Clover improvement for stout-smeared 2+1 flavour SLiNC fermions: perturbative results

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QCDSF collaboration

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Outline

Introduction
  Action for 2+1

Calculation
  Results for $c_{SW}$ and $\kappa_c$
  Mean field improvement
  Point operators

Summary
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Total action

Current simulations with $2 + 1$ flavours require a careful choice of lattice representations of fermions and gluons

**QCDSF collaboration:**

\[
S^{\text{total}}(U, U, \psi; c_{\text{SW}}, \kappa, c_i) = S_{\text{SLiNC}} + S_G(U; c_i)
\]

**SLiNC action** = Stout Link Non-perturbative Clover

\[
S_{\text{SLiNC}} = S_F(U, U, \psi; c_{\text{SW}}, \kappa)
\]
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**SLiNC action=Stout Link Non-perturbative Clover**

\[ S_{\text{SLiNC}} = S_F(U, U, \psi; c_{SW}, \kappa) \]
Fermionic part

Clover action with stout smeared links $U$ in the hopping term

$$S_F(U, U, \psi; c_{SW}, \kappa) = \sum_{x} \left\{ \overline{\psi}(x) \psi(x) - \kappa \overline{\psi}(x) U^\dagger_\mu(x - \hat{\mu}) [1 + \gamma_\mu] \psi(x - \hat{\mu}) - \kappa \overline{\psi}(x) U_\mu(x + \hat{\mu}) [1 - \gamma_\mu] \psi(x + \hat{\mu}) + \frac{i}{2} \kappa c_{SW} \overline{\psi}(x) \sigma_{\mu\nu} F_{\mu\nu}(U, x) \psi(x) \right\}$$
Fermionic part

where [Morningstar/Peardon]

\[ U_\mu(x) = \exp\{iQ_\mu(x)\} U_\mu(x) \]

\[ Q_\mu(x) = \frac{\omega}{2i} [VU^\dagger - UV^\dagger - \frac{1}{N_c} \text{Tr}(VU^\dagger - UV^\dagger)]_{\mu} \]

\( V_\mu \) is the sum of all staples around \( U_\mu \).

Terms at \( O(\omega) \):

\[ VU^\dagger U \quad \quad \quad \quad UU^\dagger U \quad \quad \quad \quad \text{Tr}(U) \]
Fermionic part

Benefits:  
- UV-filtering → improving chiral behavior of clover fermions  
- UV-filtering → suppressing unwanted tadpole contributions  
- Stout smearing → fat link remains automatically in the gauge group  

Choices:  
- $\omega \approx 0.1$ → mild smearing  
- $F_{\text{clover}}(U)$ unsmeared → fermionic matrix remains not too extended
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Gauge part

Symanzik improved gauge action:

\[
S_G(U; c_i) = \frac{6}{g^2} \left[ c_0 \sum_{\text{plaquette}} \frac{1}{3} \text{Re} \text{Tr} (1 - U_{\text{plaquette}}) + 
\right. \\
\left. c_1 \sum_{\text{rectangle}} \frac{1}{3} \text{Re} \text{Tr} (1 - U_{\text{rectangle}}) \right]
\]

with \( c_1 = -1/12 \), \( c_0 + 8c_1 = 1 \), \( \beta = \frac{6}{g^2} c_0 \)
Gauge part

Benefits:
- Six-link gauge actions $\rightarrow \mathcal{O}(a^2)$ improvement
- Six-link gauge actions $\rightarrow$ better phase behavior for 2+1

\textbf{JLQCD}
- Tree-level Symanzik $\rightarrow \Lambda^{\overline{\text{MS}}}/\Lambda^{\text{latt}} \approx \mathcal{O}(1)$
- One-loop corrections $\Delta c_i^{(1)}$ to the $c_i \rightarrow \Delta c_i^{(1)} \approx -0.01$, $\Delta c_2^{(1)} \approx -0.00006$ Zhao et al. [2007]
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Parameters of SLiNC

