A test of first order scaling in $N_f=2$ QCD: progress report

G. Cossu
Pisa University & SNS

XXVI International Symposium on Lattice Field Theory
Williamsburg, Virginia, USA, July 14-19 2008

In collaboration with:
C. Bonati (Pisa), M. D'Elia (Genova),
A. Di Giacomo (Pisa), C. Pica (BNL)
A test of first order scaling in $N_f=2$ QCD: progress report

Outline

✔ The order of the chiral transition in $N_f=2$ QCD and the QCD phase diagram
✔ Prediction from effective models and previous literature
✔ Present evidence from lattice simulations
✔ (Extremely!) Preliminary new results
✔ Conclusion and discussion
A test of first order scaling in $N_f=2$ QCD: progress report

Phase diagram

- Perturbative regime
- Deconfinement
- Chiral symmetry restored
- Axial U(1) effectively restored

Crossover or true phase transition?

Nature of the critical line?

$T_c$?

Non perturbative regime
- Confinement
- Chiral symmetry breaking
- Axial U(1) problem

First order

Perturbative regime
- Color superconductivity
- Deconfined quark matter?
The order of the transition at $\mu_B=0$ is of great importance: the existence of a critical endpoint at $T_E$ stems from the hypothesis of a crossover at the massless, $\mu=0$ theory.

There is a general tendency to accept the crossover scenario in the real QCD case ($N_f=2+1$ with physical quark masses): it has been shown (Y. Aoki, Z. Fodor, S. D. Katz and K. K. Szabo, Phys. Lett. B 643, 46 (2006); Nature 443, 675 (2006)) that the susceptibility of a possible order parameter for the transition (the chiral condensate) does not show any signal of growing with the spatial volume, till $L_s \sim 6$ fm.

However is interesting to deeply address the problem because experimental evicences still lack.  

Two flavor QCD in the light quark mass limit is an interesting case:  
- there are theoretical predictions about the order (eff. Chiral models)  
- no definite answer from lattice simulations yet

Model predictions for chiral $N_f=2$

- $U_A(1)$ anomaly effective (no light $\eta'$, $c \neq 0$): effective model has a fixed point, i.e. second order transition in the $O(4)$ universality class or a first order.
- $U_A(1)$ anomaly not effective (light $\eta'$, $c = 0$): no $O(4)$ stable f. p.

F. Basile, A. Pelissetto, E. Vicari, 2005 $\implies$ $U(2)_L \otimes U(2)_R/U(2)_V$ or first order

Second order in the chiral limit $\implies$ crossover at small quark masses

First order in the chiral limit $\implies$ first order in a small region around the chiral point
Determining the order: strategies

- Try the easiest (?) thing: look for metastabilities and double peak structure of the order parameter and of the energy density around the transition, i.e. coexistence of phases, which is a clear signature for first order. This search failed in the past leading to a preference for the second order scenario.

- Perform an accurate **Finite Size Scaling** analysis of various thermodynamical quantities around the chiral critical point to extract critical indexes ($O(2)$ is expected rather than $O(4)$ for the staggered fermion formulation).

<table>
<thead>
<tr>
<th></th>
<th>$Y_h$</th>
<th>$Y_t$</th>
<th>$\nu$</th>
<th>$\alpha$</th>
<th>$\gamma$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$O(4)$</td>
<td>1.336(25)</td>
<td>2.487(3)</td>
<td>0.748(14)</td>
<td>-0.24(6)</td>
<td>1.479(94)</td>
</tr>
<tr>
<td>$O(2)$</td>
<td>1.496(20)</td>
<td>2.485(3)</td>
<td>0.668(9)</td>
<td>-0.005(7)</td>
<td>1.317(38)</td>
</tr>
<tr>
<td>1st Order</td>
<td>3</td>
<td>3</td>
<td>1/3</td>
<td>1</td>
<td>1</td>
</tr>
</tbody>
</table>
Finite Size Scaling

Approaching the transition the correlation length of the order parameter $\xi$ goes large and one can write the following scaling ansatzs:

- **free energy density** $\Longrightarrow L/kT \approx L_s^{-d} \phi (\tau L_s^{1/\nu}, am_q L_s y_h)$
- **specific heat** $\Longrightarrow C_V - C_0 \approx L_s^{\alpha/\nu} \phi_c (\tau L_s^{1/\nu}, am_q L_s y_h)$
- **order parameter suscept.** $\Longrightarrow \chi \approx L_s^{\gamma/\nu} \phi_\chi (\tau L_s^{1/\nu}, am_q L_s y_h)$

**Technical difficulties:**
- Simulations on large volumes and with light quark masses are necessary for a reliable f.s.s. analysis $\Longrightarrow$ huge computational power required
- f.s.s. behavior is given in terms of two different scales (two scaling variables).

