Light pseudoscalar masses and decay constants with a mixed action

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Christopher Aubin and Ruth Van de Water
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Mixed action simulations

Work done with Christopher Aubin and Ruth Van de Water

These simulations use MILC lattices (with 2+1 Asqtad staggered quarks in the sea sector) and domain wall quarks in the valence sector.

Advantages

- A large number of ensembles with different volumes, sea quark masses and lattice spacings exist and are publicly available.
- The existing ensembles have 2+1 flavors of light sea quarks ($m_{\text{strange}}/10$ for the lightest quarks)
- The good chiral properties of the valence sector make things much simpler than the staggered case. There are only two additional parameters (over pure domain wall) that appear at one loop in the mixed action ChPT for $m_{\pi}, f_{\pi},$ and $B_K$. They can both be obtained from spectrum calculations.
- NPR can be carried through in the same way as in domain wall.
Mixed action calculations

In 1-loop Mixed Action $\chi$PT only two parameters beyond those of domain-wall:

\[ m_{dw}^2 = 2\mu_{dw}(m_v + m_{res}) , \]
\[ m_I^2 = 2\mu_{stag}m_s + a^2 \Delta_I , \]
\[ m_{mix}^2 = \mu_{dw}(m_v + m_{res}) + \mu_{stag}m_s + a^2 \Delta_{mix} , \]
## Run parameters

### Table 1: Lattice parameters

<table>
<thead>
<tr>
<th>$a$(fm)</th>
<th>$a\hat{m}/am_s$</th>
<th>$L$(fm)</th>
<th>$m_\pi L$</th>
<th>$10/g^2$</th>
<th>Lat Dim</th>
<th># Conf s</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\approx 0.12$</td>
<td>0.02/0.05</td>
<td>2.4</td>
<td>6.2</td>
<td>6.79</td>
<td>$20^3 \times 64$</td>
<td>117</td>
</tr>
<tr>
<td>$\approx 0.12$</td>
<td>0.01/0.05</td>
<td>2.4</td>
<td>4.5</td>
<td>6.76</td>
<td>$20^3 \times 64$</td>
<td>220</td>
</tr>
<tr>
<td>$\approx 0.12$</td>
<td>0.007/0.05</td>
<td>2.4</td>
<td>3.8</td>
<td>6.76</td>
<td>$20^3 \times 64$</td>
<td>268</td>
</tr>
<tr>
<td>$\approx 0.12$</td>
<td>0.005/0.05</td>
<td>2.9</td>
<td>3.8</td>
<td>6.76</td>
<td>$24^3 \times 64$</td>
<td>216</td>
</tr>
<tr>
<td>$\approx 0.12$</td>
<td>0.01/0.03</td>
<td>2.4</td>
<td>4.5</td>
<td>6.76</td>
<td>$20^3 \times 64$</td>
<td>160</td>
</tr>
<tr>
<td>$\approx 0.09$</td>
<td>0.0124/0.031</td>
<td>2.4</td>
<td>5.8</td>
<td>7.11</td>
<td>$28^3 \times 96$</td>
<td>198</td>
</tr>
<tr>
<td>$\approx 0.09$</td>
<td>0.0062/0.031</td>
<td>2.4</td>
<td>4.1</td>
<td>7.09</td>
<td>$28^3 \times 96$</td>
<td>210</td>
</tr>
<tr>
<td>$\approx 0.09$</td>
<td>0.0031/0.031</td>
<td>3.4</td>
<td>4.2</td>
<td>7.08</td>
<td>$40^3 \times 96$</td>
<td>38</td>
</tr>
</tbody>
</table>
The residual mass

circles: a=0.12fm; squares: a=0.09fm

\begin{align*}
    r_1 m_{res} &= 0.005/0.05 \\
    am_l/am_s &= 0.007/0.05 \\
    am_l/am_s &= 0.01/0.05 \\
    am_l/am_s &= 0.02/0.05 \\
    am_l/am_s &= 0.0062/0.031 \\
    am_l/am_s &= 0.0124/0.031
\end{align*}
Determining the splitting $\Delta_{\text{mix}}$
Scalar bubble prediction

\[ B(t) = \frac{\mu^2}{3L^3} \sum_k \left[ \frac{2}{9} \frac{e^{-(\omega_{\nu\nu} + \omega_{\eta I})t}}{\omega_{\nu\nu} \omega_{\eta I}} \frac{(m_{SI}^2 - m_{UI}^2)^2}{(m_{\nu\nu}^2 - m_{\eta I}^2)^2} \right. \\
- \frac{e^{-2\omega_{\nu\nu}t}}{\omega_{\nu\nu}^2} \left[ \frac{3m_{\nu\nu}^2(m_{\nu\nu}^2 - 2m_{\eta I}^2) + 2m_{SI}^4 + m_{UI}^4}{3(m_{\eta I}^2 - m_{\nu\nu}^2)^2} \right] \\
- \frac{e^{-2\omega_{\nu\nu}t}}{2\omega_{\nu\nu}^4} (\omega_{\nu\nu}t + 1) \frac{(m_{UI}^2 - m_{\nu\nu}^2)(m_{SI}^2 - m_{\nu\nu}^2)}{m_{\eta I}^2 - m_{\nu\nu}^2} + \frac{3}{2} \frac{e^{-2\omega_{\nu\nu}t}}{\omega_{\nu\nu}^2} + \frac{3}{4} \frac{e^{-2\omega_{\nu s}t}}{\omega_{\nu s}^2} \right] \\

Prelovsek, PR D73, 014506 (2006), hep-lat/0510080
Scalar bubble prediction vs. data

\[ C(t) \]

\[ am_t / am_s = 0.007 / 0.05 \]

- \[ am_{val} = 0.01 \]
- \[ am_{val} = 0.02 \]
- \[ am_{val} = 0.03 \]
Scalar bubble prediction vs. data

- $a_m/a_s = 0.007/0.05$, $a_{val} = 0.01$
- $a_m/a_s = 0.0062/0.031$, $a_{val} = 0.0062$
Approach to chiral fits

We have generated data with relatively high statistics so that we can resolve a correlation matrix and obtain reliable confidence levels in fits.