Summary of parameters:

- $c_i$, $\omega$, $c_{SW}$, number of smearing steps ($n_{smear}$)

- $c_i$, $\omega$, $n_{smear}$: certain freedom

but:

$c_{SW}$ has to be tuned to cancel $O(a)$ scaling violation
(if $n_{smear}$ is small)
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Perturbative O(a) improvement

First determinations of $c_{SW}$ in one-loop have been published by:

*Wohlert[1987]* (twisted antiperiodic b.c., plaquette action)

*Lüscher and Weisz[1996]* (Schrödinger functional, plaquette action)

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*Torrero[2008]* (NSPT, talk at Lattice08)

This talk:
off-shell Green function from SLiNC action
Perturbative $O(a)$ improvement

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This talk:
off-shell Green function from SLiNC action
qqg-Vertex

Looking for quantity $\rightarrow$ one-loop information for $c_{SW}$

Quark-quark-gluon-vertex ($V_\mu$): it contains to lowest order the improvement parameter $c_{SW} \rightarrow$ one-loop calculation sufficient

$$V_\mu(p_1, p_2, c_{SW}) = -ig \gamma_\mu - g^{1/2} a \mathbf{1}(p_1 + p_2)_\mu$$

$$+ c_{SW} \left( ig \frac{1}{2} a \sigma_{\mu\alpha}(p_1 - p_2)_\alpha + O(a^2) \right).$$

with

$$c_{SW} = 1 + g^2 c_{SW}^{(1)}$$

Strategy: Calculate the full three-point function $V_{qqg}^\mu$ to one-loop and demand that all $O(a)$ terms cancel $\rightarrow c_{SW}^{(1)}$
qqg-Vertex

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$$+ c_{SW} i g \frac{1}{2} a \sigma_{\mu\alpha}(p_1 - p_2)_\alpha + \mathcal{O}(a^2).$$

with

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Strategy: Calculate the full three-point function $V_{qqg}^\mu$ to one-loop and demand that all $\mathcal{O}(a)$ terms cancel → $c_{SW}^{(1)}$
**Off-shell "benefit"**

Calculating the qqqg-vertex off-shell $\rightarrow$ additional improvement of the quark field is necessary:

$$\psi_*(x) = \left(1 + a c_D \vec{\nabla} + a i g c_{\text{NGI}} A(x)\right) \psi(x)$$

$$c_D = -\frac{1}{4} \left(1 + g^2 c_D^{(1)}\right) + \mathcal{O}(g^4)$$

QCDSF [2001]

$$c_{\text{NGI}} = g^2 c_{\text{NGI}}^{(1)} + \mathcal{O}(g^4)$$

introduced by Martinelli et al. [2001] $\rightarrow$ first result in one-loop
Calculating the qqg-vertex off-shell → additional improvement of the quark field is necessary:

\[ \psi_\star(x) = \left( 1 + a c_D \vec{D} + a i g c_{\text{NGI}} A(x) \right) \psi(x) \]

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Feynman diagrams

Figure: One-loop diagrams contributing to the amputated quark-quark-gluon vertex
Example: qqggg-Vertex and stout smearing

\[
V_{\alpha \beta \gamma}^{abc}(p_2, p_1, k_1, k_2, k_3, \omega) = \frac{1}{6} a^2 g^3 \sum_{\mu} \left\{ W_{1\mu}(p_2, p_1) \left[ F_{\alpha \beta \gamma \mu}^{abc}(k_1, k_2, k_3) + \text{cyclic perm.} \right] - 6 \omega W_{2\mu}(p_2, p_1) \left[ T_{s\alpha}^{abc} V_{\alpha \mu}(k_1) g_{\beta \gamma \mu}(k_2, k_3) + \text{cyclic perm.} \right] \right\}.
\]

