Neutral Kaon Mixing beyond the Standard Model with Domain Wall Fermions

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for the RBC and UKQCD collaborations

Lattice 2008 Williamsburg
1 Motivation

2 Non-perturbative Renormalisation

3 Matrix Elements

4 Summary
1 Motivation

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4 Summary
In the Standard Model: effective operator with \((V - A)(V - A)\) structure from integrating out \(W\)s in box diagram

- FCNCs from theories beyond the SM constrained to be small
- Model independent studies use the *mass insertion approximation*
- Lattice QCD can provide matrix elements for a full operator basis
In the Standard Model: effective operator with \((V - A)(V - A)\) structure from integrating out \(W\)s in box diagram

FCNCs from theories beyond the SM constrained to be small

model independent studies use the *mass insertion approximation*

Lattice QCD can provide matrix elements for a full operator basis
Operators and RG running

- operator basis

\[ Q_1 = \bar{s}^a \gamma_\mu P_L \ d^a \bar{s}^b \ \gamma_\mu P_L \ d^b \]
\[ Q_2 = \bar{s}^a \gamma_\mu P_L \ d^a \bar{s}^b \ \gamma_\mu P_R \ d^b \]
\[ Q_3 = \bar{s}^a \ P_L \ d^a \bar{s}^b \ P_R \ d^b \]
\[ Q_4 = \bar{s}^a \ P_L d^a \bar{s}^b \ P_L \ d^b \]
\[ Q_5 = \bar{s}^a \ \sigma_{\mu\nu} P_L \ d^a \bar{s}^b \ \sigma^{\mu\nu} P_L \ d^b \]

- related to “SUSY” basis by Fierz identities

- continuum QCD: only Wilson coefficients of \( Q_2, Q_3 \) and \( Q_4, Q_5 \) mix

\[ \mu \frac{d}{d\mu} \tilde{C}(\mu) = \begin{pmatrix}
\gamma_1 & \gamma_{22} & \gamma_{23} \\
\gamma_{32} & \gamma_{33} & \\
\gamma_{44} & \gamma_{45} & \\
\gamma_{54} & \gamma_{55}
\end{pmatrix} \tilde{C}(\mu) \]
Operators and RG running

- operator basis

\[
Q_1 = \bar{s}^a \gamma_\mu P_L d^a \bar{s}^b \gamma_\mu P_L d^b \\
Q_2 = \bar{s}^a \gamma_\mu P_L d^a \bar{s}^b \gamma_\mu P_R d^b \\
Q_3 = \bar{s}^a P_L d^a \bar{s}^b P_R d^b \\
Q_4 = \bar{s}^a P_L d^a \bar{s}^b P_L d^b \\
Q_5 = \bar{s}^a \sigma_{\mu\nu} P_L d^a \bar{s}^b \sigma^{\mu\nu} P_L d^b
\]

- related to “SUSY” basis by Fierz identities
- continuum QCD: only Wilson coefficients of \(Q_2, Q_3\) and \(Q_4, Q_5\) mix
- chiral symmetry essential for reduced mixing
- Domain Wall Fermions well suited for this problem
Outline

1. Motivation
2. Non-perturbative Renormalisation
3. Matrix Elements
4. Summary
5 × 5 matrix needed to renormalise operator basis

\[ \Lambda_{ij} \equiv (\Gamma_i)^{ABCD}_{\alpha\beta\gamma\delta} (P_j)^{BADC}_{\beta\alpha\delta\gamma} \]

\( \Gamma \) amputated four-point vertex function, \( P \) projector
renormalisation condition

\[ \frac{1}{Z_q^2} Z(\mu) = \Lambda_{\text{tree}} \cdot \Lambda^{-1}(p^2 = \mu^2) \]

trade \( Z_q \) for \( Z_A \):

\[ \frac{1}{Z_A^2} Z = \Lambda_{\text{tree}} \cdot \Lambda^{-1}/\Lambda_A \]
Gauge-fixed Momentum Sources