Using SU(3) chiral perturbation theory in order to interpolate about the strange quark mass and extrapolate in the light quark mass. We are using one-loop SU(3) mixed action $\chi$PT and higher order analytic terms.

Plans to investigate SU(2) $\chi$PT and 2-loop corrections.

Separate fits to $m_{\pi}^2/m_q$ and $f_\pi$, where leading order $\mu$ is taken from linear fits to $m_{\pi}^2$ data, evaluated in region of data, rather than chiral limit. $f_\pi$ evaluated at physical pion point.
$m^2_{\pi}/m_q$ chiral fit

\[ \chi^2/\text{dof}=0.58, \text{C.L.}=0.97 \]
$f_\pi$ chiral fit

\[ \chi^2/\text{dof}=1.09, \text{C.L.}=0.33 \]
$f_\pi$ chiral fit (compared w/ MILC)
# Fit results

<table>
<thead>
<tr>
<th>type of $f_\pi$ fit</th>
<th>$\chi^2$/d.o.f.</th>
<th>C.L.</th>
</tr>
</thead>
<tbody>
<tr>
<td>NNLO analytic</td>
<td>1.09</td>
<td>33%</td>
</tr>
<tr>
<td>No NNLO</td>
<td>3.78</td>
<td>$3 \times 10^{-14}$</td>
</tr>
<tr>
<td>No NLO logs</td>
<td>2.43</td>
<td>$8 \times 10^{-6}$</td>
</tr>
<tr>
<td>No FV</td>
<td>1.66</td>
<td>1%</td>
</tr>
<tr>
<td>No splittings</td>
<td>1.33</td>
<td>10%</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>type of $m_\pi^2/m_q$ fit</th>
<th>$\chi^2$/d.o.f.</th>
<th>C.L.</th>
</tr>
</thead>
<tbody>
<tr>
<td>NNLO analytic</td>
<td>0.58</td>
<td>97%</td>
</tr>
<tr>
<td>No NNLO</td>
<td>4.51</td>
<td>$7 \times 10^{-19}$</td>
</tr>
<tr>
<td>No NLO logs</td>
<td>2.06</td>
<td>$4 \times 10^{-4}$</td>
</tr>
<tr>
<td>No FV</td>
<td>0.91</td>
<td>61%</td>
</tr>
<tr>
<td>No splittings</td>
<td>0.82</td>
<td>75%</td>
</tr>
</tbody>
</table>
Convergence of SU(3) $\chi$PT

\[
(f_{\text{SU}(3)} r_1)^{1/2} \quad (m_x + m_y + 2m_{\text{res}}) r_1^{1/2}
\]

- **LO**
- **NLO**
- **All orders**
$m^2_\pi/m_q$ with a higher cut on masses

$\chi^2$/dof$=1.14$, C.L.$=0.24$
\( f_{\pi} \) with a higher cut on mass

\[
\chi^2 / \text{dof} = 1.25, \ C.L. = 0.12
\]
$f_K$ determination

\[(f \pi_r) / 2^{1/2}\]

\[(m_x + m_y + 2m_{\text{res}}) r_1 / 2\]

- full QCD $K^+$
- full QCD $\pi$
- MILC $f_\pi$
- MILC $f_K$
## Preliminary decay constant error budget

<table>
<thead>
<tr>
<th>source</th>
<th>$f_K$</th>
<th>$f_\pi$</th>
<th>$f_K / f_\pi$</th>
</tr>
</thead>
<tbody>
<tr>
<td>statistics</td>
<td>1.2</td>
<td>1.6</td>
<td>1.5</td>
</tr>
<tr>
<td>input $r_1$</td>
<td>1.6</td>
<td>2.0</td>
<td>0.3</td>
</tr>
<tr>
<td>chiral/continuum extrap</td>
<td>3.1</td>
<td>3.1</td>
<td>1.4</td>
</tr>
<tr>
<td>finite volume</td>
<td>0.3</td>
<td>0.9</td>
<td>0.9</td>
</tr>
<tr>
<td><strong>total error</strong></td>
<td><strong>3.7</strong></td>
<td><strong>4.1</strong></td>
<td><strong>2.3</strong></td>
</tr>
</tbody>
</table>

Preliminary error budget for the decay constants and their ratio. Uncertainties are quoted as a percentage. The total combines systematic errors with statistical errors in quadrature.
Preliminary result for $|V_{us}|$

$f_K/f_\pi = 1.185(18)(20)$ where the first error is statistical and the second is systematic. The MILC value is $f_K/f_\pi = 1.197(3)(^{+6}_{-13})$.


$|V_{us}| = 0.2269(51)$

This is consistent with other recent determinations of $|V_{us}|$ and with unitarity constraints, given the latest $|V_{ud}|$. 
For the future

Investigate alternative fits: SU(2) $\chi$PT, SU(3) 2-loop logarithms in order to study convergence of the chiral expansion, and potentially decrease systematic errors.

Move on to $B_K$. We have $B_K$ data on the same lattices, and the analysis is in progress.