\[
F_{\alpha \beta \gamma \mu}^{abc}(k_1, k_2, k_3) = T_{s\alpha}^{abc} f_{\alpha \beta \gamma \mu}^{(1)}(k_1, k_2, k_3) + T_{a\alpha}^{abc} (f_{\alpha \beta \gamma \mu}^{(2)}(k_1, k_2, k_3) - f_{\alpha \gamma \beta \mu}^{(2)}(k_1, k_3, k_2)) + \left( T_{s\alpha}^{abc} - \frac{1}{N_c} g^{abc} \right) f_{\alpha \beta \gamma \mu}^{(3)}(k_1, k_2, k_3),
\]

\[
f_{\alpha \beta \gamma \mu}^{(1)}(k_1, k_2, k_3) = \frac{1}{2} V_{\alpha \mu}(k_1, \omega) V_{\beta \mu}(k_2, \omega) V_{\gamma \mu}(k_3, \omega),
\]

\[
f_{\alpha \beta \gamma \mu}^{(2)}(k_1, k_2, k_3) = \frac{1}{2} V_{\alpha \mu}(k_1, \omega) V_{\beta \mu}(k_2, \omega) \delta_{\gamma \mu} - \frac{1}{2} \delta_{\alpha \mu} \delta_{\beta \mu} V_{\gamma \mu}(k_3, \omega) + 6 \omega \delta_{\alpha \beta} \left[ c_{\mu}(k_1 - k_2) c_{\beta}(2k_3 + k_1 + k_2) \delta_{\gamma \mu} + s_{\mu}(k_3) s_{\gamma}(k_3 + 2k_1) \delta_{\beta \mu} \right],
\]

\[
f_{\alpha \beta \gamma \mu}^{(3)}(k_1, k_2, k_3) = 2 \omega \delta_{\beta \gamma} \left[ (3 w_{\alpha \mu}(k_1, k_2 + k_3) + v_{\alpha \mu}(k_1 + k_2 + k_3)) \delta_{\alpha \beta} + 12 s_{\beta}(k_1) s_{\alpha}(k_2) s_{\alpha}(k_3) (s_{\beta}(k_1 + k_2 + k_3) \delta_{\alpha \mu} - s_{\alpha}(k_1 + k_2 + k_3) \delta_{\beta \mu}) \right].
\]
Example: qqggg-Vertex and stout smearing

Notation:

\[ T^{abc}_{ss} = \{ T^a, \{ T^b, T^c \} \}, \quad T^{abc}_{aa} = [T^a, [T^b, T^c]], \quad T^{abc}_{sa} = \{ T^a, [T^b, T^c] \} \]

\[ s_\mu(k) = \sin \left( \frac{a}{2} k_\mu \right), \quad c_\mu(k) = \cos \left( \frac{a}{2} k_\mu \right), \quad s^2(k) = \sum_\mu s^2_\mu(k), \]

\[ s^2(k_1, k_2) = \sum_\mu s_\mu(k_1 + k_2) s_\mu(k_1 - k_2) \equiv s^2(k_1) - s^2(k_2) \]

\[ W_{1\mu}(p_2, p_1) = i c_\mu(p_2 + p_1) \gamma_\mu + r s_\mu(p_2 + p_1) \]

\[ W_{2\mu}(p_2, p_1) = i s_\mu(p_2 + p_1) \gamma_\mu - r c_\mu(p_2 + p_1) \]

\[ V_{\alpha\mu}(k, \omega) = \delta_{\alpha\mu} + 4 \omega v_{\alpha\mu}(k) \]

\[ v_{\alpha\mu}(k) = s_\alpha(k) s_\mu(k) - \delta_{\alpha\mu} s^2(k) \]

\[ g_{\alpha\beta\mu}(k_1, k_2) = \delta_{\alpha\beta} c_\alpha(k_1 + k_2) s_\mu(k_1 - k_2) - \]

\[ \delta_{\alpha\mu} c_\alpha(k_2) s_\beta(2k_1 + k_2) + \delta_{\beta\mu} c_\beta(k_1) s_\alpha(2k_2 + k_1) \]