- Landau gauge fixed configurations
- source: 4-d volume source with phase factor $e^{2\pi i p \cdot x}$
- vertices calculated at sink position

- much smaller statistical errors than point sources
- small number of momenta
- different $O((ap)^4)$ errors for momenta with same $p^2$
  $\Rightarrow$ Dirk Brömmel’s talk on Friday afternoon
Mixing Matrix

**diagonal elements**

![Graph showing the relationship between \( p^2 \) and \( \Lambda_{\pi}/\Lambda_4 \).](image)

- larger \( p^2 \) dependence for some operators
Mixing Matrix

off-diagonal elements, $Q_3$

- block diagonal structure like in the continuum
off-diagonal elements, $Q_5$

- block diagonal structure like in the continuum
\[ \Lambda_A - \Lambda_V \text{ splitting} \]

- difference between \( \Lambda_A(p^2) \) and \( \Lambda_V(p^2) \) leads to systematic error
- way out: RI-MOM with non-exceptional momentum configurations
  → Chris Kelly’s talk in a few minutes
  → Yasumich Aoki’s talk on Wednesday afternoon
Removing the Perturbative Running

- RG evolution of the Wilson coefficients, $\vec{C}(\mu_2) = U(\mu_2, \mu_1) \vec{C}(\mu_1)$
- $U$ depends on anomalous dimension (matrix) $\gamma$, known at NLO

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Iwasaki gauge action with $\beta = 2.25$, Domain Wall fermion action
- $32^3 \times 64 \times 16$ lattices (new!)
- lattice spacing: $a^{-1} = 2.42(4) \text{ GeV} \times \frac{0.47 \text{ fm}}{r_0}$ (statistical error only)
- $m_{\text{res}} = 0.00066(2)$ (preliminary)

<table>
<thead>
<tr>
<th>$m_l$</th>
<th>$m_S$</th>
<th>$m_\pi$</th>
<th>Renorm.</th>
<th>Matrix Elements</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.004</td>
<td>0.03</td>
<td>$\sim 300 \text{ MeV}$</td>
<td>0.004</td>
<td>0.002, 0.004, 0.006, 0.008</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td>0.025, 0.03</td>
</tr>
<tr>
<td>0.006</td>
<td>0.03</td>
<td>$\sim 365 \text{ MeV}$</td>
<td>0.006</td>
<td>&quot;</td>
</tr>
<tr>
<td>0.008</td>
<td>0.03</td>
<td>$\sim 420 \text{ MeV}$</td>
<td>(0.008)</td>
<td>&quot;</td>
</tr>
</tbody>
</table>
gauge-fixed wall sources
use of fermionic boundary conditions: $p + a$ at $t = 0$, $p - a$ at $t = 64$
currently $O(100)$ measurements for each mass, will go up to 200
statistical errors $O(1\%)$ at lowest kaon mass
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Raw Data

$B_4$, unitary masses

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Mass Dependence

- $B_K$ style normalisation quark mass dependent for $B_2 - B_5$

\[
\langle \bar{K} | Q_i | K \rangle = N_i m_K^2 F_K^2 B_i, \quad N_i = \frac{8}{3}, \frac{-4}{3}R, 2R, \frac{5}{3}R, -4R
\]

\[
R = \left( \frac{m_K}{m_s' + m_d'} \right)^2
\]

B4
Mass Dependence

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$$R = \left( \frac{m_K}{m_s' + m_d'} \right)^2$$

- plan: use partially quenched Heavy Meson ChPT (SU(2))
- first fix LO LECs from fits in the pion sector
- treat the kaon as heavy relative to the pion
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status report on project about neutral kaon mixing beyond the SM
2+1 flavour DWF with small quark masses and very small chiral symmetry breaking
renormalisation of the chosen operator basis in the RI-MOM scheme
reduced operator mixing due to (lattice) chiral symmetry
preliminary results on the matrix elements from the new 32^3 ensembles
analysis in framework of partially quenched ChPT is on the way
scaling study with existing coarser DWF lattices planned