1 - e^{-E_0'}) \sum_{j=t+1}^{t_1} C(j) \]

\[ = \sum_n c_n \left[ 1 + \frac{1 - e^{-E_0'}}{e^{En} - 1} \right] \left( e^{-En} - e^{-E_nt_1} \right) \]

\[ = \sum_n c'_n \left( e^{-En} - e^{-E_nt_1} \right) \]
Filtering

Filter out the backwards propagating state prior to diagonalization

\[ C(t) = \sum_n c_n e^{-E_n t} + b e^{-E'_0 (T-t)} \]

\[ E'_0 \int_t^{t_1} dt' C(t') = \sum_n \frac{E'_0}{E_n} c_n (e^{-E_n t} - e^{-E_n t_1}) - b \left( e^{-E'_0 (T-t)} + e^{-E'_0 (T-t_1)} \right) \]

\[ C_{\text{filt}}(t, t_1) = C(t) - C(t_1) + (1 - e^{-E'_0}) \sum_{j=t+1}^{t_1} C(j) \]

\[ = \sum_n c_n \left[ 1 + \frac{1 - e^{-E'_0}}{e^{E_n} - 1} \right] (e^{-E_n t} - e^{-E_n t_1}) \]

\[ = \sum_n c'_n (e^{-E_n t} - e^{-E_n t_1}) \]
The filtered correlators look like:

\[ \sum_{n} c'_n e^{-E_n t} + K \]

Diagonalize

Fit to \( A e^{-E t} + K \)

Compute the effective mass:

\[ M_{\text{eff}} = \log \left( \frac{C(t) - K}{C(t + 1) - K} \right) \]
Does the filter matter?

Does filtering change the diagonalization?
Look at the overlap between filtered and unfiltered eigenvectors
Does the filter matter?

Does filtering change the diagonalization?
Look at the overlap between filtered and unfiltered eigenvectors.
Does the filter matter?

Does filtering change the diagonalization?
Look at the overlap between filtered and unfiltered eigenvectors

Filtering may be needed for higher excited states.
$G_{1u}$ Preliminary Results

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![Graph showing $M_{at}/a_t$ vs. $t/a_t$ with ground state indicated.](image)
$G_{1u}$ Preliminary Results

\[ M_{a_t} \]

\begin{align*}
0.2945(94) \\
0.3232(92) \\
0.5024(128) \\
0.5314(72) \\
0.5316(109)
\end{align*}

1st Excited State

The diagram shows the 1st excited state over time $t/a_t$. The data points, represented by blue lines with error bars, indicate the variation of $M_{a_t}$ with time, highlighting the trend towards the excited state.
$G_{1u}$ Preliminary Results

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2nd Excited State
Preliminary Results

$M_{a_t}$

0.2945(94)
0.3232(92)
0.5024(128)
0.5314(72)
0.5316(109)

3rd Excited State

$t/a_t$
$G_{1u}$ Preliminary Results

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4th Excited State

![Graph showing $M_{at}$ vs. $t/a_t$]
$N_f = 2$ Nucleon Spectrum

Nucleon Mass Spectrum ($N_f=2$)

$\pi = 360$ MeV/c²

A. C. Lichtl - July 2008
Outlook

- Analyze $G_2$ and $H$ irreps with the filter
- Refine fitting and filtering - evaluate systematics
- $\Delta$ spectrum
- Other lattices - different volumes and pion masses
More filtering

Before the filter:

\[ e^{-Ht} + e^{-\bar{H}(T-t)} \]

After the filter:

\[ e^{-Ht} + Ce^{-Ht_1} \]

Eigenvalues:

\[ e^{-Et} + Ce^{-Et_1} \]