\[ w_{\alpha\mu}(k_1, k_2) = s_\alpha(k_1 + k_2) s_\mu(k_1 - k_2) - \delta_{\alpha\mu} s^2(k_1, k_2), \quad w_{\alpha\mu}(k, 0) = v_{\alpha\mu}(k) \]
Results: $c_{SW}$

\begin{equation*}
c_{SW} = 1 + g^2 c_{SW}^{(1)}
\end{equation*}

\begin{equation*}
c_{SW}^{(1)} = C_F \left( 0.116185 + 0.828129 \omega - 2.455080 \omega^2 \right) \\
+ N_c \left( 0.013777 + 0.015905 \omega - 0.321899 \omega^2 \right)
\end{equation*}

coincides for $\omega = 0$ with Aoki, Kuramashi [2003]
Results: $\kappa_c$

Additive mass renormalization

$$am_0 = \frac{1}{2\kappa_c} - 4 = \frac{g^2 C_F}{16\pi^2} \frac{\Sigma_0}{4} \rightarrow \kappa_c = \frac{1}{8} \left(1 - \frac{g^2 C_F}{16\pi^2} \frac{\Sigma_0}{4}\right)$$

SLiNC action + quark self energy ($\Sigma(p = 0)$) → $\kappa_c$:

$$\kappa_c = \frac{1}{8} \left[1 + g^2 C_F \left(0.037730 - 0.662090\omega + 2.668543\omega^2\right)\right].$$

$$\omega = 0.088689 \rightarrow \kappa_c = \frac{1}{8}$$
Quark field improvement results

\[ \psi_\star(x) = \left( 1 + a \ c_\text{D} \ \vec{D} + a \ i \ g \ c_{\text{NGI}} \ A(x) \right) \psi(x) \]

\[ c_\text{D} = -\frac{1}{4} \left( 1 + g^2 \ c^{(1)}_\text{D} \right) + \mathcal{O}(g^4) \]

\[ c^{(1)}_\text{D} = C_F \ (0.037614 + 0.011755 \xi - 0.835571 \omega + 3.418757 \omega^2) \]

(\(\xi\) - covariant gauge parameter)

\[ c_{\text{NGI}} = g^2 \ c^{(1)}_{\text{NGI}} + \mathcal{O}(g^4) \]

\[ c^{(1)}_{\text{NGI}} = N_c \ (0.002395 - 0.010841 \omega) \]
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Mean field improvement

Bare coupling constant $g^2$ leads to a poor approximation:
- $g^2$ is large in most quantities
- perturbative series converges poorly

Two ideas combined
(1) Calculate each quantity in a simple mean field approximation
   → Re-express the perturbative result as the mean field result multiplied by a perturbative correction factor
   → One-loop correction term should be small
(2) Bare coupling $g^2$ → “boosted” coupling constant $g_{MF}^2$
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Mean field improvement and stout smearing

Express with two mean fields:

\( u_0 \) - a mean value for the unsmeared link

\( u_S \) - a mean value for smeared links

\[
\kappa_c(g^2) \to \kappa_c^{MF}(g_{MF}^2, u_S) = \frac{u_S^{pert}(g_{MF}^2)}{u_S} \kappa_c(g_{MF}^2)
\]

\[
c_{SW}(g^2) \to c_{SW}^{MF}(g_{MF}^2, u_S, u_0) = \frac{u_S}{u_0^4} \frac{u_0^{pert,4}(g_{MF}^2)}{u_S^{pert}(g_{MF}^2)} c_{SW}(g_{MF}^2)
\]

with

\[
u_S^{pert} = 1 - \frac{g^2 C_F}{16\pi^2} k_S(\omega), \quad u_0^{pert} = 1 - \frac{g^2 C_F}{16\pi^2} k_S(\omega = 0)
\]
Mean field improvement and stout smearing

Express with two mean fields:

- $u_0$ - a mean value for the unsmeared link
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$$\kappa_c(g^2) \rightarrow \kappa_c^{MF}(g_{MF}^2, u_S) = \frac{u_S^{pert}(g_{MF}^2)}{u_S} \kappa_c(g_{MF}^2)$$

$$c_{SW}(g^2) \rightarrow c_{SW}^{MF}(g_{MF}^2, u_S, u_0) = \frac{u_S}{u_0^4} \frac{u_0^{pert,4}(g_{MF}^2)}{u_S^{pert}(g_{MF}^2)} c_{SW}(g_{MF}^2)$$

with

$$u_S^{pert} = 1 - \frac{g^2 C_F}{16\pi^2} k_S(\omega), \quad u_0^{pert} = 1 - \frac{g^2 C_F}{16\pi^2} k_S(\omega = 0)$$
Mean field improvement: $c_{SW}$

\[
\begin{align*}
  c_{SW}^{Sym} &= 1 + g^2 \times \\
  &\left[ C_F \left( 0.116185 + 0.828129 \omega - 2.455080 \omega^2 \right) \\
  &+ N_c \left( 0.013777 + 0.015905 \omega - 0.321899 \omega^2 \right) \right].
\end{align*}
\]

\[\beta = 6.0, \ u_S = 0.9497, \ u_0 = 0.8644:\]

\[
\begin{align*}
  c_{SW}^{Sym} &= 1.4484 \rightarrow c_{SW}^{Sym, MF} = 1.8678 \leftrightarrow c_{SW}^{Sym, NP} = 2.137
\end{align*}
\]
Mean field improvement: $c_{SW}$

\[
c_{SW}^{Sym} = 1 + g^2 \times \left[ C_F \left( 0.116185 + 0.828129 \omega - 2.455080 \omega^2 \right) \\
+ N_c \left( 0.013777 + 0.015905 \omega - 0.321899 \omega^2 \right) \right].
\]

\[
\downarrow
\]

\[
c_{SW}^{Sym, MF} = \frac{u_S}{u_0^4} \left\{ 1 + g_{MF}^2 \times \left[ C_F \left( -0.0211635 + 0.115961 \omega + 0.685247 \omega^2 \right) \\
+ N_c \left( 0.013777 + 0.015905 \omega - 0.321899 \omega^2 \right) \right] \right\}
\]

$\beta = 6.0, u_S = 0.9497, u_0 = 0.8644$:

$c_{SW}^{Sym} = 1.4484 \rightarrow c_{SW}^{Sym, MF} = 1.8678 \leftrightarrow c_{SW}^{Sym, NP} = 2.137$
\[
\kappa_c^{\text{Sym}} = \frac{1}{8} \left[ 1 + g^2 C_F \times \left( 0.037730 - 0.662909 \omega + 2.668543 \omega^2 \right) \right]
\]

\[
\kappa_c^{\text{Sym},MF} = \frac{1}{8 u_S} \left[ 1 + g_{MF}^2 C_F \times \left( -0.008053 + 0.0500781 \omega - 0.471784 \omega^2 \right) \right]
\]

\[
\beta = 6.0, u_S = 0.9497, u_0 = 0.8644 : \\
\kappa_c^{\text{Sym}} = 0.1245 \rightarrow \kappa_c^{\text{Sym},MF} = 0.1276 \leftrightarrow \kappa_c^{\text{Sym},NP} = 0.124356
\]
\[
\kappa^\text{Sym}_c = \frac{1}{8} \left[ 1 + g^2 C_F \times 
\left( 0.037730 - 0.662090 \omega + 2.668543 \omega^2 \right) \right]
\]

\[\downarrow\]

\[
\kappa^\text{Sym, MF}_c = \frac{1}{8 u_S} \left[ 1 + g^2_{\text{MF}} C_F \times 
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\[\beta = 6.0, u_S = 0.9497, u_0 = 0.8644: \]

\[
\kappa^\text{Sym}_c = 0.1245 \rightarrow \kappa^\text{Sym, MF}_c = 0.1276 \leftrightarrow \kappa^\text{Sym, NP}_c = 0.124356
\]
Choice of $g_{MF}^2$ or $\frac{\Lambda_{lat}^M}{\Lambda_{MS}^M}$

