Dual quark condensate and dressed Polyakov loops

Falk Bruckmann (Univ. of Regensburg)

Lattice 2008, William and Mary

with Ereke Bilgici, Christian Hagen and Christof Gattringer

Motivation

**QCD at finite temperature: confinement and chiral symmetry breaking**

- **Polyakov loop**: \( \mathcal{P}(\vec{x}) = \mathcal{P} \exp \left(i \int_0^\beta dx_0 A_0(x_0, \vec{x}) \right) \), \( \beta = 1/k_B T \)

- order parameter for confinement:
  related to the free energy of a single quark
  confined phase: \( \langle \text{tr}_c \mathcal{P} \rangle = 0 \quad (F_{\text{quark}} \to \infty) \)
spectral density $\rho(\lambda)$ of the Dirac operator (in background $A_\mu$):

order parameter of chiral symmetry:

$$\rho(0) \sim \langle \bar{\psi} \psi \rangle \ldots \text{chiral condensate}$$

Banks-Casher
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Is there an underlying mechanism connecting the two?
does confinement leave a trace in the Dirac spectrum?
quarks should know that they are confined!

⇒ dressed Polyakov loops as a new order parameter
The idea

work on the lattice (regulator)

- Polyakov loop: \( \mathcal{P}(x) \equiv \prod_{\tau=1}^{N_0} U_0(x_0 + \tau, \vec{x}) \)
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- Dirac operator, here staggered

\[
D(x, y) \equiv \frac{1}{2a} \sum_{\mu} \eta_{\mu}(x) \left[ U_{\mu}(x) \delta_{x+\hat{\mu}, y} - h.c. \right]
\]

hopping by one link

\( \Rightarrow \) \( D^l(x, x) \supset \) products of links along closed loops of length \( l \), at \( x \)

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Kogut, Susskind

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how to distinguish Polyakov loops from ‘trivially closed’ loops?

- phase ‘twisted’ boundary conditions, as a tool:

$$\psi(x_0 + \beta, \vec{x}) = z \psi(x_0, \vec{x}), \quad z = e^{i\phi}$$

imag. chem. potential

realized by $U_0 \to zU_0$ at some time slice

$\Rightarrow$ Polyakov loops: $\mathcal{P} \to z\mathcal{P}$, trivial loops stay the same

Kogut, Susskind

Gattringer ’06

Falk Bruckmann
$\mathcal{P}$ itself turned out to be not suitable (UV dominated) \hspace{1cm} \text{FB et al. ’06}

propagator: \hspace{1cm} \text{cf. Synatschke, Wipf, Wozar ’07}

$$\text{tr} \frac{1}{m + D_\phi} = \frac{1}{m} \sum_{l=0}^{\infty} \frac{(-1)^l}{m^l} \text{tr}(D_\phi)^l \quad \ldots \text{all powers of } D_\phi$$
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\[
= \frac{1}{m} \sum_{\text{loops of length } l} \frac{(\pm 1)}{(2am)^l} \text{tr}_c \prod_{\text{loop}} U_\mu(x) e^{i\phi q(\text{loop})}
\]

\( q(\text{loop}) \in \mathbb{Z} \): how many times the loop winds around \([0, \beta]\)
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project onto particular winding \( q \):

\[
\frac{1}{2\pi} \int_{0}^{2\pi} d\phi \ e^{-i\phi q}
\]

let’s specify to a single winding \( q = 1 \) like the Polyakov loop:
A new observable

\[ \tilde{\Sigma}_1 = \int_0^{2\pi} \frac{d\phi}{2\pi} e^{-i\phi} \frac{1}{V} \langle \text{tr} \frac{1}{m + D_\phi} \rangle = \frac{1}{mV} \sum_{\text{loops}} \frac{(\pm 1)}{(2am)^l} \langle \text{tr}_c \prod_l U_\mu(x) \rangle \]

of length \( l \), winding once

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- dual condensate
- dressed Polyakov loops

- massless limit:

\[ \lim_{m \to 0} \lim_{V \to \infty} \tilde{\Sigma}_1 = \int_0^{2\pi} \frac{d\phi}{2\pi} e^{-i\phi} \rho(0) \phi \]

- dual chiral condensate
  \[ \rho(0) \sim \langle \bar{\psi}\psi \rangle \]
  (integrated over phase bc.s)

- massive limit:

\[ \lim_{m \to \infty} \tilde{\Sigma}_1 \sim \langle \text{tr}_c P \rangle \]

- thin Polyakov loop (shortest)
  detours suppressed by \( 2am \)
$\Sigma_1$ is an order parameter

numerical results (quenched):