No clear answer from previous literature (see our works for references).
$O(4)$ and $O(2)$ are ruled out by our data. Notice that $\alpha<0$ for $O(4)$.
Previous work: first order check

An approximate check for a first order scaling on the collected data was performed.

Indication that the transition could be first order. So we decided to give a chance to this hypothesis. But:

• where is the growth linear with the spatial volume expected for a first order transition at fixed mass?
• where are the double peaks?
Further tests: $1^{\text{st}}$ order direct check


- Chiral susceptibility shows deviations, possibly due to the large mass range explored (up to 0.1) which could be outside the scaling region.
- Good scaling of the specific heat: not only the peak heights but also the widths are well described by the first order hypothesis.
Consider again the scaling law $C_V - C_0 \approx L_s^{\alpha/\nu} \phi_c(\tau L_s^{1/\nu}, \tau a m_q L_s y_h)$.

- Continuous transition $\implies$ $L_s$ dependence must cancel as $L_s \to \infty$ at finite $m_q$. The scaling function can be expanded in terms of $1/(a m_q L_s y_h)$: the leading term must be $1/(a m_q L_s y_h)^{\alpha/\nu y_h}$ $\implies$ no discontinuity (no latent heat) at finite $m_q$.
- First order chiral transition $\implies$ a first order singularity is expected also at some $m_q \neq 0$, leading to a non-zero latent heat: we can allow for a constant term in the expansion in powers of $1/(a m_q L_s y_h)$
  \[ C_V - C_0 \sim a m_q^{-1} \phi_c(\tau V) + V \phi'_c(\tau V) \]

In the second case the relative weight of the singular to the regular contribution is not known apriori, may be very small for small volumes and weak first order transitions.
There are various possibilities:

- There is really a first order transition which however is so weak that metastabilities will not show up but on very large, still unexplored volumes.
- We observe the “wrong” critical indexes because the scaling region around the chiral point is so small that the “correct” $O(4)$ indexes will not show up but at very small, still unexplored quark masses.

In order to clarify the issue, we have judged worth dedicating a large numerical effort to a run at $am_q=0.01335$ on a $48^3 \times 4$ lattice (thanks to apeNEXT!)

That corresponds to $m_\pi \sim$ twice the physical value and to a spatial size $\sim 13$-$14$ fm.
We collected $\sim 30k$ trajectories in total.
A test of first order scaling in $N_f=2$ QCD: progress report

Preliminary results - Histories

Spatial Plaquette

5.2718

G.Cossu (Pisa University)
Preliminary results - Histories

Spatial Plaquette

5.2719
A test of first order scaling in $N_f=2$ QCD: progress report

Preliminary results - Histories

Spatial Plaquette

5.2720
Preliminary results - Histograms

Normalized Histograms
Spatial Plaquette

L = 16, 5.2710
L = 16, 5.2720

G.Cossu (Pisa University)
Preliminary results - Histograms
Preliminary results - Histograms

Normalized Histograms
Spatial Plaquette

L = 16, 5.2710
L = 16, 5.2720
L = 32, 5.2715
L = 32, 5.2720
L = 48, 5.2720
L = 48, 5.2716
L = 48, 5.2719
A test of first order scaling in N_f=2 QCD: progress report

Preliminary results - Histograms

Dated half June 2008.

G.Cossu (Pisa University)
A test of first order scaling in $N_f=2$ QCD: progress report

Preliminary results - Histograms

Spatial Plaquette Histogram

$m_q=0.01335$

- Black: 5.2718
- Red: 5.2719
- Blue: 5.2720
A test of first order scaling in $N_f=2$ QCD: progress report

Preliminary results - Susceptibility

Shrinks with the correct factor but doesn't grow. We need more statistics. We hope to completely clarify this issue in the next months.
Discussion and conclusions

Conclusion 1: With present UV cutoff effects ($N_t=4$, non-improved action) and within the present quark mass range a second order chiral transition in the $O(4)$ (and $O(2)$ and $U(2)_L \otimes U(2)_R/U(2)_V$) seems to be excluded.

Conclusion 2: First order critical indexes seem to be preferred. Preliminary: we have some signals (to be confirmed!!) for a first order bistability at $am_q=0.01335$, however the bistability does not show up until $L_s=12/T \sim 13–14$ fm. LOW STATISTICS! Needs more investigation!

Our results have been obtained with a quite large lattice spacing $N_t=4$, 0.3 fm (lattice spacing) and with a non-improved action. If our results will be confirmed on $N_t=6$ and/or using an improved lattice action, then the crossover scenario must be changed.
A test of first order scaling in $N_f=2$ QCD: progress report

Thank you!