The natural choice

$$g_{MF}^2 = \frac{g^2}{u_0^4}.$$ 

We have the relation (e.g. Kawai, Seo [1981])

$$\frac{1}{g_{MS}^2(\mu)} - \frac{1}{g_{MF}^2(a)} = 2b_0 \left( \log \frac{\mu}{\Lambda_{MS}} - \log \frac{1}{a\Lambda_{lat}^M} \right)$$

$$= 2b_0 \log(a\mu) + d_g + N_f d_f + \frac{k_u}{3\pi^2}$$

giving

$$\frac{\Lambda_{lat}^M}{\Lambda_{MS}^M} = \exp \left( \frac{d_g + N_f d_f + k_u/3\pi^2}{2b_0} \right).$$

($k_u = k_S(\omega = 0)$)
Choice of $g^2_{MF}$ or $\frac{\Lambda_{lat}^{MF}}{\Lambda_{MS}}$

We have

\[ d_g = -0.2361 \text{ (Hasenfratz et al.[1980])} \]

\[ d_f = 0.0314917 \text{ (Booth et al.[2001])}, \text{ independent of } \omega \]

\[ k_u = 0.732525 \pi^2 \]

\[ \rightarrow \frac{\Lambda_{lat}^{MF}}{\Lambda_{MS}} = 2.459 \]
SLiNC and point operators

Expected that renormalization (Z-) factors closer to unity

Z-factors for \( \mathcal{O} = \bar{\psi} 1 \psi, \bar{\psi} \gamma_5 \psi, \bar{\psi} \gamma_\mu \psi, \bar{\psi} \gamma_5 \gamma_\mu \psi \)

General one-loop form

\[
Z_\mathcal{O} = 1 - \frac{g^2 C_F}{16\pi^2} \left( \gamma_\mathcal{O} \log(a^2 \mu^2) + B_\mathcal{O} \right)
\]

Mean field improving program:

\[
Z_{\mathcal{O}}^{MF} = u_S(1 - \frac{g_{MF}^2 C_F}{16\pi^2} \left( \gamma_\mathcal{O} \log(a^2 \mu^2) + B_\mathcal{O} - k_S(\omega) \right)
\]
\[ \mathcal{O} = \bar{\psi} 1 \psi = S \]

\[ B_S = 15.0747 - 168.341 \omega + 242.254 \omega^2 \]

\[ \omega \equiv 0 \]

unsmeared

\[ \omega \equiv 0.1 \]

smeared

Mean field improvement:

\[ \beta = 6.0, u_S = 0.9497, u_0 = 0.8644, \omega = 0.1 : \]

\[ Z_S = 0.9907 \rightarrow Z_S^{MF} = 0.9564 \]
\[ \mathcal{O} = \bar{\psi} 1 \psi = S \]

\[ \mathcal{B}_S = 15.0747 - 168.341 \omega + 242.254 \omega^2 \]
\[ \omega = 0 \quad \implies 15.0747 \quad \text{unsmeared} \]
\[ \omega = 0.1 \quad \implies 0.663069 \quad \text{smeared} \]

**Mean field improvement:**

\[ \beta = 6.0, \ u_S = 0.9497, \ u_0 = 0.8644, \ \omega = 0.1 : \]
\[ Z_S = 0.9907 \rightarrow Z^\text{MF}_S = 0.9564 \]
\[ \mathcal{O} = \bar{\psi} \gamma_5 \psi = P \]

\[ B_P = 19.1500 - 267.462\omega + 1065.55\omega^2 \]

\[ \omega = 0 \quad 19.1500 \quad \text{unsmeared} \]

\[ \omega = 0.1 \quad 3.0593 \quad \text{smeared} \]