$\Sigma_1$ as a function of temperature for $m = 100\,\text{MeV}$
$\tilde{\Sigma}_1 \equiv \int_0^{2\pi} d\phi \frac{1}{2\pi} e^{-i\phi} \frac{1}{V} \langle \text{tr} \frac{1}{m + D_\phi} \rangle = \int_0^{2\pi} d\phi \frac{1}{2\pi} e^{-i\phi} \frac{1}{V} \langle \sum_i \frac{1}{m + \lambda^{(i)}_\phi} \rangle$

truncate the sum: IR dominance expected since $\lambda$ in denominator!
confirmed by lattice data (if $m$ not too large):

![Graph showing individual and accumulated contributions for $m = 100$ MeV and $m = 1$ GeV]
how is a vanishing/finite Polyakov loop built up by the eigenvalues?
respond differently to bc.s in confined and deconfined phase

\[
\frac{1}{V} \langle \sum_i \frac{1}{m + \lambda^{(i)}_\phi} \rangle \text{ as a function of } \phi \text{ for real } \mathcal{P}
\]

nonvanishing cos $\phi$-part only in the deconfined phase $\Rightarrow \tilde{\Sigma}_1 \neq 0$

non-real $\mathcal{P}$: the plot is shifted by $\pm 2\pi/3$
\[\Rightarrow \text{periodicity } 2\pi/3, \text{ known from imag. } \mu\]

Lombardo et al.
How about the chiral condensate?

remember:

\[ \tilde{\Sigma}_1 \xrightarrow{m \to 0, V \to \infty} \int_0^{2\pi} d\phi \ e^{-i\phi} \rho(0) = \int_0^{2\pi} d\phi \ e^{-i\phi} \langle \bar{\psi}\psi \rangle \]

- confined phase:
  \[ \langle \bar{\psi}\psi \rangle \neq 0, \text{ but independent of } \phi \quad \Rightarrow \quad \text{vanishing } \tilde{\Sigma}_1 \]
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deconfined phase:

\[ \langle \bar{\psi}\psi \rangle = 0 \text{, spectral gap: } \rho(0) = 0 !? \]
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- deconfined phase:
  \[ \langle \bar{\psi} \psi \rangle = 0 \ , \text{ spectral gap: } \rho(0) = 0 !? \]
  no: \( \rho(0)_{\text{periodic}} \neq 0 \) for real \( P \)
  always one bc. where \( \rho(0) \neq 0 \)

\[ \langle \bar{\psi} \psi \rangle_\phi \sim \delta(\phi + \phi_P) \quad \Rightarrow \quad \text{nonvanishing } \tilde{\Sigma}_1 \]
for all \( T > T_c \)

Gattringer, Schaefer '03
Center symmetry

The deconfinement transition of pure gauge theory can be described as spontaneous breaking of the center symmetry:

- The action is invariant under
  \[ U_0 \rightarrow z U_0 \quad \text{at some time slice,} \quad z \in \text{center}(SU(3)) \]
- The Polyakov loop changes as
  \[ \text{tr}_c \mathcal{P} \rightarrow z \text{tr}_c \mathcal{P} \]
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  \[ \Rightarrow \text{therefore order parameter for confinement} \]

- all functions of the form
  \[ \int_0^{2\pi} \frac{d\phi}{2\pi} e^{-i\phi} f(D_\phi) \]

  transform this way, thus are order parameters for center symm.
Generalisation: Locally resolved Polyakov loops

so far: $\sum_x P(x) \rightarrow$ eigenvalues $\lambda^{(i)}_\phi$

now: $P(x) \rightarrow$ eigenvalues $\lambda^{(i)}_\phi$ and eigenvectors $\psi^{(i)}_\phi$
Generalisation: Locally resolved Polyakov loops

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- static quark potential $V_{q\bar{q}}(|\vec{x} - \vec{y}|) \sim \ln\langle \text{tr} \mathcal{P}(\vec{x}) \text{tr} \mathcal{P}(\vec{y}) \rangle$

SU(2):

$\Rightarrow$ string tension preserved by a truncated mode sum
mechanism not fully clear

Synatschke, Wipf, Langfeld '08

Bilgici, Gattringer '08
the response of Dirac spectra to different temporal bc.s contains information about confinement

the dressed Polyakov loop $\tilde{\Sigma}_1$ is a novel deconfinement order param. that relates the dual chiral condensate to the thin Polyakov loop

...and is dominated by IR modes

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outlook:

- random matrix theory description of $D_\phi$  Bruckmann, Verbaarschot in progr.
- gauge group $G(2)$: no nontrivial center  Gattringer, Maas in progr.
- full QCD and 4-fermi deformation (Sinclair): $T_{\chi_{sb}} \neq T_{\text{deconf}}$
  how in the formalism?!