Mean field improvement:

\[ \beta = 6.0, u_S = 0.9497, u_0 = 0.8644, \omega = 0.1 : \]

\[ Z_P = 0.9569 \rightarrow Z_P^{MF} = 0.8990 \]
\[ \mathcal{O} = \bar{\psi} \gamma_5 \psi = P \]

\[ B_P = 19.1500 - 267.462 \omega + 1065.55 \omega^2 \]

\[ \omega = 0 \quad 19.1500 \quad \text{unsmeread} \]
\[ \omega = 0.1 \quad 3.0593 \quad \text{smeared} \]

Mean field improvement:

\[ \beta = 6.0, u_S = 0.9497, u_0 = 0.8644, \omega = 0.1 : \]

\[ Z_P = 0.9569 \rightarrow Z_P^{MF} = 0.8990 \]
$\mathcal{O} = \bar{\psi} \gamma_\mu \psi = V$

\[ B_V = 11.9106 - 170.763 \omega + 754.029 \omega^2 \]
\[ \begin{align*}
\omega = 0 & : 11.9106 \quad \text{unsmeared} \\
\omega = 0.1 & : 2.37464 \quad \text{smeared}
\end{align*} \]

Mean field improvement:

\[ \beta = 6.0, u_S = 0.9497, u_0 = 0.8644, \omega = 0.1 : \]

\[ Z_V = 0.9666 \rightarrow Z_V^{MF} = 0.9154 \leftrightarrow Z_V^{NP} = 0.889 \]
\[ \mathcal{O} = \bar{\psi} \gamma_\mu \psi = V \]

\[
\begin{align*}
B_V &= 11.9106 - 170.763\omega + 754.029\omega^2 \\
\omega &\equiv 0 \quad 11.9106 \quad \text{unsmeared} \\
\omega &\equiv 0.1 \quad 2.37464 \quad \text{smeared}
\end{align*}
\]

**Mean field improvement:**

\[
\beta = 6.0, u_S = 0.9497, u_0 = 0.8644, \omega = 0.1 : \\
Z_V = 0.9666 \rightarrow Z_V^{MF} = 0.9154 \leftrightarrow Z_V^{NP} = 0.889
\]
\[ \mathcal{O} = \bar{\psi} \gamma_5 \gamma_\mu \psi = A \]

\[ B_A = 10.7165 - 127.200 \omega + 342.380 \omega^2 \]

- \( \omega = 0 \) \( \equiv 10.7165 \) unsmeared
- \( \omega = 0.1 \) \( \equiv 1.42034 \) smeared

Mean field improvement:

\[ \beta = 6.0, u_S = 0.9497, u_0 = 0.8644, \omega = 0.1 : \]

\[ Z_A = 0.9800 \rightarrow Z_{A}^{MF} = 0.9383 \]
\[ \mathcal{O} = \bar{\psi} \gamma_5 \gamma_\mu \psi = A \]

\[ B_A = 10.7165 - 127.200 \omega + 342.380 \omega^2 \]

\[ \omega = 0 \implies 10.7165 \text{ unsmeared} \]

\[ \omega = 0.1 \implies 1.42034 \text{ smeared} \]

**Mean field improvement:**

\[ \beta = 6.0, u_S = 0.9497, u_0 = 0.8644, \omega = 0.1 : \]

\[ Z_A = 0.9800 \rightarrow Z_A^{MF} = 0.9383 \]
Summary

- We have introduced the SLiNC action as a base for future 2+1 simulations
- Using standard perturbation theory we have calculated one-loop non-amputated Green’s function related to the qqg-vertex with SLiNC fermions
- The result is used to determine the improvement coefficient $c_{SW}$ including stout smearing
- We determined the quark field improvement coefficients $c_D$ and $c_{NGI}$
- Using SLiNC and quark self energy we determined $\kappa_c$ also
- On-shell we have reproduced earlier results for non-smeared links
- Mean field improvement for smeared links has been discussed
- With SLiNC fermions we calculated the one-loop corrections to point operators
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